# $\mathcal{P}_1$ Elements

## Triangle

| vertex ● | edge → |
|---|---|
| 1: $(0,0)$ | 1: $\{1,2\}$ |
| 2: $(1,0)$ | 2: $\{2,3\}$ |
| 3: $(0,1)$ | 3: $\{3,1\}$ |
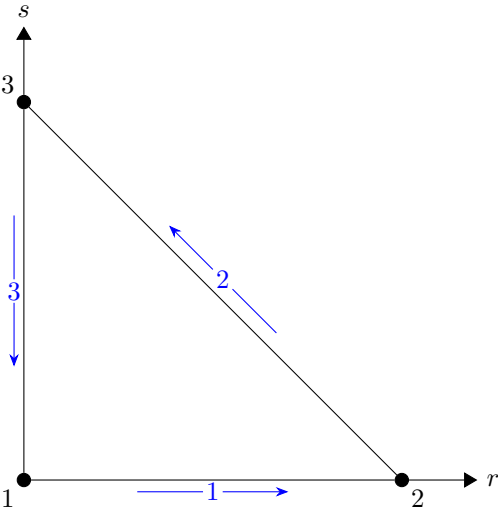
## Quadrilateral

| vertex ● | edge → |
|---|---|
| 1: $(0,0)$ | 1: $\{1,2\}$ |
| 2: $(1,0)$ | 2: $\{2,3\}$ |
| 3: $(1,1)$ | 3: $\{3,4\}$ |
| 4: $(0,1)$ | 4: $\{4,1\}$ |
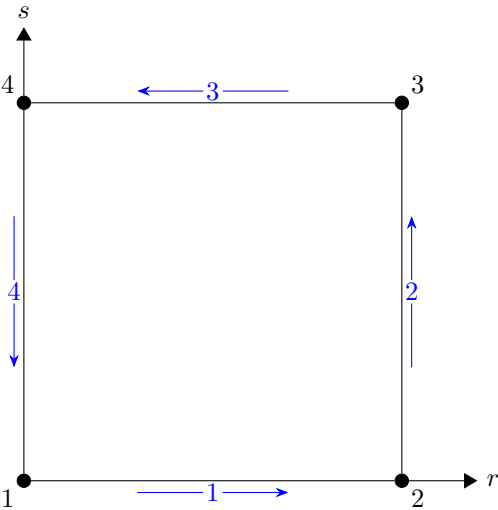
# Tetrahedron

| vertex ● | edge → | face ↻ |
|---|---|---|
| 1: $(0,0,0)$ | 1: $\{1,2\}$ | 1: $\{1,2,3\}$ |
| 2: $(1,0,0)$ | 2: $\{2,3\}$ | 2: $\{1,4,2\}$ |
| 3: $(0,1,0)$ | 3: $\{3,1\}$ | 3: $\{2,4,3\}$ |
| 4: $(0,0,1)$ | 4: $\{1,4\}$ | 4: $\{3,4,1\}$ |
| | 5: $\{2,4\}$ | |
| | 6: $\{3,4\}$ | |
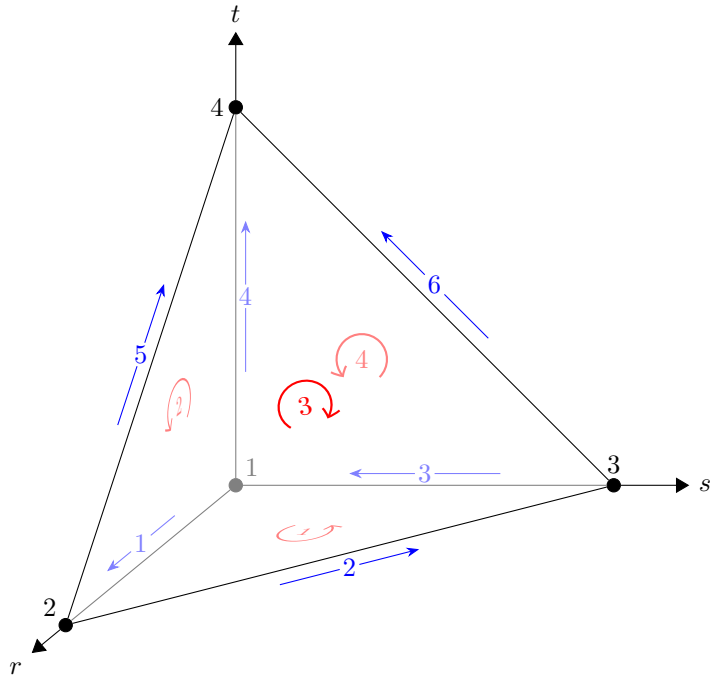
# Pyramid

