

Modelling potential environmental impacts of science activity in Antarctica

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Abstract

We use GPS data collected on a science expedition in Antarctica to estimate hiking functions for the speed at which humans traverse terrain differentiated by slope and by ground cover (moraines and rock). We use the estimated hiking functions to build weighted directed graphs as a representation of specific environments in Antarctica. From these we estimate using a variety of graph metrics—particularly betweenness centrality—the relative potential for human environmental impacts arising from scientific activities in those environments. We also suggest a simple approach to planning science expeditions that might allow for reduced impacts in these environments.

1 Introduction

Overview of Antarctic science: when and where, its intensity etc. Background on international treaties, etc.

Relevant findings as to human impacts in Antarctica. Note that in this environment even ‘leave only footprints’ is likely impacting the environment in significant ways.

Overview of sections ahead.

2 Our approach and related work

We chose to explore the question of where human impacts are likely to be strongest using an approach closely related to work on patterns of human movement in archaeology (Verhagen et al. 2019) where likely and potential movement paths of humans across landscapes have been used to infer the settlement structure and human geography of large-scale landscapes. Closely related work in biology investigates the structure and geography of animal transportation networks (Perna and Latty 2014). Both approaches rely on the idea that humans or animals move around in an environment in time or energy efficient ways. These approaches rely on hiking functions that relate speed of movement across a terrain to its slope.

Hiking functions must be applied in some context where locations across a landscape are connected to one another. Because hiking functions are asymmetric, with estimated speed of movement up slopes different than estimated speeds down the same slope, landscape must be represented in a way that allows for this asymmetry. We therefore represent terrain in our landscapes as directed graphs (or network) of connections between locations regularly distributed in planimetric space across the landscape of interest. Because the graphs are directed the costs associated with movement between two locations can be different depending on the direction of movement. Additionally, we associate with locations (i.e., vertices in the graph) the ground cover at the location, which also affects the speed at which it can be traversed. Because the ground cover in the Antarctic environments under study can be broadly categorised into only two types, moraine and rock, we use the ground cover of a location to switch between two estimated hiking functions, rather than the more widely used approach of penalising movement on different ground covers by applying cost factors. We consider previous work on hiking functions and directed graphs in more detail below.

2.1 Hiking functions

Prisner and Sui (2023) provide an overview of a variety of functions that have been used to model how hiking times and speeds vary with terrain slope. They consider longstanding rules of thumb (Naismith 1892), and

later modifications thereof (Langmuir 1984), along with more recent such guidance from the Swiss and German Alpine Clubs (Winkler et al. 2010; Deutscher und Österreichischer Alpenverein 2011). These functions estimate the time taken to travel 1km, referred to as *pace*, based on slope expressed as *rise over run*, that is change in elevation divided by horizontal distance. They are all piecewise functions with sharp changes in estimated pace at specific slopes.

Alongside these hiking pace functions Prisner and Sui (2023) also present hiking speed functions (generally referred to as simply *hiking functions*) from Tobler (1993 generally considered the first hiking function) and more recent, related but more firmly empirically grounded alternatives offered by Márquez-Pérez et al. (2017), Irmischer and Clarke (2018), and Campbell et al. (2019). Another hiking function not discussed by Prisner and Sui (2023) is presented by Rees (2004). These hiking functions are all continuous in the slope of the terrain so that $v = f(\theta)$, where v is the speed, and θ is the slope. They can all be parameterised to control the maximum speed attainable, the slope at which maximum speed is attained (expected to be a shallow downhill slope), and the rate at which speed falls off with increasing slope.

The functional forms of some hiking functions are shown in Table 1 and graphed in Figure 1.

