Modeling and Optimization of Renewable-Energy Sharing among Base Stations as a Minimum-Cost-Maximum-Flow Problem

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***Abstract*—Energy sharing among energy harvesting base stations (BSs) has the potential to improve the utilization of the harvested renewable energy. However, not much work has been done to optimize the power sharing among BSs while considering the topology of the cellular network and the distance-dependent power loss (DDPL) in the transmission lines. In this paper, we propose two power sharing optimization algorithms: the min-cost-max-flow (MCMF) algorithm and the max-flow (MF) algorithm. The MCMF algorithm optimizes the power sharing considering the DDPL, and therefore shares the power over much shorter distances whereas the MF algorithm optimizes the power sharing without considering the DDPL. Our numerical results show that for cellular networks with moderate DDPL value, the MCMF saves up to 10%, 22%, and 30% more power than the MF algorithm for 5, 10, and 15 BSs uniformly distributed in a square of unit length where every pair of BSs can share power, respectively. In contrast, for cellular networks with very high or very low DDPL value, the performance difference between the two algorithms is negligible. In addition, the performance gain of MCMF over MF increases with the BS density.**

***Index Terms*—Cellular network, min-cost-max-flow algorithm, max-flow algorithm, energy harvesting, power sharing**

I. Introduction

BSs equipped with renewable energy harvesting devices, e.g., solar cells, have become increasingly attractive to cellular network operators [1]. The amount of harvest power varies over time and space resulting in power surpluses or power deficits at the BSs over time. To make efficient use of the harvested power, power should be transmitted from surplus BSs to deficit BSs via transmission lines [2]. For instance, 80% of power can be saved if power sharing is conducted between two BSs with anti-correlated power profiles [3].

Nonetheless, these existing works about power sharing have not considered the effects of the distance-dependent power loss (DDPL) along the transmission lines on the performance of BS power sharing sufficiently. Transmitting power over longer distances will result in higher resistive power losses. In addition, most papers considered sharing power among only a few BSs, e.g., two BSs in [3], and modeled the resistive power loss independent of the distance between the BSs or independent of the spatial topology of the cellular network. This has motivated us to incorporate the DDPL in the transmission lines and to expand the power sharing scenario to a dense cellular network, where power should be shared among nearby BSs wherever possible.

In this paper, we propose a min-cost-max-flow (MCMF) algorithm and a max-flow (MF) algorithm to optimize the power sharing among BSs. The DDPL in the transmission lines is considered in the formulation and optimization of the MCMF algorithm. We will investigate the performance gap between both algorithms for different values of unit power loss per unit length, for different values of BS densities, and for different values of maximum power surpluses/deficits at the BSs.

II. System Model

We consider $N \in \mathbb{N}$ uniformly distributed BSs in a unit square (cf. Fig. 1), which are denoted as BS_i, $i \in \{1, ..., N\}$. Each BS is equipped with an energy harvesting device, e.g., a solar cell, as well as a main grid connection.

The difference between the power generation and consumption of BS_i is denoted as the weight W_i and given in units of power. A surplus (deficit) in power at BS_i is indicated by a positive (negative) weight W_i. The objective is to balance out the power in the network by transmitting power from surplus BSs to deficit BSs so that the total power drawn from the main grid by the deficit BSs is minimized. A BS_i with $W_i = 0$ will not take part in the power sharing scheme. We assume that the weights W_i are discretely uniformly distributed in the set $\{-W, -W+1, ..., W-1, W\}$, $W \in \mathbb{N}$. Hence, the probability that BS_i experiences a surplus/deficit of p units of power is given by

$$P(W_i = p) = \frac{1}{2W+1}, \qquad p \in \{-W, -W+1, ..., W\}. \tag{1}$$

The set of surplus BSs is denoted as BS^+, and the set of deficit BSs is denoted as BS^-, i.e.,

$$\begin{aligned} BS^+ &= \{i \,|\, W_i > 0,\ i \in \{1, ..., N\}\}, \\ BS^- &= \{i \,|\, W_i < 0,\ i \in \{1, ..., N\}\}. \end{aligned} \tag{2}$$

BSs can be connected by a transmission line in a cellular network. As depicted in Fig. 2, the network is represented

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by a neighboring graph, where vertices denote BSs and edges denote transmission lines. Two BSs can share power between each other only if they are connected by an edge. BSs that are connected by an edge are referred to as neighboring BSs.

