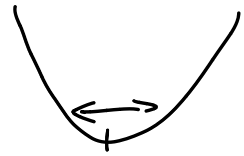
HW-1 ideas

Limit theorems, law of large numbers

$$X = \frac{x_1 + x_2 + \dots x_N}{N}$$

$I(x)$



$\langle x \rangle$

$$P(X=x) \sim e^{-N I(x)}$$

e^{-N} $\qquad$ $I(x) \simeq I''(x_0)(x-x_0)^2$

$$p(f) = e^{-N(f-1/2)^2}$$

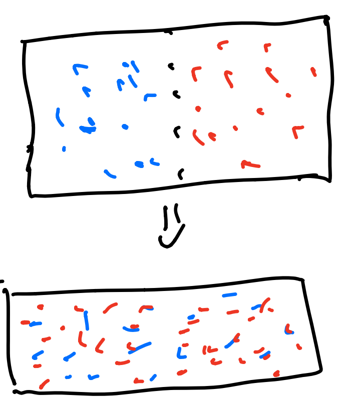
$$B_N(n) = \frac{N!}{n!(N-n)!}$$

$$\ln N! \approx \int_1^N \ln x \, dx = N\ln N - N$$

$$\ln B_N(n) = -N\left(f\ln f + (1-f)\ln(1-f) + \ln 2\right)$$

$-N$ $\qquad$ $I(f)$

1



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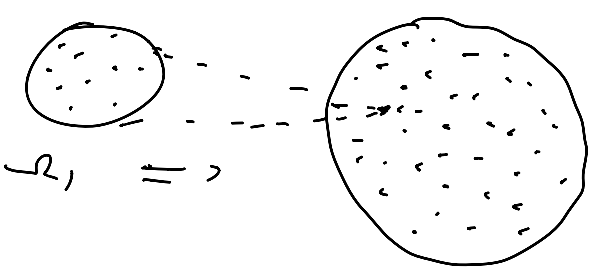
$$S = k_B \ln \Omega(N,V,E)$$

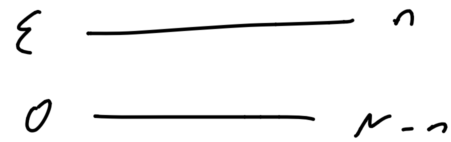


$\Omega(N,V,E)$

$\Omega_1 \implies \Omega_2$

$S_2 > S_1$

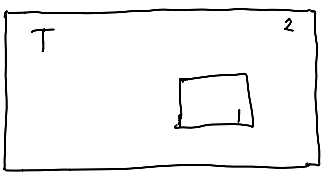
$$P = \frac{1}{\Omega}$$



Lattice models:

N, n $\Omega = \frac{N!}{n!(N-n)!}$ $E = n\varepsilon$

(N, V, E) (N, V, T)
(N, P, T)



$E_0 = E_1 + E_2$ total energy distributed between E_1 and E_2

$$\Omega(E_0) = \sum_{\varepsilon=0}^{E_0} \underbrace{\Omega_1(\varepsilon)\,\Omega_2(E_0-\varepsilon)}$$

$$\ln \Omega_2(E_0-\varepsilon) \approx \ln \Omega_2(E_0) - \frac{\partial \ln \Omega_2(E_0-\varepsilon)}{\partial(E_0-\varepsilon)}\varepsilon$$

$$= \ln \Omega_2(E_0) - \varepsilon\left(\frac{\partial \ln \Omega_2(E_2)}{\partial E_2}\right) + \dots$$

$\beta = \frac{1}{k_B T}$

$$= \ln \Omega_2(E_0) - \frac{\varepsilon}{k_B T} = \frac{S_2(E_0)}{k_B} - \beta\varepsilon$$

$$\Omega(E_0)\, e^{-S_2'(E_0)} = \sum_{\varepsilon} \Omega_1(\varepsilon)\, e^{-\frac{\varepsilon}{k_B T}}$$

