## $\mathcal{O}_{11}$ -rational Points and Matrices for d=11 case

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## 1 $\mathcal{O}_{11}$ -rational point representatives

Below,  $v = \sqrt{11}$ , and  $\tau = \frac{1+i\sqrt{11}}{2}$ .

Depth	$\mathcal{O}_{11} ext{-Rational Points}$
1	(0,0)
3	$(\frac{\tau}{3}, \frac{2}{3}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{5}{3}v)$
4	(0,v)
5	$(\frac{1}{5} + \frac{1}{5}\tau, \frac{3}{5}v), (\frac{2}{5} + \frac{2}{5}\tau, \frac{2}{5}v), (\frac{1}{5} + \frac{2}{5}\tau, \frac{1}{5}v), (\frac{3}{5} + \frac{1}{5}\tau, \frac{1}{5}v)$
9	
	$(0,\frac{2}{3}v),(0,\frac{4}{3}v),(\frac{\tau}{3},\frac{1}{3}v),(\frac{\tau}{3},\frac{1}{3}v),(\frac{1}{3}+\frac{2}{3}\tau,\frac{1}{3}v),(\frac{1}{3}+\frac{2}{3}\tau,v),(\frac{1}{9}+\frac{5}{9}\tau,v),(\frac{2}{9}+\frac{1}{9}\tau,\frac{7}{9}v),(\frac{4}{9}+\frac{2}{9}\tau,\frac{10}{9}v),\\ (\frac{1}{3}+\frac{2}{9}\tau,\frac{5}{3}v),(\frac{1}{3}+\frac{5}{9}\tau,\frac{13}{9}v),(\frac{2}{3}+\frac{1}{9}\tau,\frac{1}{9}v)$
11	$(0, \frac{2j}{11}v), 1 \le j \le 10$
12	$(\frac{\tau}{3}, \frac{2}{3}v), (\frac{2}{3} + \frac{1}{3}\tau, 0)$
15	$ \begin{array}{c} (0,\frac{2}{3}v), (0,\frac{4}{3}v), (\frac{7}{3},\frac{1}{3}v), (\frac{1}{3}+\frac{2}{3}\tau,\frac{1}{3}v), (\frac{1}{3}+\frac{2}{5}\tau,\frac{1}{3}v), (\frac{1}{2}+\frac{2}{9}\tau,\frac{1}{9}v), (\frac{2}{9}+\frac{1}{9}\tau,\frac{1}{9}v), (\frac{4}{9}+\frac{2}{9}\tau,\frac{10}{9}v), \\ (\frac{1}{3}+\frac{2}{9}\tau,\frac{5}{5}v), (\frac{1}{3}+\frac{2}{9}\tau,\frac{1}{9}v), (\frac{1}{2}+\frac{1}{9}\tau,\frac{1}{9}v), (\frac{4}{9}+\frac{2}{9}\tau,\frac{10}{9}v), \\ (\frac{1}{3}+\frac{2}{9}\tau,\frac{5}{5}v), (\frac{1}{3}+\frac{2}{9}\tau,\frac{1}{9}v), (\frac{1}{3}+\frac{1}{9}\tau,\frac{1}{9}v), (\frac{1}{4}+\frac{2}{9}\tau,\frac{10}{9}v), (\frac{1}{2}+\frac{2}{9}\tau,\frac{10}{9}v), \\ (\frac{1}{1}v,1\leq j\leq 10) \\ (\frac{1}{5}+\frac{1}{15}\tau,\frac{1}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{5}v), (\frac{2}{5}+\frac{1}{15}\tau,\frac{1}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{1}{15}v), (\frac{1}{5}+\frac{4}{15}\tau,\frac{2}{15}v), \\ (\frac{1}{15}+\frac{1}{15}\tau,\frac{1}{3}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{1}{3}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{1}{3}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{1}{3}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{1}{5}v), \\ (\frac{1}{15}+\frac{1}{15}\tau,\frac{3}{3}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), \\ (\frac{1}{15}+\frac{1}{15}\tau,\frac{3}{3}v), (\frac{1}{2}+\frac{1}{15}\tau,\frac{3}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{15}v), \\ (\frac{1}{5}+\frac{1}{15}\tau,\frac{3}{3}v), (\frac{2}{3}+\frac{1}{24}\tau,\frac{3}{4}v), (\frac{1}{2},\frac{4}{4}v), (\frac{2}{2},\frac{4}{4}v), (\frac{2}{2},\frac{4}{4}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{4}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2}v), (\frac{1}{2}+\frac{4}{2}\tau,\frac{4}{2$
