IB9KC0 Financial Econometrics Group Project

March 23, 2021

Task 1

A) In this section we will be going long in the Goldman Sachs (GS) stock while shorting the Intel (INTC) stock. We will then subtract the returns in the INTC stock from the returns in the GS stock to obtain the returns for our long-short strategy. Computing the five minute returns for each interval for each day we obtain the following table.

Table 1: Summary Statistics of Long-Short Continuous Returns

Average	Min	5% Percentile	95% Percentile	Max
-0.00017	-7.3584	-0.4062	0.4086	5.4950

The results show the average return of the long short portfolio over five minute intervals was almost zero. This is somewhat expected as we would expect a long term average return only to be visible over a longer time period such as months or years, rather than in five minute increments. As the minimum and maximum values are extreme, we wanted to see the exact intervals in which they occur to see if there was any explanation to these values.

Table 2: Occurrence of Maximum and Minimum Long-Short Returns

	Date	Interval
Minimum	18/09/2008	12:35-12:40
Maximum	29/09/2008	14:15-14:20

Both of these values occur in September 2008, the month in which the global financial crisis was at its peak. In particular, the minimum return occurred in the same week that Lehman Brothers went bankrupt, which would have had a significant impact on the Goldman Sachs stock price. Also with markets being so volatile, having the largest return in this period is also not surprising due to volatility clustering. During times of crisis, the large fluctuations in stock prices are often followed by large rebounds to adjust for a market over-correction and uncertainty.

B) To calculate the realised beta we will differentiate between returns due to jumps and those that are diffusive for both the market and the long-short portfolio. We will then calculate an estimator for realised beta for each day according to the following formula.

$$\widehat{R\beta_t} \equiv \frac{\widehat{RCov_t}}{TV_{m,t}} = \frac{\sum_{i=1}^{n} r^c_{ls,t,i} r^c_{m,t,i}}{\sum_{i=1}^{n} (r^c_{m,t,i})^2} \tag{1}$$

Where $\widehat{R\beta_t}$ is the estimated realised beta for day t , $r^c_{ls,t,i}$ are the diffusive returns of the long-short portfolio, $r^c_{m,t,i}$ are the diffusive returns of the market for day t at 5-minute time i.

Having separated the returns for the market and long-short portfolio into their respective continuous and discontinuous returns we obtain the indices common to both where the returns are continuous in order to make a fair comparison to calculate the estimator for the realised beta above. We obtain the following results.

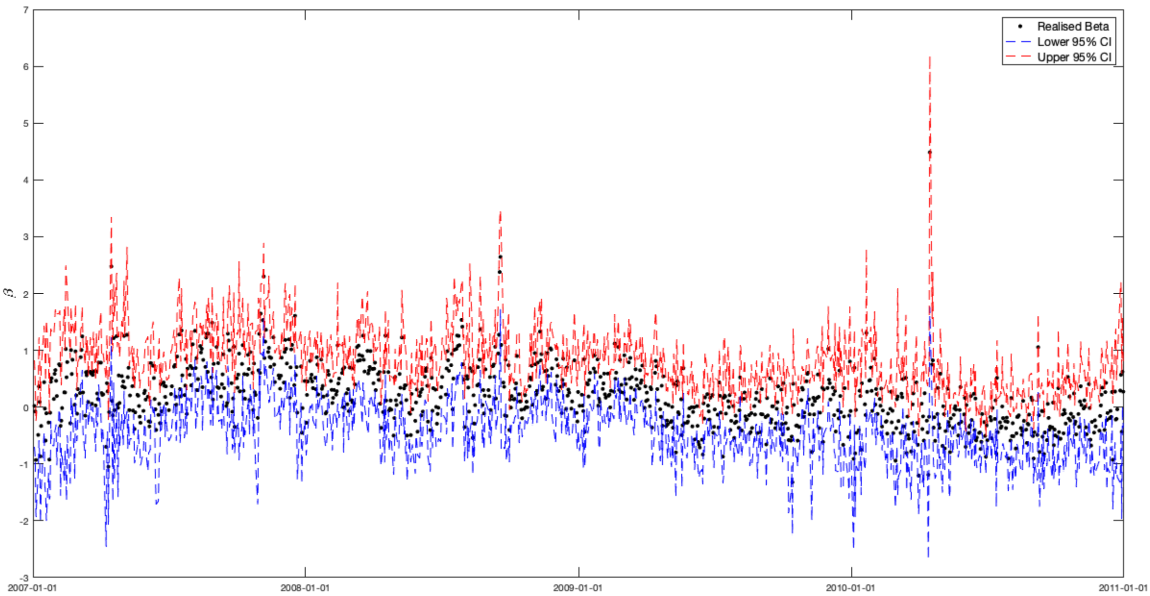
Average	Min	5th Percentile	95th Percentile	Max
0.1942	-1.3221	-0.5822	1.0632	4.4855

