To the use of Sellmeier formula

by Volkmar Brückner

Senior Experten Service (SES) Bonn and HfT Leipzig, Germany

Abstract

Based on dispersion of pure silica we proposed a general Sellmeier formula for various dopants like GeO_2, P_2O_5, B_2O_3 and F suitable for calculation of dispersion in glass fibers.

1 Introduction

Already in 1871 Wolfgang Sellmeier [1] found that the refractive index dependence on wavelength is very similar in many transparent and non-transparent media in visible and near infrared range. It can be described by the well-known Sellmeier formula

$$n(\lambda) = \sqrt{1 + \sum_{i=1}^{M} A_i \cdot \frac{\lambda^2}{\lambda^2 - \lambda_i^2}} \qquad \text{Eq. 1}$$

For M= 3 Eq. 1 was named 3-term Sellmeier equation. That means measuring the refractive index of the medium at least at 6 different wavelengths we are able to calculate 6 Sellmeier constants A_1, A_2, A_3, λ_1, λ_2 and λ_3 and to approximate the dispersion curve.

Today Sellmeier formula is widely used in optical science and optical industry to describe and characterize the dispersion in glasses and crystals. And many companies which offer optical materials deliver their products together with the corresponding Sellmeier coefficients, e.g. Schott [2]. As an example we can use Sellmeier constants of the most popular glass BK7 [2] to describe the dispersion curve. In practical cases it is often necessary to know dispersion $n(\lambda)$ roughly – that's why we tried to develop a more general Sellmeier formula which can be used for many materials like a rule of thumb.

2 Glasses for fiber optics

For fibers with core and cladding(s) we need special glasses, which have very low losses of about 0.2 dB/km (at 1550 nm) combined with well-known refractive indices. To prepare special fibers and/or calculate their transmission properties the refractive indexes of core (n_{core}) and cladding (n_{clad}) as well as the corresponding dispersion properties should be flexible and well-known. In fiber optics, pure silica (SiO_2) with its specific dependence of refractive index on wavelength serves as the basic material.

2.1 Dopants of glasses

For a long time [3] it is known that the refractive index can be changed by additives (dopants). To increase the refractive index, one has to add, e.g., GeO_2, TiO_2 or P_2O_5, to decrease it, e.g., B_2O_3 or fluorine (F).

Following [3] we developed empiric formulas to describe this dependence on concentration d (in Mol%) of dopants:

for GeO_2 $\quad n_{Ge}(d) = 1.457 + 0.0016 \cdot d$ Eq. 2

for TiO_2 $\quad n_{Ti}(d) = 1.457 + 0.0051 \cdot d$ Eq. 3

for P_2O_5 $\quad n_P(d) = 1.457 + 0.0009 \cdot d$ Eq. 4

for B_2O_3 $\quad n_B(d) = 1.457 - 0.0008 \cdot d + 0.00003 \cdot d^2$ Eq. 5

for F $n_F(d) = 1.457 - 0.004 \cdot d + 0.00015 \cdot d^2$ Eq. 6

Results are depicted in Fig. 1. As reference index we used the refractive index in SiO_2 at $\lambda = 633$ nm.

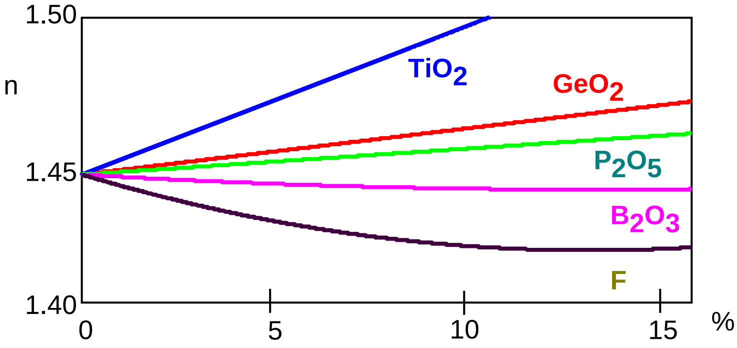


Fig. 1 Changes of refractive index in glass by additives

To describe the dependence of refractive index on wavelengths for different dopants as function of dopants concentration we used data from [4] and obtained the Sellmeier parameters given in Tab. 1.