Sharing power between two BSs will result in some power loss as heat along the transmission line known as resistive heating. The power loss P_{loss} in the transmission line can be calculated by Ohm's law and the formula for the transmission line resistance [4] as follows

$$P_{\text{loss}} = I^2 \cdot \rho \cdot \frac{l}{A}, \tag{3}$$

where I represents the current traveling through the transmission line, ρ represents the resistivity of the transmission line, l represents the length of the transmission line, and A represents the cross-sectional area of the transmission line.

We assume that the current is independent of the transmission line length. Hence, the power loss in the transmission line is proportional to its length. The power loss coefficient $L(i,j)$ of the edge between BS_i and BS_j is defined as

$$L(i,j) = \min(1, ||BS_i - BS_j|| \cdot C), \tag{4}$$

where $||BS_i - BS_j||$ is the Euclidean distance in units of length between BS_i and BS_j, and C is the unit power loss per unit length in the transmission line, which encapsulates the parameters from (3) as $C = \frac{I^2 p}{A}$. $L(i,j)$ is truncated to 1 because it is not possible to lose more than the available power.

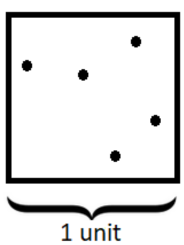
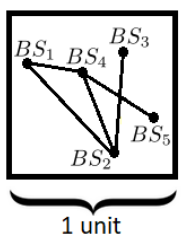


Fig. 1: Illustration of the considered cellular network, with $N = 5$ BSs uniformly distributed in a unit square.



i	W_i
1	1
2	- 4
3	- 2
4	2
5	- 1

Fig. 2: The neighboring graph representation of the cellular network with example surplus/deficit power parameters W_i given for the BSs.

III. The MCMF Problem

A. Optimization Objective

We use the denotation (i,j) for the edge between the surplus BS_i and the deficit BS_j, the denotation $(i,j) \in E_n$ to indicate that BS_i and BS_j are connected by a transmission line and the denotation $f\big((i,j)\big)$ for the power flow on the edge from the surplus BS_i to the deficit BS_j in units of power.

The optimization objective is to minimize the total power M drawn from the main grid by the deficit BSs as follows

$$M = \min_f \Big\{ \underbrace{\sum_{j \in BS^-} |W_j| - \sum_{\substack{i \in BS^+ \\ j \in BS^- \\ (i,j) \in E_n}} f\big((i,j)\big)}_{\text{deficit power that cannot be balanced out}} + \underbrace{\sum_{\substack{i \in BS^+ \\ j \in BS^- \\ (i,j) \in E_n}} L(i,j) f\big((i,j)\big)}_{\text{power loss in transmission lines}} \Big\} \tag{5}$$

subject to

Power flow out of the surplus BSs:

$$W_i \geq \sum_{\substack{j \in BS^- \\ (i,j) \in E_n}} f\big((i,j)\big), \quad \forall i \in BS^+ \tag{6}$$

Power flow into the deficit BSs:

$$|W_j| \geq \sum_{\substack{i \in BS^+ \\ (i,j) \in E_n}} f\big((i,j)\big), \quad \forall j \in BS^-. \tag{7}$$

There are two scenarios in which deficit BSs have to draw power from the main grid. On the one hand, some deficit BSs may not have neighboring surplus BSs that have sufficient power to balance out their power deficits. On the other hand, even if the neighboring surplus BSs have sufficient power to balance out the power deficits, due to the power losses in the transmission lines, the received powers at the deficit BSs are below the required deficit powers, so that the deficit BSs have to offset these differences by drawing main grid power.