Table 3: Summary Statistics of daily Realised Betas

We note how the average realised beta for the long-short portfolio is less than one but greater than zero, hence on average the daily long short beta is indicated the portfolio is less risky than the market, but is not market neutral. However this is not always the case as we see how 5% of the daily realised betas are greater than or equal to 1.0632, with a maximum realised beta of 4.4855, indicating that the long-short portfolio could be extremely risky compared to the market during certain periods.

C) In order for a strategy to be market neutral it would have to have a beta of zero. Here we will plot the realised betas and their 95% confidence intervals obtained using bootstrapping.

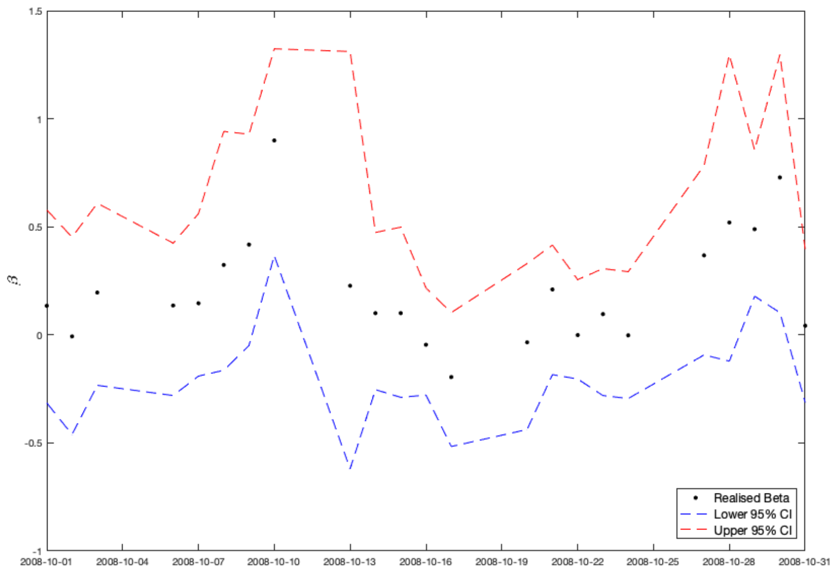
Figure 1: Plot of Trajectory of Realised Betas with 95% Confidence Intervals



Although the majority of the realised betas are between -1 and 1, due to the confidence intervals being somewhat wide and the realised betas fluctuating away from 0, visually it does not appear that the long-short position is market neutral. If the realised betas were in a horizontal line from 0 with much narrower confidence intervals it would be an indication of market-neutrality, which is not the case here.

D)

Figure 2: Plot of Trajectory of Realised Beta with 95% Confidence Intervals for October 2008



Here we are looking at the realised betas and their 95% confidence intervals at a more granular level for October 2008. Although some days do have a realised beta of close to zero, the 95% confidence intervals for these days can range from [-0.5, 1.3]. With the realised betas varying to some degree with the confidence intervals being somewhat wide we cannot conclude that the beta for the long short portfolio is equal to zero, making the strategy market neutral. If the 95% confidence intervals were narrower such as between [-0.1,0.1] we could propose that the strategy may be tending towards market neutrality before carrying out any more tests. However with the majority of the confidence interval upper bounds being less than 1, our long-short portfolio will have offered some protection from large swings in returns in the market during the turbulent times of 2008.