	$ \begin{pmatrix} (15 & 15), (15) & 15), (15 & 15), (5) &$
16	$(0, \frac{1}{2}v), (0, \frac{3}{2}v), (\frac{\tau}{2}, \frac{1}{4}v), (\frac{\tau}{2}, \frac{3}{4}v), (\frac{1}{2}, \frac{1}{4}v), (\frac{1}{2}, \frac{3}{4}v), (\frac{1}{2} + \frac{1}{2}\tau, \frac{1}{4}v), (\frac{1}{2} + \frac{1}{2}\tau, \frac{3}{4}v)$
20	$(\frac{1}{5} + \frac{1}{5}\tau, \frac{8}{5}v), (\frac{2}{5} + \frac{2}{5}\tau, \frac{7}{5}v), (\frac{1}{5} + \frac{2}{5}\tau, \frac{6}{5}v), (\frac{3}{5} + \frac{1}{5}\tau, \frac{6}{5}v)$
23	$ \begin{array}{c} (\frac{1}{23} + \frac{6}{23}\tau, \frac{25}{23}v), (\frac{2}{23} + \frac{12}{23}\tau, \frac{8}{23}v), (\frac{3}{23} + \frac{18}{23}\tau, \frac{41}{23}v), (\frac{4}{23} + \frac{1}{23}\tau, \frac{5}{23}v), (\frac{5}{23} + \frac{7}{23}\tau, \frac{45}{23}v), (\frac{6}{23} + \frac{13}{23}\tau, \frac{43}{23}v) \\ (\frac{8}{23} + \frac{2}{23}\tau, \frac{20}{23}v), (\frac{9}{23} + \frac{8}{23}\tau, \frac{29}{23}v), (\frac{12}{23} + \frac{3}{23}\tau, \frac{45}{23}v), (\frac{13}{23} + \frac{9}{23}\tau, \frac{34}{23}v), (\frac{16}{23} + \frac{4}{23}\tau, \frac{34}{23}v), (\frac{1}{23} + \frac{9}{23}\tau, \frac{32}{23}v), (\frac{1}{23} + \frac{9}{$
	$ \left( \frac{8}{23} + \frac{2}{23}\tau, \frac{20}{23}v), \left( \frac{9}{23} + \frac{8}{23}\tau, \frac{29}{23}v), \left( \frac{12}{23} + \frac{3}{23}\tau, \frac{45}{23}v), \left( \frac{13}{23} + \frac{9}{23}\tau, v), \left( \frac{16}{23} + \frac{4}{23}\tau, \frac{34}{23}v), \left( \frac{1}{23} + \frac{9}{23}\tau, \frac{37}{23}v), \left( \frac{1}{23} + \frac{9}{23}\tau, \frac{37}{23}v, \frac{1}{23}v, $
	$(\frac{23}{23} + \frac{13}{23}\tau, \frac{13}{23}v), (\frac{13}{23} + \frac{23}{23}\tau, \frac{13}{23}v), (\frac{13}{23} + \frac{23}{23}\tau, \frac{13}{23}v), (\frac{13}{23} + \frac{23}{23}\tau, \frac{13}{23}v), (\frac{13}{23} + \frac{17}{23}\tau, \frac{17}{23}v), (\frac{13}{23} + \frac{2}{23}\tau, \frac{17}{23}v), (\frac{13}{23} + \frac{17}{23}\tau, $
25	$(0, \frac{2}{\epsilon}v), (0, \frac{6}{\epsilon}v), (\frac{6}{\epsilon}v), (\frac{1}{\epsilon} + \frac{1}{\epsilon}\tau, \frac{1}{\epsilon}v), (\frac{1}{\epsilon} + \frac{1}{\epsilon}\tau, \frac{7}{\epsilon}v), (\frac{1}{\epsilon} + \frac{1}{\epsilon}\tau, \frac{9}{\epsilon}v), (\frac{2}{\epsilon} + \frac{2}{\epsilon}\tau, 0),$
	$(\frac{2}{5} + \frac{2}{5}\tau, \frac{4}{5}v), (\frac{2}{5} + \frac{2}{5}\tau, \frac{6}{5}v), (\frac{2}{5} + \frac{2}{5}\tau, \frac{8}{5}v), (\frac{3}{5} + \frac{1}{5}\tau, v), (\frac{3}{5} + \frac{1}{5}\tau, \frac{9}{5}v), (\frac{1}{5} + \frac{2}{5}\tau, v),$
	$(\frac{1}{5} + \frac{2}{5}\tau, \frac{7}{5}v), (\frac{1}{5} + \frac{2}{5}\tau, \frac{9}{5}v), (\frac{1}{25} + \frac{22}{25}\tau, \frac{23}{25}v), (\frac{2}{25} + \frac{19}{25}\tau, \frac{3}{5}v), (\frac{3}{25} + \frac{16}{25}\tau, \frac{1}{25}v), (\frac{4}{25} + \frac{13}{25}\tau, \frac{31}{25}v),$
