Blaze characteristics of echelle gratings

Sonja Engman and Peter Lindblom

A new model for the blaze function of the two in-plane mountings of echelle gratings is described and extended to account for defects in the edges of the grooves. Expressions for the relative efficiency are given. The model shows that the wavelength for the maximum blaze intensity differs slightly from the wavelength λ_0 satisfying $m\lambda_0 = d[\sin\alpha + \sin(2\theta - \alpha)]$, the difference being a function of the spectral order. The off-blaze orders of the wavelengths diffracted at the central blaze for the mounting $\alpha < \theta$ (α = angle of incidence, θ = blaze angle) lie at the minima of the blaze function for a perfect echelle. Results are given from measurements of the blaze angle and the groove edge defects of three echelles.

I. Introduction

Although the echelle grating was invented by Harrison[1] in 1949, it was not used extensively in spectral instruments until the last decade. At present many echelle spectrometer systems are on the market. Much work has gone into developing the art of ruling high-quality echelles,[2] and at present there are several suppliers of these gratings. The main blaze characteristics of the echelles were given by Harrison, and since then not much work has been done on this subject.

However, when constructing echelle spectrometers one is forced to consider the question of the accuracy of the blaze angle given by the manufacturer. Furthermore, when photoelectrical recording is used the form of the blaze function determines the spectral position of where to change from one spectral order to another.[3]

Normally the blaze efficiency of the echelle grating is assumed to be given by the envelope function

$$I \sim \left(\frac{\sin\mathcal{H}}{\mathcal{H}}\right)^2, \tag{1}$$

where

$$\mathcal{H} = \frac{\pi d \cos\theta}{\lambda}[\sin(\theta - \alpha) + \sin(\theta - \beta)]. \tag{2}$$

In Eq. (2) θ is the blaze angle, α the angle of incidence, β the angle of diffraction, d the grating constant, and λ the wavelength of the radiation. It is thus the Fraunhofer diffraction pattern given by a slit of width $d \cos\theta$ that gives the blaze characteristics of the echelle grating. Equations (1) and (2) give a fairly good description of the intensity distribution at α and β values close to the blaze angle θ. However, for in-plane mountings where the angles α and β can deviate appreciably from θ, shading by the neighboring grooves takes place. An attempt to take this effect into account has been made by Schroeder and Hilliard[4] by letting the width of the reflecting groove side vary with the angle of incidence. The relative efficiencies of echelle gratings have been treated further by Bottema[5] and Schroeder.[6]

In this paper we shall discuss a new model for the blaze function. We shall also discuss a simple method to measure the blaze angle and to get an estimate of the quality of the groove profile.

II. Blaze Function for In-Plane Mountings

In Figs. 1(A) and (B) the two in-plane mountings of echelle gratings are illustrated. Figure 1(A) represents the mounting with the angle of incidence larger than the blaze angle θ and Fig. 1(B) the mounting with the angle of incidence smaller than θ for a perfect groove shape. In both cases the width of the beam is restricted by the edge of the neighboring groove. In Ref. 4 this has been accounted for in the case $\alpha < \theta$ by replacing $d \cos\theta$ in Eq. (2) by the width of the illuminated part of the reflecting groove side. For $\alpha > \theta$ the reflection from the steep side of the groove is taken into account by assuming it to be specularly reflecting and orthogonal to the reflecting groove side. However, the steep side of the grooves is not always specularly reflecting. Thus not all echelles show a blaze on the opposite side of the grating normal at the complement blaze angle. Only one of the three gratings investigated in this paper has an opposite blaze close to the complement blaze angle.

We will discuss a model which involves the reflecting groove side and the influence of the groove edges in a way which we believe is physically more justified than

The authors are with Abo Akademi, Institutionen for Fysik, SF-20500 Turku 50, Finland.

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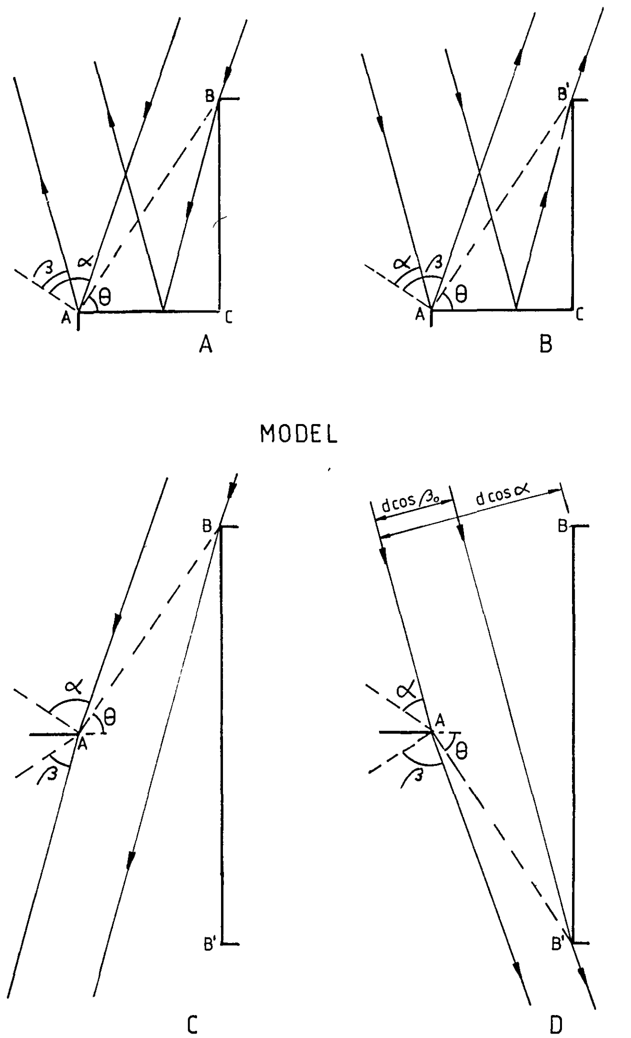

