BC-CTRL — UTF-8 Instruction Set (ℤ-Coefficients)

A paper-first control language for integer linear combinations and affine state operators.

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0. Domains and primitives

- Scalars: $z \in \mathbb{Z}$
- Vectors: $s \in \mathbb{Z}^m$
- Matrices: $A \in \mathbb{Z}^{m\times m}$
- Basis tuples: $B = (b_1, \ldots, b_k) \in \mathbb{Z}^k$
- Coefficient tuples: $c = (c_1, \ldots, c_k) \in \mathbb{Z}^k$

UTF-8 symbols are permitted (e.g. ⟨ ⟩, ℤ, ∘, →, ∈, ∑). Use ASCII hyphens (-).

1. Linear-combination instruction (BC kernel)

BC(EVAL) evaluates an integer linear combination over a chosen basis.

$$\mathrm{BC}(B;c) := \langle B{:}c\rangle = \sum_{i=1}^{k} c_i\, b_i \in \mathbb{Z}$$

BC(SPAN) describes exactly which integers the basis can generate.

Let $d := \gcd(b_1, \ldots, b_k)$. $\mathrm{Span}(B) := \{\langle B{:}c\rangle : c \in \mathbb{Z}^k\} = d\mathbb{Z}$. So $\mathrm{SpansAll}(B) \Leftrightarrow \gcd(B) = 1$.

2. State update instruction (system action)

Operators act on integer state vectors and represent system actions.

OP $O \equiv \langle A \mid u\rangle : \mathbb{Z}^m \to \mathbb{Z}^m$, $O(s) = A\, s + u$, with $A \in \mathbb{Z}^{m\times m}$, $u \in \mathbb{Z}^m$.

Optional anti-chaos bounding uses a modulus to keep values within a chosen residue system.

OPM $O \equiv \langle A \mid u \mid M\rangle$, $O(s) = (A\, s + u) \bmod M$, where $M \in \mathbb{Z}_{>0}$ (scalar) or $M \in \mathbb{Z}_{>0}^{\ m}$ (componentwise).

3. Composition instruction (action calculus)

Composition compresses action chains into a single operator.

If $O_1 = \langle A_1 \mid u_1\rangle$ and $O_2 = \langle A_2 \mid u_2\rangle$, then COMP $O_2 \circ O_1 = \langle A_2A_1 \mid A_2u_1 + u_2\rangle$.

Bounded case (same modulus M): $\langle A_2|u_2|M\rangle \circ \langle A_1|u_1|M\rangle = \langle A_2A_1 \mid A_2u_1 + u_2 \mid M\rangle$.

4. Iteration instruction (time evolution)

A system trace is defined by repeated application of operators.

STEP $s_{t+1} = O_t(s_t)$.
ITER $s_{t+n} = (O_{t+n-1} \circ \cdots \circ O_t)(s_t)$. If $O_t \equiv O$ (autonomous): $s_{t+n} = O^n(s_t)$.

5. Polynomial-as-basis instruction (optional)

Polynomials fit the same basis-coefficient pattern by taking monomials as the basis.

Let $M_n := (x^n, x^{n-1}, \ldots, x, 1)$ and $a := (a_n, \ldots, a_0) \in \mathbb{Z}^{n+1}$.
POLY $\langle M_n : a \rangle := \sum_{j=0}^{n} a_{n-j}\, x^{n-j}$.
Example (cubic): $g(x) := \langle (x^3, x^2, x, 1) : (a,b,c,d) \rangle = ax^3 + bx^2 + cx + d$.

6. Minimal paper protocol (one line per fact)

A canonical log line records time, state transition, and the acting operator.

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t: s_{t+1} = ⟨A_t | u_t | M_t⟩(s_t)
t: z_t = BC(B_t; c_t) (optional scalar control/measurement)
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