Table 1: Functional forms of hiking functions

Description	Equation	Examples
Exponential	$ae^{b \theta-c }$	Tobler (1993), Márquez-Pérez et al. (2017)
Gaussian	$ae^{-b(\theta-c)^2}$	Irmischer and Clarke (2018), Campbell et al. (2019)
Lorentz	$\frac{a}{[b+d(\theta-c)^2]}$	Campbell et al. (2019)
Quadratic	$a + b\theta + c\theta^2$	Rees (2004)

The parameterisations of the functions in Figure 1 have been chosen for

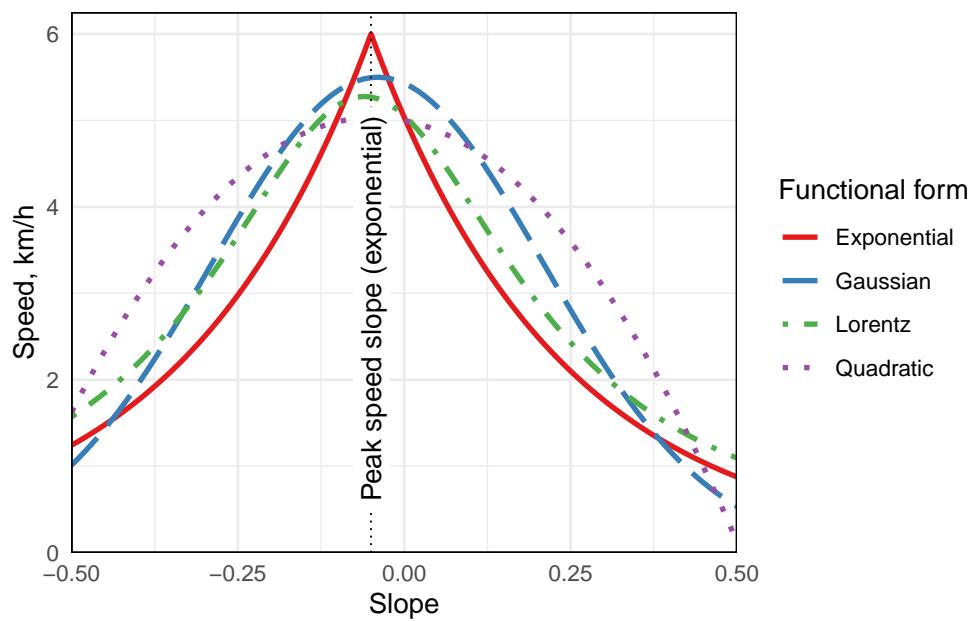


Figure 1: Example hiking functional forms: Exponential (Tobler 1993), Gaussian (Irmischer and Clarke 2018), Lorentz (Campbell et al. 2019), and Quadratic (Rees 2004).

illustrative purposes only, although the parameter values for the exponential function shown are such that $v = 6e^{-3.5|\theta|+0.05}$ as suggested in Tobler (1993). These parameters give *Tobler's hiking function* but, as has been noted elsewhere, (see Herzog 2010; Campbell et al. 2019), are based on a poorly specified fit to secondary data on hiking speeds found in Imhof (1950, 217–220). Nevertheless, these parameter values are widely applied in the literature.

All these hiking functional forms are somewhat *ad hoc*. They exhibit desirable, expected properties: a peak speed at a slope near zero, which we expect to be slightly negative (i.e., downhill), and continuously falling speeds as the slope deviates from the slope of peak speed. However, there is no theoretical basis for the specific functional forms listed in Table 1. More principled approaches might be developed based on the literature on the physiology of human movement, see e.g. **CITATIONS NEEDED HERE**. In general, approaches based on minimising energy use yield similar results to empirical speed-slope functions, although it is worth noting that they more reliably generate zig-zag or ‘switchback’ movement behaviour on steep slopes (Llobera and Sluckin 2007). However, these are hard to implement, and we have adopted empirically-derived and locally-specific hiking functions following Márquez-Pérez et al. (2017), Irmischer and Clarke (2018), and Campbell et al. (2019). This choice is based on available data and the goals of our study, where the relative cost of different potential routes in the landscape is more important than exact prediction of routes. In practice, almost any function with the peaked form of those shown in Figure 1 is suited to our requirements.

