NVE: $\boxed{S = k_B \ln \Omega(E)}$

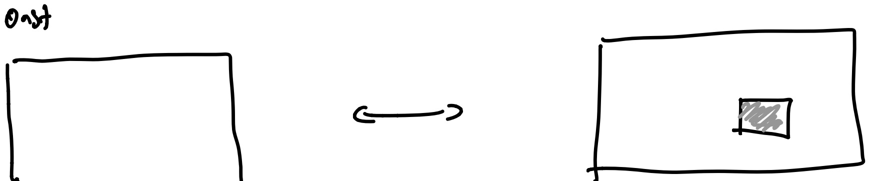
NVT: $\boxed{A = -k_B T \log Z}$

thermodynamics
e.g. macro-averages

Stat Mech.
microscopic degrees
of freedom

E = const

T = const



$\boxed{\text{isolated} \quad E(S,V,N)}$ $\overset{\text{Leg. transform}}{\Longrightarrow}$ $\boxed{E - TS = A}$

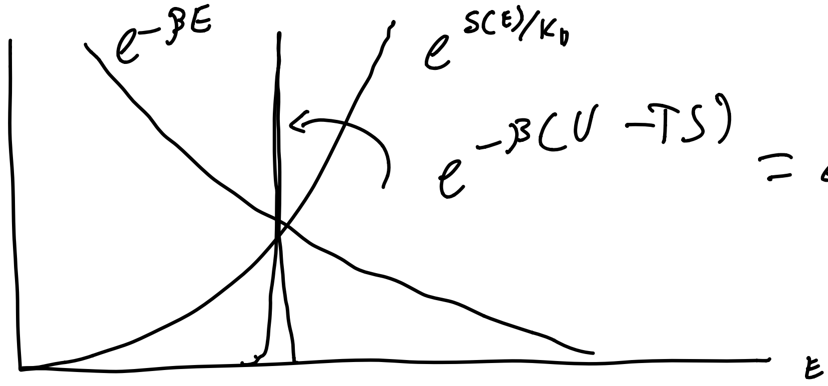
$$\boxed{-A = \max_{S}\{TS - E\} \qquad T = \left(\frac{\partial E}{\partial S}\right)}$$

$$\boxed{\Omega(E) = \int \frac{dx^N dp^N}{h^{3N}} = e^{S/k_B} \qquad E - \Delta E \le H(x,p) \le \Delta E} \overset{\text{Laplace}}{\Longrightarrow} \boxed{Z = \sum_i e^{-\beta E_i} = e^{-\beta A}}$$

$$P(E) = \frac{1}{\Omega(E)} \qquad \boxed{Z = \int \Omega(E) e^{-\beta E} dE} \qquad p = \frac{e^{-\beta E_i}}{Z}$$

$$N \to \infty \qquad \int e^{-\beta(E - TS(E))} dE \approx e^{\beta(TS(E) - E)^{\max}}$$

Large dev function $E \leftrightarrow A$



Thermodynamic equivalence of ensembles!!!

Gibbs formula of entropy

$$S = k_B \log \Omega = -k_B \log \frac{1}{\Omega}$$

$$TS = E - A$$

$$TS = \sum_i p_i E_i + \frac{1}{\beta} \log Z$$

$$TS = -\frac{1}{\beta}\sum_i p_i \log e^{-\beta E_i} + \frac{1}{\beta} \log Z \cdot \sum_i p_i$$

$$TS = -\frac{1}{\beta}\sum_i p_i \log \frac{e^{-\beta E_i}}{Z}$$

$$S = -k_B \sum_i p_i \log p_i \quad \text{Gibbs}$$

Shanon measure of information

SMI

n choice for a letter $\eta(n)$ increase

$$\eta(n) = c \log n$$

$$c = (\log 2)^{-1}$$

$$\eta(n) = \log_2 n$$

1 bit $\equiv$ amount of info from single yes-no question

$$\eta(n) = \log_2 \frac{1}{n} = \log_2 P_n$$

$$\eta(n) = -\sum_{i=1}^{n} P_i \log P_i$$

example horses, roommate-keys, coins, die, etc.