E)

In order to determine the market neutrality of this hedge fund, we run a statistical test to test a null hypothesis of zero beta at the 5% level. If 0 is not contained in the 95% confidence interval of beta, then we reject the null hypothesis. If the percentage of rejections across the 999 days is greater than 10%, we can conclude the hedge fund is not market neutral.

For our long-short portfolio 734 of the 999 confidence intervals for the daily realised beta contained zero. Hence we had 265 rejections of the null hypothesis. With an average number of rejections of 0.3610. Hence we have 36% of the confidence intervals satisfying our rejection criteria. Since this is greater than the 10% rule, we would reject the null hypothesis of market neutrality for the long short portfolio.

This result should not be too surprising as both GS and INTC are among the top 100 companies by weight in the S&P 500. Hence taking a position in only both of these companies is unlikely to be diverse enough to be market neutral, especially in volatile years such as that in 2008.

Task 2

In this task we will investigate the returns on our two stocks and the market separately and obtain their individual realised betas, while also investigating the correlation between the residual returns of the two stocks.

A)

Having obtained the continuous returns for both stocks and the market we obtain the following table.

Table 4: Summary Statistics of Continuous Returns

Stock	Average	Min	5th Percentile	95th Percentile	Max
GS	0.000105	-7.4746	-0.4302	0.4286	9.2862
INTC	0.000018	-2.5364	-0.3418	0.3354	5.3075
SPY	-0.000005	-2.9285	-0.2153	0.2091	3.7564

Over the 4 year period we can see that the average continuous five minute return was essentially zero. This shouldnt be surprising as we don't expect to see large returns at such a granular time-scale. We cannot yet determine whether the stocks outperformed the market as the jump process of the stocks may result in their returns being lower than the market overall. We can however note that the stocks appeared to be more volatile than the market. Comparing the difference between the max and min values, we can see that GS and INTC fluctuated by 16.7608% and 7.8439% respectively, whereas the markets value is only 6.6849%. This observation is supported by both of the stocks having a wider 90% confidence interval (95% - 5% values).

B)

Using our value for alpha of 4.5 when separating jump returns from diffusive returns, we detected 56 jumps in the market.

It is hard to interpret this number by itself. However intuitively we would expect this to be higher than the long run average number of jumps given the financial uncertainty and volatility that followed the financial crisis of 2008.

C)

To calculate the realised beta we will differentiate between returns due to jumps and those that are diffusive for the market and each stock. We will then calculate the estimator for realised beta for each day for both stocks separately, similar to what we did in the first task. Having calculated realised betas for both stocks we obtain the following table.

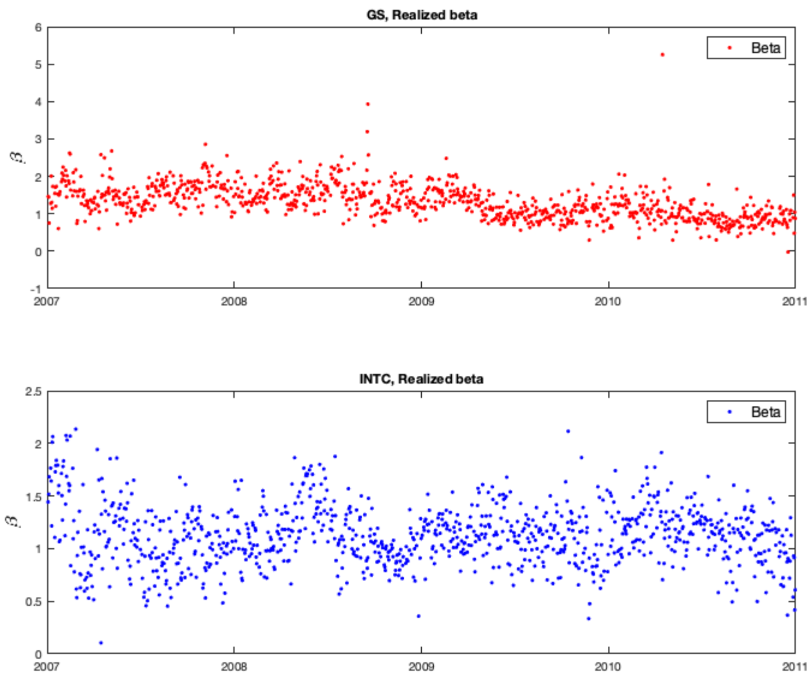
Table 5: Summary Statistics of Realised Betas