It is commonplace in many applications to also incorporate a penalty on movement contingent on land cover, especially for off-track or off-road movement. For example the speed attainable off-track in forested terrain might be only half that attainable in grasslands. Unsurprisingly, there are no widely agreed land cover penalties, but see for example, those compiled by Herzog (2020). A hiking function derived by Wood et al. (2023) includes the local gradient of the terrain (i.e., the maximum slope at each location, not the slope in the direction of movement) as a covariate in the estimated function. It is possible that this kind of approach including other spatial covariates,

such as land cover (which would be a categorical variable in most cases) in the estimation of complex hiking functions might be more widely applied in future work. In our application, because there are only two kinds of navigable land cover—moraine (or gravel) and rock—we chose instead to estimate two hiking functions, one for each land cover, and estimate movement costs conditional on the land cover at each location. This also has the advantage of allowing for differing effects of slope on speed due to land cover, where for example, gravel might allow more rapid movement on the level surfaces, but more rapidly reduce speed on slopes. Details of this approach are reported and discussed in Section 4.1.

2.2 Representing the landscape

As has been noted, hiking functions are usually asymmetric, with the highest attainable speed at a slight downhill slope. This asymmetry means that unless analysis is focused on assessing movement cost from a single origin or to a single destination, it is necessary to represent the landscape in a way that can accommodate asymmetry. We therefore represent the landscape as a directed graph $G(V, E)$ of vertices and directed edges connecting them. In this representation, graph vertices v_i are locations with associated elevation and land cover. Vertices minimally have spatial coordinates (x_i, y_i) , an elevation z_i , and a land cover, C_i :

$$v_i = (x_i, y_i, z_i, C_i) \quad (1)$$

Graph edges $e_{ij} = (v_i, v_j)$ are directed connections between vertices for which a change in elevation between the start and end vertex can be calculated, and a slope derived, based on the elevation difference, and the horizontal distance between the vertices. Thus the slope θ_{ij} of edge e_{ij} is given by

$$\theta_{ij} = \frac{z_j - z_i}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \quad (2)$$

The obvious way to derive such a graph from spatial data is to assign each cell

in a raster digital elevation model (DEM) to a graph vertex, so that graph vertices are arranged in a regular square grid or lattice, and this approach has been widely adopted. **CITATIONS NEEDED HERE** However, this still requires a decision about how to define graph edges, which can be thought of in terms of *allowed moves* between vertices. Three possibilities on a square lattice—Rook’s, Queen’s, and Knight’s moves—are shown in Figure 2. Another option shown in the figure is to lay out a regular hexagonal grid of vertex locations and link each vertex to its six nearest neighbours.

