MATHEMATICS

SECTION A

January 27, 2024

1. Find $|AB|$,if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$.
2. Differentiate $e^{\sqrt{3x}}$, with respect to x.
3. Find the order and degree if defined of the differential equation.
$$\frac{d^2y}{d^2x} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$$
4. Find the direction cosines of a line which makes equal angles with the coordinate axes.
5. A line passes through the point with position vecto $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in the cartesian form.
6. Find :
$$\int \sqrt{3 - 2x - x^2}dx$$
7. If $A = \begin{bmatrix} p & 2 \\ 2 & p \end{bmatrix}$ and $|A^3| = 125$,then find values of p.
8. Examine whether the operation $*$ defined on$\mathbb{R}$, the set of all real numbers, by $a * b = \sqrt{a^2 + b^2}$ is a binary operation or not, and if it is a binary operation,find whether it is associative or not.
9. $\left|\overrightarrow{a}\right| = 2,\left|\overrightarrow{b}\right| = 7$ and $\overrightarrow{a} \times \overrightarrow{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$,find the angle between $\left|a\right|$ and $\left|b\right|$.
10. Find the volume of a cuboid whose edges are given by $-3\hat{i} + 7\hat{j} + 5\hat{k}$, $-5\hat{i} + 7\hat{i} - 3\hat{k}$ and $7\hat{i} - 5\hat{j} - 3\hat{k}$.
11. A coin is tossed 5 times. what is the probablity of getting (i) 3 heads,(ii) at most 3 heads?
12. Find the probability distribution of X, the number of heads in a simultaneous toss of two coins.
13. Find the general solution of the differentail equation $\frac{dy}{dx} = e^{x+y}$.
14. Using properties of determinants, show that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3\ (a+b+c)(ab+bc+ca)$
15. Let $f : N \to Y$ be a function defined as
$$f(x) = 4x + 3, \tag{1}$$
Where $Y = y \in N : y = 4x + 3, for some x \in N$. Show that f is invertible.Find its inverse.
16. If $x\sqrt{1+y}$+ $y\sqrt{1+x}$=0 and $x \neq y$, prove that $\frac{dy}{dx} = -\frac{1}{(x+1)^2}$..

17. If $(\cos x)^{y} = (\sin y)^{x}$, find $\frac{dy}{dx}$.

18. Find:

$$\int \frac{x^2 + x + 1}{(x + 2)\left(x^2 + 1\right)} dx$$

19. Solve the differential equation :

$$x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$$

20. If $(a + bx)\, e^{\frac{y}{x}} = x$, then prove that

$$x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2.$$

21. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of its edge is 12 cm ?

22. Find the cartesian and vector equations of the plane passing through the points $A\,(2, 5, -3)$,$B\,(-2, -3, 5)$,$C\,(5, 3, -3)$.

23. Find the point on the curve $y^2 = 4x$,which is nearest to the point $(2, -8)$.

24. Find: $\int_1^3 \left(x^2 + 2 + e^{2x}\right)dx$ as the limit of sums.

25. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$,$y = 3x + 1$,$x = 4$.

26. Find the inverse of the following matrix,using elementary transformations : $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

27. There are three coins. One is a two-headed coin, another is a baised coin that comes up heads 75% of the time and the thrid is an unbaised coin.One of the three coins is chosen at random and tossed. If it shows heads,What is the probability that it is the two-headed coin ?

28. Find the equation of the line passing through $(2, 0, 3)$,$(1, 1, 5)$ and $(3, 2, 4)$.Also, find their point of intersection.