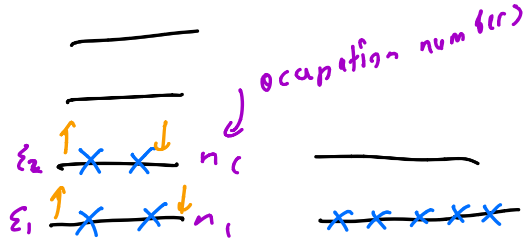
ideal systems

$$\hat{H} = \hat{H}_1 + \hat{H}_2 \qquad Z = Z_1 Z_2$$

$$P(E_i) = \frac{e^{-\beta E_i}}{Z}$$



$$P(n_i, \varepsilon_i) = \frac{e^{-\beta(\varepsilon_i - \mu n_i)}}{\Xi}$$

$$p(x,y) \longrightarrow \int p(x,y)\, e^{ikx} e^{ik'y}\, dx\,dy = G(k,k')$$

$$p(x) \longrightarrow \int p(x)\, e^{ikx}\, dx = \langle e^{ikx} \rangle = G(k)$$

$$\Xi = \prod_{i=1}^{N} \Xi_i$$

occupation numbers

$$\Xi_i = \sum_{n_i}{}' e^{-\beta(n_i \varepsilon_i - \mu n_i)}$$

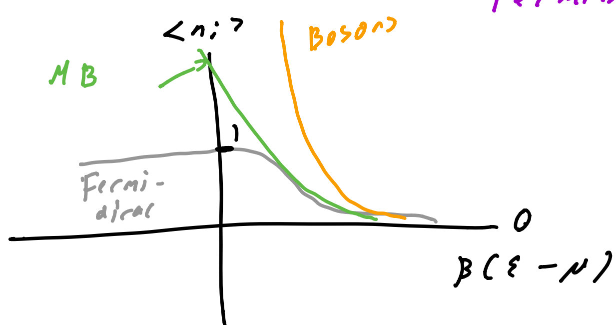
orbital energies

$$E = \sum_i{}' n_i \varepsilon_i$$

$$\langle n_i \rangle = \frac{1}{e^{\beta(\varepsilon_i - \mu)} - 1} \qquad \langle n_i \rangle = \frac{1}{e^{\beta(\varepsilon_i - \mu)} + 1}$$

Bosons $\qquad\qquad$ Fermions

$$p = \frac{\langle n_i \rangle}{N}$$



$\mu \to -\infty$ classical limit

$$\langle n_i \rangle = \frac{1}{e^{\beta(\varepsilon_i - \mu)} \pm 1} \boxed{= e^{-\beta\varepsilon_i} e^{\beta\mu}}$$

classic approx

$$\sum_i \langle n_i \rangle = N = e^{\beta\mu} \sum e^{-\beta\varepsilon_i}$$

$$\boxed{e^{\beta\mu} = \frac{N}{\sum e^{-\beta\varepsilon_i}}}$$

classic approx

$$p_i = \frac{\langle n_i \rangle}{N} = \frac{e^{-\beta \varepsilon_i}}{\sum_i' e^{-\beta \varepsilon_i}}$$

$$PV = k_B T \log \Xi \qquad A = -k_B T \log Z$$

$$E = TS - pV + \mu N$$

$$\log Z = -\beta \mu N + \log \Xi$$

$$\log \Xi = \sum_i \log\left(1 \pm e^{\beta(\mu - \varepsilon_i)}\right) \approx \sum_i' e^{+\beta(\mu - \varepsilon_i)} =$$

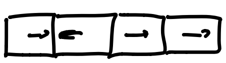
$$= \sum' \langle n_i \rangle = N$$

$$\log Z = -\beta \mu N + N$$

$$\log Z = \overbrace{-N \log N + N \log \sum' e^{-\beta \varepsilon_i}} + N$$

$$Z = \frac{1}{N!}\left(\sum_i e^{-\beta \varepsilon_i}\right)^N$$

Phonon



$$U = \sum_i \sum_j \frac{\partial^2 U}{\partial x_i \partial y_j}(x_i - \bar{x}_i)(y_j - \bar{y}_j)$$

normal coords $q = Ax$

$$A^{-1} K A = g$$

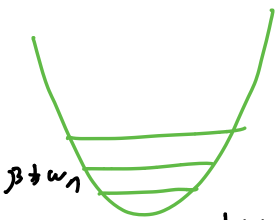
$V(q_1, q_2, \ldots q_N) = \sum_i' g_i q_i^2$ Normal modes!

