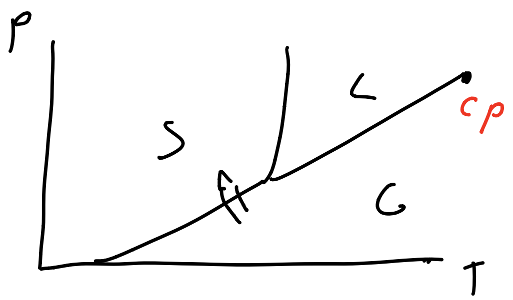
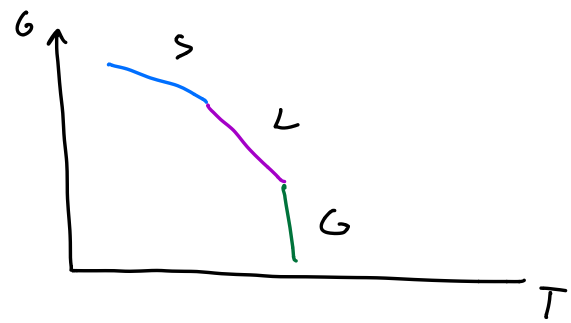
Phase transitions

stat mech point of view





$$dG = -S\,dT + V\,dp$$

Where does singularity come from?

$$Z = \sum_i e^{-\beta E_i} \qquad A = -k_B T \log Z$$

$$N \to \infty \qquad N/V = \text{const}$$

$$\chi = \left(\frac{\partial x}{\partial X}\right)_y = \beta \langle \delta x^2 \rangle \longrightarrow \infty$$

order-disorder transition

symmetry breaking

divergence of fluctuation

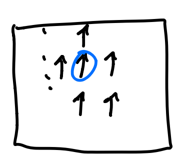
Ising model

$$H = -J \sum_{\langle ij \rangle} s_i s_j - h \sum_i s_i$$

$J > 0 \quad \uparrow\uparrow\uparrow \qquad h > 0 \quad s = +1$

$\uparrow \cdot \uparrow \cdot \uparrow \cdot \uparrow$

nearest neighbour interactions

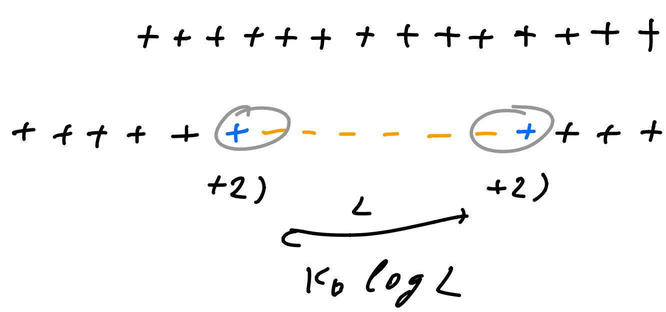


$z = 2$ 1D

$z = 4$ 2D

$z = 6$ 3D

$$Z = \sum_{\{s\}} e^{-\beta H(\{s\})} = \sum_{\langle ij \rangle}' e^{\beta J \sum' s_i s_j + \beta h \sum' s_i}$$



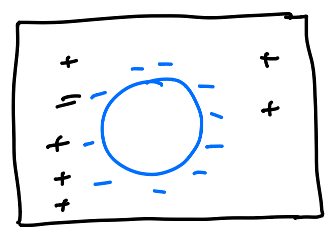
$E = -NJ$

$E = -NJ + 4J$

$T = 0$

$E \mathrel{+}= 4\pi J L$ $\qquad \sim N^{1/2}$

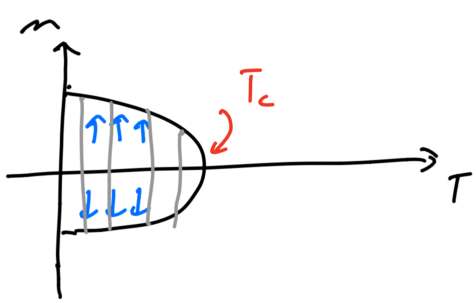
$P \sim e^{-\beta 4\pi J L}$ $\qquad \sim N^{2/3}$

