

CONSTANT DIAMETER OR EQUICHORDAL CURVES

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The famous American physicist Richard P. Feynman, who disappeared in 1988, was an inexhaustible source of inspiration for anyone who knew him personally, because of his driving enthusiasm when dealing with any kind of problem. His books are filled with acute observations and problems, often associated to jokes in order to test the smartness of the reader. In one of them⁽¹⁾, at the pp.167-168, Feynman tells us that, when investigating the causes of the tragic accident of the Challenger space shuttle, it happened to him to consider the properties of what can be called constant diameter curves. He even shows a sketch, drawn by his hands, of a curve of such kind.

In the mathematical literature these curves are referred as equichordal⁽²⁾ curves. A historical example is the “limaçon” studied by Étienne Pascal, the father of the famous Blaise. The name “limaçon” was given by G. P. de Roberval (1602-1675), a B. Pascal’s contemporary and friend, who also proposed the concept of the generalized conchoids to which category these curves belong.

Recently a long lasting problem related to them was solved⁽³⁾ using techniques from dynamical systems.

These curves should not be confused with the constant width^(4,5) curves but it seems that in Feynman’s description they are not clearly distinguished. Talking about the roundness of the rocket booster sections of the Challenger he writes “ NASA gave me all the numbers on how far out of round the sections can get...The numbers were measurements taken along three diameters, every 60 degrees. But three matching diameters won’t guarantee that things will fit; six diameters, or any other number of diameters, won’t do, either.”

First of all it is a bit odd that NASA technicians would believe that three diameters could determine the circularity of a section. It was well known⁽⁵⁾ that in practice you need a precision circular shape to compare with.

Leaving alone the tricky mechanical problems let consider the mathematical aspects. To what diameter did one refer? Is it the distance between two parallel tangents to the border or is it the chord of the curve of Fig.17⁽¹⁾? He doesn’t explain.

If you use a gauge caliber to measure the diameter you get the distance between two parallel tangent planes but if you want to measure the chord of the curve of Fig.17 you have to know the position of the equichordal point.

Actually there is a dissimilarity between the description Feynman gives and the sketch he draws. Furthermore he cites the example of a Reuleaux - type triangle that is a constant width curve but not an equichordal curve as that of Fig.17!

Dulcis in fundo he tells us a story of when he was a kid and saw in a museum a mechanism composed by gears whose shapes were constant diameter curves turning on shafts that wobbled but that made a gear rack move perfectly horizontal. It is more probable that he saw gears whose shapes were constant width curves, that are more appropriate for this kind of task⁽⁵⁾, but not equichordal curves. The story told by Feynman seems unbelievable in many senses and this feeling is reinforced by the fact that his description always refers to constant width curves and only in Fig.17 he sketches an equichordal curve never cited explicitly in the discourse.

Was he joking or simply confused?

Feynman was used to make jokes related to physics (see e.g. B. F. Chao⁽⁶⁾, A. Ruina⁽⁷⁾ and M. Kuzyk⁽⁷⁾) so that the one cited here could be considered an example concerning mathematics.

Anyway, as a further proof that the properties of the cited curves are doomed to ingenerate confusion one can see the article by B. Kawohl⁽⁸⁾ where the author, in connection with constant width curves, cites (at p.21) the wrong Feynman's book for the wrong reason!

References

- 1) R.P.Feynman," What do you care what other people think?", W.W. Norton & Co., N.Y. - London, 1988.
- 2) M. Rychlik, "The Equichordal Point Problem", Elec. Res. Announcements Amer. Math. Soc. **2**, no. 3 (1996), 108-123.
- 3) M. Rychlik, "A complete solution to the Equichordal Problem of Fujiwara, Blaschke, Rothe, and Weitzenböck", *Inventiones Mathematicae* **129**, issue 1 (1997), 141-212.
- 4) D. Hilbert, S. Cohn-Vossen, "Geometria Intuitiva", Boringhieri, Turin, reprint 1967.
- 5) M. Gardner, "Giochi Matematici", vol. 4th, Sansoni, Florence, 2nd reprint 1979.
- 6) B. F. Chao, "Feynman's Dining Hall Dynamics", *Physics Today* **42**, no. 2 (1989), p.15.

- 7) A. Ruina, M. Kuzik, “Feynman: Wobbles, Bottles and Ripples”, *Physics Today* **42**, no. 11 (1989), 127-130.
- 8) B. Kawohl, “Symmetry or not?”, *The Mathematical Intelligencer* **20**, no.2 (1998), 16-22.
- 9) A. Ricotta, “Constant-Diameter Curves”, *The Mathematical Intelligencer* **25**, no. 4, 2003, 4-5.