Tab. 1 Sellmeier coefficients of glasses with different dopants [4]

Material	Name in Fig. 2	A_1	λ_1	A_2	λ_2	A_3	λ_3
SiO_2	Si	0.696750	0.069066	0.408218	0.115662	0.890815	9.900559
13,5% GeO_2 + 86,5% SiO_2	Ge	0.711040	0.064270	0.451885	0.129408	0.704048	9.425478
9,1% P_2O_5 + 90,9% SiO_2	P	0.695790	0.061568	0.452497	0.119921	0.712513	8.656641
13,3% B_2O_3 + 86,7% SiO_2	B	0.690618	0.061900	0.401996	0.123662	0.898817	9.098960
1% F + 99% SiO_2	F	0.691116	0.068227	0.399166	0.116460	0.890423	9.993707

One can see from in Fig. 2 that the dispersion shapes for different dopants are very similar and only shifted by an offset Δn. It should be noted, however, that other dopants, e.g. silica with 16.9% Na_2O <u>and</u> 32.5% B_2O_3 lead to dispersion curves with other shapes. This can be advantageously used for suitable core-cladding compositions to manage dispersion in the fiber.

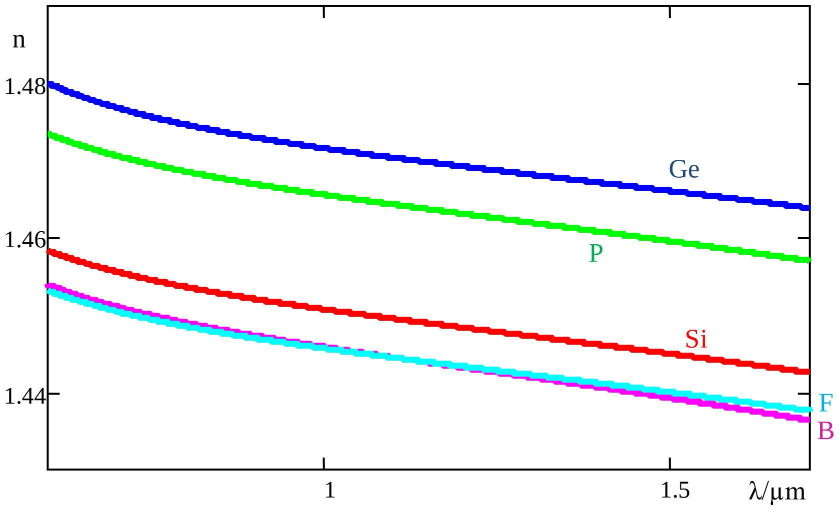


Fig. 2 Dispersion in glasses with different dopants

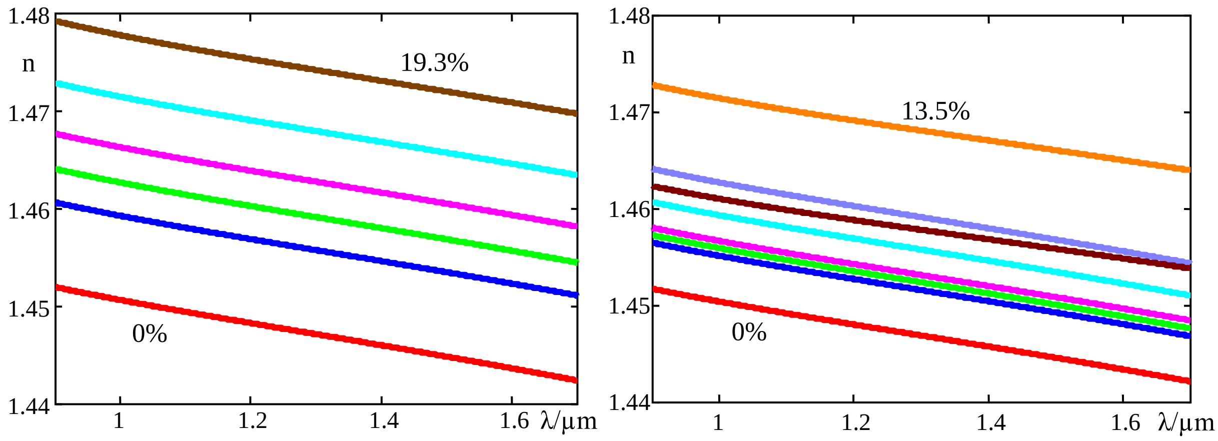
2.2 Variation of concentration of GeO_2 dopants

There exist a huge number of papers which provide the Sellmeier coefficients and the corresponding dispersion for various undoped and doped glasses. Many authors studied the dispersion in silica (SiO_2) doped with GeO_2. In Tab. 2 we give the data of [5] = source 1, and [6] = source 2.