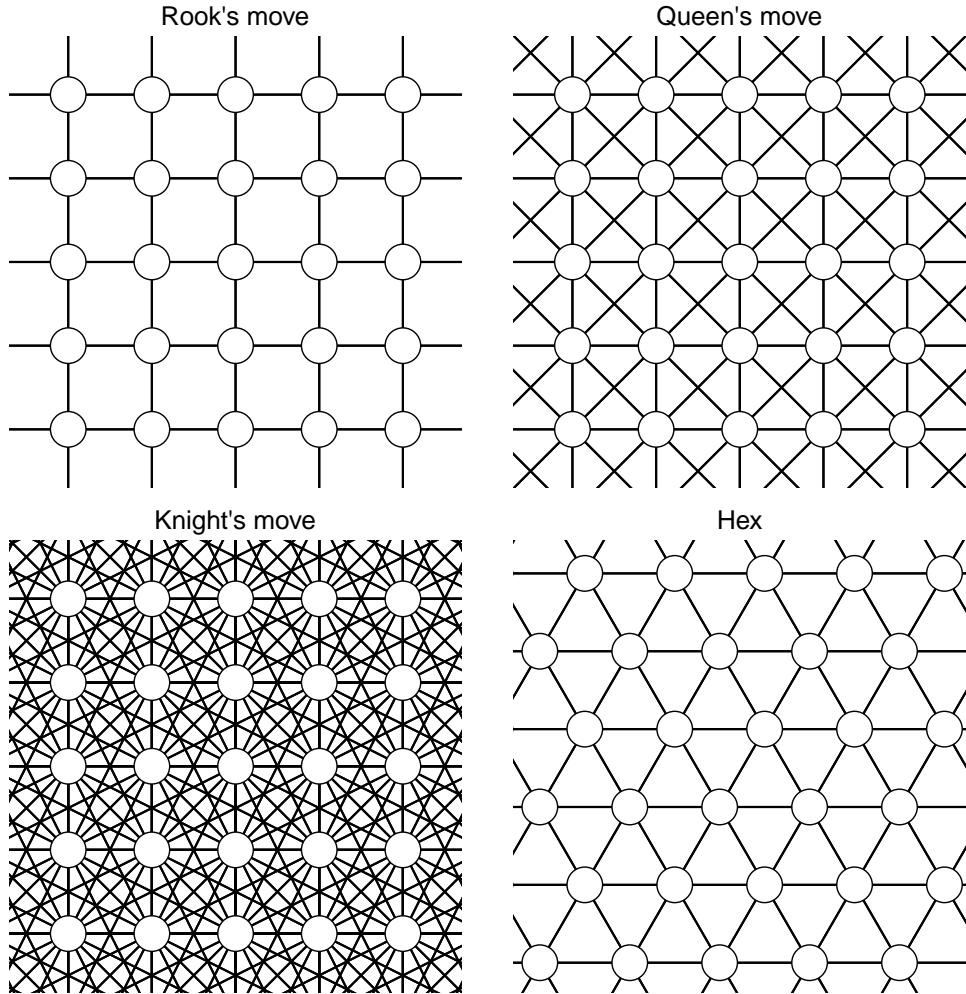


Figure 2: Possible graph lattices.

The advantage of a square lattice is that it is easy to derive vertex elevations assuming a DEM at the desired lattice spacing is available. A hexagonal lattice on the other hand usually requires that elevation values be interpolated from a DEM to the lattice locations.

Both square and hexagonal lattices lead to geometric artifacts when the allowed moves are used to determine contours of equal travel time or *isochrones* from an origin point on a flat surface with uniform movement speeds. In this situation, the Rook's move produces diamond-shaped isochrones, while the Queen's move produces octagons. In general isochrones will be polygons with as many sides as the number of neighbours of each vertex. While the 16-gons of the Knight's move lattice are likely to be close enough to circular for many purposes, the resulting graph is denser than the alternatives. Etherington (2012) suggests that by combining results from randomly generated planar graphs such geometric artifacts can be removed, but this substantially increases the computational requirements. Another computationally intensive approach might draw on methods for estimating geodesics on complex triangular meshes (Martínez et al. 2005). Because these geometric effects of graph structure are masked when we introduce varying movement speeds due to the slope of each edge, we do not consider these more complex approaches necessary in our application. Nevertheless, more circular base isochrone shapes are to be preferred, all other things equal.

A further complication when a lattice includes edges of varying length as in the Queen's and Knight's move lattices, is bias in the estimation of slopes leading to lower estimated movement costs for longer edges. This is because longer edges (e.g., the knight's moves) may ‘jump’ across intervening segments of varying slope, smoothing them to a single slope estimate. Such smoothing will usually result in shorter edge traversal times along such edges than those that would accumulate along intervening sections of varying slope. This problem is discussed in a slightly different context by Campbell et al. (2019, 96–98).

Any choice of graph structure is necessarily a compromise, and estimates of movement rates are always an approximation (Goodchild 2020). We consider the differing edge lengths introduced by all the square lattices other

than the Rook’s case to be problematic, and favour the hexagonal lattice structure over the simple square lattice because a base hexagonal isochrone is preferable to a diamond shape.