Fig. 1. Principal outline of the two mountings of echelle gratings ($\alpha > \theta$ and $\alpha < \theta$) together with their associated models for diffraction at the groove edges. (A) and (B) reflected in the horizontal groove side give (C) and (D), respectively. For the mounting with $\alpha > \theta$ the diffraction takes place in a slit of width d formed by the edges A and B prior to reflection from the horizontal groove side (C). For the mounting with $\alpha > \theta$ the diffraction takes place in the edges after reflection from the horizontal groove side (D). The notations used in the text are defined in this figure.

in previous models. Looking at Figs. 1 one can see that the diffraction in the groove takes place not in the reflecting groove side but in the edges of the two neighboring grooves. This means that for the mounting $\alpha > \theta$ the intensity distribution is given by diffraction in the slit formed by the edges A and B prior to reflection from the groove surface, whereas for $\alpha < \theta$ it is given by diffraction in the edges A and B after reflection from the groove surface. Reflecting the edge B in the groove side AC gives us the models shown in Figs. 1(C) and (D) for the description of the blaze function. The mounting $\alpha > \theta$ will thus be represented by diffraction in the slit AB and the mounting $\alpha < \theta$ by diffraction in the slit AB'.

Applying the Fresnel-Kirchhoff diffraction formula to Fraunhofer diffraction in the slits AB and AB' of the two mountings, we obtain with the notations of Figs. 1

$$I \sim \left[\frac{\sin\mathcal{H}}{\mathcal{H}}\left(\frac{\cos\alpha + \cos(2\theta - \beta)}{2\cos\alpha}\right)\right]^2 \tag{3}$$

for the mounting $\alpha > \theta$, where

$$\mathcal{H} = \frac{\pi d}{\lambda}[\sin(2\theta - \beta) - \sin\alpha]. \tag{4}$$

Writing $\beta = (2\theta - \alpha) + \Delta\beta \equiv \beta_0 + \Delta\beta$ we can rewrite Eq. (4) in the form

$$\mathcal{H} = -\frac{\pi d\cos\alpha}{\lambda}\left[\sin\Delta\beta + 2\tan\alpha\sin^2\left(\frac{\Delta\beta}{2}\right)\right]\cdot \tag{5}$$

Denoting by m_0 the spectral order of the wavelength λ_0 diffracted so that $\beta = \beta_0 = 2\theta - \alpha$, we can write Eq. (4) in the form

$$\mathcal{H} = -\pi\left(\frac{\cos\alpha}{\cos\beta}\right)\left(\frac{m\lambda - m_0\lambda_0}{\lambda}\right)\left\{1 + \left(\frac{\tan\beta_0 + \tan\alpha}{2\cos\beta_0}\right)\right.$$
$$\left.\times\left(\frac{m\lambda - m_0\lambda_0}{d}\right) + O\left[\left(\frac{m\lambda - m_0\lambda_0}{d}\right)^2\right]\right\}, \tag{6}$$

where λ is an arbitrary wavelength diffracted in order m.

For the mounting $\alpha < \theta$ we get the corresponding expression

$$I \sim \left\{\frac{\sin\mathcal{H}}{\mathcal{H}}\left[\frac{\cos\beta + \cos(2\theta - \alpha)}{2\cos\beta_0}\right]\right\}^2, \tag{7}$$

where

$$\mathcal{H} = \frac{\pi d}{\lambda}[\sin(2\theta - \alpha) - \sin\beta]. \tag{8}$$

Putting again $\beta = \beta_0 + \Delta\beta$ we can rewrite Eq. (8) as

$$\mathcal{H} = -\frac{\pi d\cos\beta_0}{\lambda}\left[\sin\Delta\beta - 2\tan\beta_0\sin^2\left(\frac{\Delta\beta}{2}\right)\right]\cdot \tag{9}$$

Rewriting Eq. (7) in the quantities m_0, λ_0, m, and λ we get the following simple expression:

$$\mathcal{H} = \pi\left(\frac{m_0\lambda_0 - m\lambda}{\lambda}\right)\cdot \tag{10}$$

