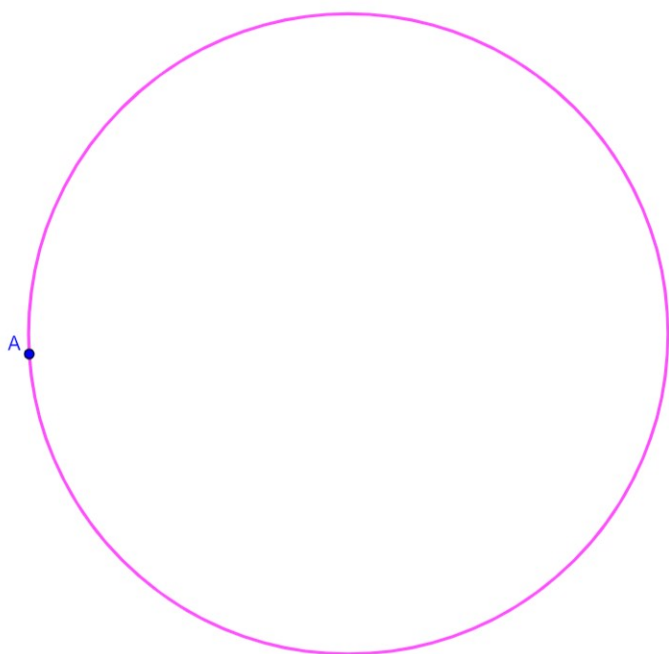


Is this object a circle?

Suppose we fish some device out of the ocean with a face whose perimeter is given by a simple closed curve, S . To what degree can we determine if S is a circle by measuring chords and diameters?

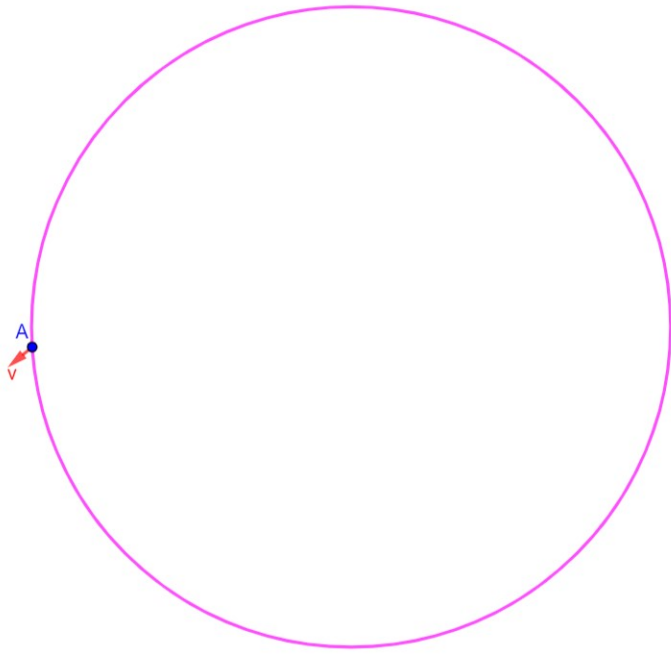
If S ends up being a circle, an important piece of information about S would be its radius. If S is a circle, we can obtain the radius of S as follows.