Having settled on a hexagonal lattice for the graph structure, at a chosen resolution we set out a regular hexagonal grid of locations across the study area, and assign to each point an elevation by interpolation from a DEM. Because the DEM is at finer resolution than the hexagonal lattice, the choice of interpolation method is not a major concern. Our approach applies bilinear interpolation based on elevation values in the DEM cell the graph vertex falls in and its four orthogonal neighbours. Based on the difference in elevation of the vertices at each end of each edge we estimate a slope using Equation 2, and also traversal times using our estimated hiking function. The important point here is that our graph is *directed* so that different traversal times t_{ij} and t_{ji} are estimated for moving between vertex v_i and v_j depending on the direction of travel.

As noted in the previous section, we estimate two hiking functions, one for moraine and one for rock land cover. When an edge connects locations of two different land cover types the estimated traversal time is the mean of the traversal times for each land cover.

3 Data sources

3.1 Antarctic geospatial data

Geospatial data for Antarctica were obtained from sources referenced in Cox et al. (2023b), Cox et al. (2023a), and Felden et al. (2023). The study area was defined to be the Skelton and Dry Valleys basins, as defined by the NASA Making Earth System Data Records for Use in Research Environments (MEaSUREs) project (Mouginot and University Of California Irvine 2017) and shown in Figure 3a. The Skelton basin was included because while the expedition GPS data was ostensibly collected in the McMurdo Dry Valleys, it actually extends into that basin as shown in Figure 3b. Elevation data from the Reference Elevation Model of Antarctica project Howat et al. (2022), and geology from GeoMAP Cox et al. (2023b) are shown in Fig-

ure 3c. The five largest areas of contiguous non-ice surface geology across the study area shown in Figure 3d were chosen to be the specific sites for more detailed exploration using the methods set out in this paper. These range in size from around 320 to 2600 square kilometres.

3.2 GPS data from an expedition

FRASER: Timeline, devices used, and associated protocols for scientists while on site.

GPS data were processed to make them better suited for use in the estimation of hiking functions.

The first processing step was to confirm the plausibility of the data, particularly the device-generated speed distance between fixes, and elevations associated with fixes. The challenges of post-processing GPS data are well documented and relate to issues with GPS drift which can lead to estimated non-zero movement speeds as a result of noise in the signal. The raw GPS data included distance since last fix, speed, and elevation estimates and it was determined in all cases that the device generated results for these measurements were likely to be more reliable than post-processing the raw latitude-longitude fixes to calculate the values.

The second processing step was to remove fixes associated with faster movement on other modes of transport than walking. Wood et al. (2023) cite a number of previous works that base detection of trip segments based on recorded speeds. This method was trivially applicable to our data to a limited degree as scientists arrive at the expedition base camp and occasionally also travel on helicopters on trips to more remote experimental sites.

The third, more challenging processing step was to deal with sequences of fixes associated with non-purposeful movement when scientists were in or around base camp, at rest stops, or at experimental sites. Crude filters removed fixes with low recorded distances between fixes (less than 2.5 metres), high turn angles at the fix location (greater than 150°), and fixes recorded on ice-covered terrain, but this did not clean the data sufficiently for further analysis. An additional filtering step was to count fixes (across all scientists)