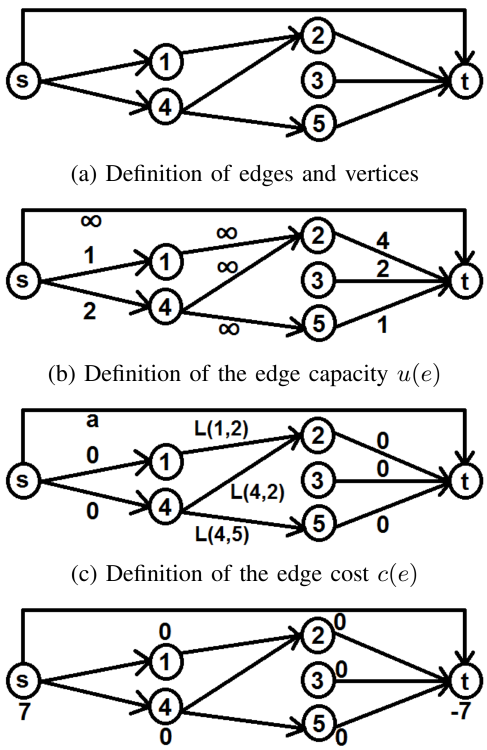
B. Definition of the flow network

In the following subsections, we will show the conversion of a neighboring graph (cf. Fig. 2) and the optimization objective (cf. (5)-(7)) into a flow network $G = (V, E, \mathrm{s}, \mathrm{t})$ and the MCMF problem (cf. (13)-(16)), where V, E, s, and t denote the set of vertices, the set of edges, the source vertex, and the sink vertex of the flow network, respectively. The conversion steps in Figs. 3(a)-3(d) depict the conversion of Fig. 2 into a flow network as an example. We use the denotation $e = (i,j)$ to represent the directed edge e from vertex i to vertex j in the flow network. The optimization in form of an MCMF problem and an MF problem can be efficiently solved in $\mathcal{O}(|E| \log |E| (|E| + |V| \log |V|))$ (Orlin's algorithm [5]) and $\mathcal{O}(|V|^2 |E|)$ (general push-relabel algorithm [6]), respectively, where $|E|$ and $|V|$ denote the number of edges and vertices in the flow network. In practice, network simplex algorithms are often very efficient as well [5].

1) Edges and Vertices (cf. Fig. 3(a)): Each surplus BS is connected from the source vertex to the surplus BS by a directed edge. These edges are denoted as source edges E_{s}. Each deficit BS is connected from the deficit BS to the sink vertex by a directed edge. These edges are denoted as sink edges E_{t}. If an edge exists between a surplus BS and a deficit BS in the neighboring graph, then the edge is replaced by a directed edge from the surplus BS to the deficit BS in the flow network. These edges are denoted as transmission edges E_{BS}. The edges and vertices in the flow network are defined as follows

$$\begin{aligned} E_{\mathrm{s}} &= \{(\mathrm{s}, j) \,|\, j \in BS^+\}, \\ E_{\mathrm{t}} &= \{(i, \mathrm{t}) \,|\, i \in BS^-\}, \\ E_{\mathrm{BS}} &= \{(i,j) \,|\, (i,j) \in E_n; i \in BS^+; j \in BS^-\}, \\ E &= E_{\mathrm{s}} \cup E_{\mathrm{t}} \cup E_{\mathrm{BS}} \cup (\mathrm{s}, \mathrm{t}), \\ V &= \{1, 2, ..., N\} \cup \mathrm{s} \cup \mathrm{t}. \end{aligned} \tag{8}$$

The power transmitted from BS_i to BS_j is modeled as a flow along the path $\mathrm{s} - BS_i - BS_j - \mathrm{t}$ in the flow network. To complete the conversion into a flow network, edge capacities $u(e)$, edge costs $c(e)$ and vertex weights $w(i)$ will be defined in the following subsections.



(a) Definition of edges and vertices

(b) Definition of the edge capacity $u(e)$

(c) Definition of the edge cost $c(e)$

(d) Definition of the vertex weight $w(v)$

Fig. 3: Conversion of the neighboring graph in Fig. 2 into a flow network

2) Edge Capacity $u(e)$ (cf. Fig. 3(b)): The capacity $u(e)$ of an edge e represents the maximum power that can pass through this edge. In compliance with (6) and (7), we set the capacities of the edges $e = (\mathrm{s}, j)$ $(j \in BS^+)$ to W_j and the capacities of the edges $e = (i, \mathrm{t})$ $(i \in BS^-)$ to $|W_i|$. We assume that the power generated by a typical energy harvesting device at a BS is relatively small with respect to the capacity of a typical transmission line. Hence, the capacities of the transmission edges are set to infinity for simplicity. The edge capacity is thus given by

$$u(e = (i,j)) = \begin{cases} W_j & e \in E_\mathrm{s} \\ |W_i| & e \in E_\mathrm{t} \\ \infty & e \in E_\mathrm{BS}. \end{cases} \tag{9}$$

