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The Cloned Dynamics Approach for Lyapunov Exponents Calculation

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DESCRIPTION

This repository contains scripts developed using MATLAB and C languages which are used to implement the Cloned Dynamics methodology. This approach aims to compute the Lyapunov exponents associate to smooth and non-smooth nonlinear dynamical systems.

Summary

1 History

The Cloned Dynamics approach was proposed by Diogo Coutinho Soriano (https://www.diogosoriano.com/) and me (my personal ResearchGate page), with the help and oriented by our professors and friends, Dr. Romis Attux, Dr. Ricardo Suyama and Dr. Marconi Kolm Madrid, during the very beginning of our PhD, in 2008, inspired by the thesis of a good friend, Prof. Dr. Reinaldo Gonçalves Nogueira, titled "*Técnicas Alternativas de Reconhecimento de Caos em Sistemas com Dinâmica Complexa: Análise de um Sistema com Descontinuidade*" (http://repositorio.unicamp.br/jspui/handle/REPOSIP/260285; in portuguese). In his work, Prof. Reinaldo proposed an implementation which was capable to compute the Lyapunov spectrum of non-smooth nonlinear dynamical system, particularly, systems subject to impacts (Figure 1). The system studied was a pendulum with restrictions.

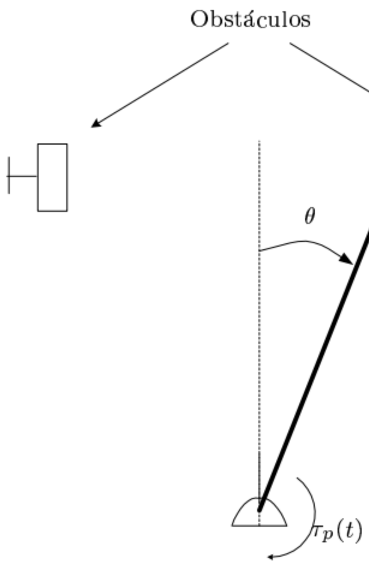

Figure 1: $\tau_p(t)$ represents the torque generated by the motor.

The main idea of the Cloned Dynamics approach is to compute the Lyapunov exponents (or the Lyapunov spectrum). To improve its understanding, consider, for instance, a continuous time third order nonlinear dynamical system, as the Duffing and the Van der Pol oscillators [Guckenheimer and Holmes, 2000] and/or the Chua's circuit [Fazanaro et al., 2010; Parker and Chua, 1989]. Initially, one has to make copies (clones) of the original dynamical system; in this case, three copies. Each copy is perturbed by a infinitesimal δ. The four systems - the original one and the clones - are all evaluated during a small time interval. In the end, the Gram-Schimidt orthonormalization needs to be applied to correct the 'collapse effect' which can be observed due to the expansion of the most divergence direction. The Cloned Dynamics approach was proposed in [Soriano et al., 2012b] to study the effects of the FitzHugh-Nagumo neuronal model driven by a square wave external function.

2 How To Cite

The Cloned Dynamics approach was used do develop some works. The main results were published in some important journals, such as Chaos [Fazanaro et al., 2013], Communications in Nonlinear Science and Numerical Simulation [Fazanaro et al., 2016; Soriano et al., 2018], International Journal of Bifurcation and Chaos [Soriano et al., 2012a, 2018] and others [Fischer et al., 2020], and also in a few conferences and symposiums as [Fazanaro et al., 2010, 2012].

To cite the Cloned Dynamics approach, please use the following:

```
@ARTICLE{article:SorianoFazanaro2012,
  author  = {Soriano, D.C.; Fazanaro, F.I.; Suyama, R.; Oliveira, J.R.; Attux, R.; Madrid, M
      .K.},
  title   = {{A method for Lyapunov spectrum estimation using cloned dynamics and its
      application to the discontinuously-excited FitzHugh-Nagumo model}},
  journal = {Nonlinear Dynamics},
  year    = {2012},
  volume  = {67},
  pages   = {413--424},
  number  = {1},
  month   = {Jan},
  doi     = {10.1007/s11071-011-9989-2}
}
```

3 Organization of the Repository

The repository is organized based on the following folder structure:

- **01.add_to_MATLAB_path:** this folder contains some auxiliary functions as the implementation of 4th order Runge-Kutta integration. Some functions were obtained from the MATLAB Central. It is necessary to add this scripts in the MATLAB folder.