$\log_2 27 \simeq 4.755$ but real is 1.3 bits

$\frac{70}{250} \cdot \log_2 27 \simeq 1.3$ bits   70 given to guess 250 long paragraph two kittens : arrival of new info $\log_2 4 - \log_2 3$

Monty python example : $-\frac{1}{5}\log\frac{1}{5} - \frac{4}{5}\log\frac{4}{5}$

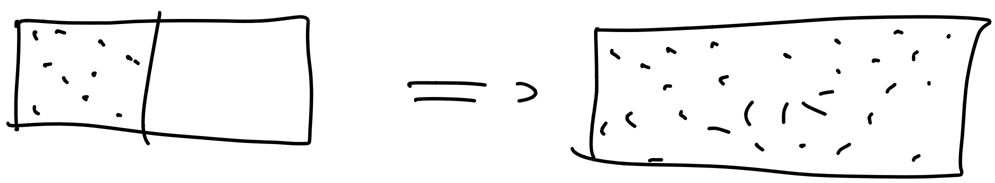
~ 66 bits

Units

1 bit/molecule = $k_B \log 2 = 9.57 \cdot 10^{-24}$ J/K.mol

1 bit/mole = $R \log 2 = 5.7628\ J \cdot K^{-1} mol^{-1}$

1 $J \cdot K^{-1}$ mol = 0.17352 bit/molecule



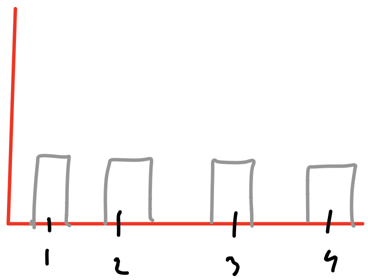
$\Delta S = R \log 2$ entropy increase? N_A bits

$$S = -\sum' p_i \log p_i$$

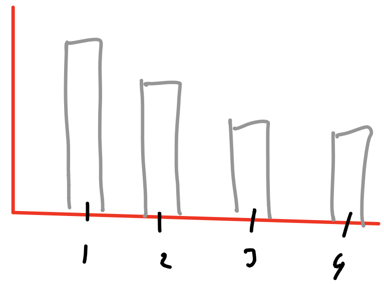
uncertainty over picking microstate

or # Yes/No question to identify state.

(1) $S'(\text{flat dist}) > S'(\text{non-flat dist})$



$p_1 = p_2 = p_3 = p_4 = 1/4$



$p_1 > p_2 > p_3 > p_4$

$f = f^* = 1/2$

Why entropy increase?

$$P(f) = e^{-N I(f)} \qquad I(f) = -f \log f - (1-f)\log(1-f)$$

Large dev function is Entropy! or information!!
System tends to the greatest uncertainty $\equiv$ highest prob

$\boxed{\text{Entropy is a functional of prob of distribution. Systems evolve towards prob dis over microstates that maximize their entropy given any constraints}}$

MaxEnt

Principle of insufficient reason
maximum entropy principle
fair allocation principle

$\sum p_i E_i = E \quad \sum N_i p_i = N \quad \sum p_i = 1$

$S = -\sum p_i \log p_i - \lambda\left(\sum p_i - 1\right)$

$\frac{\partial S}{\partial p_i} = 0 \quad -1 - \log p_i - \lambda = 0$

$\log p_i = -1 - \lambda$

$p_i = e^{\lambda'} \quad \sum p_i = 1$

$\sum_i \left(e^{\lambda'}\right) = e^{\lambda'} \Omega$

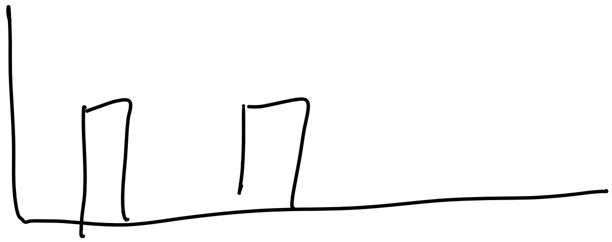
$p_i = \frac{1}{\Omega}$ # states

$\sum p_i E_i = E \quad \sum p_i = 1$

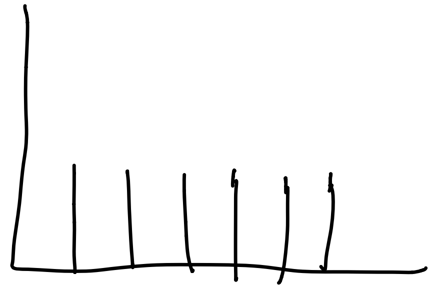
$-1 - \log p_i - \lambda - \beta E_i$

$p_i \sim e^{-\beta E_i}$ Boltzmann distribution

constrained maximization



S_1



S_2

rigged coin and rigged die

$\langle x \rangle = 0.8$

$\langle x \rangle \approx 5.5$