hydrograph\_scale

<https://github.com/tclarkin/hydrograph_scale>

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# Hydrograph Scaling

Hydrographs can be scaled using one of four different methods:

1. Peak Scaling
2. Volume Scaling
3. Peak & Volume Scaling
4. Balanced Scaling
5. Beard Balanced
6. Peak Only Scaling
7. Modified Peak and Volume Scaling

The theory and mathematics for each of these methods of scaling are described below. Before the descriptions, however, let’s establish our mathematical notation. In general, uppercase symbols represent an array of values, while lowercase symbols represent a single value from an array. For example, represents ordinate from the hydrograph array . Mean values are represented by a bar, , and are generally used to represent volumes in terms of average flow. Commonly used subscripts are:

= return period

= input hydrograph

= input hydrograph relative to input hydrograph peak

= portion of hydrograph being scaled

= portion of hydrograph previously scaled

= output (scaled) hydrograph

## Peak Scaling

The concept of peak scaling is simple: multiply the entire hydrograph by the ratio of the desired peak over the current peak. First, the ratio of the desired peak and input hydrograph peak is determined:

(1.1.1)

where represents the peak for the return period of interest and represents the peak of the input hydrograph, . The entire input hydrograph is then multiplied by this ratio to produce the scaled hydrograph:

(1.1.2)

## Volume Scaling

The concept of volume scaling is nearly identical to peak scaling: multiply the entire hydrograph by the ratio of the desired volume over the input hydrograph volume. First, the ratio of the desired volume and input hydrograph volume is determined:

(1.2.1)

where represents the average flow for the return period of interest and represents the average flow of the input hydrograph, . The entire input hydrograph is then multiplied by this ratio to produce the scaled hydrograph:

(1.2.2)

## Peak & Volume Scaling

The concept of peak and volume scaling is scaling the hydrograph to the peak, then correcting the rest of the hydrograph ordinates to have the desired volume. First, peak scaling was conducted by multiplying the entire input hydrograph by the ratio of the desired peak over the input hydrograph peak, as described in Section 1.1.

Next, volume scaling was conducted. The end goal was volume scaling to create a scaled hydrograph, , such that the resulting average flow of the scaled hydrograph, , is equal to the average flow of the desired frequency volume, . Thus,

(1.3.1)

To develop the volume scaling function for , we began with the equation of a line:

(1.3.2)

where is the dependent variable and is the independent variable related by a slope and a y-axis intercept of . For our application, is a single ordinate, , from the scaled hydrograph, and is a single ordinate, , from the input array, . Thus, equation (1.3.2) became:

(1.3.3)

Because we did not want to scale the peak and we do not want the rest of the hydrograph to be discontinuous from the peak, we made two assumptions:

Assumption 1:

Assumption 2: when

where represent an ordinate from the raw input hydrograph, , and represents an ordinate value relative to the peak, . Using these assumptions, equation (1.3.3) became:

(1.3.4)

Inserting equation (1.3.4) into equation (1.3.1) and expanding the mean calculation resulted in the following:

(1.3.5a)

(1.3.5b)

(1.3.5c)

where represents a specific ordinate in , and is the number of ordinates in . To solve for , the resulting equation was:

(1.3.6)

The solved value of was then used with equation (1.3.4) to calculate the ordinates for the scaled portion of the hydrograph.

## Balanced Scaling

Balanced scaling uses the same concept as peak & volume scaling, except with volume scaling modified for use with multiple durations. First, peak scaling was conducted by multiplying the entire input hydrograph by the ratio of the desired peak over the input hydrograph peak, as described in Section 1.1.

Next, volume scaling was conducted. The end goal was volume scaling to create a scaled hydrograph, , such that the resulting average flow of the scaled hydrograph, , is equal to the average flow of the desired frequency volume, . Because multiple durations were being used, we assumed that was comprised of two segments: and , where is the portion of the input hydrograph being scaled and is the portion of the hydrograph previously scaled (and not currently being scaled). Thus, the base equation is:

(1.4.1)

To develop the volume scaling function for , we began with the equation of a line:

(1.4.2)

where is the dependent variable and is the independent variable related by a slope and a y-axis intercept of . For our application, is a single ordinate, , from the scaled hydrograph, and is a single ordinate, , from the input array, . Thus, equation (1.4.2) became:

(1.4.3)

Because we did not want to scale the previously scaled portion of the hydrograph and we do not want the hydrograph to be discontinuous, we made two assumptions:

Assumption 1:

Assumption 2: when

where represent an ordinate from the portion of the raw input hydrograph being scaled, , and represents an ordinate value relative to the peak of that same portion, . Using these assumptions, equation (1.3.3) became:

(1.4.4)

Inserting equation (1.4.4) into equation (1.4.1) resulted in the following:

(1.4.5a)

(1.4.5b)

(1.4.5c)

where represents a specific ordinate in , and and are the number of ordinates in and , respectively. To solve for , the resulting equation was:

(1.4.6)

The solved value of was then used with equation (1.4.4) to calculate the ordinates for the scaled portion of the hydrograph.

## Beard Balanced

The original balanced hydrograph scaling developed by Leo Beard uses a similar concept, but with flat multipliers for each duration. First, peak scaling was conducted by multiplying the peak flow ordinate by the ratio of the desired peak over the input hydrograph peak.

Next, volume scaling was conducted. The end goal was volume scaling to create a scaled hydrograph, , such that the resulting average flow of the scaled hydrograph, , is equal to the average flow of the desired frequency volume, . As in Section 1.4, we assumed that was comprised of two segments: and , where is the portion of the input hydrograph being scaled and is the portion of the hydrograph previously scaled (and not currently being scaled). Thus, the base equation is:

(1.5.1)

To develop the volume scaling function for , we assume a scaling factor is applied:

(1.5.2)

Inserting equation 1.5.2 into equation 1.5.1 yields:

(1.5.3)

To solve for , the resulting equation was:

(1.5.4)

The solved value of was then used with equation (1.5.2) to calculate the ordinates for the scaled portion of the hydrograph. This process is repeated for each duration being considered.

## Peak Only Scaling

One weakness of linear (and log) scaling, is the tendency to overinflate the baseflow at the beginning and end of the hydrograph. To work around this, we developed a similar (but inverted) method to that presented in section 1.3, but inverted to preserve baseflow ().

To develop the volume scaling function for , we began with the equation of a line:

(1.6.1)

where is the dependent variable and is the independent variable related by a slope and a y-axis intercept of . For our application, is a single ordinate, , from the scaled hydrograph, and is a single ordinate, , from the input array, . Thus, equation (1.6.2) became:

(1.6.2)

Because we did not want to scale the baseflow, but we do want to scale the peak to a defined value, we made three assumptions:

Assumption 1:

Assumption 2: when

Assumption 3: when

where represent an ordinate from the raw input hydrograph, , represents an ordinate value relative to the minimum flow,. Using assumptions 1 and 2, equation (1.6.2) became:

(1.6.3)

We then solve for by incorporating assumption 3:

(1.6.4)

The solved value of was then used with equation (1.3.3) to calculate the ordinates for the scaled portion of the hydrograph.

## Modified Peak and Volume Scaling

One weakness the peak and volume scaling presented in Section 1.3, is the tendency to have negative flows because only peaks are preserved during the volume scaling. To work around this, a modified methodology was developed. First, the peak was scaled according to the peak only method described in Section 1.6.

To develop the volume scaling function for , we began with a second order equation:

(1.7.1)

where is the dependent variable, is the independent variable, and , , and are unknown factors. For our application, is a single ordinate, , from the scaled hydrograph, and is a single ordinate, , from the input array, . Thus, equation (1.7.2) became:

(1.7.2)

Because we did not want to scale the peak or the baseflow, we make the same three assumptions:

Assumption 1:

Assumption 2: when

Assumption 3: when

where represent an ordinate from the raw input hydrograph, , represents an ordinate value relative to the minimum flow,. Using assumptions 1 and 2, equation (1.7.2) became:

(1.7.3)

We then use assumption 3 to develop an estimate for :

(1.7.4)

We then use equation 1.3.1 combined with equation 1.7.4 into equation 1.7.3 and solve for

(1.7.5a)

(1.7.5b)

(1.7.5c)

(1.7.5d)

(1.7.5e)

With equation 1.7.5e, we can solve for. With solved, we can solve for and ultimately use equation 1.7.3 to scale all of the ordinates in the hydrograph to match the desired volume () while maintaining the preset baseflow and peak flow.