Stock	Average	Min	5th Percentile	95th Percentile	Max
GS	1.3226	-0.0203	0.6901	2.0857	5.2521
INTC	1.1271	0.1050	0.6706	1.6300	2.1351

We can see that the GS stock has a higher daily beta on average than the INTC stock. This implies that given an increase (decrease) in the returns of the market by 1%, you would expect the GS stock to increase(decrease) by 1.32%. This higher beta often means that the GS stock is a relatively risky investment compared to that of INTC. This is supported by our previous observation that GS stock seemed to exhibit higher levels of volatility compared with the INTC and SPY returns when comparing maximum and minimum values. We also note that for one day GS had a beta of 5.25 which could lead to amplified price movements in the GS price if the market fluctuates wildly.

D) Here we calculated the realised betas for both stocks and obtained summary statistics for both.

Figure 3: Realised Betas for GS and INTC from 2007 to 2011



Looking at the GS plot, it is evident the stock is riskier than the market with the majority of beta values being greater than 1. We also see a number of very large positive betas greater than 3.

Looking at the INTC plot, the trend is less clear, however it appears to be slightly riskier than the market with more beta values being greater than 1. However, the large positive beta values are much less frequent than with the GS plot (only a handful of betas in INTC plot above 2), suggesting overall it is less risky than GS.

The realised betas for GS had some larger spikes than INTC during 2008 and 2010. The 2008 spike can be explained by the financial crash, which left the financial services sector in turmoil. This period was catalysed by the crash of Lehman Brothers on 15th September 2008, hence we see the major spikes towards the back end of 2008. The 2010 spike can be explained by reports emerging in late April that GS was under a criminal investigation by Federal Prosecutors relating to the 2008 housing market crash. In July 2010, they were ordered to pay $550 million by the SEC. All these reports and criminal charges had a considerable effect on the GS stock price., specific to GS rather than the market.

E)

Having calculated the residual correlations we obtained the following summary statistics.