3) Edge Cost $c(e)$ (cf. Fig. 3(c)): The cost $c(e)$ of an edge e represents the power loss in the transmission line due to resistive heating. The costs of the virtual edges in E_s as well as in E_t are set to 0. The cost of the transmission edge from BS_i to BS_j is equivalent to the power loss coefficient $L(i,j)$ in the transmission line defined in (4). The edge cost is given by

$$c(e = (i,j)) = \begin{cases} 0 & e \in E_\mathrm{s} \cup E_\mathrm{t} \\ L(i,j) & e \in E_\mathrm{BS}. \end{cases} \tag{10}$$

4) Vertex Weight $w(i)$ (cf. Fig. 3(d)): The weight $w(i)$ of a vertex i represents the net power flow out or into the vertex. All deficit BSs together require $|W^-|$ units of power, therefore the weight of the sink vertex is set to W^-. The weight of the source vertex is set to $|W^-|$ because, even if the supply is greater, the sink does not need more power than $|W^-|$ units. The weights of all other vertices are set to 0 because they pass on the power flow from the source to the sink. The definition of the vertex weight is summarized as follows

$$w(i) = \begin{cases} W^- & i = \mathrm{t} \\ |W^-| & i = \mathrm{s} \\ 0 & i \in V \setminus \{\mathrm{s}, \mathrm{t}\}. \end{cases} \tag{11}$$

5) (s, t) Edge (cf. Figs. 3(a)-3(d)): The maximum value of an s-t-flow is equal to the minimum capacity of an s-t-cut in a flow network [7]. Due to the weights of the source s and the sink t, the maximum flow value in the defined flow network is smaller or equal to $|W^-|$. The maximum flow value is smaller if and only if a minimum cut with capacity smaller than $|W^-|$ exists.

We can ensure that the maximum flow value always equals $|W^-|$ by adding a virtual (s, t) edge connecting the source and sink directly with a capacity of infinity and a cost of $a > 1$ $(a \in \mathbb{N})$. This virtual edge ensures that no minimum cut with capacity smaller than $|W^-|$ would occur. The trivial flow of passing $|W^-|$ units of power through this virtual edge is a feasible flow. Therefore, any other maximum flow will have a maximum flow value of $|W^-|$ as well. The purpose of this virtual edge is to solve the MCMF problem with the linear program described in Section IV, which requires that the complete flow of $|W^-|$ units of power can be passed through the network.

Flow through this virtual edge represents the deficit power, which cannot be balanced out, and thus has to be drawn from the main grid. There are two reasons why power cannot be balanced out. On the one hand, the total power surplus (W^+) may be smaller than the total power deficit $(|W^-|)$. On the other hand, the flow network could be sparse, so that some deficit BSs may not have neighboring surplus BSs that have sufficient power to balance out their power deficit (cf. BS_3 in Fig. 3(b)).

Because every $\mathrm{s} - BS_i - BS_j - \mathrm{t}$ path has a cost of smaller or equal to 1, the MCMF algorithm only passes flow over the virtual edge when there is no other possible path to pass it through the network.

The definition of the (s, t) edge is summarized as follows

$$\begin{aligned} &u(e = (\mathrm{s}, \mathrm{t})) = \infty, \\ &c(e = (\mathrm{s}, \mathrm{t})) = a > 1 \;\; (a \in \mathbb{N}). \end{aligned} \tag{12}$$

IV. Optimizing the MCMF Problem

We use the Graph::minCost(Graph G) function implemented in the MuPAD notebook of the Symbolic Math Toolbox in MATLAB to solve the problem. More specifically, it solves the following linear programming problem given in (13)-(16).