| vertex ● | edge → | face ↻ |
|---|---|---|
| 1: $(0,0,0)$ | 1: $\{1,2\}$ | 1: $\{1,2,3,4\}$ |
| 2: $(1,0,0)$ | 2: $\{2,3\}$ | 2: $\{1,5,2\}$ |
| 3: $(1,1,0)$ | 3: $\{3,4\}$ | 3: $\{2,5,3\}$ |
| 4: $(0,1,0)$ | 4: $\{4,1\}$ | 4: $\{3,5,4\}$ |
| 5: $(\frac{1}{2},\frac{1}{2},1)$ | 5: $\{1,5\}$ | 5: $\{4,5,1\}$ |
| | 6: $\{2,5\}$ | |
| | 7: $\{3,5\}$ | |
| | 8: $\{4,5\}$ | |
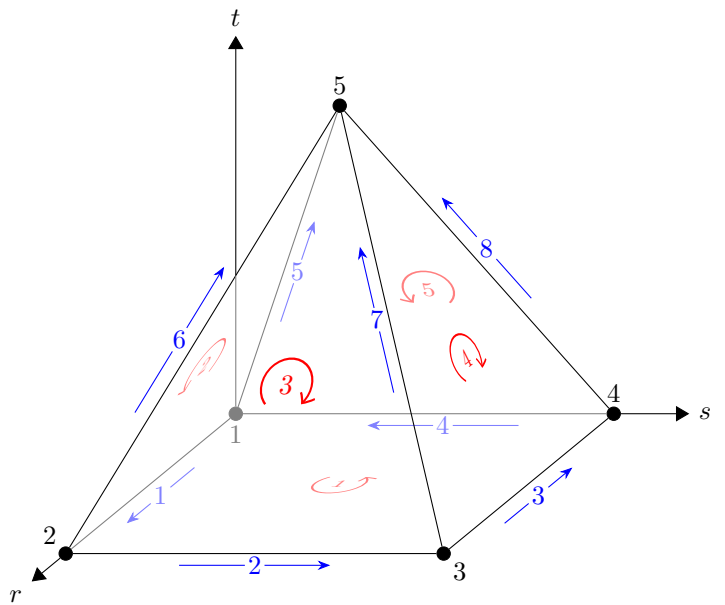
# Prism