Tab. 2 Sellmeier coefficients of silica for various GeO_2 concentrations

GeO_2 (mole%)	source	Sellmeier constant					
		A_1	A_2	A_3	λ_1	λ_2	λ_3
0.0	1	0.6961663	0.4079426	0.8974794	0.0684043	0.1162414	9.896161
0.0	2	0.6965325	0.4083099	0.8968766	0.0660932	0.1181101	9.896160
3.1	1	0.7028554	0.4146307	0.8974540	0.0727723	0.1143085	9.896161
3.5	1	0.7042038	0.4160032	0.9074049	0.0514415	0.1291600	9.896156
4.1	1	0.6867178	0.4348151	0.8965658	0.0726752	0.1151435	10.002398
5.8	1	0.7088876	0.4206803	0.8956551	0.0609053	0.1254514	9.896162
6.3	2	0.7083925	0.4203993	0.8663412	0.0853842	0.1024839	9.896175
7.0	1	0.6869829	0.4447950	0.7907351	0.0780876	0.1155184	10.436628
7.9	1	0.7136824	0.4254807	0.8964226	0.0617167	0.1270814	9.896161
8.7	2	0.7133103	0.4250904	0.863198	0.0831439	0.1079664	9.896131
11.2	2	0.7186243	0.4301997	0.8543265	0.0634539	0.1277683	9.896181
13.5	1	0.73454395	0.4271083	0.8210340	0.0869769	0.1119519	10.48654
15.0	2	0.7249180	0.4381220	0.8221368	0.0871572	0.1078145	9.896197
19.3	2	0.7347008	0.4461191	0.8081698	0.0764679	0.1246081	9.896203

In Fig. 3 dispersion curves are given separately for two sources of Sellmeier coefficients. Increasing refractive index corresponds to increasing concentration of dopants between 0 – 19.3% GeO_2 (Fig. 3a) and 0 – 13.5% GeO_2 (Fig. 3b). As wavelength range we considered the range of fiber optical windows (0.9 – 1.7 µm).



(a) (b)

Fig. 3 Dispersion in silica for different GeO_2 concentration from sources 2 (a) and 1 (b)

As one can see in Fig. 3 nearly all dispersion curves have the same shape. The only difference is an offset in refractive index Δn. For example we calculated the difference Δn between maximum

(19.3 % in Fig. 3a, 13.5% in Fig. 3b) and minimum (0%) concentrations of dopants GeO_2 (Fig. 4).

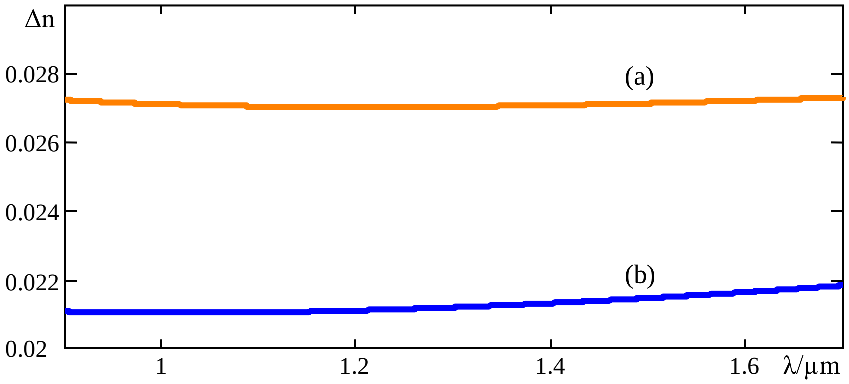


Fig. 4 Maximum difference Δn at GeO_2 concentrations corresponding to Fig. 3a and b

Furthermore; at all wavelengths we found a linear dependence of refractive index on GeO_2 concentration calculated by Sellmeier formula. As an example, this dependence is illustrated for 1300 nm in Fig. 5 separately for refractive indexes from source 1 and source 2. A slight difference was only found in the slope (0.00157 /% for source 1 and 0.00142/% for source 2).

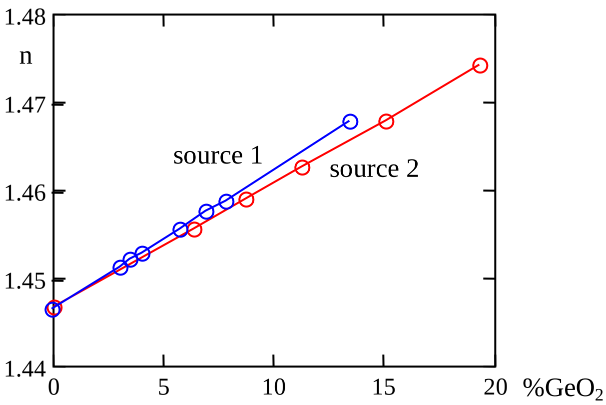


Fig. 5 Dependence of refractive index on concentration of dopants at λ = 1300 nm

Having this behavior in mind we suggest an approximation of these dispersion curves by a general formula. It starts from dispersion of pure silica, and we have to add the offset in agreement with dopants concentration. To this end we have to select a “reference” wavelength, where our choice is 1300 nm. That means our general formula is based on Sellmeier equation of pure silica:

$$n_{Si}(\lambda) = \sqrt{1+\sum_{i=1}^{3} A_i \cdot \frac{\lambda^2}{\lambda^2-\lambda_i{}^2}} \quad \text{with} \quad A_i = \begin{pmatrix} 0.6961663 \\ 0.4079426 \\ 0.8974794 \end{pmatrix} \quad \text{and} \quad \lambda_i = \begin{pmatrix} 0.068404 \\ 0.1162414 \\ 9.896161 \end{pmatrix} \qquad \text{Eq. 7}$$

Then we have to add only an offset depending on the concentration of dopants (d_{Ge}) using the slope of curve in Fig. 5 (we used the curve corresponding to source 2). For GeO_2 dopants we get the following equation:

$$n_{Ge}(\lambda) = n_{Si}(\lambda) + 1.4145 \cdot 10^{-} \qquad \text{Eq. 8}$$

Calculations using this equation are in very good agreement with data described in the literature (see). It works perfectly in the wavelength range used in optical communication technologies like DWDM (1500 – 1570 nm). Here the maximum error is less than $10^{-4.}$

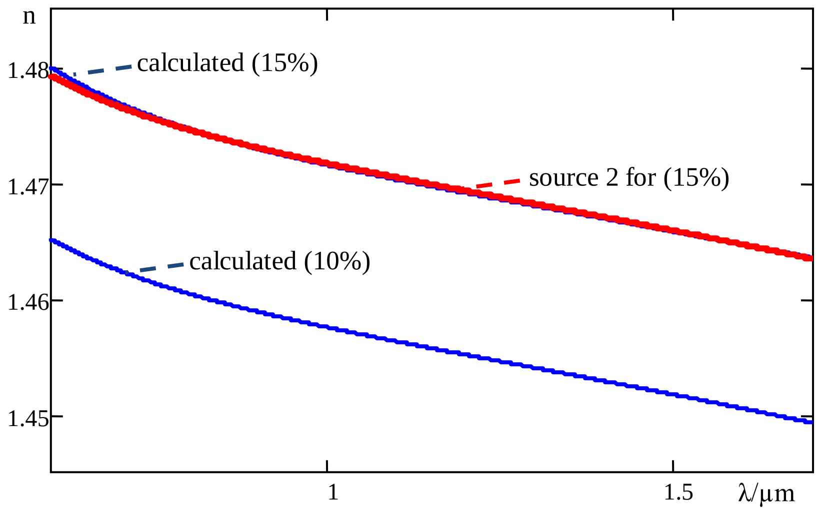


Fig. 6 Dispersion curves for d_{Ge} = 10% and 15% GeO_2 calculated by using **Eq**. 8 and curve corresponding to source 2 for d_{Ge} = 15%

For practical use it is often useful to use derivatives of refractive index, i.e. group index $n_g(\lambda) = n(\lambda) - \frac{dn(\lambda)}{d\lambda}$ and group velocity dispersion (GVD) resulting in the material dispersion parameter $D_{Mat}(\lambda) = \frac{d}{d\lambda}\left(\frac{n_g(\lambda)}{c}\right) = -\frac{\lambda}{c}\frac{d^2n(\lambda)}{d\lambda^2}$. For pure silica it is depicted in Fig. 7. Note that $D_{Mat}(\lambda)$ is exactly the same for all dopants and concentrations discussed here.

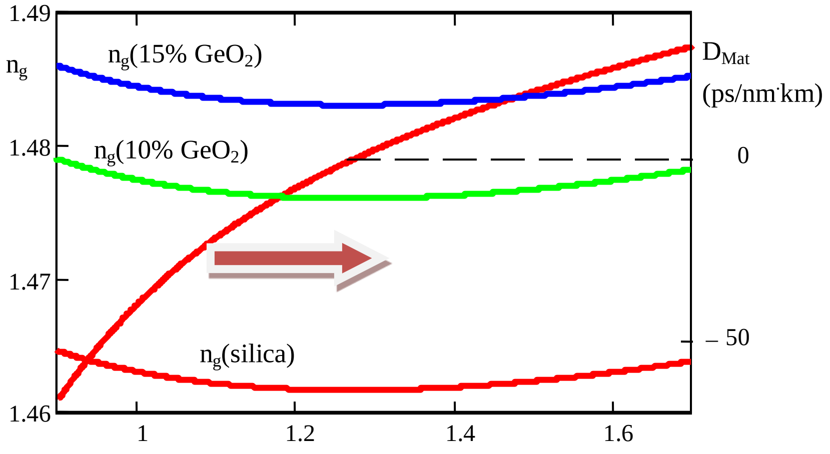


Fig. 7 Group index n_g in doped and undoped silica and dispersion parameter D_{Mat} in pore silica