The factors $[\cos\alpha + \cos(2\theta - \beta)]$ and $[\cos\beta + \cos(2\theta - \alpha)]$ appearing in Eqs. (3) and (7) are due to the skewness of the slits AB and AB' in regard to the incident light (see Ref. 7). These factors are normally omitted when considering diffraction in a slit. In our model these factors have been included as they will have a distorting effect on the blaze function due to the large tilt of the slits. To get $I = 1$ at $\beta = \beta_0 = 2\theta - \alpha$ and the envelope function comparable to previous models, these factors have been divided by $2\cos\alpha$ for $\alpha > \theta$ and $2\cos\beta_0$ for $\alpha < \theta$. The main feature, however, is still given by the $\sin\mathcal{H}/\mathcal{H}$ function. The two cases are quite different, and some general consequences can immediately be seen.

The wavelength λ_0 diffracted in order m_0 in the central blaze is also diffracted in the orders $m_0 + k$ ($k = \pm1, \pm2, \ldots$). For the mounting $\alpha < \theta$, Eq. (10) then gives

$$\mathcal{H} = k\pi \tag{11}$$

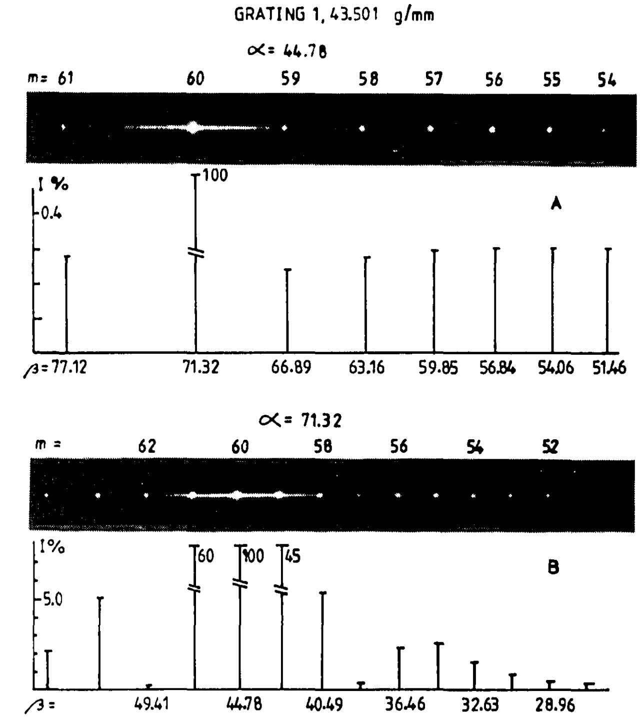

Fig. 2. Diffracted orders of the laser beam for grating 1, recorded at such angles of incidence that order 60 is diffracted in the central blaze for the two mountings [$\alpha < \theta$ (A); $\alpha > \theta$ (B)]. This grating is the best of the three gratings tested. As seen from (A) the off-blaze orders are close to the diffraction minima, indicating a small groove edge defect. The diagrams under the photographs show the relative intensities of the orders as calculated from the round edge model using the fitted values for the blaze angle θ and the groove edge correction parameter R. The calculated intensities seem to describe the intensity variations of the orders fairly well. Note the very low calculated intensities of the off-blaze orders for the mounting with $\alpha < \theta$ (A). This grating has opposite blazes at $\theta \approx 25.3$ and $\theta \approx 22.6$ deg.