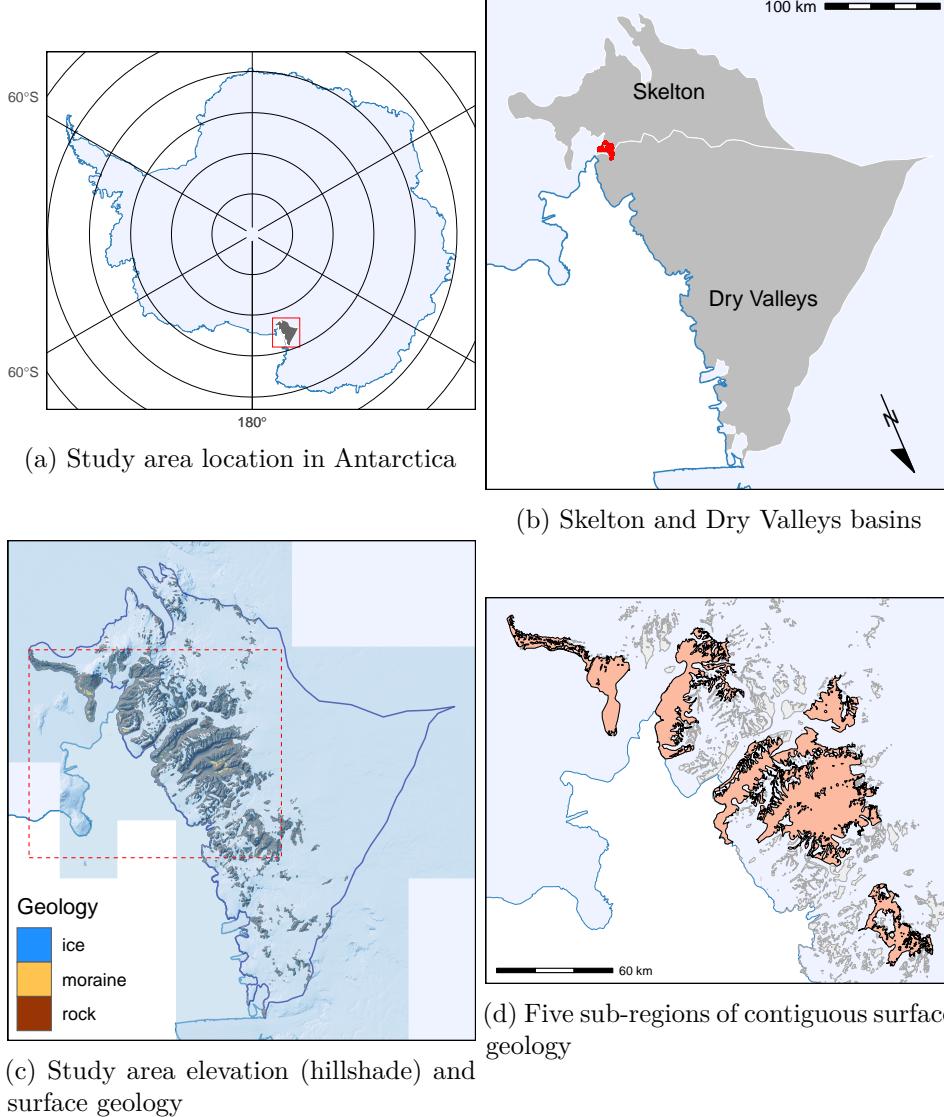
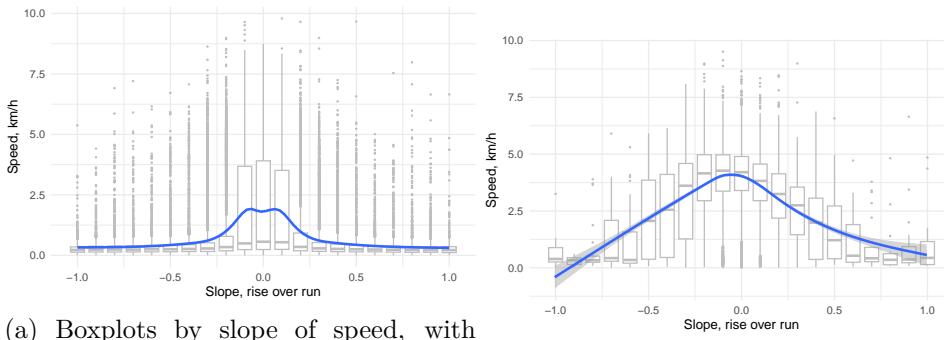


Figure 3: The study area.

in square grid cells and remove all fixes in grid cells with more than 50 fixes.

