In[•]:=
```
Integrate[
 Integrate[k * Exp[-1 * k * Sqrt[(x^2 + y^2)]], {x, x0, x0 + 1}, Assumptions ->
   ({Element[k, Reals], Element[x0, Reals], Element[y0, Reals], x0 > 0, y0 > 0, k > 0})],
 {y, y0, y0 + 1}, Assumptions -> ({Element[k, Reals], Element[x0, Reals],
    Element[y0, Reals], x0 > 0, y0 > 0, k > 0})
]
```

Out[•]=
$$\text{Integrate}\left[\text{Integrate}\left[e^{-k\sqrt{x^2+y^2}}\ k,\ \{x,\ x0,\ 1+x0\},\ \text{Assumptions}\to\{k\in\mathbb{R},\ x0\in\mathbb{R},\ y0\in\mathbb{R},\ x0>0,\ y0>0,\ k>0\}\right],\ \{y,\ y0,\ 1+y0\},\ \text{Assumptions}\to\{k\in\mathbb{R},\ x0\in\mathbb{R},\ y0\in\mathbb{R},\ x0>0,\ y0>0,\ k>0\}\right]$$

In[•]:=
```
Integrate[
 Integrate[Sqrt[(x^2 + y^2)], {x, x0, x0 + a}, Assumptions ->
   ({Element[a, Reals], Element[x0, Reals], Element[y0, Reals], x0 > 0, y0 > 0, a > 0})],
 {y, y0, y0 + a}, Assumptions -> ({Element[a, Reals], Element[x0, Reals],
    Element[y0, Reals], x0 > 0, y0 > 0, a > 0})
]
```

Out[•]=

$$\frac{1}{24}\left(8\,x0\,y0\,\sqrt{x0^2+y0^2}-8\,a\,y0\,\sqrt{a^2+2\,a\,x0+x0^2+y0^2}-\right.$$
$$8\,x0\,y0\,\sqrt{a^2+2\,a\,x0+x0^2+y0^2}-8\,a\,x0\,\sqrt{a^2+x0^2+2\,a\,y0+y0^2}-$$
$$8\,x0\,y0\,\sqrt{a^2+x0^2+2\,a\,y0+y0^2}+8\,a^2\,\sqrt{2\,a^2+2\,a\,x0+x0^2+2\,a\,y0+y0^2}+$$
$$8\,a\,x0\,\sqrt{2\,a^2+2\,a\,x0+x0^2+2\,a\,y0+y0^2}+8\,a\,y0\,\sqrt{2\,a^2+2\,a\,x0+x0^2+2\,a\,y0+y0^2}+$$
$$8\,x0\,y0\,\sqrt{2\,a^2+2\,a\,x0+x0^2+2\,a\,y0+y0^2}+2\,x0^3\,\text{ArcSinh}\left[\frac{y0}{x0}\right]-2\,x0^3\,\text{ArcSinh}\left[\frac{a+y0}{x0}\right]+$$
$$4\,y0^3\,\text{Log}\left[x0+\sqrt{x0^2+y0^2}\right]+2\,x0^3\,\text{Log}\left[-y0+\sqrt{x0^2+y0^2}\right]+4\,x0^3\,\text{Log}\left[y0+\sqrt{x0^2+y0^2}\right]-$$
$$4\,y0^3\,\text{Log}\left[a+x0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-a^3\,\text{Log}\left[-y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-$$
$$3\,a^2\,x0\,\text{Log}\left[-y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-3\,a\,x0^2\,\text{Log}\left[-y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-$$
$$x0^3\,\text{Log}\left[-y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-5\,a^3\,\text{Log}\left[y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-$$
$$15\,a^2\,x0\,\text{Log}\left[y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-15\,a\,x0^2\,\text{Log}\left[y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-$$
$$5\,x0^3\,\text{Log}\left[y0+\sqrt{a^2+2\,a\,x0+x0^2+y0^2}\right]-4\,a^3\,\text{Log}\left[x0+\sqrt{a^2+x0^2+2\,a\,y0+y0^2}\right]-$$
$$12\,a^2\,y0\,\text{Log}\left[x0+\sqrt{a^2+x0^2+2\,a\,y0+y0^2}\right]-12\,a\,y0^2\,\text{Log}\left[x0+\sqrt{a^2+x0^2+2\,a\,y0+y0^2}\right]-$$
$$4\,y0^3\,\text{Log}\left[x0+\sqrt{a^2+x0^2+2\,a\,y0+y0^2}\right]-2\,x0^3\,\text{Log}\left[-a-y0+\sqrt{a^2+x0^2+2\,a\,y0+y0^2}\right]-$$
$$4\,x0^3\,\text{Log}\left[a+y0+\sqrt{a^2+x0^2+2\,a\,y0+y0^2}\right]+4\,a^3\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$12\,a^2\,y0\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$12\,a\,y0^2\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$4\,y0^3\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$a^3\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$3\,a^2\,x0\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$3\,a\,x0^2\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$x0^3\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$5\,a^3\,\text{Log}\left[a+y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$15\,a^2\,x0\,\text{Log}\left[a+y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$15\,a\,x0^2\,\text{Log}\left[a+y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$\left.5\,x0^3\,\text{Log}\left[a+y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]\right)$$