	$ \begin{pmatrix} \frac{6}{25} + \frac{7}{25}\tau, \frac{23}{25}v), \begin{pmatrix} \frac{8}{25} + \frac{1}{25}\tau, \frac{41}{25}v), \begin{pmatrix} \frac{13}{25} + \frac{11}{25}\tau, \frac{19}{25}v), \begin{pmatrix} \frac{14}{25} + \frac{8}{25}\tau, \frac{8}{5}v), \begin{pmatrix} \frac{16}{25} + \frac{2}{25}\tau, \frac{14}{25}v), \begin{pmatrix} \frac{1}{25} + \frac{11}{25}\tau, \frac{9}{5}v), \\ \frac{2}{25} + \frac{22}{25}\tau, \frac{6}{5}v), \begin{pmatrix} \frac{3}{25} + \frac{8}{25}\tau, \frac{27}{25}v), \begin{pmatrix} \frac{4}{25} + \frac{19}{25}\tau, \frac{41}{25}v), \begin{pmatrix} \frac{6}{25} + \frac{16}{25}\tau, \frac{8}{25}v, \end{pmatrix}, \begin{pmatrix} \frac{16}{25} + \frac{2}{25}\tau, \frac{9}{25}v, \begin{pmatrix} \frac{1}{25} + \frac{11}{25}\tau, \frac{9}{25}v, \end{pmatrix} \\ \frac{2}{25} + \frac{22}{25}\tau, \frac{6}{25}v), \begin{pmatrix} \frac{3}{25} + \frac{1}{25}\tau, \frac{19}{25}v, \end{pmatrix} $
	$\left[ \frac{(\frac{2}{25} + \frac{22}{25}\tau, \frac{6}{5}v), (\frac{3}{25} + \frac{8}{25}\tau, \frac{27}{25}v), (\frac{4}{25} + \frac{19}{25}\tau, \frac{41}{25}v), (\frac{6}{25} + \frac{16}{25}\tau, \frac{8}{25}v), (\frac{7}{25} + \frac{2}{25}\tau, \frac{9}{25}v), (\frac{8}{25} + \frac{13}{25}\tau, \frac{31}{25}v), \right]$
	$\left(\frac{12}{25} + \frac{7}{25}\tau, \frac{9}{5}v\right), \left(\frac{14}{25} + \frac{4}{25}\tau, \frac{36}{25}v\right), \left(\frac{16}{25} + \frac{1}{25}\tau, \frac{33}{25}v\right)$
27	$(\frac{1}{9} + \frac{2}{27}\tau, \frac{47}{27}v), (\frac{1}{9} + \frac{11}{27}\tau, \frac{1}{9}v), (\frac{1}{9} + \frac{20}{27}\tau, \frac{49}{27}v), (\frac{2}{9} + \frac{4}{27}\tau, \frac{26}{27}v), (\frac{2}{9} + \frac{13}{27}\tau, \frac{1}{27}v), (\frac{4}{9} + \frac{8}{27}\tau, \frac{30}{27}v),$
	$(\frac{3}{9} + \frac{1}{27}\tau, \frac{3}{3}v), (\frac{3}{9} + \frac{10}{27}\tau, \frac{41}{27}v), (\frac{1}{9} + \frac{3}{27}\tau, \frac{1}{27}v), (\frac{1}{3}, \frac{3}{9}v), (\frac{1}{3}, \frac{11}{9}v), (\frac{1}{3} + \frac{2}{9}\tau, \frac{1}{3}v),$
	$(\frac{1}{3} + \frac{2}{9}\tau, v), (\frac{1}{3} + \frac{1}{3}\tau, \frac{1}{9}v), (\frac{1}{3} + \frac{1}{3}\tau, \frac{1}{9}v), (\frac{1}{3} + \frac{1}{9}\tau, \frac{1}{9}v), ($
	$(\frac{1}{3} + \frac{1}{6}\tau, \frac{1}{6}v), (\frac{1}{3} + \frac{1}{6}\tau, \frac{1}{9}v), (\frac{1}{27} + \frac{27}{27}\tau, \frac{1}{6}v), (\frac{1}{27} + \frac{27}{27}\tau, \frac{1}{9}v), (\frac{1}{27} + \frac{27}{27$
	$\left(\frac{8}{27} + \frac{13}{27}\tau, \frac{13}{27}v\right), \left(\frac{11}{27} + \frac{1}{27}\tau, \frac{35}{27}v\right), \left(\frac{13}{27} + \frac{11}{27}\tau, \frac{13}{27}v\right), \left(\frac{17}{27} + \frac{4}{27}\tau, \frac{17}{27}v\right), \left(\frac{22}{27} + \frac{2}{27}\tau, \frac{32}{27}v\right), \left(\frac{1}{1} + \frac{5}{9}\tau, \frac{1}{3}v\right), \left(\frac{1}{1} + \frac{5}{17}\tau, \frac{1}{3}v\right), \left(\frac{1}{1} + \frac{1}{17}\tau, \frac{1}{17}v\right), \left(\frac{1}{17}\tau, \frac{1}{17}\tau, \frac{1}{17}v\right), \left(\frac{1}{17}\tau, \frac{1}{17}\tau, \frac{1}{17}v\right), \left(\frac{1}{17}\tau, \frac{1}{17}\tau, \frac{1}{17}\tau, \frac{1}{17}v\right), \left(\frac{1}{17}\tau, \frac{1}{17}v\right), \left(1$