Table 6: Summary Statistics of Residual Correlation

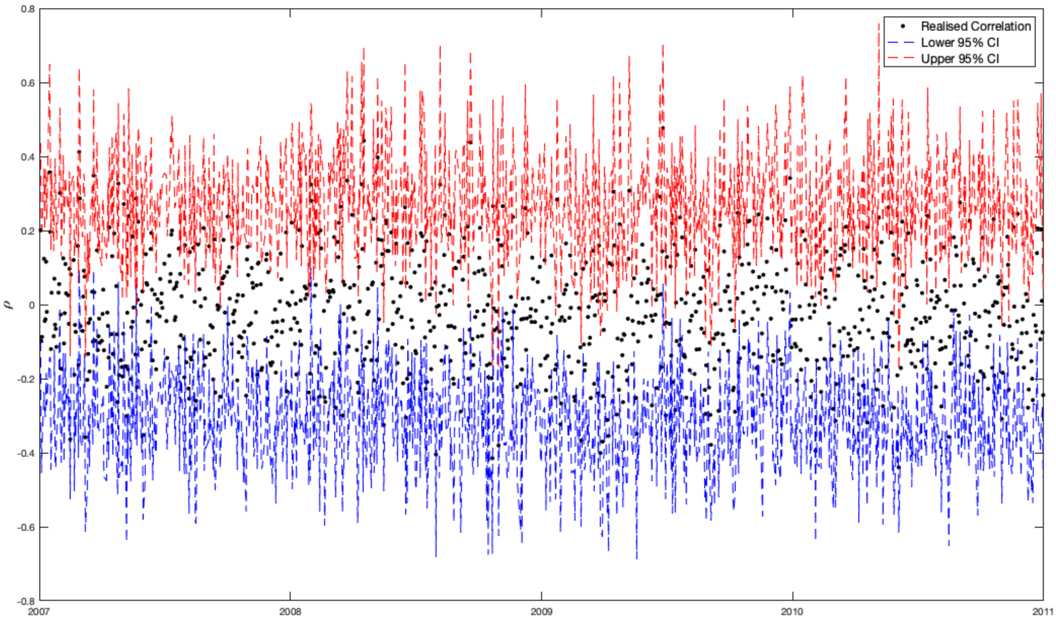
Average	Min	5th Percentile	95th Percentile	Max
-0.0230	-0.4394	-0.2522	0.2225	0.4762

Correlation between stocks can often involve more parameters than just the market returns. For example we might expect that a stock will have higher correlation with a stock within the same sector, than it would with a stock in a different sector. Given that INTC (Intel Corporation) and GS (Goldman Sachs) are in different industries, we may expect the average of their daily residual correlations to be low. As we can see from the data, this is indeed the case with a mean value of -0.023. Although the max residual correlation value does seem high it is likely that this is an outlier due to the 95th percentile being equal to 0.2225 with an average correlation of essentially zero.

F)

Here we will compute 95% confidence intervals for the realised correlation between the residuals using bootstrapping with $k_n = 11$ with 1000 simulations. The following is a plot of the daily realised correlations and their corresponding 95% confidence intervals.

Figure 4: Plot of Trajectory of Residual Realised Correlations with 95% Confidence Intervals

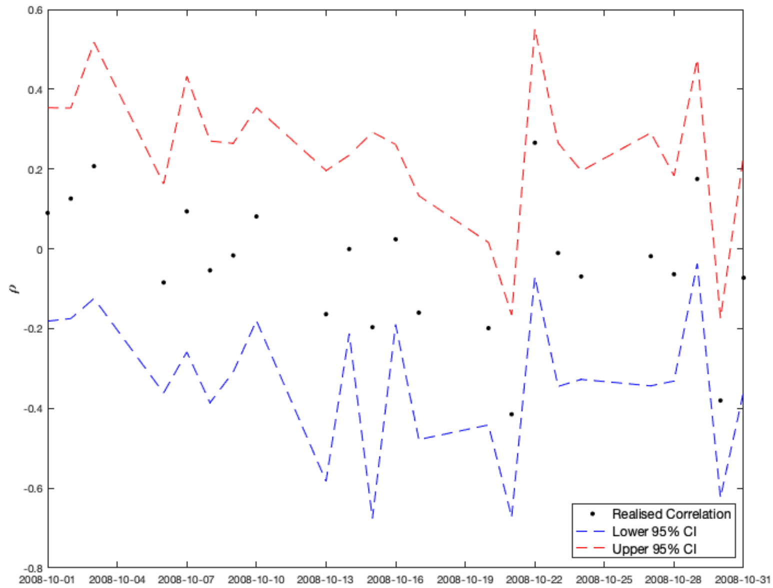


As expected, (given the low value we calculated in the previous question) the realised correlation between the residual returns for most of the sample space is close to zero. This should not be too surprising as both GS and INTC belong to different industries. The market would be one of the biggest things they have in common that would affect their stock price, while trends in the tech or investment banking industry are only likely to have a visible effect on one of the two stocks. If for example our second stock was that of J.P. Morgan we would assume that the residual correlation between these two stocks would be higher in absolute value and more than likely positive due to both GS and JPM being very similar in terms of the services they provide and the different factors that affect both of their performance.