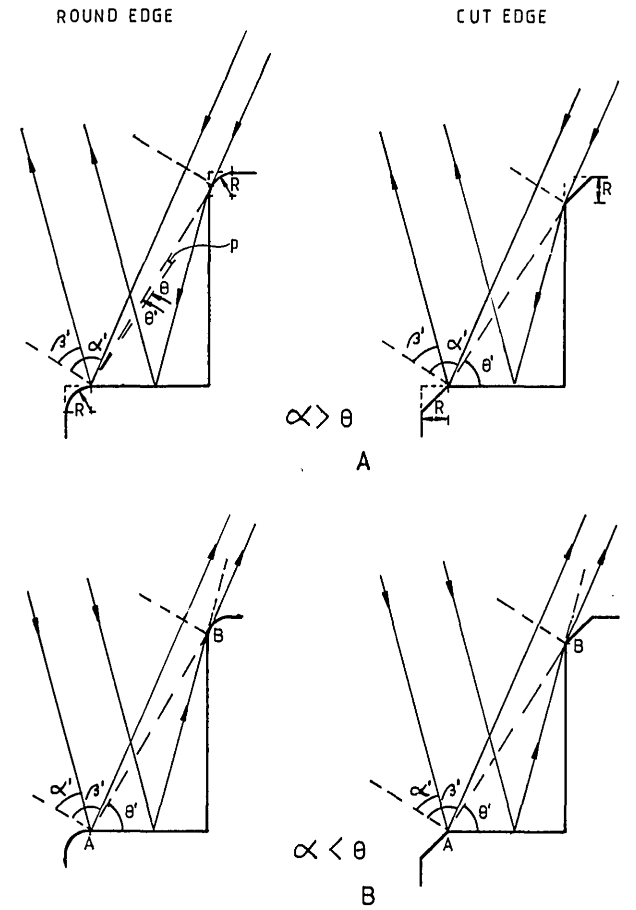

Fig. 3. Principal outline of the two models used for the correction of groove edge defects through one parameter R. The parameter R represents the radius of curvature of the edge for the round edge model and the depth of a 45-deg cut of the edge for the cut edge model. In both cases the width of the reflecting groove side is reduced by R. In the blaze functions the quantities α, β_0, β, θ, and d have been replaced by $\alpha' = \alpha + p$, $\beta_0' = \beta_0 + p$, $\beta' = \beta + p$, $\theta' = \theta + p$, and d', where d' is the distance AB. The quantities d' and p depend on R according to Eqs. (5)–(7).

showing that these orders lie at the diffraction minima of the blaze function.

For the mounting $\alpha > \theta$ we get from Eq. (6)

$$\mathcal{H} = -\pi k \left(\frac{\cos\alpha}{\cos\beta}\right)\left\{1 + \frac{\tan\beta_0 + \tan\alpha}{2\cos\beta_0}\left(\frac{k\lambda_0}{d}\right) + O\left[\left(\frac{k\lambda_0}{d}\right)^2\right]\right\}. \qquad (12)$$

Equation (12) shows that as α deviates more and more from β an increasing number of intense orders falls within the blaze region. Thus we can conclude that the two mountings of the echelle grating give quite different blaze characteristics. In Figs. 2(A) and (B) photographs of the orders of the He–Ne laser line diffracted by an echelle grating are shown for the two mountings. Note that for $\alpha < \theta$ the central order is very strong, and all other orders are very close to the minima of the blaze function. The fact that they do not lie exactly on the minima is due to grating defects and will be discussed below. The photograph of the mounting with $\alpha > \theta$ shows the increasing number of intense orders within the blaze as the difference between α and β increases. Other data are in the figure captions.

IV. Correction for Groove Edge Defects

A ruled grating will naturally not have the perfect groove shape shown in Figs. 1. Among other possible defects, the edge of the grooves will not be completely sharp. This implies that the reflecting groove side will be slightly less than $d \cos\theta$. In our model this means that the edge A in Figs. 1 is slightly shifted toward the steep groove side resulting in a wider blaze function of the grating.

To obtain an estimate of the magnitude of the edge defect by using only one parameter, we have assumed the edges to be either curved having an average radius of curvature R or cut, as shown in Figs. 3. This is simply a way through which the edge defects can be accounted for in the blaze function and should not be interpreted as the real shape of the grooves. As seen from Figs. 3 the reflecting part of the groove side is reduced by R. The blaze function is now given by diffraction in the slit AB of width d', which for the round edge model is a function of the direction of the incident radiation. Furthermore, the inclination of the slit AB is increased by a small quantity p. In Eqs. (3), (4), (7), and (8) we should now substitute $\alpha' = \alpha + p, \beta_0' = \beta_0 + p, \beta' = \beta + p, \theta' = \theta + p$, and d' instead of $\alpha, \beta_0, \beta, \theta$, and d. For $\mathcal{H}$ we get

$$\mathcal{H} = \frac{\pi d'}{\lambda} [\sin(2\theta - \beta + p) - \sin(\alpha + p)] \tag{13}$$

for $\alpha > \theta$ and

$$\mathcal{H} = \frac{\pi d'}{\lambda} [\sin(2\theta - \alpha + p) - \sin(\beta + p)] \tag{14}$$

for $\alpha < \theta$.

From Figs. 3 one can deduce approximate expressions for d' and p. For the round edge model we get

$$d' = d\left[1 - \frac{R}{d}(\cos\alpha + \sin\theta)\right],$$

$$p = \frac{R}{d}(\sin\alpha - \cos\theta) \tag{15}$$

for $\alpha > \theta$ and

$$d' = d\left[1 - \frac{R}{d}(\cos\beta_0 + \sin\theta)\right],$$

$$p = \frac{R}{d}(\sin\beta_0 - \cos\theta) \tag{16}$$

for $\alpha < \theta$.

For the cut edge model we get

$$d' = d\left[1 - \frac{R}{d}(\cos\theta + \sin\theta)\right],$$

$$p = \frac{R}{d}(\sin\theta - \cos\theta) \tag{17}$$

for both mountings.

V. Experimental

The minima of the blaze function given by $\sin\mathcal{H} = 0$ depend strongly on the variables in $\mathcal{H}$. Measuring the angles α and β at which the different spectral orders of the wavelength have minima makes possible the determination of the unknown parameters in $\mathcal{H}$, i.e., the blaze angle θ and the parameter R for both groove edge correction models. These minima can be recorded simply by using a laser and a turntable and observing the diffracted orders on a screen.

