

Solve equation

$$[K]\{X\} = [F]$$



State variable

$$\{X\} = \{u \quad a \quad V\}^t$$



State equations

$$S = 1/2(\mathbf{grad} + \mathbf{grad}^t)u$$

$$B = \mathbf{curl} \, a$$

$$E = -\mathbf{grad} \, V$$

Local field

$$S, \quad B, \quad E$$



Local constitutive laws

$$T = cS - e^t E - h^t B$$

$$H = -hS + \nu B$$

$$D = -eS + \epsilon E$$

Local field

$$T, \quad H, \quad D$$



Volume average

$$\begin{cases} \bar{T} &= 1/V \int T dV \\ \bar{H} &= 1/V \int H dV \\ \bar{D} &= 1/V \int D dV \end{cases}$$

Material coefficients

$$\begin{bmatrix} \bar{T} \\ \bar{H} \\ \bar{D} \end{bmatrix} = \begin{bmatrix} \tilde{C} & -\tilde{h}^t & -\tilde{e}^t \\ \tilde{h} & \tilde{\nu} & \tilde{\alpha}_H^t \\ \tilde{e} & \tilde{\alpha}_H & \tilde{\epsilon} \end{bmatrix} \begin{bmatrix} \bar{S} \\ \bar{B} \\ \bar{E} \end{bmatrix}$$