Although the confidence intervals between both stocks here are fairly wide, the majority of them are within the bounds of -0.4 and 0.4 which for such large companies that would have common exposures to things such as foreign exchange fluctuations and inflation is not an extremely large correlation.

G)

Figure 5: Trajectory of Residual Realised Correlations with 95% Confidence Intervals for Oct 2008



The residual correlation confidence intervals between the two stocks for this period do not indicate any extreme additional correlation during the market crash of 2008 compared to other time periods within the sample. This should not be surprising as most of the common market moves due to volatile markets during this time will be captured by the realised beta and market returns for this time period, which we have theoretically removed using our method of calculating the realised residuals. Once again with the confidence intervals being so wide with most of them containing zero it is difficult to say visually that there is a statistically significant non-zero correlation between the residual returns.

H)

Here we will consider the the hypothesis $H_0 : \rho_{e,t} = 0$ versus the alternative $H_1 : \rho_{e,t} \neq 0$.

Our figures for each of the proportions calculated from our confidence intervals were as follows:

Table 7: Proportions for Hypothesis Test

Proportion	Value
ρ_0	3.8056
ρ_+	0.0357
ρ_-	0.1230

As the value for ρ_0 is 3.8056 this indicates that for 959 days out of our sample of 999 trading days of returns the 95% confidence interval of the residual correlation contained zero.

As the value for ρ_+ is 0.0357 this indicates that for 9 days out of our sample of 999 trading days of returns the upper and lower bounds of the 95% confidence interval of the residual correlation were greater than zero.

As the value for ρ_- is 0.1230 this indicates that for 31 days out of our sample of 999 trading days of returns the upper and lower bounds of the 95% confidence interval of the residual correlation were greater less zero.

Since p_0 is so large we fail to reject the null hypothesis that the residual correlation is equal to zero.

Task 3

A)

For Δ_n small enough, each jump time $t \in T$ will lie in a distinct discrete interval of size Δ_n. Thus, to each $t_p \in$ T there is a unique index i$_p$ such that $t_p \in ((i_p - 1)\,\Delta_n, i_p\Delta_n]$.

Now rewrite the model as

$$\Delta Y_{t_p} = \begin{pmatrix} \beta^+ & \beta^- \end{pmatrix} \begin{pmatrix} (\Delta Z_{t_p})_+ \\ (\Delta Z_{t_p})_- \end{pmatrix}$$

and the estimation of β is

$$\begin{pmatrix} \widehat{\beta^+} \\ \widehat{\beta^-} \end{pmatrix} = \begin{pmatrix} \frac{\sum_{p=1}^{P_n} \left(\Delta_{i_p}^n X_1\right)\left(\Delta_{i_p}^n X_2\right) 1_{(\Delta_{i_p}^n X_1 \geq 0)}}{\sum_{p=1}^{P_n} \left(\Delta_{i_p}^n X_1\right)^2 1_{(\Delta_{i_p}^n X_1 \geq 0)}} \\ \frac{\sum_{p=1}^{P_n} \left(\Delta_{i_p}^n X_1\right)\left(\Delta_{i_p}^n X_2\right) 1_{(\Delta_{i_p}^n X_1 \leq 0)}}{\sum_{p=1}^{P_n} \left(\Delta_{i_p}^n X_1\right)^2 1_{(\Delta_{i_p}^n X_1 \leq 0)}} \end{pmatrix} = \begin{pmatrix} \frac{\sum_{a=1}^{A_n} (\Delta_{i_a}^n X_1)(\Delta_{i_a}^n X_2)}{\sum_{a=1}^{A_n} (\Delta_{i_a}^n X_1)^2} \\ \frac{\sum_{b=1}^{B_n} \left(\Delta_{i_b}^n X_1\right)\left(\Delta_{i_b}^n X_2\right)}{\sum_{b=1}^{B_n} \left(\Delta_{i_b}^n X_1\right)^2} \end{pmatrix}$$

where X_1 denotes the market return, X_2 denotes the specific stock return, a denotes the index of positive jump return, b denotes the index of negative jump return and $A_n + B_n = P_n$.