2.3 Variation of concentrations of other dopants

Unfortunately, until now here are not available sufficient experimental data to make similar calculations for other dopants (P_2O_5, B_2O_3 and F). We can use only make use of dopants concentrations given in Tab. 1.

Again we selected 1300 nm as reference wavelength. In this case the general formula is based again on dispersion of silica and we have to add an offset depending on concentration of dopants. The slope can be obtained from Fig. 2 at 1300 nm. Thus we get the following formulas:

for P_2O_5 $\quad n_P(\lambda) = n_{Si}(\lambda) + 1.652 \cdot 10^{-3} \cdot d_P$ Eq. 9

for B_2O_3 $\quad n_B(\lambda) = n_{Si}(\lambda) - 3.760 \cdot 10^{-4} \cdot d_B$ Eq. 10

for F $\quad n_F(\lambda) = n_{Si}(\lambda) - 4.665 \cdot 10^{-3} \cdot d_F$ Eq. 11

At λ = 633 nm equations 8 – 11 are in rather good agreement with results of Fig. 1 corresponding empirical equations 2 and 4 – 6. Note that group index and group velocity dispersion behavior is in general the same as depicted in Fig. 7.

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2.4 Use of generalized formulas

Often there is another task: To create and produce special (mainly single mode) fibers one has to select material for core and cladding(s) to achieve a certain numerical aperture $NA = \sqrt{n_{core}^2 - n_{clad}^2}$ (application-specific fibers). For example, we assume NA = 0.14 is required in standard single-mode fibers SMF-28 of Corning Glass company. Using equations 8 – 11 we are able to calculate for this fiber n_{clad} at given values of n_{core} and NA, or n_{core} at given values of n_{clad} and NA, respectively. For calculation of necessary concentration of dopants to realize a certain refractive index n_1 we developed the following formulas (note that n_1 should be larger than n_{Si} for GeO_2 and P_2O_5 and less than n_{Si} for B_2O_3 and F):

for GeO_2: $\quad d_{Ge}(\lambda) = \dfrac{n_1(\lambda) - n_{Si}(\lambda)}{1.4145 \cdot 10^{-3}}$ Eq. 12

for P_2O_5: $\quad d_P(\lambda) = \dfrac{n_1(\lambda) - n_{Si}(\lambda)}{1.652 \cdot 10^{-3}}$ Eq. 13

for B_2O_3: $\quad d_B(\lambda) = \dfrac{n_{Si}(\lambda) - n_1(\lambda)}{3.760 \cdot 10^{-4}}$ Eq. 14

for F: $\quad d_F(\lambda) = \dfrac{n_{Si}(\lambda) - n_1(\lambda)}{4.665 \cdot 10^{-3}}$ Eq. 15

Examples for calculations using equations 12 – 14 are given in Tab. 3. Here we used NA = 0.14 and λ = 1550 nm.

Tab. 3 Calculations of core and cladding refractive indexes

Core				Cladding			
n_{core}	dopants	concen-tration (%)	equation	n_{cladd}	dopants	concen-tration (%)	equ.
1.4514	GeO_2	4.8	Eq. **12**	1.4446	pure SiO_2	-	Eq. 7
1.4560	GeO_2	8.5	Eq. **12**	1.4493	GeO_2	3.7	Eq. **12**
1.4514	P_2O_5	4.1	Eq. **13**	1.4446	pure SiO_2	-	Eq. 7
1.4560	GeO_2	8.5	Eq. **12**	1.4493	P_2O_5	2.8	Eq. **13**
1.4446	pure SiO_2	-	Eq. 7	1.4514	B_2O_3	18.0	Eq. **14**
1.4446	pure SiO_2	-	Eq. 7	1.4514	F	1.5	Eq. 15

This algorithm can be used for preparation of application-specific glass fibers. At the same time one can use the corresponding Sellmeier formulas to describe material, waveguide and chromatic dispersion [7].

All calculations have been performed by the program MathCad. Corresponding programs may be downloaded from the same online-plus website at Vieweg+Teubner publisher.

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