The minima are determined by

$$\mathcal{H} = l\pi \qquad (l = \pm 1, \pm 2, \ldots), \tag{18}$$

where l is the order of the minima. We have made the least squares fits of θ and R to Eq. (18) based on several measured minima using Eqs. (13), (15), and (17) for the mounting $\alpha > \theta$ and Eqs. (14), (16), and (17) for $\alpha < \theta$.

Three gratings have been measured and fitted to both correction models. The results are summarized in Table I. As seen the 43.5- and 54.5-g/mm gratings are good echelles, whereas the 30-g/mm grating is poor. The difference in quality between the gratings can also be seen from the photographs in Figs. 2, 4, and 5. The illustrations for $\alpha < \theta$ show the significance of the groove edge correction [Figs. 2(A), 4(A), 5(A)]. If the gratings were perfect, all the orders off the central blaze would lie on the minima of the blaze function. Figure 4(A) shows that the 43.5-g/mm grating is the best in this respect, and it also has the smallest groove edge correction. In Figs. 2, 4, and 5 the relative intensities of the different orders have been calculated from the round edge model using the fitted values of θ and R. As seen, the groove edge correction parameter affects strongly the relative intensities of the off-blaze orders. According to the measurements in this work both groove edge correction models are equally applicable.

Table I. Grating Specifications

	Blaze angle (deg)	Corr. param. (μm)	Orders of recorded minima
GRATING 1, 43.501 g/mm			
Round edge correction			
$\alpha > \theta$	58.06	0.5	58–60
$\alpha < \theta$	58.16	0.3	57, 58, 60–62
Cut edge correction			
$\alpha > \theta$	58.06	0.5	58–60
$\alpha < \theta$	58.16	0.3	57, 58, 60–62
GRATING 2, 54.529 g/mm			
Round edge correction			
$\alpha > \theta$	44.73	1.2	36–41
$\alpha < \theta$	44.75	1.0	31–37, 39–41
Cut edge correction			
$\alpha > \theta$	44.74	1.2	36–41
$\alpha < \theta$	44.73	1.2	31–37, 39–41
GRATING 3, 30.003 g/mm			
Round edge correction			
$\alpha > \theta$	62.05	3.7	89, 90, 92, 93
$\alpha < \theta$	Too poor minima for visual recording		
Cut edge correction			
$\alpha > \theta$	62.07	3.9	89, 90, 92, 93
$\alpha < \theta$	Too poor minima for visual recording		

The quality of the groove shape of an echelle can simply be tested qualitatively by letting a laser beam be diffracted at $\alpha < \theta$, adjusting the grating to diffract 1 order in the central blaze and observing the location of the off-blaze orders in regard to the diffraction minima seen in the scattered light. The difference between the fitted blaze angles for the two mountings of the 43.5-g/mm grating is probably due to groove shape defects that the correction models do not account for.

VI. Shape of the Blaze Function

Figures 6 show the blaze function calculated for grating 2, with $R = 0, 1$, and $2\ \mu$m. As seen the blaze function is widened and the minima shifted due to the groove edge defect. The position of the maximum intensity is slightly shifted as seen from the expanded plots of the top region in Figs. 6. Differentiation of Eqs. (3) and (7) gives that the wavelength λ_p of the maximum intensity in order m_0 for $\alpha > \theta$ is approximately given by

$$\frac{\lambda_p}{\lambda_0} = 1 + \frac{3}{2\pi^2 m_0^2}\left(\frac{\sin\alpha}{\cos^2\alpha}\right)\left(\frac{\cos\beta_0}{\cos\alpha}\right)(\sin\alpha + \sin\beta_0) \tag{19}$$

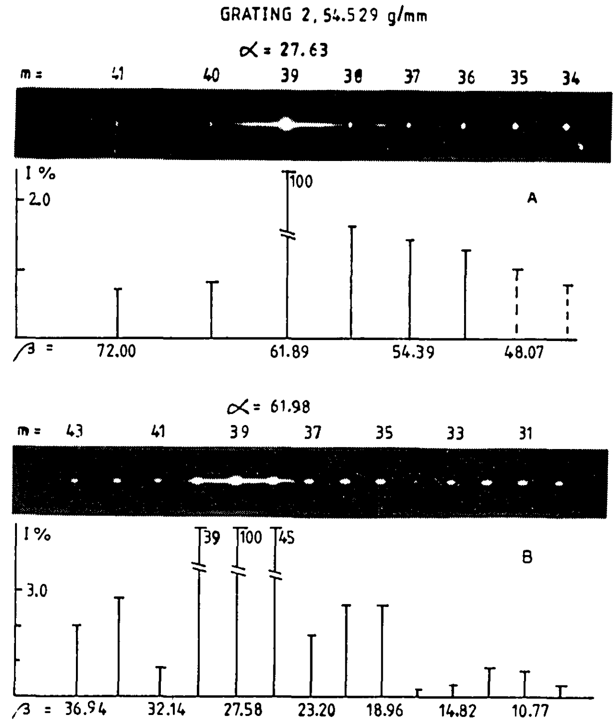

Fig. 4. Same as Fig. 2, but for grating 2, recorded at such angles of incidence that order 39 is diffracted in the central blaze. This grating is not as good as grating 1 as can be seen by comparing the photograph of (A) with that of Fig. 2(A). Compared to grating 1 the larger groove edge defect of this grating (see Table I) results in higher relative intensities of the off-blaze orders. The model cannot be expected to give correct intensities for the orders 34 and 35 in (A) as these orders are diffracted at an angle β close to the blaze angle. This grating has no distinct opposite blaze.

