MATHEMATICS

SECTION A

January 29, 2024

1 Matrix

1. Find $|AB|$,if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.

2. If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$, then find values of p.

3. Using properties of determinants, show that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3\ (a+b+c)(ab+bc+ca)$

4. Find the inverse of the following matrix, using elementary transformations : $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

2 Differentiation

5. Differentiate $e^{\sqrt{3x}}$, with respect to x.

6. Find the order and degree if defined of the differential equation.

$$\frac{d^2y}{d^2x} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

7. Find the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

8. Solve the differential equation :

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

9. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and $x \neq y$, prove that $\dfrac{dy}{dx} = -\frac{1}{(x+1)^2}$

10. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$.

3 Vectors

11. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in the cartesian form.

12. $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $|a|$ and $|b|$.

13. Find the volume of a cuboid whose edges are given by $-3\hat{i}+7\hat{j}+5\hat{k}$, $-5\hat{i}+7\hat{i}-3\hat{k}$ and $7\hat{i}-5\hat{j}-3\hat{k}$.
14. Find the direction cosines of a line which makes equal angles with the coordinate axes.
15. Find the cartesian and vector equations of the plane passing through the points $A\,(2,5,-3)$, $B\,(-2,-3,5)$, $C\,(5,3,-3)$.
16. Find the equation of the line passing through $(2,0,3)$, $(1,1,5)$ and $(3,2,4)$.Also, find their point of intersection.

4 Optimization

17. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of its edge is 12 cm ?

5 Functions

18. Let $f : N \to Y$ be a function defined as

$$f(x) = 4x + 3, \tag{1}$$

Where $Y = y \in N : y = 4x + 3, for some x \in N$. Show that f is invertible. Find its inverse.

6 Integration

19. Find:

$$\int \frac{x^2+x+1}{(x+2)\left(x^2+1\right)}dx$$

20. Find :

$$\int \sqrt{3-2x-x^2}dx$$

21. Find: $\int_1^3 \left(x^2+2+e^{2x}\right)dx$ as the limit of sums.
22. Using integration, find the area of the triangular region whose sides have the equations $y = 2x+1$, $y = 3x+1$, $x = 4$.
23. If $(a+bx)\,e^{\frac{y}{x}} = x$, then prove that

$$x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx}-y\right)^2.$$

7 Probability

24. A coin is tossed 5 times. what is the probablity of getting (i) 3 heads,(ii) at most 3 heads.
25. There are three coins. One is a two-headed coin, another is a baised coin that comes up heads 75% of the time and the third is an unbaised coin.One of the three coins is chosen at random and tossed. If it shows heads,What is the probability that it is the two-headed coin ?
26. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.

8 Intersection of conics

27. Find the point on the curve $y^2 = 4x$, which is nearest to the point $(2,-8)$.