- **02.MATLAB_ode45:** in this folder, it is possible to find the implementations of some of the most important dynamical systems, as the Duffing Oscillator, the Chua oscillator, the FitzHugh-Nagumo neuronal model, and THE most important dynamical system of all, the one that initiate the study of the chaotic behaviour, the dynamical system proposed by Lorenz [1963]. Others will be upload as soon as possible. It was employed the MATLAB language and the ode45 integrator. There are comments and descriptions inside the scripts to improve the understanding of each code.

  Considering the implementation of the Duffing dynamical model[1] - refer to [Guckenheimer and Holmes, 2000; Parker and Chua, 1989] for further analysis of this model -, and the Chua's oscillator circuit implementation [Matsumoto et al., 1985], the scripts are organize as follows:

  (1) `Duffing1989.m`: the dynamical system. This function is used to obtain the time evolution.

  (2) `Duffing1989_ClDyn.m`: this function defines the original dynamical systems and the clones.

  (3) `Duffing1989_TanMap.m`: this function is used to compute the Lyapunov exponents employing the classical Tangent Map methodology. See [Parker and Chua, 1989; Wolf et al., 1985] for further details.

  (4) `ode45_prog11a.m`: plot the time series. It could be interesting to test some different values of the parameters "to feel" the behaviour of the dynamical system. Usually, the transient period must be ignored during the analysis, specially when the Poincaré section is considered. For further information, see [Guckenheimer and Holmes, 2000; Parker and Chua, 1989].

  (5) `ode45_prog21a_PoincareZ.m`: build the Poincaré section. Basically, when one state variable is equal to a value, the others are sampled. For further details, see [Parker and Chua, 1989]. When it is done considering the parameter variation, then it can be obtained the bifurcation diagram.

  (6) `ode45_prog31a_Lyap_TanMap.m`: calculate the Lyapunov exponents using the classical Tangent Map methodology.

  (7) `ode45_prog32a_Lyap_ClDyn.m`: calculate the Lyapunov exponents using the Cloned Dynamics approach. To understand how to set the parameters of the algorithm, refer to the reference [Soriano et al., 2012b].

  (8) `ode45_prog41a_bifurc_vEpsilon_Lyap_TanMap.m`: computes the Lyapunov spectrum considering the variation of the Epsilon (ε) parameter, using the Tangent Map methodology.

[1] Note that the organization of the remaining dynamical systems is similar.

One must to refine the parameter step variation, i.e. decrease the value of such variation, resulting in a more elegant plot. The computational cost will increase as the number of parameter variation increase. Complementary discussion could be found in [Fazanaro et al., 2016].

(9) `ode45_prog42a_bifurc_vEpsilon_Lyap_ClDyn.m`: same as the previous one however using the Cloned Dynamics approach.

(10) `ode45_prog43a_bifurc_vGamma_Lyap_ClDyn.m`: same as the previous ones however considering the variation of the Gamma (γ) parameter.

(11) `ode45_prog44a_bifurc_vOmega_Lyap_ClDyn.m`: analogous to the previous ones. It considers the variation of the Omega (ω) parameter.

(12) `ode45_prog45a_bifurc_vAlpha_var_PoincareZ.m`: implements the bifurcation diagram for the Poincaré section. Considers the variation of one parameter (in this case, the α) and for each step, constructs the section. Here, it is possible to observe when the oscillation period is doubled until the chaotic behaviour occurs.