| vertex ● | edge → | face ↻ |
|---|---|---|
| 1: $(0,0,0)$ | 1: $\{1,2\}$ | 1: $\{1,2,3\}$ |
| 2: $(1,0,0)$ | 2: $\{2,3\}$ | 2: $\{6,5,4\}$ |
| 3: $(0,1,0)$ | 3: $\{3,1\}$ | 3: $\{1,4,5,2\}$ |
| 4: $(0,0,1)$ | 4: $\{4,5\}$ | 4: $\{2,5,6,3\}$ |
| 5: $(1,0,1)$ | 5: $\{5,6\}$ | 5: $\{3,6,4,1\}$ |
| 6: $(0,1,1)$ | 6: $\{6,4\}$ | |
| | 7: $\{1,4\}$ | |
| | 8: $\{2,5\}$ | |
| | 9: $\{3,6\}$ | |
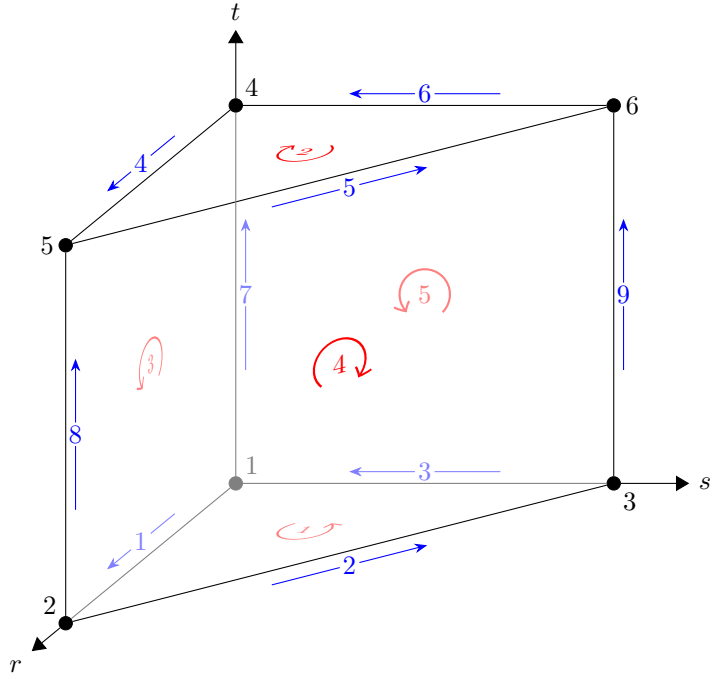
# Hexahedron



| vertex ● | edge → | face ↻ |
|---|---|---|
| 1: $(0,0,0)$ | 1: $\{1,2\}$ | 1: $\{1,2,3,4\}$ |
| 2: $(1,0,0)$ | 2: $\{2,3\}$ | 2: $\{3,2,6,7\}$ |
| 3: $(1,1,0)$ | 3: $\{3,4\}$ | 3: $\{5,8,7,6\}$ |
| 4: $(0,1,0)$ | 4: $\{4,1\}$ | 4: $\{1,4,8,5\}$ |
| 5: $(0,0,1)$ | 5: $\{5,6\}$ | 5: $\{2,1,5,6\}$ |
| 6: $(1,0,1)$ | 6: $\{6,7\}$ | 6: $\{4,3,7,8\}$ |
| 7: $(1,1,1)$ | 7: $\{7,8\}$ | |
| 8: $(0,1,1)$ | 8: $\{8,5\}$ | |
| | 9: $\{1,5\}$ | |
| | 10: $\{2,6\}$ | |
| | 11: $\{3,7\}$ | |
| | 12: $\{4,8\}$ | |
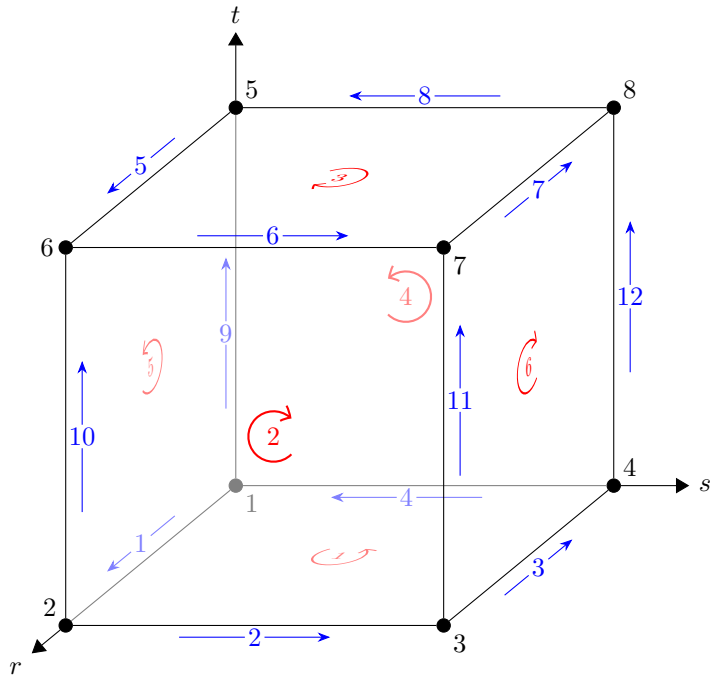
# $\mathcal{P}_2$ Elements