In[ ]:= **StandardForm[FullSimplify[%22]]**

Out[ ]//StandardForm=

$$\frac{1}{24}\left(8\,x0\,y0\sqrt{x0^2+y0^2}-8\,a\,y0\sqrt{(a+x0)^2+y0^2}-8\,x0\,y0\sqrt{(a+x0)^2+y0^2}-\right.$$
$$8\,a\,x0\sqrt{x0^2+(a+y0)^2}-8\,x0\,y0\sqrt{x0^2+(a+y0)^2}+8\,a^2\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}+$$
$$8\,a\,x0\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}+8\,a\,y0\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}+$$
$$8\,x0\,y0\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}+2\,x0^3\,\text{ArcSinh}\left[\frac{y0}{x0}\right]-2\,x0^3\,\text{ArcSinh}\left[\frac{a+y0}{x0}\right]+$$
$$4\,y0^3\,\text{Log}\left[x0+\sqrt{x0^2+y0^2}\right]-4\,y0^3\,\text{Log}\left[a+x0+\sqrt{(a+x0)^2+y0^2}\right]-$$
$$3\,a^2\,x0\left(\text{Log}\left[-y0+\sqrt{(a+x0)^2+y0^2}\right]+5\,\text{Log}\left[y0+\sqrt{(a+x0)^2+y0^2}\right]\right)-$$
$$3\,a\,x0^2\left(\text{Log}\left[-y0+\sqrt{(a+x0)^2+y0^2}\right]+5\,\text{Log}\left[y0+\sqrt{(a+x0)^2+y0^2}\right]\right)-$$
$$12\,a^2\,y0\,\text{Log}\left[x0+\sqrt{x0^2+(a+y0)^2}\right]-$$
$$12\,a\,y0^2\,\text{Log}\left[x0+\sqrt{x0^2+(a+y0)^2}\right]-4\,y0^3\,\text{Log}\left[x0+\sqrt{x0^2+(a+y0)^2}\right]-$$
$$a^3\left(\text{Log}\left[-y0+\sqrt{(a+x0)^2+y0^2}\right]+5\,\text{Log}\left[y0+\sqrt{(a+x0)^2+y0^2}\right]+4\,\text{Log}\left[x0+\sqrt{x0^2+(a+y0)^2}\right]\right)-$$
$$x0^3\left(-2\,\text{Log}\left[-y0+\sqrt{x0^2+y0^2}\right]-4\,\text{Log}\left[y0+\sqrt{x0^2+y0^2}\right]+\text{Log}\left[-y0+\sqrt{(a+x0)^2+y0^2}\right]+\right.$$
$$5\,\text{Log}\left[y0+\sqrt{(a+x0)^2+y0^2}\right]+2\,\text{Log}\left[-a-y0+\sqrt{x0^2+(a+y0)^2}\right]+$$
$$\left.4\,\text{Log}\left[a+y0+\sqrt{x0^2+(a+y0)^2}\right]\right)+4\,a^3\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$12\,a^2\,y0\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$12\,a\,y0^2\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$4\,y0^3\,\text{Log}\left[a+x0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$a^3\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$3\,a^2\,x0\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$3\,a\,x0^2\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$x0^3\,\text{Log}\left[-a-y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]+$$
$$\left.5\,(a+x0)^3\,\text{Log}\left[a+y0+\sqrt{2\,a^2+x0^2+y0^2+2\,a\,(x0+y0)}\right]\right)$$