This left one persistent concern: an over-representation of consecutive fixes recorded at exactly the same elevation, resulting in many fixes with estimated slopes of exactly 0, and leading to a clearly evident dip in estimated movement speeds at 0 slope (Figure 4a). It is likely that these fixes are associated with GPS device drift, so it was decided to remove all fixes where estimated slope was exactly 0. Figure 4b shows the improvement in even a crudely estimated hiking function derived from local scatterplot (LOESS) smoothing. Note that such functions are likely overfitted and not used further in our analysis where we favour more easily parameterised functions such as those discussed in Section 2.1.



(a) Boxplots by slope of speed, with smoothed estimated hiking function showing a ‘dip’ due to over-representation of 0 slope fixes
(b) After filtering the estimated hiking function no longer has a dip.

Figure 4: GPS data and crudely estimated hiking functions before and after filtering the to remove fixes associated iwht non-purposive movement.

4 Methods and results

4.1 Hiking functions

We fit three alternative functional forms to the cleaned GPS data: exponential (Tobler 1993), Gaussian (following Irmischer and Clarke 2018), and Lorentz (following Campbell et al. 2019) using the Levenburg-Marquardt algorithm (Moré 1978) as provided by the `nlsLM` function in the `minpack.lm` R package (Elzhov et al. 2022). The raw data and fitted curves are shown

in Figure 5.

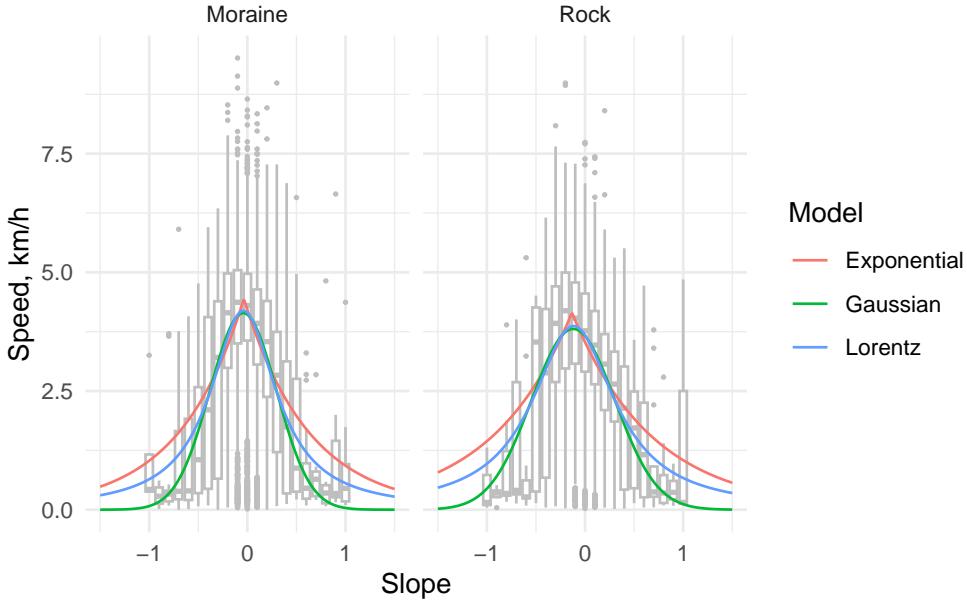


Figure 5: Three possible hiking functions applied to GPS data split by land cover.

The Lorentz function offers a marginal improvement in the model fit in comparison with the Gaussian function, while both are clearly better than the exponential form. However, the improvement offered by the Lorentz function over the Gaussian is marginal: residual standard error 1.489 vs 1.491 on Moraine, and 1.487 vs 1.488 on Rock, and inspection of the curves shows that estimated hiking speeds for the Gaussian functions are much closer to a plausible zero on very steep slopes. We therefore chose to adopt Gaussian hiking functions for the remainder of the present work.