## Triangle

| vertex ● | mid-edge ○ |
|---|---|
| 1: $(0,0)$ | 4: $(\frac{1}{2},0)$ |
| 2: $(1,0)$ | 5: $(\frac{1}{2},\frac{1}{2})$ |
| 3: $(0,1)$ | 6: $(0,\frac{1}{2})$ |
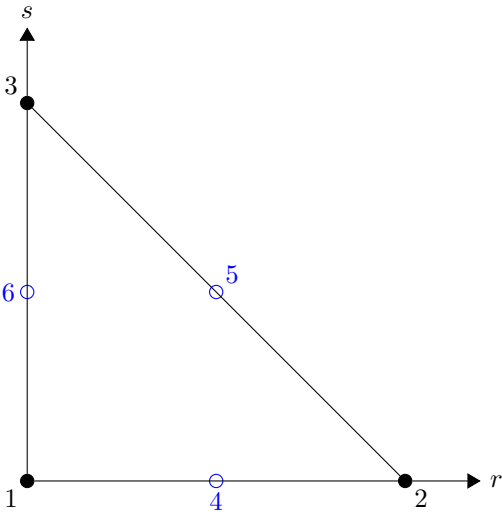
## Quadrilateral

| vertex ● | mid-edge ○ | mid-face □ |
|---|---|---|
| 1: $(0,0)$ | 5: $(\frac{1}{2},0)$ | 9: $(\frac{1}{2},\frac{1}{2})$ |
| 2: $(1,0)$ | 6: $(1,\frac{1}{2})$ | |
| 3: $(1,1)$ | 7: $(\frac{1}{2},1)$ | |
| 4: $(0,1)$ | 8: $(0,\frac{1}{2})$ | |
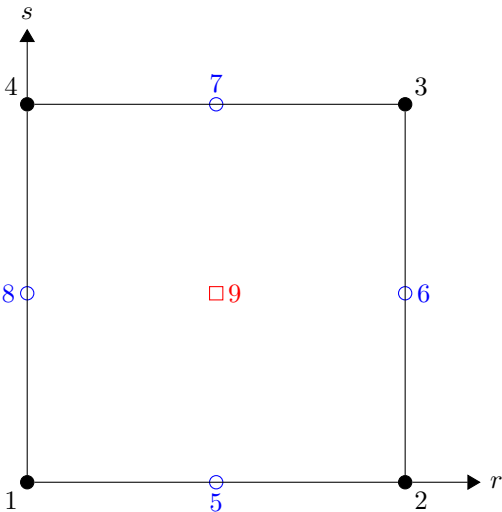
# Tetrahedron