The output of the algorithm is the optimal power flow $\hat{f}$ over all edges in the flow network so that the power travels over the shortest distances in the cellular network. The optimal power flow $\hat{f}(e)$ over the transmission edge $e = (i, j) \in E_{\mathrm{BS}}$ represents the optimal flow of $\hat{f}(e)$ units of power from BS_i to BS_j. The linear program is summarized as follows

$$\hat{f} = \arg\min_{f} \Big\{ \sum_{e \in E} c(e) \cdot f(e) \Big\} \tag{13}$$

subject to

Capacity constraints: $f(e) \leq u(e), \quad \forall e \in E$ (14)

Skew symmetry:

$$f(e = (i,j)) = -f(-e = (j,i)), \quad \forall e \in E \tag{15}$$

Flow conservation and required flow:

$$\sum_{j \in V} f(e = (i,j)) = w(i), \quad \forall i \in V, \tag{16}$$

where $e \in E$, $i, j \in V$, s is the source vertex, t is the sink vertex, $f(e)$ is the power flow over edge e, $c(e)$ is the cost of edge e, $u(e)$ is the capacity of edge e, and $w(i)$ is the weight of vertex i.

The original optimization objective (5) is equivalent to (17), which calculates the total power drawn from the main grid by the BSs of the MCMF algorithm denoted as M^{MCMF}. It consists of the power passing through the virtual (s, t) edge denoted as M_1^{MCMF}, and the power lost in the transmission lines denoted as M_2^{MCMF}, i.e.,

$$M^{\mathrm{MCMF}} = \underbrace{\hat{f}(e = (\mathrm{s}, \mathrm{t}))}_{M_1^{\mathrm{MCMF}}} + \underbrace{\sum_{e \in E \setminus (\mathrm{s},\mathrm{t})} c(e) \cdot \hat{f}(e)}_{M_2^{\mathrm{MCMF}}}. \tag{17}$$

V. The MF Problem

This section derives an MF algorithm to optimize the power sharing in the same flow network. In contrast to the MCMF algorithm, the MF algorithm does not consider the power loss in the transmission lines during the optimization, leading to a lower computational complexity. As a result, the power flow in the MF algorithm may travel longer distances and thus may experience a higher power loss in the transmission lines than in the MCMF algorithm.

We use the same linear program (13)-(16) for the MF algorithm but with the modified cost function $\tilde{c}$ given as

$$\tilde{c}(e) = \begin{cases} 0 & e \in E_{\mathrm{s}} \cup E_{\mathrm{t}} \cup E_{\mathrm{BS}} \\ b > 0 \ \ (b \in \mathbb{N}) & e = (\mathrm{s}, \mathrm{t}). \end{cases} \tag{18}$$

Because every $\mathrm{s} - BS_i - BS_j - \mathrm{t}$ path has now a cost of 0, the MF algorithm only passes flow over the virtual (s, t) edge when there is no other possible path to pass it through the network. Because every $\mathrm{s} - BS_i - BS_j - \mathrm{t}$ path has the same cost in (18), the optimal flow generated by (13)-(16) is now a random maximum flow. The total power drawn from the main grid by the deficit BSs of the MF algorithm is denoted as M^{MF} and is calculated by using the original cost function c together with the generated random maximum flow $\hat{f}$. It consists of the power passing through the virtual (s, t) edge denoted as M_1^{MF}, and the power lost in the transmission lines denoted as M_2^{MF}, i.e.,

$$M^{\mathrm{MF}} = \underbrace{\hat{f}(e = (\mathrm{s}, \mathrm{t}))}_{M_1^{\mathrm{MF}}} + \underbrace{\sum_{e \in E \setminus (\mathrm{s},\mathrm{t})} c(e) \cdot \hat{f}(e)}_{M_2^{\mathrm{MF}}}. \tag{19}$$

The performance gap Δ between the two proposed algorithms is defined as follows

$$\Delta = M^{\mathrm{MF}} - M^{\mathrm{MCMF}}. \tag{20}$$

In the following section, we will classify the cellular networks which should be optimized by the MCMF algorithm or the MF algorithm, respectively.

VI. Numerical Results

If not stated differently, we use a BS density of $N = 5$ BSs in a unit square, a maximum power surplus/deficit of $W = 4$, and a unit power loss per unit length of $C \in \{0, 0.2, ..., 3.8, 4\}$ to numerically evaluate the performance of both algorithms. Both algorithms are run 10000 times to derive their average performance. The superscript ^{ENG} and ^{CNG} are added to the parameters if an edgeless neighboring graph and a complete neighboring graph were used, respectively. An edgeless neighboring graph and a complete neighboring graph correspond to no pair of BSs and every pair of BSs is connected by a transmission line, respectively.