Now focus on $\widehat{\beta^+}$

$$\begin{aligned} \widehat{\beta^+} &= \frac{\sum_{a=1}^{A_n} (\Delta_{i_a}^n X_1)\,(\Delta_{i_a}^n X_2)}{\sum_{a=1}^{A_n} \left(\Delta_{i_a}^n X_1\right)^2} \\ &= \frac{\sum_{a=1}^{A_n} (\Delta_{i_a}^n X_1)\,(\Delta_{i_a}^n X_2 - \beta\Delta_{i_a}^n X_1 + \beta\Delta_{i_a}^n X_1)}{\sum_{a=1}^{A_n} \left(\Delta_{i_a}^n X_1\right)^2} \\ &= \frac{\sum_{a=1}^{A_n} (\Delta_{i_a}^n X_1)\,(\Delta_{i_a}^n X_2 - \beta\Delta_{i_a}^n X_1)}{\sum_{a=1}^{A_n} \left(\Delta_{i_a}^n X_1\right)^2} + \beta \\ &= \frac{\sum_{a=1}^{A_n} (\Delta_{i_a}^n X_1)\,(\Delta_{i_a}^n E)}{\sum_{a=1}^{A_n} \left(\Delta_{i_a}^n X_1\right)^2} + \beta \end{aligned} \tag{2}$$

where $\Delta_{i_a}^n E$ is the error process for the positive jump return.

Now write the error process in continuous part and discontinuous part

$$\begin{aligned} \Delta_{i_a}^n E &= \Delta_{i_a}^n X_2 - \beta\Delta_{i_a}^n X_1 \\ &= \Delta_{i_a}^n X_2^c + \Delta_{i_a}^n X_2^d - \beta\Delta_{i_a}^n X_1^c - \beta\Delta_{i_a}^n X_1^d \\ &= (\Delta_{i_a}^n X_2^c - \beta\Delta_{i_a}^n X_1^c) + (\Delta_{i_a}^n X_2^d - \beta\Delta_{i_a}^n X_1^d) \end{aligned}$$

Because there is an assumption for the jump regression model, if at time t there is a jump in $X_{1,t}$, then at that same instant, $X_{2,t}$ cannot have an idiosyncratic jump (jump independent of the market index). It means $\Delta_{i_a}^n X_2^d - \beta\Delta_{i_a}^n X_1^d = 0$. Then

$$\begin{aligned} \Delta_{i_a}^n E &= \Delta_{i_a}^n X_2^c - \beta\Delta_{i_a}^n X_1^c \\ &= \Delta_{i_a}^n E^c \end{aligned}$$

By the formula

$$\begin{aligned} E_t^c - E_{t-\Delta n}^c &= \int_{t-\Delta_n}^t \sqrt{ce_s} dw_s \\ &\approx \sqrt{ce_t}\sqrt{\Delta n} Z \end{aligned}$$

where ce is the local variance process related to the error process and Z is a standard normal random variable.

The error process can be written as

$$\begin{aligned}\Delta^n_{i_a} E &= \Delta^n_{i_a} E^c \\ &= \sqrt{ce_{t_a}}\sqrt{\Delta n} Z_a\end{aligned} \tag{3}$$

where t_a is the time of positive jump return.

