The Cloned Dynamics Approach for Lyapunov Exponents Calculation

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DESCRIPTION

This repository contains scripts developed using MATLAB and C languages which are used to implement the Cloned Dynamics methodology. This approach aims to compute the Lyapunov exponents associate to smooth and non-smooth nonlinear dynamical systems.

1 History

The Cloned Dynamics approach was proposed by Diogo Coutinho Soriano (https://www.diogosoriano.com/) and me, with the help and oriented by our professors and friends, Dr. Romis Attux, Dr. Ricardo Suyama and Dr. Marconi Kolm Madrid, during the very beginning of our PhD, in 2008, inspired by the thesis of a good friend, Prof. Dr. Reinaldo Gonçalves Nogueira, titled "*Técnicas Alternativas de Reconhecimento de Caos em Sistemas com Dinâmica Complexa: Análise de um Sistema com Descontinuidade*" (http://repositorio.unicamp.br/jspui/handle/REPOSIP/260285; in portuguese). In his work, Prof. Reinaldo proposed an implementation which was capable to compute the Lyapunov spectrum of non-smooth nonlinear dynamical system, particularly, systems subject to impacts (Figure 1). The system studied was a pendulum with restrictions.

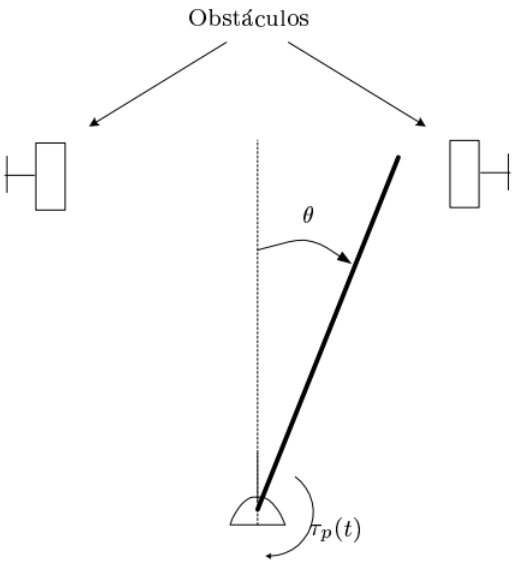

Figure 1: $\tau_p(t)$ represents the torque generated by the motor.

The main idea of the Cloned Dynamics approach is to compute the Lyapunov exponents (or the Lyapunov spectrum). To improve its understanding, consider, for instance, a continuous time third order nonlinear dynamical system, as the Duffing and the Van der Pol oscillators [Guckenheimer and Holmes, 2000] and/or the Chua's circuit [Fazanaro et al., 2010; Parker and Chua, 1989]. Initially, one has to make copies (clones) of the original dynamical system; in this case, three copies. Each copy is perturbed by a infinitesimal δ. The four systems - the original one and the clones - are all evaluated during a small time interval. In the end, the Gram-Schimidt orthonormalization needs to be applied to correct the 'collapse effect' which can be observed due to the expansion of the most divergence direction. The Cloned Dynamics approach was proposed in [Soriano et al., 2012b] to study the effects of the FitzHugh-Nagumo neuronal model driven by a square wave external function.

2 How To Cite

The Cloned Dynamics approach was used do develop some works. The main results were published in some important journals, such as Chaos [Fazanaro et al., 2013], Communications in Nonlinear Science and Numerical Simulation [Fazanaro et al., 2016; Soriano et al., 2018], International Journal of Bifurcation and Chaos [Soriano et al., 2012a, 2018], and in a few conferences and symposiums as [Fazanaro et al., 2010, 2012].

To cite the Cloned Dynamics approach, please use the following:

```
@ARTICLE{article:SorianoFazanaro2012,
  author  = {Soriano, D.C.; Fazanaro, F.I.; Suyama, R.; Oliveira, J.R.; Attux, R.; Madrid, M
     .K.},
  title   = {{A method for Lyapunov spectrum estimation using cloned dynamics and its
     application to the discontinuously-excited FitzHugh-Nagumo model}},
  journal = {Nonlinear Dynamics},
  year    = {2012},
  volume  = {67},
  pages   = {413--424},
  number  = {1},
  month   = {Jan},
  doi     = {10.1007/s11071-011-9989-2}
}
```

3 Organization of the Repository

The repository is organized based on the following folder structure:

- **01.add_to_MATLAB_path:** this folder contains some auxiliary functions as the implementation of 4th order Runge-Kutta integration. It is necessary to add this scripts in the MATLAB folder.
- **02.MATLAB_ode45:** in this folder, it is possible to find the implementations of some of the most important dynamical systems, as the Duffing Oscillator, the Chua oscillator, THE most important dynamical system of all, the one proposed by Lorenz [1963], the FitzHugh-Nagumo neuronal model, and others. It was employed the MATLAB language and the ode45 integrator. There are comments and descriptions inside the scripts to improve the understanding of each code.

  Considering the implementation of the Duffing dynamical model[1], the scripts are organize as follows:

  - `Duffing1989.m`: the dynamical system. This function is used to obtain the time evolution.
  - `text`
- **03.MATLAB_ode45_parfor:** the computation of the Lyapunov exponents is usually very computational expensive. Here, it was implemented the computation of the Lyapunov spectrum using the Parallel computational toolbox of the MATLAB. It is necessary to adjust the number of cores used during each simulation.
- **04.MATLAB_RK4:**
- **05.MATLAB_RK4_parfor:**
- **06.C** as a great friend of mine, André Luiz Delai (http://lattes.cnpq.br/4609660221170758), usually says, “C is the mother language of all”. In this folder, there are some implementations using the C language, basically, only to compute the Lyapunov exponents and the Lyapunov spectrum. As discussed in the work [Fazanaro et al., 2016], it is possible to improve the computational cost to (i.e. decrease the total amount of time to) realize such computation when compare to the MATLAB implementation.
- **07.Python:** under development. It will contain the same scripts detailed before yet using the Python 3.8 language and the matplotlib.

[1] Note that the organization of the remaining dynamical systems is similar.

4 Versions

- **xx / June / 2020:**
  - v01 - initial version.

References

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