Fig. 4 shows the average total power drawn from the main grid of the MCMF algorithm ($M^{\mathrm{MCMF\_CNG}}$) and the MF algorithm ($M^{\mathrm{MF\_CNG}}$) versus unit power loss per unit length (C) on a complete neighboring graph, respectively. The MCMF algorithm saves more grid power than the MF algorithm for any given $C > 0$ because it takes into account the power loss in the transmission lines in the optimization. As a result, the power flow in the network travels over shorter distances in the MCMF algorithm and is therefore subject to a smaller power loss than in the MF algorithm. The performance gap (Δ) between the two algorithms is greater for moderate C than for very large or very small C. Hence, the higher complexity of running an MCMF algorithm compared to an MF algorithm can be justified if C is moderate. Because no power is lost on the transmission lines for $C = 0$, $M^{\mathrm{MCMF\_CNG}}$ is equal to $M^{\mathrm{MF\_CNG}}$, $M_1^{\mathrm{MCMF\_CNG}}$, and $M_1^{\mathrm{MF\_CNG}}$ (red circle in Fig. 4).

$M^{\mathrm{MCMF\_CNG}}$ and $M^{\mathrm{MF\_CNG}}$ are bounded by the solid black horizontal lower bound line corresponding to the $M_1^{\mathrm{MCMF\_CNG}} = M_1^{\mathrm{MF\_CNG}}$ value and the solid black horizontal upper bound line corresponding to the $M^{\mathrm{MCMF\_ENG}} = M^{\mathrm{MF\_ENG}}$ value. The lower and upper bound are horizontal lines because $M_1^{\mathrm{MCMF\_CNG}}$, $M_1^{\mathrm{MF\_CNG}}$, $M^{\mathrm{MCMF\_ENG}}$, and $M^{\mathrm{MF\_ENG}}$ are independent of C. These two horizontal lines correspond to the extreme points of the cellular network behavior where all BSs behave like one single mega BS corresponding to the solid black horizontal lower bound line in Fig. 4 and all BSs behave like isolated BSs corresponding to the solid black horizontal upper bound line in Fig. 4.

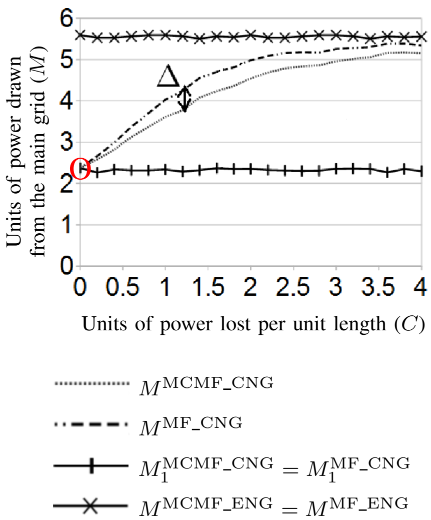


Fig. 4: Average total power drawn from the main grid of the MCMF algorithm ($M^{\mathrm{MCMF\_CNG}}$) and the MF algorithm ($M^{\mathrm{MF\_CNG}}$) versus unit power loss per unit length.

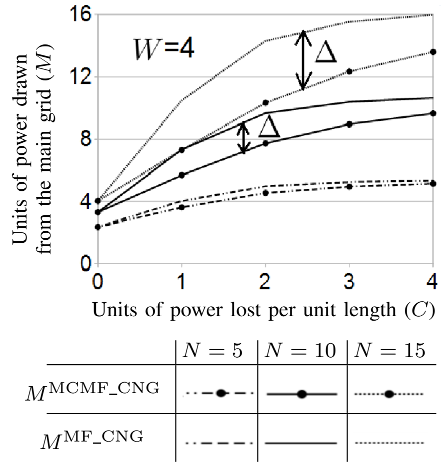


Fig. 5: Average total power drawn from the main grid of the two proposed algorithms versus unit power loss per unit length for different number of BSs (N).