$$\hat{H} = \sum_i h_i(q)$$

$$h = \frac{p^2}{2\mu} + \frac{(\mu \omega q)^2}{2\mu} \qquad E = k_B T$$

$$Z = \int \frac{dp\,dq}{h} e^{-\beta h(p,q)} \qquad C_v = k_B$$

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega$$



$$Z = \sum_{n=0}^{\infty} e^{-\beta\left(\frac{1}{2}+n\right)\hbar\omega} = e^{-\frac{\beta\hbar\omega}{2}} \sum_{n=0}^{\infty} e^{-\beta\hbar\omega n}$$

$1 + x + \dots x^n =$

$$= \frac{1}{1-x}$$

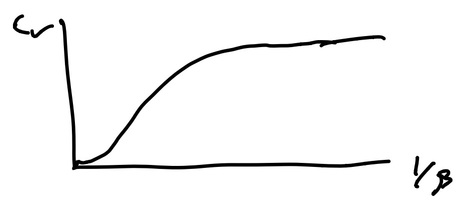
$|x| < 1$

$$Z = \frac{e^{-\beta\frac{\hbar\omega}{2}}}{1 - e^{-\beta\hbar\omega}} = \left(e^{\beta\frac{\hbar\omega}{2}} - e^{-\frac{\beta\hbar\omega}{2}}\right)^{-1}$$

$$E = -N \frac{\partial \log Z}{\partial \beta} \qquad C_v = \left(\frac{\partial E}{\partial T}\right)_V$$

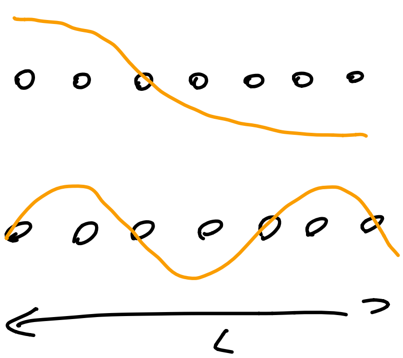
$$C_v = N k_B \left(\frac{\hbar\omega}{k_B T}\right)^2 \frac{e^{\beta\hbar\omega}}{\left(e^{\beta\hbar\omega} - 1\right)^2}$$

$\beta\hbar\omega \gg 1$
low temp



$$C_v \sim T^3$$

— Debye model of solids —



$\lambda = 2L \qquad 2L/2 \dots 2L/n$

$$K = \frac{2\pi}{\lambda} \qquad \frac{n\pi}{L}$$

$$\int_0^{\omega} D(\omega)\,d\omega = \int_{p \le p(\omega)} \frac{dr\,dp}{h^3} = \frac{V}{h^3}\int_0^{p(\omega)} 4\pi p^2\,dp$$

$D(\omega)d\omega$ # normal modes ω, $\omega + d\omega$ range

$$p(\omega) = \hbar k = \frac{\hbar\omega}{c}$$

Dispersion relation

$$\frac{d}{d\omega} \qquad D(\omega) = \frac{V}{h^3} 4\pi p^2 \frac{dp}{d\omega}$$

$$D(\omega) = \frac{1}{2\pi^2} V \frac{\omega^2}{c^3}$$

$$D(\omega) = \frac{1}{2\pi^2} V \frac{\omega^2}{c^3} \theta(\omega_D - \omega)$$

$$E = \int_0^{\omega} D(\omega) \left(\frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\beta\hbar\omega}-1} \right) d\omega$$

$$C_V = \int_0^{\omega} D(\omega) \left(\left(\frac{\hbar\omega}{k_B T}\right)^2 \frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega}-1)^2} \right) d\omega$$

$$E \sim T^4 \qquad C_V \sim T^3$$

Black-body radiation

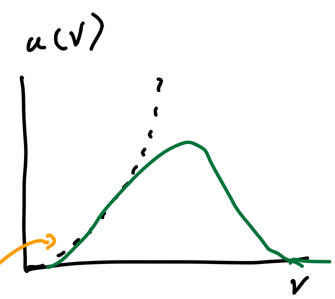
$$Z(\beta) = \prod_i \sum_{n_i=0}^{\infty} e^{-\beta n_i \hbar\omega} = \prod_i \frac{1}{1-e^{-\beta n_i \hbar\omega}}$$

$\nu^2 d\nu$

$\omega^2 d\omega$

$\nu^2 \cdot k_B T$

classical



$$E_\omega = \frac{\hbar\omega}{e^{\beta\hbar\omega}-1}$$

$$E = \int \underbrace{D(\omega) E(\omega) d\omega}$$

$$u(T,\omega) = \frac{\omega^2}{\pi^2 c^3} \frac{\hbar\omega}{e^{\beta\hbar\omega}-1}$$