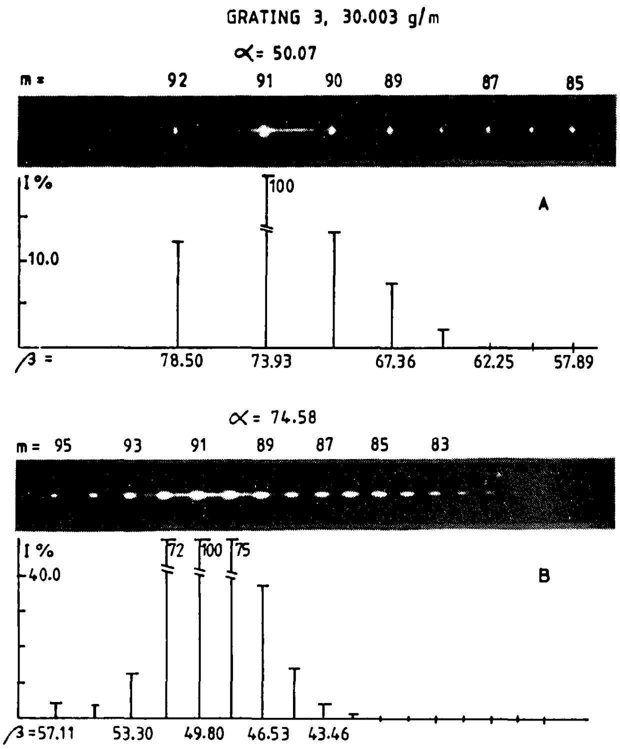

Fig. 5. Same as Fig. 2, but for grating 3, recorded at such angles of incidence that order 91 is diffracted in the central blaze. This grating has a large groove edge defect (Table I) resulting in a broad blaze. The off-blaze orders of (A) lie far from the diffraction minima and have thus high relative intensities. This grating also has several other blazes, the strongest at $\theta = 16.3$ deg, indicating a poor groove shape.

with the peak intensity

$$I_p = 1 + \frac{3}{2\pi^2}\left(\frac{\sin\alpha}{\cos^2\alpha}\right)^2\left(\frac{\lambda_0}{d}\right)^2 . \tag{20}$$

For $\alpha < \theta$ we get

$$\frac{\lambda_p}{\lambda_0} = 1 - \frac{3}{2\pi^2 m_0^2}\left(\frac{\sin\beta_0}{\cos^2\beta_0}\right)(\sin\alpha + \sin\beta_0), \tag{21}$$

$$I_p = 1 + \frac{3}{2\pi^2}\left(\frac{\sin\beta_0}{\cos^2\beta_0}\right)^2\left(\frac{\lambda_0}{d}\right)^2 . \tag{22}$$

As before λ_0 is the wavelength satisfying the grating equation for $\beta_0 = 2\theta - \alpha$. As seen $\lambda_p > \lambda_0$ for $\alpha > \theta$ and $\lambda_p < \lambda_0$ for $\alpha < \theta$. Including the groove edge corrections, Eqs. (19)–(22) are approximately ($\alpha > \theta$)

$$\frac{\lambda_p}{\lambda_0} = 1 + \frac{3}{2}\left(\frac{d}{\pi d' m_0}\right)^2\left(\frac{\sin\alpha + \sin\beta_0}{\cos\alpha}\right) \times \frac{\cos\beta_0}{\cos\alpha}\,[\tan\alpha + p(1 + 3\tan^2\alpha)], \tag{23}$$

$$I_p = 1 + \frac{3}{2\pi^2}\left(\frac{\lambda_0}{d'}\right)^2 \frac{\sin\alpha}{\cos^3\alpha}\,[\tan\alpha + p(1 + 4\tan^2\alpha)], \tag{24}$$

and ($\alpha < \theta$)

$$\frac{\lambda_p}{\lambda_0} = 1 - \frac{3}{2}\left(\frac{d}{\pi d' m_0}\right)^2\left(\frac{\sin\alpha + \sin\beta_0}{\cos\beta_0}\right) \times [\tan\beta_0 + p(1 + 3\tan^2\beta_0)], \tag{25}$$

$$I_p = 1 + \frac{3}{2\pi^2}\left(\frac{\lambda_0}{d'}\right)^2 \frac{\sin\beta_0}{\cos^3\beta_0}\,[\tan\beta_0 + p(1 + 4\tan^2\beta_0)]. \tag{26}$$