1. Pick a point on S , label this point A .

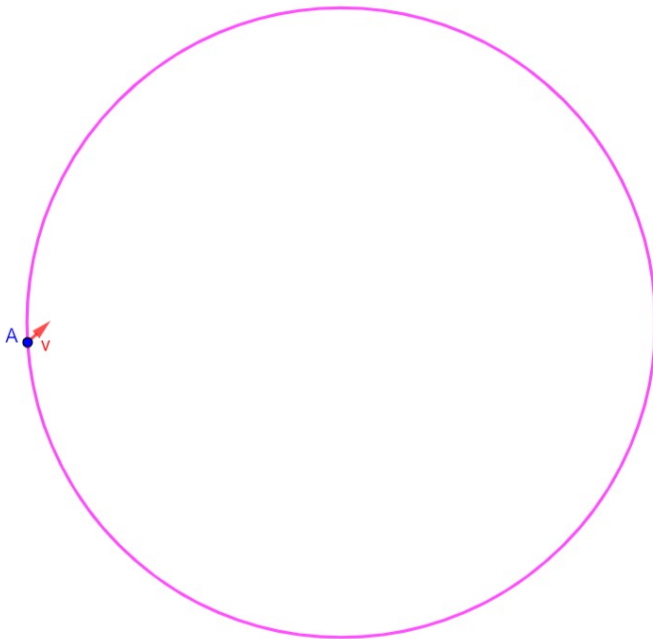


If S is a circle, then S is a simple closed curve that exists in a 2-dimensional subspace of \mathbb{R}^3 . Since a 2D subspace of \mathbb{R}^3 is isomorphic to \mathbb{R}^2 , the Jordan curve theorem implies S divides this subspace into two regions, a region interior to S and a region exterior to S .

2. Pick a direction, that is, a vector, v , that lands you in the region interior to S and construct the ray with origin A emanating in the direction of v . For example,

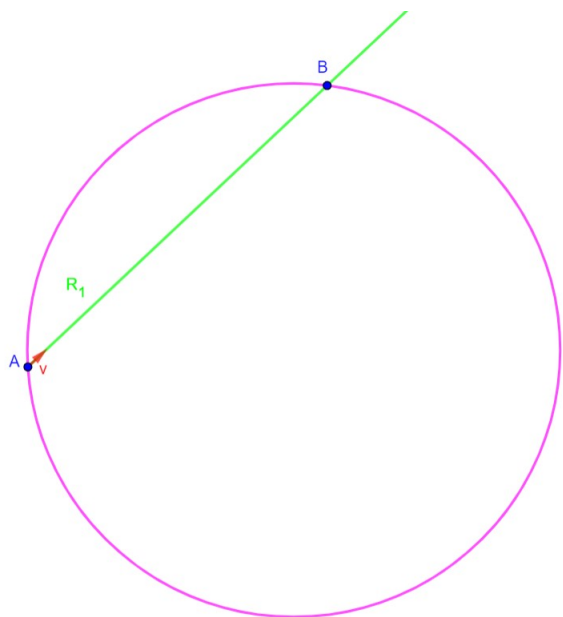


is not a valid direction, whereas

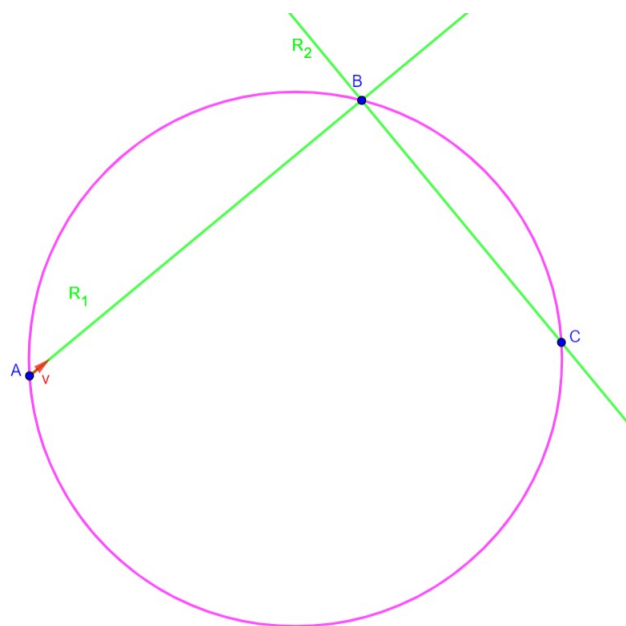


is a valid direction.

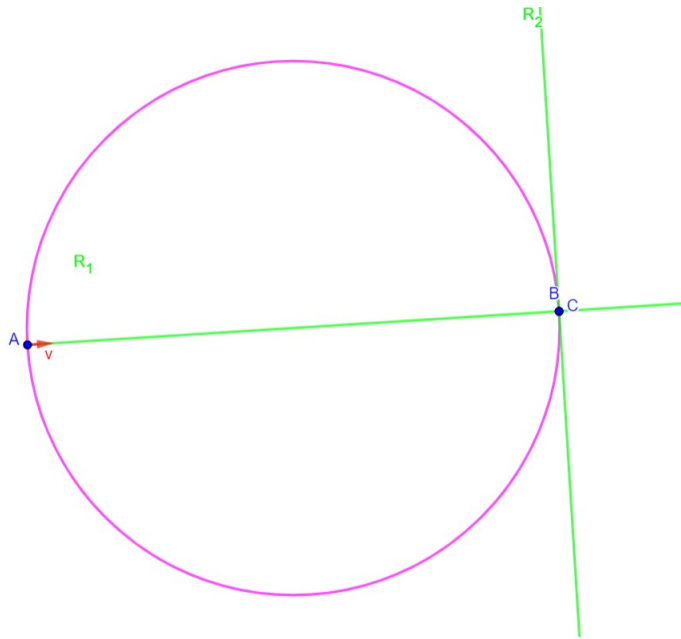
We then obtain the ray, R_1 , and the intersection point, B .