In[•]:=
```
FullSimplify[%22 /. {a → 0.5}]
```

Out[•]=
$$
\begin{aligned}
&0.333333\, x0\, y0 \sqrt{x0^2+y0^2} - 0.166667\, y0 \sqrt{0.25 + x0\,(1.+x0) + y0^2} - \\
&0.333333\, x0\, y0 \sqrt{0.25 + x0\,(1.+x0) + y0^2} - 0.166667\, x0 \sqrt{0.25 + x0^2 + y0\,(1.+y0)} - \\
&0.333333\, x0\, y0 \sqrt{0.25 + x0^2 + y0\,(1.+y0)} + 0.0833333 \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)} + \\
&0.166667\, x0 \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)} + 0.166667\, y0 \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)} + \\
&0.333333\, x0\, y0 \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)} + 0.166667\, y0^3 \operatorname{Log}\left[x0 + \sqrt{x0^2+y0^2}\right] - \\
&0.166667\, y0^3 \operatorname{Log}\left[0.5 + x0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] - \\
&0.00520833 \operatorname{Log}\left[-y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] - \\
&0.0260417 \operatorname{Log}\left[y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] - \\
&0.166667\,(0.5 + 1.\, y0)^3 \operatorname{Log}\left[x0 + \sqrt{0.25 + x0^2 + y0\,(1.+y0)}\right] + \\
&0.0208333 \operatorname{Log}\left[0.5 + x0 + \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)}\right] + \\
&y0\,(0.125 + (0.25 + 0.166667\, y0)\, y0) \operatorname{Log}\left[0.5 + x0 + \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)}\right] + \\
&0.00520833 \operatorname{Log}\left[-0.5 - 1.\, y0 + \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)}\right] + \\
&0.0260417 \operatorname{Log}\left[0.5 + y0 + \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)}\right] + \\
&x0 \left(-0.03125 \operatorname{Log}\left[-y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] - \right. \\
&\quad 0.15625 \operatorname{Log}\left[y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] + x0 \left(-0.0625 \right. \\
&\qquad \operatorname{Log}\left[-y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] - 0.3125 \operatorname{Log}\left[y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] + \\
&\qquad x0 \left(0.0833333 \operatorname{ArcSinh}\left[\frac{y0}{x0}\right] - 0.0833333 \operatorname{ArcSinh}\left[\frac{0.5 + y0}{x0}\right] + \right. \\
&\qquad\quad 0.0833333 \operatorname{Log}\left[-y0 + \sqrt{x0^2+y0^2}\right] + 0.166667 \operatorname{Log}\left[y0 + \sqrt{x0^2+y0^2}\right] - 0.0416667 \operatorname{Log}\left[\right. \\
&\qquad\quad \left. -y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] - 0.208333 \operatorname{Log}\left[y0 + \sqrt{0.25 + x0\,(1.+x0) + y0^2}\right] - \\
&\qquad\quad 0.0833333 \operatorname{Log}\left[-0.5 - 1.\, y0 + \sqrt{0.25 + x0^2 + y0\,(1.+y0)}\right] - \\
&\qquad\quad \left. 0.166667 \operatorname{Log}\left[0.5 + y0 + \sqrt{0.25 + x0^2 + y0\,(1.+y0)}\right]\right)\Big) + \\
&\quad (0.03125 + (0.0625 + 0.0416667\, x0)\, x0) \operatorname{Log}\left[-0.5 - 1.\, y0 + \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)}\right] + \\
&\quad \left.(0.15625 + (0.3125 + 0.208333\, x0)\, x0) \operatorname{Log}\left[0.5 + y0 + \sqrt{0.5 + x0\,(1.+x0) + y0\,(1.+y0)}\right]\right)
\end{aligned}
$$