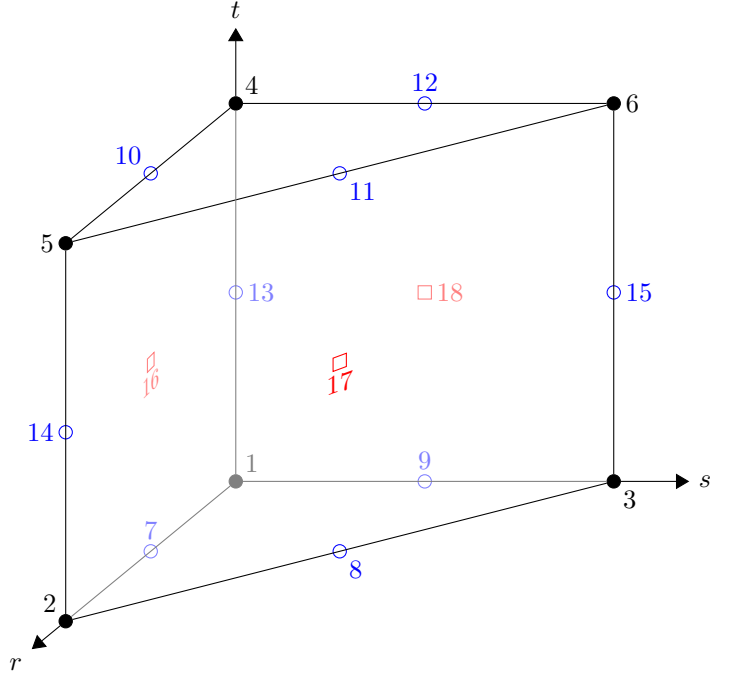| vertex ● | mid-edge ○ |
|---|---|
| 1: $(0,0,0)$ | 5: $(\frac{1}{2},0,0)$ |
| 2: $(1,0,0)$ | 6: $(\frac{1}{2},\frac{1}{2},0)$ |
| 3: $(0,1,0)$ | 7: $(0,\frac{1}{2},0)$ |
| 4: $(0,0,1)$ | 8: $(0,0,\frac{1}{2})$ |
| | 9: $(\frac{1}{2},0,\frac{1}{2})$ |
| | 10: $(0,\frac{1}{2},\frac{1}{2})$ |
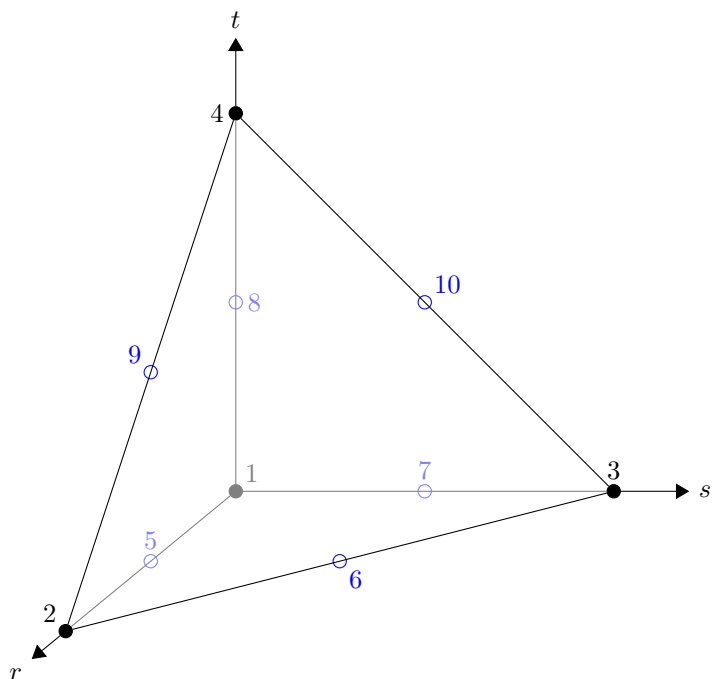
# Pyramid