	$(\frac{1}{9} + \frac{1}{9}7, \frac{1}{3}0), (\frac{1}{9} + \frac{1}{9}7, \frac{1}{9}0), (\frac{1}{9} + \frac{1}{9}7, \frac{1}{9}0), (\frac{1}{3}, \frac{1}{9}0), (\frac{1}{3}, \frac{1}{9}0), (\frac{1}{3} + \frac{1}{3}7, \frac{1}{9}0), (\frac{1}{3} + \frac{1}{2}\tau, \frac{16}{9}v)$
31	$\frac{(3+3)\cdot 9^{-7}(3+3)\cdot 9^{-7}(9+9)\cdot 9^{-7}(9+9)\cdot 9^{-7}(9+9)\cdot 9^{-7}}{(\frac{1}{21}+\frac{3}{21}\tau,\frac{49}{21}v),(\frac{2}{21}+\frac{6}{21}\tau,\frac{10}{21}v),(\frac{3}{21}+\frac{9}{21}\tau,\frac{7}{21}v),(\frac{4}{21}+\frac{12}{21}\tau,\frac{40}{21}v),(\frac{5}{21}+\frac{15}{21}\tau,\frac{47}{21}v),(\frac{6}{21}+\frac{18}{21}\tau,\frac{28}{21}v).}$
	$(\frac{31}{21} + \frac{31}{21}\tau, \frac{31}{21}v), (\frac{31}{21} + \frac{31}{21}\tau, \frac{31}{21}v), (\frac{31}2 + \frac{31}{21}\tau, \frac{31}{21}v), (\frac{31}{21} + \frac{31}{21}\tau, \frac{31}{21$
	$\left(\frac{21}{31} + \frac{31}{31}\tau, \frac{53}{31}v\right), \left(\frac{21}{31} + \frac{31}{31}\tau, \frac{50}{31}v\right), \left(\frac{23}{31} + \frac{37}{31}\tau, \frac{21}{31}v\right), \left(\frac{31}{21} + \frac{37}{31}\tau, \frac{31}{31}v\right), \left(\frac{31}{21} + \frac{14}{31}\tau, \frac{12}{21}v\right), \left(\frac{31}{21} + \frac{21}{21}\tau, \frac{27}{21}v\right), \left(\frac{31}{21} + \frac{31}{21}\tau, \frac{31}{21}v\right), \left($
	$\left[\begin{array}{cccccccccccccccccccccccccccccccccccc$
	$ \left[ \begin{array}{cccccccccccccccccccccccccccccccccccc$

Depth	$\mathcal{O}_{11}$ -Rational Points
33	$ \begin{array}{c} (\frac{1}{3} + \frac{2}{3}\tau, \frac{1}{33}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{7}{33}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{13}{33}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{19}{33}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{25}{33}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{31}{33}v), \\ (\frac{1}{3} + \frac{2}{3}\tau, \frac{37}{33}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{43}{33}v), (\frac{1}{3} + \frac{2}{3}\tau, \frac{61}{33}v), (\frac{\tau}{3}, \frac{43}{33}v), (\frac{\tau}{3$
36	$(0,\frac{1}{3}v),(0,\frac{5}{3}v),(\frac{\tau}{3},0),(\frac{\tau}{3},\frac{4}{3}v),(\frac{1}{3}+\frac{2}{3}\tau,0),(\frac{1}{3}+\frac{2}{3}\tau,\frac{4}{9}v),(\frac{1}{9}+\frac{5}{9}\tau,0),(\frac{2}{9}+\frac{1}{9}\tau,\frac{16}{9}v),(\frac{4}{9}+\frac{2}{9}\tau,\frac{1}{9}v),\\ (\frac{1}{3}+\frac{2}{9}\tau,\frac{2}{3}v),(\frac{1}{3}+\frac{5}{9}\tau,\frac{4}{9}v),(\frac{2}{3}+\frac{1}{9}\tau,\frac{10}{9}v)$
37	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$

## 2 Matrices sending $\infty$ to $\mathcal{O}_2$ -rational points

Below,  $\tau = \frac{1+i\sqrt{11}}{2}$ .  $I_0 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, A_{3,1} = \begin{bmatrix} -4 - \tau & 1 - 2\tau & -1 + 2\tau \\ -1 & -1 & 0 \\ -1 + \tau & -1 & 1 \end{bmatrix}, A_{3,2} = \begin{bmatrix} -5 & -3 + 2\tau & 2 - \tau \\ -2 + \tau & 1 + \tau & -\tau \\ \tau & 1 & -1 \end{bmatrix}, A_{4,1} = \begin{bmatrix} -1 + 2\tau & 0 & 6 \\ 0 & 1 & 0 \\ 2 & 0 & 1 - 2\tau \end{bmatrix}, A_{4,1} = \begin{bmatrix} -1 + 2\tau & 0 & 6 \\ 0 & 1 & 0 \\ 2 & 0 & 1 - 2\tau \end{bmatrix}$  $A_{5,1} = \begin{vmatrix} -1 - \tau & -2 & -1 + \tau \\ -1 & -1 + \tau & 1 \\ -2 + \tau & \tau & 1 \end{vmatrix}, A_{5,2} = \begin{vmatrix} -\tau & 1 & 1 \\ -2 & -\tau & 1 - \tau \\ -2 + \tau & -1 & -1 - \tau \end{vmatrix}, A_{5,3} = \begin{vmatrix} -1 & -1 & \tau \\ -1 + \tau & \tau & 2 \\ 1 + \tau & 1 & 2 - \tau \end{vmatrix},$  $A_{5,4} = \begin{bmatrix} -1 & -1 & \tau \\ \tau & 1+\tau & 2-\tau \\ 1+\tau & 2 & 1-\tau \end{bmatrix}, A_{9,1} = \begin{bmatrix} -1+2\tau & 0 & 4 \\ 0 & 1 & 0 \\ 3 & 0 & 1-2\tau \end{bmatrix}, A_{9,2} = \begin{bmatrix} -2+4\tau & 0 & -7 \\ 0 & 1 & 0 \\ 3 & 0 & -1+2\tau \end{bmatrix},$  $A_{9,3} = \begin{bmatrix} -1 + \tau & 1 & 1 \\ \tau & 0 & 1 \\ 3 & 1 - \tau & -\tau \end{bmatrix}, A_{9,4} = \begin{bmatrix} -2 + 3\tau & -5 - \tau & 1 - 2\tau \\ \tau & -1 & 0 \\ 3 & -2 + 2\tau & -2 \end{bmatrix}, A_{9,5} = \begin{bmatrix} -3 + \tau & -2 & -1 - 3\tau \\ 1 + 2\tau & -1 + \tau & -7 + 2\tau \\ 3 & \tau & -2 + 3\tau \end{bmatrix},$  $A_{9,6} = \begin{bmatrix} -4 + 3\tau & -2 - \tau & -1 - \tau \\ 1 + 2\tau & -3 + \tau & -2 \\ 3 & \tau & -1 + \tau \end{bmatrix}, A_{9,7} = \begin{bmatrix} -3\tau & 2\tau & -2 + \tau \\ -2 - \tau & 2 + \tau & \tau \\ -3 + \tau & 2 - \tau & 1 \end{bmatrix}, A_{9,8} = \begin{bmatrix} -1 - 2\tau & 4 & 4 \\ -1 & 1 & 0 \\ -3 + \tau & 1 - 2\tau & 1 - 2\tau \end{bmatrix},$  $A_{9,9} = \begin{bmatrix} -1 - 3\tau & 7 - 2\tau & -2 + 3\tau \\ -2 & 1 - \tau & \tau \\ -3 + \tau & -1 - 2\tau & 3 \end{bmatrix}, A_{9,10} = \begin{bmatrix} -7 + 4\tau & -5 + \tau & 4 + \tau \\ \tau & 0 & 1 \\ 2 + \tau & \tau & 1 - \tau \end{bmatrix}, A_{9,11} = \begin{bmatrix} -7 + 3\tau & 8 - 5\tau & 4 - 5\tau \\ -1 + 2\tau & -2\tau & -2 - \tau \\ 2 + \tau & -3 - \tau & -3 \end{bmatrix},$  $A_{9,12} = \begin{bmatrix} -1 & 0 & 0 \\ 1+\tau & -1 & 0 \\ 2+\tau & -2+\tau & -1 \end{bmatrix}, A_{11,1} = \begin{bmatrix} -1 & 0 & 1-2\tau \\ 0 & 1 & 0 \\ -1+2\tau & 0 & -12 \end{bmatrix}, A_{11,2} = \begin{bmatrix} -2 & -2\tau & 3 \\ 0 & -1 & 1-\tau \\ -1+2\tau & -6+\tau & 1-3\tau \end{bmatrix},$  $A_{11,3} = \begin{bmatrix} -3 & 0 & 1 - 2\tau \\ 0 & -1 & 0 \\ -1 + 2\tau & 0 & -4 \end{bmatrix}, A_{11,4} = \begin{bmatrix} -4 & 0 & 1 - 2\tau \\ 0 & -1 & 0 \\ -1 + 2\tau & 0 & -3 \end{bmatrix}, A_{11,5} = \begin{bmatrix} -5 & 0 & -1 + 2\tau \\ 0 & 1 & 0 \\ -1 + 2\tau & 0 & 2 \end{bmatrix},$  $A_{11,6} = \begin{bmatrix} -6 & 0 & 1 - 2\tau \\ 0 & -1 & 0 \\ -1 + 2\tau & 0 & -2 \end{bmatrix}, A_{11,7} = \begin{bmatrix} -7 & 0 & -2 + 4\tau \\ 0 & 1 & 0 \\ -1 + 2\tau & 0 & 3 \end{bmatrix}, A_{11,8} = \begin{bmatrix} -8 & 0 & -3 + 6\tau \\ 0 & 1 & 0 \\ -1 + 2\tau & 0 & 4 \end{bmatrix},$  $A_{11,9} = \begin{bmatrix} -9 & 0 & 4 - 8\tau \\ 0 & 1 & 0 \\ -1 + 2\tau & 0 & -5 \end{bmatrix}, A_{11,10} = \begin{bmatrix} -10 & 0 & -1 + 2\tau \\ 0 & -1 & 0 \\ -1 + 2\tau & 0 & 1 \end{bmatrix}, A_{12,1} = \begin{bmatrix} -3 - \tau & 2 - 3\tau & 10 - 5\tau \\ -2 & -1 - \tau & 3 - 4\tau \\ -2 + 2\tau & -4 & -7 - 3\tau \end{bmatrix},$  $A_{12,2} = \begin{bmatrix} -\tau & -1 & 1 \\ -2+2\tau & 2+\tau & -1-\tau \\ 2\tau & 4 & -2-\tau \end{bmatrix}, A_{15,1} = \begin{bmatrix} -1-4\tau & -6 & 6 \\ -1 & -1 & 0 \\ -4+\tau & -1+2\tau & 1-2\tau \end{bmatrix}, A_{15,2} = \begin{bmatrix} 3-3\tau & 1-\tau & 4+\tau \\ -3-2\tau & -\tau & 4-2\tau \\ -4+\tau & -2 & -2\tau \end{bmatrix}, A_{15,2} = \begin{bmatrix} -1-4\tau & -6 & 6 \\ -1 & -1 & 0 \\ -4+\tau & -1+2\tau & 1-2\tau \end{bmatrix}, A_{15,2} = \begin{bmatrix} 3-3\tau & 1-\tau & 4+\tau \\ -3-2\tau & -\tau & 4-2\tau \\ -4+\tau & -2 & -2\tau \end{bmatrix}$  $A_{15,3} = \begin{bmatrix} -2\tau & -4 & -10 + 3\tau \\ -2 & -2 + \tau & -1 + 4\tau \\ -4 + \tau & -1 + 3\tau & 5 + 5\tau \end{bmatrix}, A_{15,4} = \begin{bmatrix} 1 - 4\tau & -\tau & -10 + \tau \\ -3 - \tau & 0 & -4 + 3\tau \\ -4 + \tau & -1 & 1 + 3\tau \end{bmatrix}, A_{15,5} = \begin{bmatrix} -2 - \tau & -1 + 2\tau & -1 + 2\tau \\ -1 & 1 & 0 \\ -3 + 2\tau & 4 & 4 \end{bmatrix},$  $A_{15,6} = \begin{bmatrix} -8 - 4\tau & -2 + 4\tau & -1 + 2\tau \\ -2 & 1 & 0 \\ -3 + 2\tau & 2 & 1 \end{bmatrix}, A_{15,7} = \begin{bmatrix} -5 - 4\tau & 1 - 2\tau & 7 - 3\tau \\ -4 & -1 & 1 - 2\tau \\ -3 + 2\tau & -1 & -2 - \tau \end{bmatrix}, A_{15,8} = \begin{bmatrix} -1 - \tau & -\tau & 1 \\ -2 + \tau & -2 + \tau & -\tau \\ -3 + 2\tau & -1 - \tau \end{bmatrix},$ 

$$A_{15,9} = \begin{bmatrix} -7+7&7&7&3r\\ 1+2r&1&3-r\\ 1+2r&1&3-r \end{bmatrix}, A_{15,13} = \begin{bmatrix} -4+7&-2+7&1+r\\ 1+2r&1+r&2-r \end{bmatrix}, A_{25,13} = \begin{bmatrix} -6+7&1-27&1-r\\ 1+2r&-1&-1 \end{bmatrix},$$

$$A_{15,12} = \begin{bmatrix} -3&2-2r&3-2r\\ 1+2r&-4&-4 \end{bmatrix}, A_{15,13} = \begin{bmatrix} -10+5r&-3+4r&7+3-2r\\ 3+r&1&2-2r \end{bmatrix}, A_{35,14} = \begin{bmatrix} -1+7&1-2r&1-r\\ 1+2r&1&-5-5r \end{bmatrix},$$

$$A_{15,15} = \begin{bmatrix} -2+7&-1&-1&-1&-1\\ 1+r&0&-2+r\end{bmatrix}, A_{15,16} = \begin{bmatrix} -9+6r&-9-r&10+r\\ 2+r&-1+2r&1-2r \end{bmatrix}, A_{15,16} = \begin{bmatrix} -9+6r&-9-r&10+r\\ 2+r&-1-2r&1&-2r \end{bmatrix}, A_{15,16} = \begin{bmatrix} -3+6r&-1+2r&-9-5r\\ 3+r&-1+2r&1-2r \end{bmatrix}, A_{15,16} = \begin{bmatrix} -3+6r&-1+2r&1-2r\\ 3+r&-1+2r&1-2r \end{bmatrix}, A_{15,16} = \begin{bmatrix} -3+6r&-1+2r&1-2r\\ 3+r&-1+2r&1-2r \end{bmatrix}, A_{15,16} = \begin{bmatrix} -3+3r&2-2r&-1-r\\ 4&0&1-2r \end{bmatrix}, A_{16,2} = \begin{bmatrix} -3+6r&-9-r&10+r\\ 4&0&-1+2r \end{bmatrix}, A_{16,3} = \begin{bmatrix} -2+7&1-2r&1-2r\\ 2+r&-1-2r&1-2r \end{bmatrix}, A_{16,4} = \begin{bmatrix} -3+3r&2-2-r&-1-r\\ 4&2-r&-2+3r \end{bmatrix}, A_{16,5} = \begin{bmatrix} -3+3r&2-2-r&1-r\\ 4&2-r&-2+3r \end{bmatrix}, A_{16,6} = \begin{bmatrix} -2+3r&3+3r&3-r\\ 2+2r&-2-r&2-r \end{bmatrix}, A_{16,6} = \begin{bmatrix} -3+3r&2-r&1-2r\\ 4&2-r&-2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -2+3r&3+3r&2-2r\\ 4&2-r&-2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -2+3r&3+3r&3-r\\ 4&2r&-2-r&2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -3+3r&2-r&1-3r\\ 