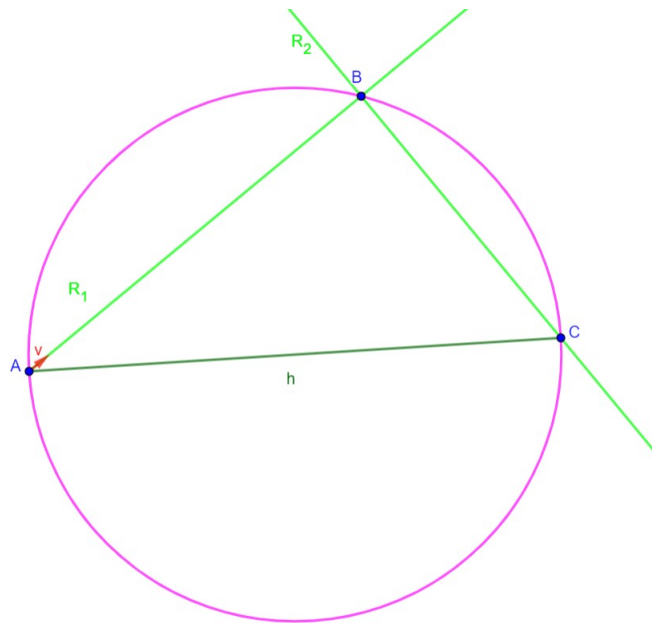
3. At the point B , construct the line, R_2 , passing through B , perpendicular to R_1 . Label the point of intersection of R_2 and S that is not equal to B , C .



In the event that there is not another point, C , intersecting S , different from B , then R_2 is tangent to S at B . In this case, AB gives a diameter of S , and $r = \frac{1}{2}\text{length}(AB)$ gives the radius of S .

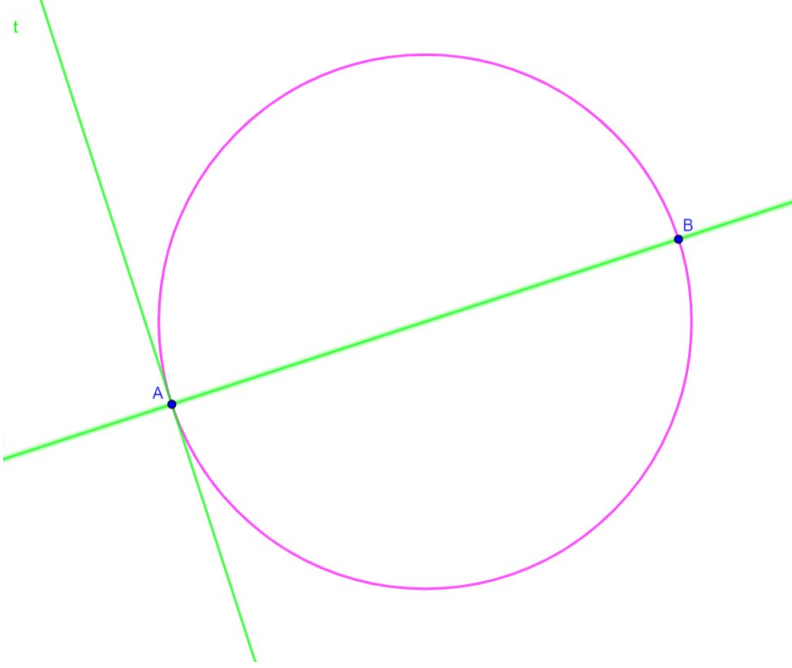


Otherwise, construct the line segment AC . Since the inscribed angle, $\angle ABC$, is 90° by construction, the central angle with inscribed points A and C must be 180° . Hence AC is a diameter of S .