In the expanded diagrams of Figs. 6 the shifts of the maxima of the blaze function are illustrated for grating 2. The peak wavelength λ_p deviates slightly from λ_0, the deviation being negligible for large spectral orders as can be seen from Eqs. (19), (21), (23), and (25). For example, $\lambda_0 = 2794.57$ Å, $m_0 = 94$, $\alpha = 37$ deg, and $\beta_0 = 56$ deg give $\lambda_p = 2794.48$ Å. However, at large blaze angles and fairly low orders ($m < 50$) the shift of the peak position should be considered, especially when measuring the blaze angle by localizing the maximum intensity of the blaze. The generally accepted opinion that the maximum intensity from an echelle grating is obtained when the diffraction angle satisfies the condition for specular reflection from the groove side is according to our model only approximately true.

VII. Relative Echelle Efficiencies

The new model has implications on the relative echelle efficiency E_r as defined in Ref. 5. We use Eq. (8) of Ref. 5 to calculate E_r from our model. We start with the model for an ideal groove profile without edge defects. For $\alpha > \theta$ we get

$$E_r(\beta) = \frac{1}{\cos\alpha \cos\beta} \left\{ \frac{\sin\mathcal{H}}{\mathcal{H}} \left[\frac{\cos\alpha + \cos(2\theta - \beta)}{2} \right] \right\}^2 , \tag{27}$$

where $\mathcal{H}$ is given by Eqs. (4)–(6).

For the case $\alpha < \theta$ we study the diffraction in the slit AB' in Fig. 1(D). The width of the beam passing through the slit is $d \cos\beta_0$, and the distribution of energy after the passage is determined by the diffraction in the slit, i.e., by the blaze function. Thus the fraction of the incident beam that contributes to the diffracted wave front is $\cos\beta_0/\cos\alpha$. This differs from the fraction $\cos\beta/\cos\alpha$ given by Bottema.[5]

We get for $\alpha < \theta$

$$E_r(\beta) = \frac{1}{\cos\alpha \cos\beta} \left[\frac{\sin\mathcal{H}}{\mathcal{H}} \left(\frac{\cos\beta + \cos(2\theta - \alpha)}{2} \right) \right]^2 . \tag{28}$$

Neglecting the peak shift discussed previously it follows from Eqs. (27) and (28) that the relative peak efficiencies at $\beta_0 = 2\theta - \alpha$ are given by

$$E_r(\beta_0) = \frac{\cos\alpha}{\cos\beta_0} \tag{29}$$

for $\alpha > \theta$ and

$$E_r(\beta_0) = \frac{\cos\beta_0}{\cos\alpha} \tag{30}$$

for $\alpha < \theta$. Equations (29) and (30) are the same as those given by Bottema.

We now proceed to study the effect of the groove edge defects on the relative efficiency. For the $\alpha > \theta$ case we note that the width of the beam subject to diffraction is $d' \cos\alpha'$ in comparison with $d \cos\alpha$ for a perfect grating. We should thus use primed quantities in the blaze function multiplied with the factor $(d' \cos\alpha'/d \cos\alpha)^2$ to get the relative efficiency:

$$E_r(\beta) = \frac{1}{\cos\alpha \cos\beta} \left(\frac{d'}{d} \right)^2 \left\{ \frac{\sin\mathcal{H}}{2\mathcal{H}} \times [\cos(\alpha + p) + \cos(2\theta - \beta + p)] \right\}^2 , \tag{31}$$

where $\mathcal{H}$ is given by Eq. (13). For the peak efficiency we get, neglecting the peak shift,

$$E_r(\beta_0) = \frac{\cos^2(\alpha + p)}{\cos\alpha \cos\beta_0} \left(\frac{d'}{d} \right)^2 . \tag{32}$$

Equation (32) can be written approximately as

$$E_r(\beta_0) = \frac{\cos\alpha}{\cos\beta_0} \left(1 - \frac{2R}{d} q \right) , \tag{33}$$

where

$$q = \tan\alpha(\sin\alpha - \cos\theta) + \cos\alpha + \sin\theta \tag{34}$$

for the round edge model and

$$q = \tan\alpha(\sin\theta - \cos\theta) + \cos\theta + \sin\theta \tag{35}$$

for the cut edge model.