(13) `ode45_prog51a_Space_Param_Epsilon_Gamma_v01_TanMap.m`: computes the space parameter, i.e. the Lyapunov exponents computation, using the Tangent Map methodology, considering the mutually variation of Epsilon (ε) and Gamma (γ). See the discussions presented in [Fazanaro et al., 2016] and, more important, in [Monti et al., 1999].

(14) `ode45_prog51b_Space_Param_Epsilon_Omega_v01_TanMap.m` same as before, considering the mutually variation of Epsilon (ε) and Omega (ω).

(15) `ode45_prog51c_Space_Param_Gamma_Omega_v01_TanMap.m`: same as before, considering the mutually variation of Gamma (γ) and Omega (ω).

(16) `ode45_prog52a_Space_Param_Epsilon_Gamma_v01_ClDyn.m`: analogous as before however using the Cloned Dynamics approach.

(17) `ode45_prog52b_Space_Param_Epsilon_Omega_v01_ClDyn.m`: idem.

(18) `ode45_prog52c_Space_Param_Gamma_Omega_v01_ClDyn.m`: idem.

- **03.MATLAB_ode45_parfor:** the computation of the Lyapunov exponents is usually very computational expensive. Here, it was implemented the computation of the Lyapunov spectrum using the Parallel computational toolbox of the MATLAB. It is necessary to adjust the number of cores used during each simulation. The scripts presented here were extensively tested with the MATLAB R2014a version however it should work without problems with newer versions.

  (1) This folder contains the scripts to compute the Lyapunov spectrum and the space parameter, all of then using the Parallel toolbox. The organization follows the description presented before.

- **04.MATLAB_RK4:** this folder contains the same implementations described for the **02.MATLAB_ode45** folder however using the 4th order Runge Kutta integration algorithm.

- **05.MATLAB_RK4_parfor:** this folder contains the same implementations described for the **02.MATLAB_ode45_parfor** folder however using the 4th order Runge Kutta integration algorithm.

- **06.C** as a great friend of mine, André Luiz Delai (http://lattes.cnpq.br/4609660221170758), usually says, "C is the mother language of all"[2]. In this folder, there are some implementations using the C language, basically, only to compute the Lyapunov exponents and the Lyapunov spectrum. As discussed in the work [Fazanaro et al., 2016], it is possible to improve the computational cost to (i.e. decrease the total amount of time to) realize such computation when compare to the MATLAB implementation.

  (1) The C codes save the results to disk, into text files. One can use any computational language to plot the results. Here, it is used and provided scripts in MATLAB.

  (2) To compile the C codes, execute the bash file - which can be found inside the folder - using the following command:

  ```
  ./compile_and_run_GCC.sh <name_of_the_C_code>
  ```

- **07.Python:** under development. It will contain the same scripts detailed before yet using the Python 3.8 language and the matplotlib. This documentation will be update as soon as possible considering this development.

[2] https://youtu.be/CYvJPra7Ebk

4 Dynamical Systems Implemented so Far

Table 1 describes the dynamical systems implemented so far (organized in alphabetical order).

Dynamical System	(A)	(B)	(C)	(D)	(E)	(F)
ChuaAdim1985						
Duffing1989	✓	✓	✓	✓	✓	
FitzHughNagumo1961						
Lorenz1963						
Rossler1976						
Rossler1979						

Table 1: Some dynamical systems implemented.

where:

(A) **MATLAB_ode45**;

(B) **MATLAB_ode45_parfor**;

(C) **MATLAB_RK4**;

(D) **MATLAB_RK4_parfor**;

(E) **C**;

(F) **Python**.

Main references related to the dynamical systems studied here:

- **ChuaAdim1985:** [Arena et al., 1995; Matsumoto et al., 1985; Parker and Chua, 1989];

- **Duffing1989:** [Guckenheimer and Holmes, 2000];

- **Lorenz1963:** [Lorenz, 1963];

- **Rossler1976:** [Rössler, 1976].

5 Document Version

- **June 17, 2020:**
  - Version v01a - initial version.
- **June 19, 2020:**
  - Version v01b - update Table 1. Small changes in the main text.

References

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