$$S(E_0) - S_2(E_0) = k_B \ln Z(\beta)$$

thermodynamics

NVT ensemble

$$S(E_0) = S_1(\bar{E}) + S_2(E_0 - \bar{E})$$

$\bar{E}$ - thermodyn. average energy

$$S_1'(\bar{E}) + S_2(E_0 - \bar{E}) - S_2(E_0) = k_B \ln Z(\beta)$$

$$S_1(\bar{E}) - \frac{\bar{E}}{T} = k_B \ln Z(\beta)$$

$$-\frac{(\bar{E} - TS)}{T} = k_B \ln Z(\beta)$$

$$-\frac{A}{T} = k_B \ln Z(\beta)$$

$$A = -k_B T \ln Z(\beta, N, V) \quad \text{NVT}$$

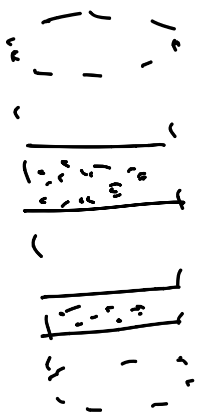
$$S = k_B \ln \Omega(N, V E) \quad \text{NVE}$$

$$Z_\beta = \sum_{\varepsilon} e^{-\beta \varepsilon} \quad \text{vs} \quad \Omega(E)$$

$$P(E) \sim \Omega(E_0 - \varepsilon)\,\Omega(\varepsilon) \sim e^{-\beta\varepsilon}$$

$$Z_\beta = \sum_i e^{-\beta\varepsilon_i} = \sum_{E_\nu} \Omega(E_\nu)\, e^{-\beta E_\nu}$$

$$Z_\beta = \int dE\, \Omega(E)\, e^{-\beta E}$$

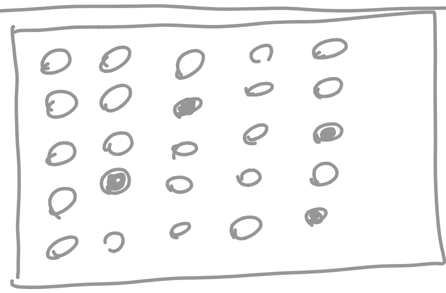


$$\frac{n(h+\Delta h)}{n(h)} \sim e^{-\beta(mg\Delta h)}$$

$$P(E) \sim \Omega(E)\, e^{-\beta E}$$

$$\sim e^{-\beta E} e^{\frac{S}{k_B}} = e^{-\beta(E - TS)}$$

$$e^{-\beta N(\varepsilon - Ts)} = e^{-\beta N I}$$



N - sites

n - defects

$$E = n\varepsilon$$

$$\Omega(E) = \frac{N!}{n!\,(N-n)!}$$

$$Z(\beta) = \sum_{n=0}^{N} \Omega(n)\, e^{-\beta n E} =$$

$$= \sum \frac{N!}{n!\,(N-n)!} e^{-\beta n E} =$$

$$= \sum \frac{N!}{n!\,(N-n)!} p^n q^{N-n} =$$

$$= \sum \frac{N!}{n!\,(N-n)!} \left(e^{-\beta\varepsilon}\right)^n 1^{N-n} =$$

$$= \left(1 + e^{-\beta\varepsilon}\right)^N$$

$$Z(\beta) = \sum_{\varepsilon_1=0}^{\varepsilon_1=\varepsilon} \sum_{\varepsilon_2=1}^{\varepsilon_2=\varepsilon} \dots \, e^{-\beta \sum \beta \varepsilon_i} =$$

$$= \left(1 + e^{-\beta\varepsilon}\right)^N = Z_1^N$$

$$P(E) = \frac{e^{-\beta E}}{\sum e^{-\beta E}} = \frac{e^{-\beta E}}{Z(\beta)}$$

$$Z(\beta) = \int \Omega(E)\, e^{-\beta E}\, dE \sim \langle e^{-\beta E} \rangle$$

$$\sum E_i\, p(E_i) = \langle E \rangle = \frac{\sum E_i\, e^{-\beta E_i}}{Z} =$$

$$= -\frac{1}{Z_\beta} \frac{\partial}{\partial \beta} Z_\beta = -\frac{\partial}{\partial \beta} \ln Z_\beta = \langle E \rangle$$

Z_β - moment generating function

$$\sum p_i E_i^2 - \left(\sum p_i E_i\right)^2 = \frac{\partial^2}{\partial \beta^2} \ln Z$$

$$= -\frac{\partial}{\partial \beta} \langle E \rangle$$

$$C_v = \frac{\partial \langle E \rangle}{\partial T}$$

$$\langle E^2 \rangle - \langle E \rangle^2 = k_B T C_v \sim O(N)$$

$$\frac{\left(\langle E^2 \rangle - \langle E \rangle^2\right)^{1/2}}{\langle E \rangle} \sim \frac{\sqrt{k_B T C_v}}{\langle E \rangle} \sim O(N^{-1/2})$$