For the $\alpha < \theta$ case the correction factor to the blaze function is $(d' \cos\beta_0'/d \cos\beta_0)^2$, giving

$$E_r(\beta) = \frac{1}{\cos\alpha \cos\beta} \left(\frac{d'}{d} \right)^2 \left\{ \frac{\sin\mathcal{H}}{2\mathcal{H}} \times [\cos(\beta + p) + \cos(2\theta - \alpha + p)] \right\}^2 . \tag{36}$$

For the relative peak efficiency we obtain

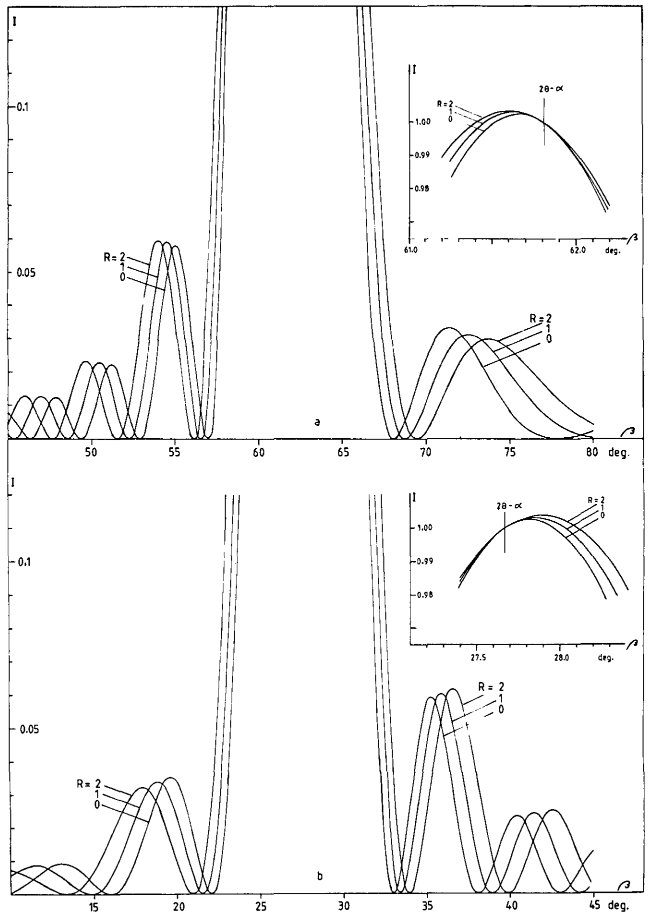

Fig. 6. Plots of the blaze function for grating 2 with $d = 18.339\ \mu m$ and $\theta = 44.74$ deg. (a) shows the case $\alpha < \theta$ with $\alpha = 27.672$ deg and (b) the case $\alpha > \theta$ with $\alpha = 61.808$ deg. In both cases the angles of incidence have been chosen so that the He–Ne laser line satisfies the grating equation at $\beta = \beta_0 = 2\theta - \alpha$. The curves represent the blaze functions calculated with $R = 0$, 1, and 2 μm from the round edge model. In the expanded diagrams of the top region the shift of the maximum intensities are illustrated.

$$E_r(\beta_0) = \frac{\cos^2(\beta_0 + p)}{\cos\alpha \cos\beta_0} \left(\frac{d'}{d} \right)^2 \tag{37}$$

or approximately

$$E_r(\beta_0) = \frac{\cos\beta_0}{\cos\alpha} \left(1 - \frac{2R}{d} q \right) , \tag{38}$$

where

$$q = \tan\beta_0(\sin\beta_0 - \cos\theta) + \cos\beta_0 + \sin\theta \tag{39}$$

for the round edge correction model and

$$q = \tan\beta_0(\sin\theta - \cos\theta) + \cos\theta + \sin\theta \tag{40}$$

for the cut edge correction model.

From Eqs. (32), (33), (37), and (38) one can see that the groove edge defect affects the relative efficiency considerably. The main reduction of the efficiency is given by the factor $(d/d')^2$. Thus the different normalizing factors for the 79- and the 31.6-g/mm gratings obtained in Ref. 4, used to compare the measured and calculated efficiencies, could be due to different groove edge defects.

VIII. Conclusions

In this paper a model has been presented for the blaze function of echelle gratings for in-plane mountings. The model shows that the two mountings with $\alpha > \theta$ and $\alpha < \theta$ have quite different blaze characteristics.

The model has also been extended to contain a one-parameter correction for groove defects assuming them to be defects in the edges of the grooves. This correction is necessary to account for the location of the minima of the blaze function. It has also been shown that by measuring the angles of incidence and diffraction for the minima of a few orders, the blaze angle and the groove edge correction parameter can be determined, the latter giving an estimate of the quality of the echelle grating. It has also been shown that groove edge defects are of decisive importance for the efficiency of the central blaze. No relative efficiency calculation for an echelle should be made without knowing the groove edge defects.

The new model also implies that the maximum intensity of the blaze is not exactly at $\beta_0 = 2\theta - \alpha$ but shifted by an amount which depends on the spectral order. A correction for the shift should be included in accurate measurements of the blaze angle through localization of the maximum blaze intensity. Such measurements should be made separately in each order.

The mounting with $\alpha < \theta$ has only one intense order of each wavelength in the spectrum, the wavelengths in the central blaze falling at the diffraction minima of the blaze function for a good echelle grating. Thus, in echelle spectrometers where reduction of stray light is of importance, the mounting with $\alpha < \theta$ could be very advantageous.

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