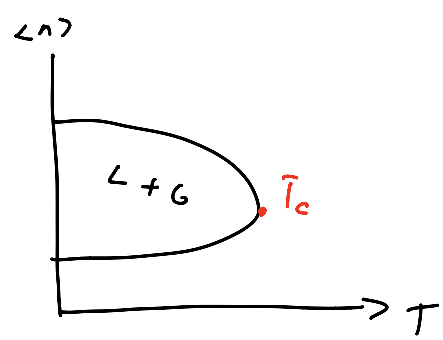


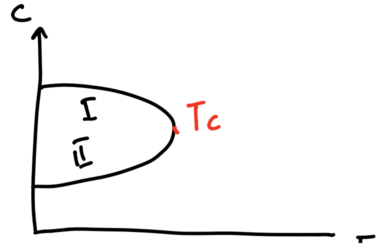
Peierl's argument

$\sim 1/r^2$ or slower

$$m = \frac{1}{N} \sum' s_i = \frac{M}{N}$$

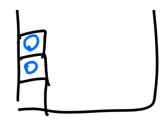


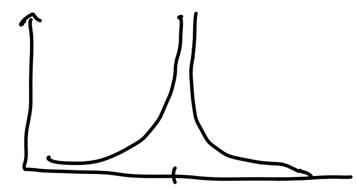




$$H = -\sum' J s_i s_j - h \sum' s_i$$

$$H = -\sum' \varepsilon^2 n_i n_j - \beta\mu \sum' n_i$$





$$\langle s(r)\, s(0) \rangle \sim e^{-r/\xi}$$

$$\langle s_i\, s_j \rangle \sim e^{-r/\xi'}$$

$$\xi \sim |T - T_c|^{-\nu}$$ critical exponent

$$H = -J \sum_i{}' \sigma_i \sigma_{i+1} - h \sum_i{}' \sigma_i$$

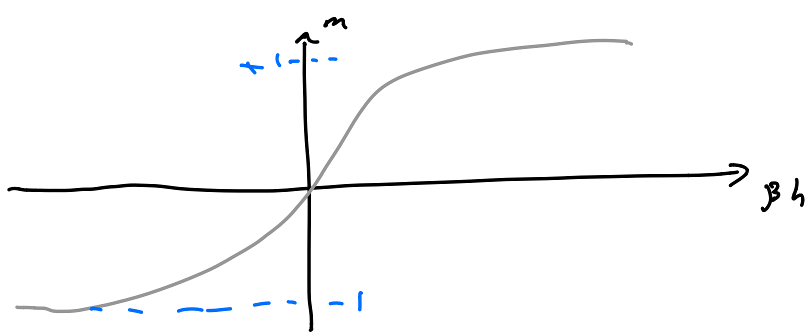
(i) open boundary

(ii) $\sigma_{N+1} = \sigma_1$

$J = 0$ $$Z = \sum_{\{\sigma\}}{}' e^{+\beta h \sum' \sigma_i} = \prod_i z_i$$

$$z_i = \sum_{\sigma_i} e^{\beta h \sigma_i} = e^{\beta h} + e^{-\beta h} = 2\cosh(\beta h)$$

$$m = \frac{\partial \log z}{\partial \beta h} = \frac{e^{\beta h} - e^{-\beta h}}{e^{\beta h} + e^{-\beta h}} = \tanh(\beta h)$$



$h = 0$ $J \neq 0$

Mean-field approx

$$\langle f(\sigma_1, \sigma_2 .. \sigma_N) \rangle = f(\langle\sigma_1\rangle, \langle\sigma_2\rangle, .. \langle\sigma_N\rangle)$$

$$s_i s_j = (s_i - m + m)(s_j - m + m) =$$

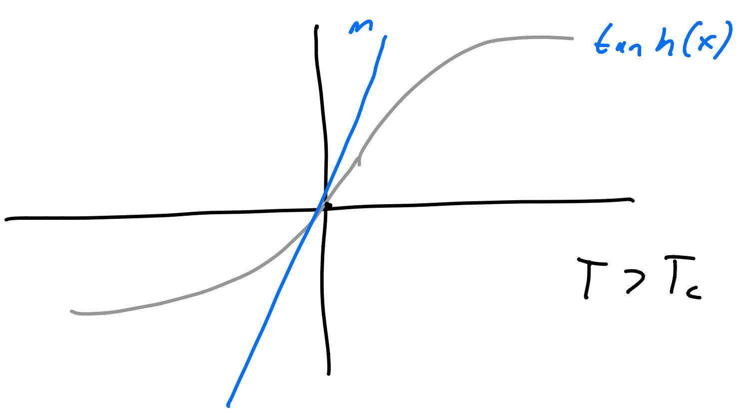
$$= \underbrace{(s_i - m)(s_j - m)} + m(s_i - m) + m(s_j - m)$$

$$H_{mf} = -J \sum_{ij} \left[m(s_i + s_j) - m^2 \right] - h \sum_i s_i$$

$$H_{mf} = \frac{1}{2} N z J m^2 - (Jzm + h) \sum_i s_i$$

$$h_{eff} = h + Jzm$$

$$m = \frac{\partial \log Z}{\partial \beta h} = \tanh(\beta J z m)$$

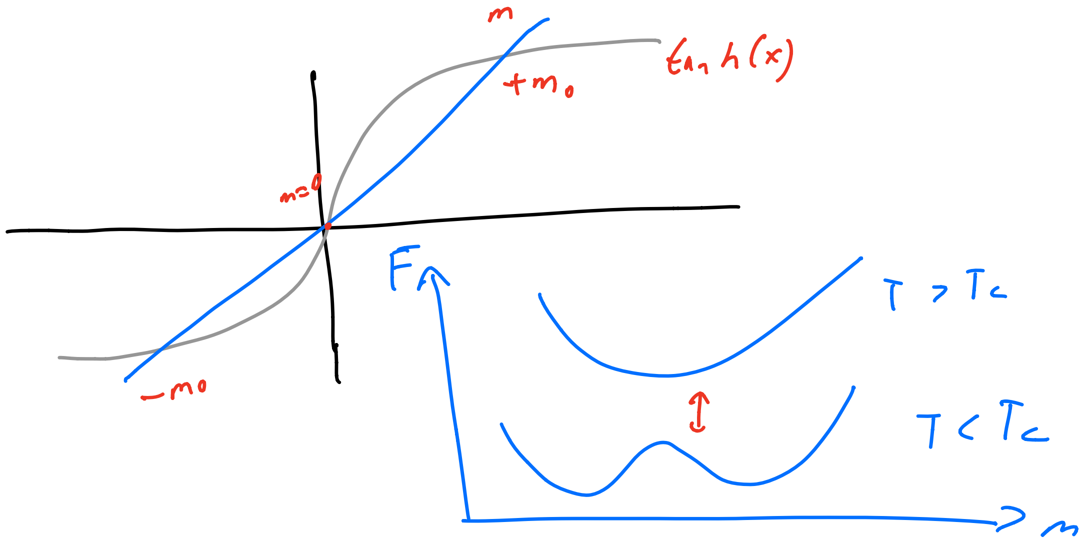


$x = \beta J z m$

$k_B T_c = Jz$

$T > T_c$

$T < T_c$



second order phase transition