We can then take $r = \frac{1}{2}length(AC)$ as the radius of S .

Another way we could find the radius of our supposed circle, S , is to pick a point, A , on S , construct the line tangent to S and A and then construct the line perpendicular to this tangent line at A . We can then label the second intersection of S with this perpendicular line as B . Since lines tangent to a circle are perpendicular to radial lines at the point of tangency, AB is a diameter of S and we may take $r = \frac{1}{2} \text{length}(AB)$ as the radius for S . We know such a point, B , will exist since S is compact and the distance function is continuous.

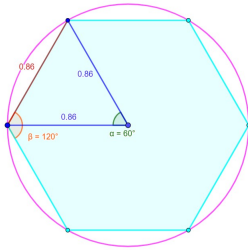


Once we have our radius, r , we then use inscribed, regular n -gons to get approximations of our circle. We can inscribe a regular n -gon in a circle, S , of radius r as follows:

1. Pick a point on S . Using the fact that an interior angle of a regular n -gon has measure $\alpha = ((\frac{n-2}{n}) \cdot 180)^\circ$, from the diameter constructed in obtaining the radius of S , rotate $(\frac{1}{2}(\frac{n-2}{n}) \cdot 180)^\circ$ in either direction.

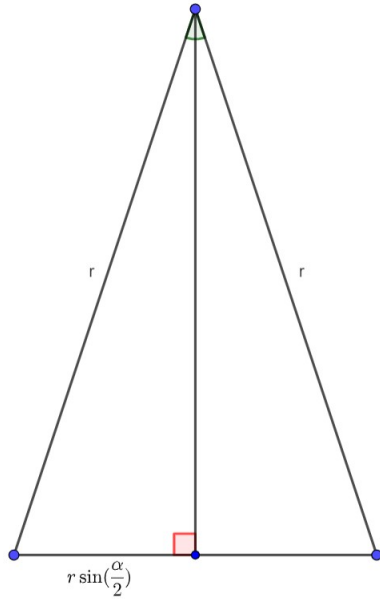
We aim to construct a side of our regular n -gon after performing this angle rotation. Note, a regular n -gon can be divided up into n equal isosceles triangles with the equal side lengths equal to the radius of our circle, and angle between these sides equal to $\frac{360^\circ}{n}$

$r = 0.86$
 $n = 6$



$$\left| \pi r^2 - r^2 n \sin\left(\frac{2\pi}{n}\right) \right| = 0.4$$

In an isosceles triangle, we can find the length of the third side according to

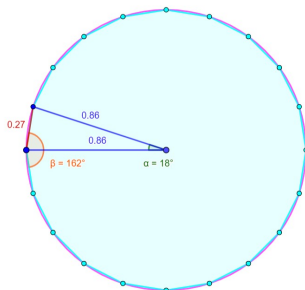
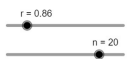


So that the length of each side in our regular n -gon is $2r \sin\left(\frac{\pi}{n}\right)$.

2. After rotating $\left(\frac{1}{2}\left(\frac{n-2}{n}\right) \cdot 180\right)^\circ$ from the diameter of S , construct a line segment of length $2r \sin\left(\frac{\pi}{n}\right)$. If S is a circle, this line segment should land back on S . If this line segment does not land back on S , you can conclude S is not a circle.

3. If the line segment constructed lands you back on S , rotate $\left(\left(\frac{n-2}{n}\right) \cdot 180\right)^\circ$ so that the terminal ray lies in the region interior to S . Construct a line segment of length $2r \sin\left(\frac{\pi}{n}\right)$. If S is a circle, this line segment should land back on S . If this line segment does not land back on S , you can conclude S is not a circle.

4. Repeat step 3 a total of $n - 2$ more times. If at any point the line segment construction does not land back on S , or at any point the line segment constructed crosses S before fully constructed, you can conclude S is not a circle. If after completing n total side constructions you do not return to your starting point, you can conclude S is not a circle.



$$\left| \pi r^2 - r^2 \frac{n}{2} \sin\left(\frac{2\pi}{n}\right) \right| = 0.04$$

