I. Deduction of the equivalent problem of DRO

For the convenience of discussion, the two-stage distributionally robust FCUC problem is reformulated in a compact form. Here, we use the uppercase to denote the matrices and the lowercase to denote the vectors.

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{y}} \quad & \boldsymbol{a}^\top \boldsymbol{x} + \sup_{\mathrm{P}_\xi \in \mathcal{G}} \mathrm{E}_{\mathrm{P}_\xi}[\mathcal{Q}(\boldsymbol{x},\xi)] \\ \text{s.t.} \quad & \boldsymbol{B}\boldsymbol{x} \le \boldsymbol{c} \end{aligned} \tag{1}$$

where the second stage objective function $\mathcal{Q}(\boldsymbol{x},\xi)$ can be expressed as:

$$\begin{aligned} \mathcal{Q}(\boldsymbol{x},\xi) = \min_{\boldsymbol{z}} \quad & \boldsymbol{d}^\top \boldsymbol{z} \\ \text{s.t.} \quad & \boldsymbol{W}\boldsymbol{z} \le \boldsymbol{h} - \boldsymbol{T}\boldsymbol{x} - \boldsymbol{M}\xi \end{aligned} \tag{2}$$

where the recourse variables $\boldsymbol{z} := \{DR_s\}$. Here, we denote $N_{\boldsymbol{z}}$ as the number of recourse variables in the second stage problem. Denote $N_{\boldsymbol{h}}$ as the number of constraints in the second stage problem.By applying the linear decision rule [1], [2], i.e., the recourse variable is linearly dependent on ξ and ν,

$$\boldsymbol{z} = \boldsymbol{z}_0 + \boldsymbol{z}_{\boldsymbol{\xi}} \cdot \boldsymbol{\xi} + \boldsymbol{z}_\nu \cdot \boldsymbol{\nu} \tag{3}$$

where $\boldsymbol{z_0} \in \mathbb{R}^{N_{\boldsymbol{z}}}$ is the intercept, $\boldsymbol{z_\xi} \in \mathbb{R}^{N_{\boldsymbol{z}} \times N_{\boldsymbol{\xi}}}$ and $\boldsymbol{z_\nu} \in \mathbb{R}^{N_{\boldsymbol{z}} \times N_{\boldsymbol{\xi}}}$ are the coefficient matrix for the random variables $\boldsymbol{\xi}$ and $\boldsymbol{\nu}$, which will be determined afterwards by the dual problem. By applying the conic duality, the proposed model given by (1) and (2) is equivalent to the following dual problem, which is a second-order cone problem (SOCP) and can be solved by off-the-shelf optimization solvers.

$$\begin{aligned} \min \quad & \boldsymbol{a}^\top \boldsymbol{x} + \alpha + \omega\overline{\sigma}^2 \\ \text{s.t.} \quad & \boldsymbol{B}\boldsymbol{x} \le \boldsymbol{c} \\ & \alpha - \boldsymbol{d}^\top \boldsymbol{z}_0 \ge -\underline{\xi}\delta + \overline{\xi}\varepsilon - \kappa + \pi + \overline{\nu}\lambda \\ & \varepsilon - \delta - 2\eta = \boldsymbol{z}_\xi^\top \boldsymbol{d} - \beta \\ & -\kappa - \pi + \lambda = \boldsymbol{z}_\nu^\top \boldsymbol{d} - \omega \\ & \|(\eta,\kappa)\|_2 \le \pi \\ & \boldsymbol{h} - \boldsymbol{T}_k\boldsymbol{x} - \boldsymbol{W}_k\boldsymbol{z}_0 \ge -\underline{\xi}\boldsymbol{\delta}_k + \overline{\xi}\boldsymbol{\varepsilon}_k \\ & \qquad\qquad - \boldsymbol{\kappa}_k + \boldsymbol{\pi}_k + \overline{\nu}^\top \boldsymbol{\lambda}_k \\ & \varepsilon_k - \delta_k - 2\eta_k = \boldsymbol{W}_k\boldsymbol{z}_\xi + \boldsymbol{M}_k \\ & -\boldsymbol{\kappa}_k - \boldsymbol{\pi}_k + \boldsymbol{\lambda}_k = \boldsymbol{W}_k\boldsymbol{z}_\nu \\ & \|\boldsymbol{\eta}_k, \boldsymbol{\kappa}_k\|_2 \le \boldsymbol{\pi}_k \\ & \omega \ge 0, \delta \ge 0, \varepsilon \ge 0, \lambda \ge 0, \boldsymbol{\delta}_k \ge 0, \boldsymbol{\varepsilon}_k \ge 0, \boldsymbol{\lambda}_k \ge 0 \end{aligned} \tag{4}$$

where the decision variables include $\{\alpha,\beta,\delta,\varepsilon,\kappa,\pi,\omega,\lambda,\eta\} \in \mathbb{R}$, $\{\delta_k,\varepsilon_k,\kappa_k,\pi_k,\lambda_k,\eta_k \mid \forall k \in 1,\cdots,N_h\} \in \mathbb{R}$, and $\boldsymbol{z_0},\boldsymbol{z_\xi},\boldsymbol{z_\nu} \in \mathbb{R}^{N_{\boldsymbol{z}}}$. $\boldsymbol{h}_k$, $\boldsymbol{T}_k$ and $\boldsymbol{M}_k$ refer to the k-th row of matrices $\boldsymbol{h}$, $\boldsymbol{T}$ and $\boldsymbol{M}$, respectively. The deduction process for the conic dual problem is conventional and can be found in [3]–[6].

References

[1] X. Chen, M. Sim, P. Sun, and J. Zhang, "A linear decision-based approximation approach to stochastic programming," *Oper. Res.*, vol. 56, no. 2, pp. 344–357, 2008.

[2] P. Xiong, P. Jirutitijaroen, and C. Singh, "A distributionally robust optimization model for unit commitment considering uncertain wind power generation," *IEEE Trans. Power Syst.*, vol. 32, no. 1, pp. 39–49, 2017.

[3] Z. Chen, M. Sim, and P. Xiong, "Robust stochastic optimization made easy with RSOME," *Manage. Sci.*, vol. 66, no. 8, pp. 3329–3339, 2020.

[4] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52–63, 2013.

[5] Y. Zhou, M. Shahidehpour, Z. Wei, Z. Li, G. Sun, and S. Chen, "Distributionally robust unit commitment in coordinated electricity and district heating networks," *IEEE Trans. Power Syst.*, vol. 35, no. 3, pp. 2155–2166, 2020.

[6] D. Bertsimas, M. Sim, and M. Zhang, "Adaptive distributionally robust optimization," *Manage. Sci.*, vol. 65, no. 2, pp. 604–618, 2019.