Now combine (1) and (2)

$$\begin{aligned}\widehat{\beta^+} - \beta^+ &= \frac{\sum_{a=1}^{A_n} (\Delta^n_{i_a} X_1) \sqrt{ce_{t_a}}\sqrt{\Delta n} Z_a}{\sum_{a=1}^{A_n} \left(\Delta^n_{i_a} X_1\right)^2} \\ \Delta_n^{-\frac{1}{2}}(\widehat{\beta^+} - \beta^+) &= \frac{\sum_{a=1}^{A_n} (\Delta^n_{i_a} X_1) \sqrt{ce_{t_a}} Z_a}{\sum_{a=1}^{A_n} \left(\Delta^n_{i_a} X_1\right)^2} \\ &= \sum_{a=1}^{A_n} w_a Z_a\end{aligned}$$

where $w_a = \frac{\Delta^n_{i_a} X_1 \sqrt{ce_{t_a}}}{\sum_{a=1}^{A_n} (\Delta^n_{i_a} X_1)^2}$

And then

$$\begin{aligned}Z_a &\sim N(0,1) \\ \Rightarrow \sum_{a=1}^{A_n} w_a Z_a &\sim N(0, \sum_{a=1}^{A_n} w_a^2) \\ \Rightarrow \Delta_n^{-\frac{1}{2}}(\widehat{\beta^+} - \beta^+) &\sim N(0, \sum_{a=1}^{A_n} w_a^2) \\ \Rightarrow \widehat{\beta^+} - \beta^+ &\sim N(0, \sum_{a=1}^{A_n} w_a^2 \Delta_n)\end{aligned}$$

Finally the asymptotic distribution of $\widehat{\beta^+}$ is

$$\Rightarrow \widehat{\beta^+} \sim N(\beta^+, \sum_{a=1}^{A_n} w_a^2 \Delta_n)$$

where $\sum_{a=1}^{A_n} w_a^2 = \sum_{a=1}^{A_n} \frac{(\Delta^n_{i_a} X_1)^2 ce_{t_a}}{\left(\sum_{a=1}^{A_n} (\Delta^n_{i_a} X_1)^2\right)^2}$

The asymptotic distribution of $\widehat{\beta^-}$ can be found by replacing the positive jump returns with the negative jump returns and follow the same way as above. It will be

$$\widehat{\beta^-} \sim N(\beta^-, \sum_{b=1}^{B_n} w_b^2 \Delta_n)$$

where $\sum_{b=1}^{B_n} w_b^2 = \sum_{b=1}^{B_n} \frac{(\Delta^n_{i_b} X_1)^2 ce_{t_b}}{\left(\sum_{b=1}^{B_n} \left(\Delta^n_{i_b} X_1\right)^2\right)^2}$

B) Having calculated the positive and negative jump betas for each stock the models for both stocks are as follows:

The model of INTC is

$$\Delta Y_{t_p} = 1.1567(\Delta Z_{t_p})_+ + 0.7624(\Delta Z_{t_p})_-$$

The model of GS is

$$\Delta Y_{t_p} = 1.2422(\Delta Z_{t_p})_+ + 1.2818(\Delta Z_{t_p})_-$$

Now use the asymptotic distribution in part A to find the 95% confidence intervals of β for upward moves and downward moves respectively.

In the model for INTC:
The confidence interval for β^+ is (0.9825,1.3309).
The confidence interval for β^- is (0.6284,0.8964).

β^+ and β^- have two completely different ranges, i.e. there is no overlap between the two intervals so we fail to accept the hypothesis $\beta^+ = \beta^-$.

It is hard to say what this means for the INTC stock but it could mean that the market is bullish on INTC and tech stocks as a whole and that it perceives that tech stocks feel less of an effect of during market downturns and are less risky during these times. But when the market as a whole is doing well then INTC becomes a riskier investment as the opportunity cost increases with better investment alternatives now prevailing in the market.

In the model for GS:
The confidence interval of β^+ is (1.0602,1.4243).
The confidence interval of β^- is (0.6554,1.9082).

β^+ and β^- have elements common to both ranges, i.e. there is an overlap between the two intervals so we fail to reject the hypothesis $\beta^+ = \beta^-$.

We note how the range of confidence interval of β^- is larger than the confidence interval of β^+. This could be due to the fact we have less negative jumps for GS while they also have a larger variance in their jump returns, thus making the interval wider.

In order to improve the study of this hypothesis we may want to have a larger sample to test on.