4&2-r&-2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4+3r&3-r&2-r\\ 4&2r&-2-r&-1 \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4+3r&3-r&2-r\\ 4+2r&-2-r&2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4+3r&3-r&2-r\\ 4+2r&-2-r&2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4-3r&-1+4r&4+3r\\ 2+2r&-2-r&2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4-3r&-1+4r&4+3r\\ 4+2r&2-2-r&2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4-3r&-1+4r&2-4-3r\\ 4+2r&2-2-r&2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4-3r&-1+4r&4-3r\\ 2+2r&-4-2r&2-2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4-3r&-1+3r&4+3r\\ 2+2r&-2-2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4-3r&-1+3r&4+3r\\ 2+2r&-2-2-r \end{bmatrix}, A_{26,6} = \begin{bmatrix} -4-3r&-1+3r&4+3r\\ 2+2r&-2-2-r \end{bmatrix}, A_{26,7} = \begin{bmatrix} -4-3r&-1+3r&4+3r\\ 4+2r&2-2-2-r \end{bmatrix}, A_{26,7} = \begin{bmatrix} -4-3r&-1+3r&4+3r\\ 4+2r&2-2-2-r \end{bmatrix}, A_{26,7} = \begin{bmatrix} -4-3r&-1+3r&4+3r\\ 4+2r&2-3-3-r \end{bmatrix}, A_{26,7} = \begin{bmatrix} -4-3r&-1+3r&4+3r\\ 3+r&-1+3r&3-3-r \end{bmatrix}, A_{2$$

$$A_{25,17} = \begin{bmatrix} -5 + 77 & 10 + 27 & 7 - 37 \\ 1 + 27 & 5 - 7 & 1 - 27 \\ 2 - 37 & 7 & 1 - 27 \\ -7 & 1 & 1 & 7 \\ 2 - 37 & 7 & 1 - 27 \\ -7 & 1 & 1 & 7 \\ -7 & 1 & 1 & 7 \\ -7 & 1 & 1 & 7 \\ -7 & 1 & 1 & 7 \\ -7 & 1 & 1 & 7 \\ -7 & 7 & 1 & 1 & 7 \\ -7 & 7 & 7 & 1 & 7 \\ -7 & 7 & 7 & 1 & 7 \\ -7 & 7 & 7 & 1 & 7 \\ -7 & 7 & 7 & 7 & -7 \\ -7 & 7 & 7 & 7 & -7 \\ -7 & 7 & 7 & 7 & -7 \\ -7 & 7 & 7 & 7 & 7 & 7 \\ -7 & 7 & 7 & 7 & 7 & 7 \\ -7 & 7 & 7 & 7 & 7 & 7 \\ -7 &$$

$$A_{27,37} = \begin{bmatrix} -13+2r & -5-6r & 7-5r \\ 3r & -5+2r & -3-r \\ 3r & -5+2r & -3-r \end{bmatrix}, A_{27,38} = \begin{bmatrix} -3-3r & -r & 1+r \\ -3+2r & -2+r & 1 \\ 3r & -2+2r & 2-r \end{bmatrix}, A_{27,39} = \begin{bmatrix} -13+r & -2-r & -1+6r \\ 3r & -1+r & 5-r \end{bmatrix}, A_{27,49} = \begin{bmatrix} -13+2r & -2-3r & -4-r \\ 3r & -1+r & 5-r \end{bmatrix}, A_{27,49} = \begin{bmatrix} -13+2r & -2-3r & -4-r \\ 3r & -1+r & 5-r \end{bmatrix}, A_{27,49} = \begin{bmatrix} -13+2r & -2-3r & -4-r \\ 3r & -1+r & 5-r \end{bmatrix}, A_{27,49} = \begin{bmatrix} -13+2r & -2-r & -3+6r & 4+r \\ -2+2r & -1-3r \end{bmatrix}, A_{27,49} = \begin{bmatrix} -16+2r & -8+6r & 4+r \\ -2+2r & -1-3r \end{bmatrix}, A_{21,39} = \begin{bmatrix} -1-4r & -16+2r & -8+6r & 4+r \\ -2+3r & -3-r & 1-r \end{bmatrix}, A_{21,39} = \begin{bmatrix} -2-r & -3+2r & 3 \\ -4+3r & -5-r & 3+r & 1-r \end{bmatrix}, A_{21,39} = \begin{bmatrix} -3-r & -1+2r & -3+2r \\ -4+3r & -2-r & -3-r & -1+3r \\ -4+3r & -2-r & -3-r & -2-r \end{bmatrix}, A_{31,49} = \begin{bmatrix} -9-5r & -1+2r & -8+6r \\ -4+3r & 1 & 3+r \end{bmatrix}, A_{31,6} = \begin{bmatrix} -4-4r & -10+2r & 10-r \\ -4+3r & -4+3r & 2+3r & -1-3r \end{bmatrix}, A_{31,49} = \begin{bmatrix} -7-6r & -3-2r & -2+4r \\ -4+3r & 2+3r & -1-3r \end{bmatrix}, A_{31,49} = \begin{bmatrix} -1-2r & 2-r & 1-3r \\ -4+3r & 1-3r & 1-r & 1 \end{bmatrix}, A_{31,19} = \begin{bmatrix} -1-3r & -1+2r \\ -4+3r & -4+2r & -2+3r \\ -4+3r & -4+2r & -3+r \end{bmatrix}, A_{31,19} = \begin{bmatrix} -1-2r & 2-r & 1-4r \\ -4+3r & -4+2r & -1+3r \\ -4+3r & -2+3r & -1+3r \end{bmatrix}, A_{31,19} = \begin{bmatrix} -1-2r & 2-r & 1-4r \\ -4+3r & -4+2r & -1+3r \\ -4+3r & -2+3r & -1+3r \end{bmatrix}, A_{31,19} = \begin{bmatrix} -1-2r & 2-r & 1-r \\ -4+3r & -2+3r & -1+3r \\ -4+3r & -2+3r & -1+3r \end{bmatrix}, A_{31,19} = \begin{bmatrix} -1-2r & 2-r & 1-r \\ -4+3r & -2+3r & -1+3r \\ -4+3r & -2+3r & -1+3r \end{bmatrix}, A_{31,19} = \begin{bmatrix} -1-2r & 2-r & 1-r \\ -4+2r & -1-r & 1 \\ -4+3r & -2+3r & -1+3r \end{bmatrix}, A_{31,19} = \begin{bmatrix} -1-2r & 2-r & 1-r \\ -4+3r & -1-3r & -1+3r \\ -4+3r & -1-3r \\$$

$$A_{36,2} = \begin{bmatrix} -5 + 10r & 0 & -9 \\ 0 & 1 & 0 \\ 6 & 0 & -1 + 2r \end{bmatrix}, A_{36,3} = \begin{bmatrix} -1 & 0 & 0 \\ 2r & 2r & 1 \end{bmatrix}, A_{36,4} = \begin{bmatrix} -5 + 8r & 13 + 3r & 10 - 5r \\ 3 & 2 - 7 \\ 3 & 4 - 4r & -3 - 2r \\ -7 & 4 - 4r & -3 + 2r & -18 + 5r \\ -8 & 2r & -5 + 8r \end{bmatrix}, A_{36,6} = \begin{bmatrix} -9 + 8r & -17 & 25 - r \\ 2 + 4r & -1 - 6r \end{bmatrix}, A_{36,7} = \begin{bmatrix} -3 - r & -1 + 2r & -1 + 2r \\ -6 + 2r & -5 + 8r \end{bmatrix}, A_{36,6} = \begin{bmatrix} -5 + 8r & 13 + 3r & 10 - 5r \\ 2 + 4r & -1 - 6r \end{bmatrix}, A_{36,1} = \begin{bmatrix} -3 - r & -1 + 2r & -1 + 2r \\ -6 + 2r & -5 + 8r \end{bmatrix}, A_{36,9} = \begin{bmatrix} -1 - r & 1 & -1 \\ -4 & -1 - 7 & r \\ -6 + 2r & -4 & 1 - r \end{bmatrix}, A_{36,10} = \begin{bmatrix} -6 + 3r & -1 + r & 11 + 8r \\ 4 + 2r & 2 & 14 - 9r \end{bmatrix}, A_{36,10} = \begin{bmatrix} -6 + 3r & -1 + r & 11 + 8r \\ 4 + 2r & 2 & 14 - 9r \end{bmatrix}, A_{37,10} = \begin{bmatrix} -6 + 3r & -1 + r & 11 + 8r \\ 4 + 2r & 2 & 14 - 9r \end{bmatrix}, A_{37,2} = \begin{bmatrix} -1 - 4r & 3 - 5r & 4 + 4r & 3 \\ -5 + 3r & -7 & -7 - 4r \end{bmatrix}, A_{37,6} = \begin{bmatrix} -6 + 7 & 5 + r & 3 + r \\ -5 + 3r & 7 & 7 & 7 \end{bmatrix}, A_{37,6} = \begin{bmatrix} -6 - 7 & -5 - 3r & 6 - r \\ -5 - 3r & 3 - 3r & 3r & 7 \end{bmatrix}, A_{37,6} = \begin{bmatrix} -6 - 5 - 7 & 1 + 6r & -6 + 12r \\ -1 - 1 & 0 & -5 + 3r & 2 + 2r & -7 \end{bmatrix}, A_{37,6} = \begin{bmatrix} -8 - 5r & 1 + 5r & -7 + 6r \\ -5 + 3r & 2 - 2r & -7 & 3 \end{bmatrix}, A_{37,6} = \begin{bmatrix} -8 - 5r & 1 + 5r & -7 + 6r \\ -5 + 3r & 2 - 2r & -7 & 3 \end{bmatrix}, A_{37,6} = \begin{bmatrix} -8 - 5r & 1 + 5r & -7 + 6r \\ -5 + 3r & 2 - 2r & -7 & 2 + r \\ -5 + 3r & 2 - 2r & -7 & 2 - 2r \end{bmatrix}, A_{37,9} = \begin{bmatrix} -2 - 3r & -7 - 7 & -3 + 4r \\ -5 + 3r & -2 + 2r & -7 & 3 - 2r \\ -5 + 3r & -2 + 2r & 3r & -7 & 2 + r \end{bmatrix}, A_{37,10} = \begin{bmatrix} -6 - 6r & 6 - 6r & 13 - 3r \\ -5 + 3r & -2 + 2r & -7 & 3 - 2r \\ -5 + 3r & -2 + 2r & 3r & -7 & 2 + r \end{bmatrix}, A_{37,10} = \begin{bmatrix} -6 - 6r & 6 - 6r & 13 - 3r \\ -5 + 3r & -2 + 2r & -3 & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2 + 2r & -3 - 3r & -2r \\ -5 + 3r & -2$$