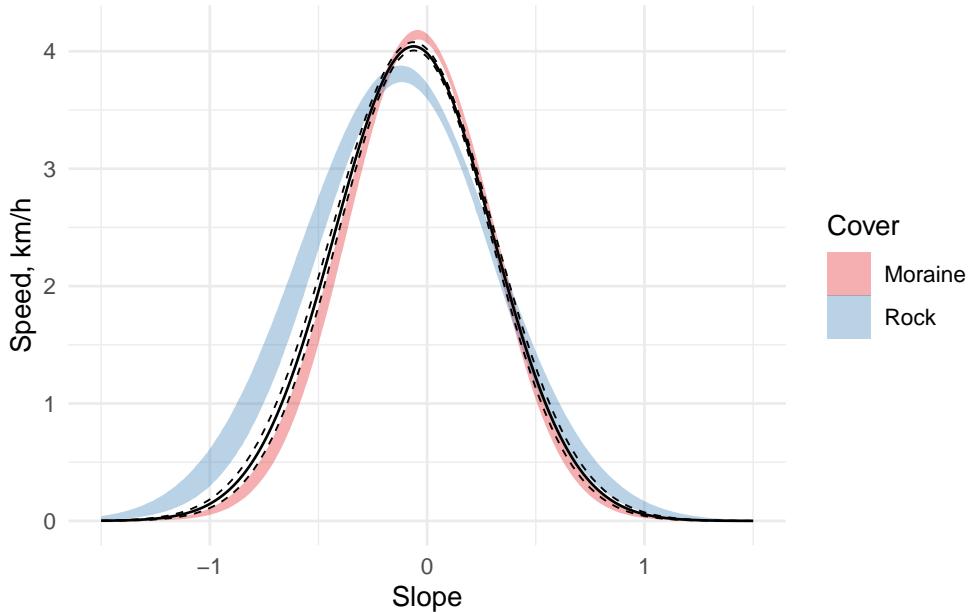
In previous work researchers have applied a ground cover penalty cost to a base hiking function to estimate traversal times. We instead, as shown, estimate different hiking functions for the two ground cover types present. The peak speed on rock is attained on steeper downhill slopes than on moraines, perhaps indicative of the greater care required on downhill gravel slopes. Meanwhile the highest speeds on level terrain are attained on moraines.

To confirm that the fitted functions are sufficiently different between the two

ground covers relative to a function fitted to all data we ran

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.....10.....20.....30.....40.....50  
.....60.....70.....80.....90.....100  
.....110.....120.....130.....140.....150  
.....160.....170.....180.....190.....200  
.....210.....220.....230.....240.....250  
.....260.....270.....280.....290.....300  
  
.....10.....20.....30.....40.....50  
.....60.....70.....80.....90.....100  
.....110.....120.....130.....140.....150  
.....160.....170.....180.....190.....200  
.....210.....220.....230.....240.....250  
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.....10.....20.....30.....40.....50  
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.....210.....220.....230.....240.....250  
.....260.....270.....280.....290.....300  
  
.....10.....20.....30.....40.....50  
.....60.....70.....80.....90.....100  
.....110.....120.....130.....140.....150  
.....160.....170.....180.....190.....200  
.....210.....220.....230.....240.....250  
.....260.....270.....280.....290.....300
```

And now we can overplot the models showing their 95% confidence intervals.



4.2 Landscapes as graphs

We developed R code (R Core Team 2024) to build graphs (i.e. networks) with hexagonal lattice structure and estimated traversal times for graph edges derived from our hiking functions. Graphs are stored as igraph package (Csárdi and Nepusz 2006; Csárdi et al. 2024) graph objects for further analysis.

How terrain is handled in graph edge cost estimation (i.e., assigning half the cost from the terrain at the vertex at each end of the edge).

`igraph` implements betweenness centrality measures (Freeman 1978; Brandes 2001), which we used to identify vertices and/or edges most likely to be traversed by minimum travel time routes across the terrain.

4.3 Betweenness centrality limited by radius.

4.4 Impact minimizing networks

Tentative proposal for impact minimizing networks based on minimum spanning trees, but noting the issue with respect to directed graphs when these

would more correctly be arborescences (Korte and Vygen 2018).

5 Discussion

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6 Conclusions

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