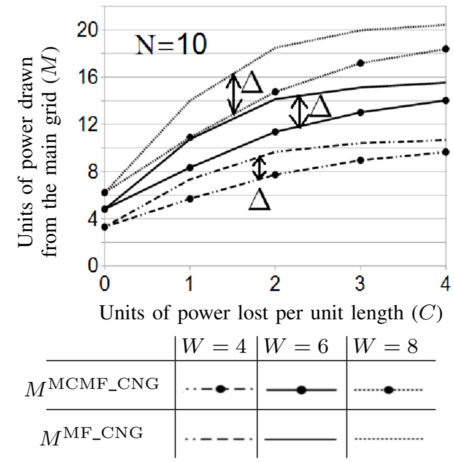


Fig. 6: Average total power drawn from the main grid of the two proposed algorithms versus unit power loss per unit length for different maximum power surpluses/deficits (W).

The power, which cannot be balanced out in the network and therefore flows on the virtual (s, t) edge, is the same in both algorithms. Hence, $M_1^{\mathrm{MCMF\_CNG}}$ is equal to $M_1^{\mathrm{MF\_CNG}}$. The total power drawn from the main grid on an edgeless neighboring graph is the same in both algorithms. Hence, $M^{\mathrm{MCMF\_ENG}}$ is equal to $M^{\mathrm{MF\_ENG}}$.

Fig. 5 shows the performance of both algorithms for different numbers of BSs (N). The performance gap (Δ) between the two algorithms increases with the number of BSs, i.e., a denser cellular network. This is because a denser cellular network offers more opportunities for power sharing between BSs, and the power savings from minimizing the distances traveled by the power flows become more significant. The MCMF algorithm saves up to 10%, 22% and 30% more power than the MF algorithm for $N = 5$, $N = 10$ and $N = 15$ BSs, respectively. The topology of the neighboring graph influences significantly Δ.

Fig. 6 shows the performance of both algorithms for different maximum power surpluses/deficits (W). The MCMF algorithm saves up to 22% more power than the MF algorithm for all three cases $W = 4$, $W = 6$ and $W = 8$. The maximum power surpluses/deficits (W) do not influence Δ as much as the BS density.

VII. Conclusion

We developed an MCMF algorithm and an MF algorithm to optimize the sharing of power among BSs with the objective of minimizing the total power drawn from the main grid by the BSs. The MCMF algorithm had a higher computational complexity but made more efficient use of the harvested power because it considered the DDPL in the transmission lines and hence shared the power over much shorter distances.

Our numerical results on a complete neighboring graph, i.e., every BS can share power with every other BS in the network, showed, that the power saving gain (Δ) of the MCMF algorithm over the MF algorithm depended on the unit power loss per unit length (C). On the one hand, Δ converged to 0% if C became very large or very small, in which case the simpler MF algorithm should be used. On the other hand, for cellular networks with moderate C, Δ increased with the BS density. In such cellular networks, the MCMF algorithm saved up to 10%, 22%, and 30% more power than the MF algorithm for 5, 10 and 15 BSs uniformly distributed in a unit square, respectively.

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References

[1] Y. Mao, Y. Luo, J. Zhang, and K. B. Letaief, "Energy harvesting small cell networks: feasibility, deployment, and operation," *IEEE Communications Magazine*, vol. 53, no. 6, pp. 94–101, Jun 2015.

[2] X. Huang and N. Ansari, "Energy sharing within EH-enabled wireless communication networks," *IEEE Wireless Communications*, vol. 22, no. 3, pp. 144–149, Jun 2015.

[3] Y. K. Chia, S. Sun, and R. Zhang, "Energy cooperation in cellular networks with renewable powered base stations," *IEEE Transactions on Wireless Communications*, vol. 13, no. 12, pp. 6996–7010, Dec 2014.

[4] H. Cole and D. Sang, *Revise AS Physics for AQA A*. Oxford, United Kingdom: Heinemann Educational Publishers, 2001, pp. 67–68.

[5] J. Vygen, "On dual minimum cost flow algorithms," *Mathematical Methods of Operations Research*, vol. 56, no. 1, pp. 101–126, Aug 2002.

[6] R. K. Ahuja, M. Kodialam, A. K. Mishra, and J. B. Orlin, "Computational investigations of maximum flow algorithms," *European Journal of Operational Research*, vol. 97, no. 3, pp. 509–542, Mar 1997.

[7] B. Korte and J. Vygen, *Combinatorial Optimization: Theory and Algorithms*, 5th ed. Berlin Heidelberg, Germany: Springer-Verlag, 2012, p. 177.