| vertex ● | mid-edge ○ | mid-face □ |
|---|---|---|
| 1: $(0,0,0)$ | 6: $(\frac{1}{2},0,0)$ | 14: $(\frac{1}{2},\frac{1}{2},0)$ |
| 2: $(1,0,0)$ | 7: $(1,\frac{1}{2},0)$ | |
| 3: $(1,1,0)$ | 8: $(\frac{1}{2},1,0)$ | |
| 4: $(0,1,0)$ | 9: $(0,\frac{1}{2},0)$ | |
| 5: $(\frac{1}{2},\frac{1}{2},1)$ | 10: $(\frac{1}{4},\frac{1}{4},\frac{1}{2})$ | |
| | 11: $(\frac{3}{4},\frac{1}{4},\frac{1}{2})$ | |
| | 12: $(\frac{3}{4},\frac{3}{4},\frac{1}{2})$ | |
| | 13: $(\frac{1}{4},\frac{3}{4},\frac{1}{2})$ | |
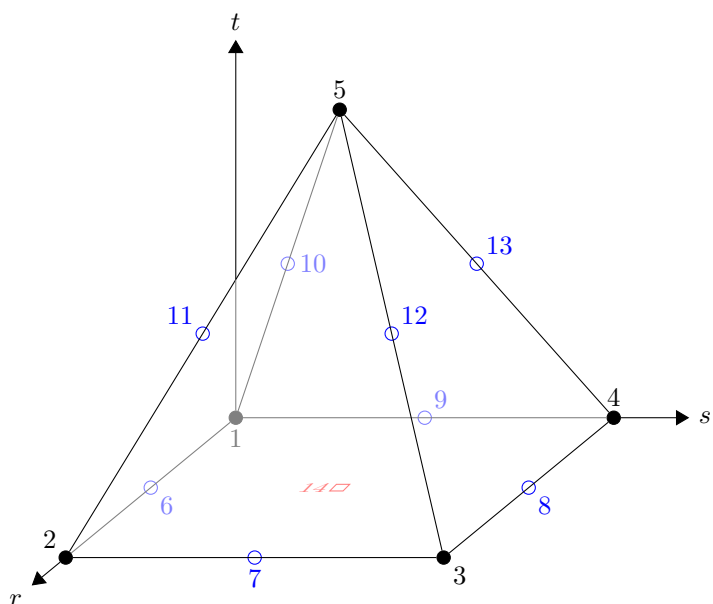
# Prism



| vertex ● | mid-edge ○ | mid-face □ |
|---|---|---|
| 1: $(0,0,0)$ | 7: $(\frac{1}{2},0,0)$ | 16: $(\frac{1}{2},0,\frac{1}{2})$ |
| 2: $(1,0,0)$ | 8: $(\frac{1}{2},\frac{1}{2},0)$ | 17: $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ |
| 3: $(0,1,0)$ | 9: $(0,\frac{1}{2},0)$ | 18: $(0,\frac{1}{2},\frac{1}{2})$ |
| 4: $(0,0,1)$ | 10: $(\frac{1}{2},0,1)$ | |
| 5: $(1,0,1)$ | 11: $(\frac{1}{2},\frac{1}{2},1)$ | |
| 6: $(0,1,1)$ | 12: $(0,\frac{1}{2},1)$ | |
| | 13: $(0,0,\frac{1}{2})$ | |
| | 14: $(1,0,\frac{1}{2})$ | |
| | 15: $(0,1,\frac{1}{2})$ | |

# Hexahedron



| vertex ● | mid-edge ○ | mid-face □ | mid-cell ● |
|---|---|---|---|
| 1: $(0,0,0)$ | 9: $(\frac{1}{2},0,0)$ | 21: $(\frac{1}{2},\frac{1}{2},0)$ | 27: $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ |
| 2: $(1,0,0)$ | 10: $(1,\frac{1}{2},0)$ | 22: $(1,\frac{1}{2},\frac{1}{2})$ | |
| 3: $(1,1,0)$ | 11: $(\frac{1}{2},1,0)$ | 23: $(\frac{1}{2},\frac{1}{2},1)$ | |
| 4: $(0,1,0)$ | 12: $(0,\frac{1}{2},0)$ | 24: $(0,\frac{1}{2},\frac{1}{2})$ | |
| 5: $(0,0,1)$ | 13: $(\frac{1}{2},0,1)$ | 25: $(\frac{1}{2},0,\frac{1}{2})$ | |
| 6: $(1,0,1)$ | 14: $(1,\frac{1}{2},1)$ | 26: $(\frac{1}{2},1,\frac{1}{2})$ | |
| 7: $(1,1,1)$ | 15: $(\frac{1}{2},1,1)$ | | |
| 8: $(0,1,1)$ | 16: $(0,\frac{1}{2},1)$ | | |
| | 17: $(0,0,\frac{1}{2})$ | | |
| | 18: $(1,0,\frac{1}{2})$ | | |
| | 19: $(1,1,\frac{1}{2})$ | | |
| | 20: $(0,1,\frac{1}{2})$ | | |