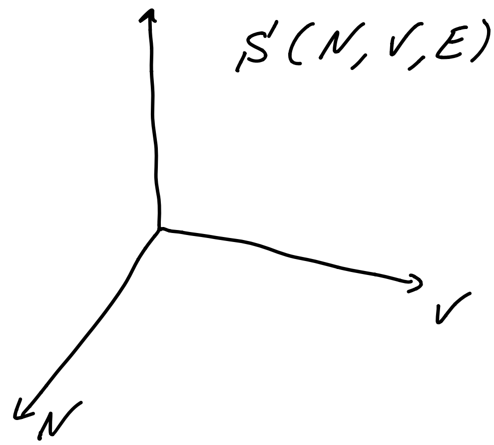
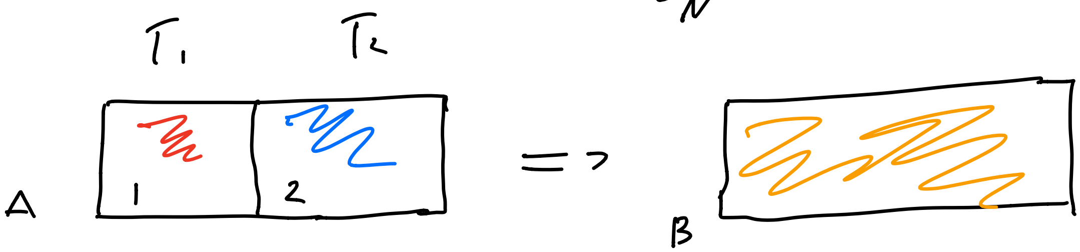
$N = 10^{23}$





A) $S(E, V, N) = S_1(E_1, V_1, N_1) + S_2(E_2, V_2, N_2)$

$\Downarrow$

B $\delta S(E|V, N) = \dfrac{\partial S_1}{\partial E_1}\delta E_1 + \dfrac{\partial S_2}{\partial E_2}\delta E_2$

$$\left(\frac{\partial S_1}{\partial E_1} - \frac{\partial S_2}{\partial E_2}\right)\delta E_1 = 0$$

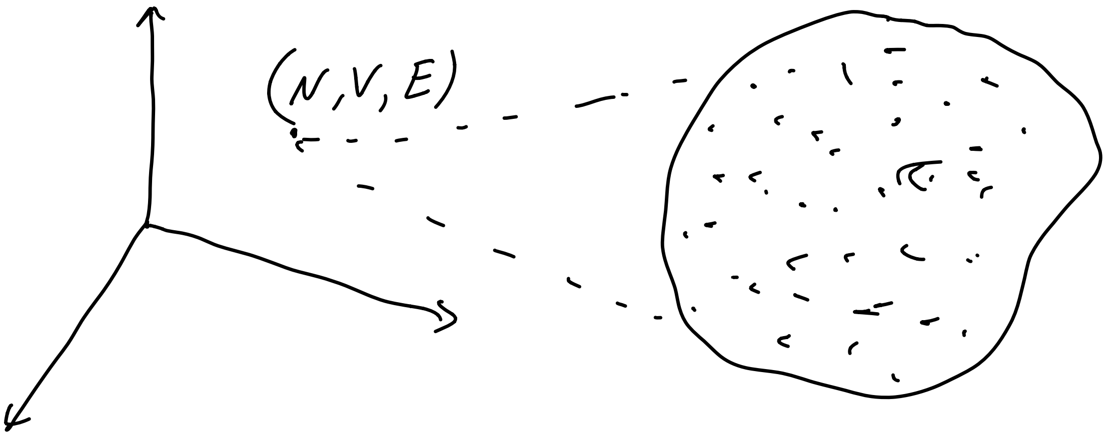
$$\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\delta E_1 = 0$$

$$T_1 = T_2 \qquad T = \left(\frac{\partial E}{\partial S}\right)_{N,V}$$

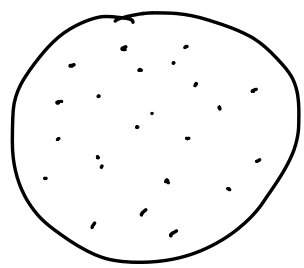
$$S = k_B \ln \Omega(E, V, N)$$

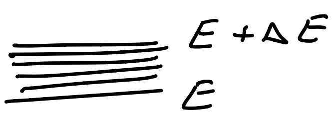
$\Omega = \Omega_1 \Omega_2$ $\quad \Omega(E,V,N)$ - # microstates consistent with E, V, N

$S = S_1 + S_2$



$\hat{H} |\psi_\nu\rangle = E_\nu |\psi_\nu\rangle$

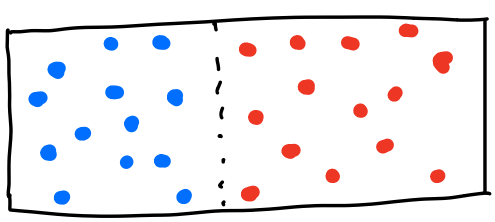




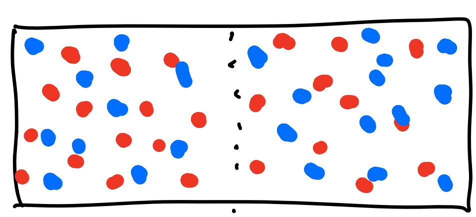
$\Gamma(x_1, \ldots x_N, p_1 \ldots p_N)$ $\qquad N \sim 10^{23}$

more is different

S_1



S_2



$$N_A = \frac{V^{\#}!}{n!\,(V^{\#}-n)!} \; \frac{V^{\#}!}{(N-n)!\,(V^{\#}-(N-n))!}$$

$$\mathcal{N}_B = \frac{V^{\#}!}{m!\,(V^{\#}-m)!}\;\frac{V^{\#}!}{(N-m)!\,(V-(N-m))!} =$$

$$P(n,m) = \mathcal{N}_A \cdot \mathcal{N}_B = \frac{V^{\#\,2N}}{2N!}\binom{N}{n}\binom{N}{m}$$

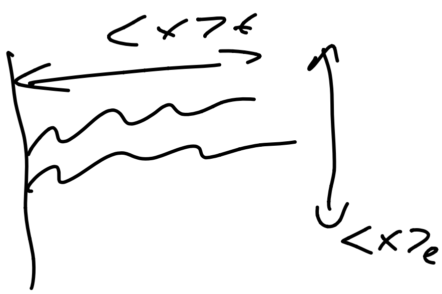
$n = \frac{1}{2}$ $\quad \langle n \rangle = pN$ $\quad\quad V^{\#} \gg n, N$

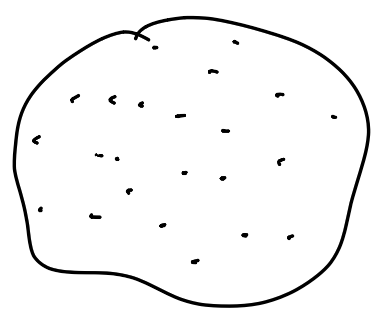
$m = \frac{1}{2}$ $\quad\quad \dfrac{V^{\#}!}{(V-m)!} = V^m$

$$\frac{P_A(\tfrac{1}{2}N+\delta)}{P_A(\tfrac{1}{2}N)} = e^{-\frac{\delta^2}{\frac{1}{2}N}} = e^{-2\delta^2/N}$$

$$N \simeq 10^{23}$$

$$S = k_B \ln \Omega(N,V,E)$$





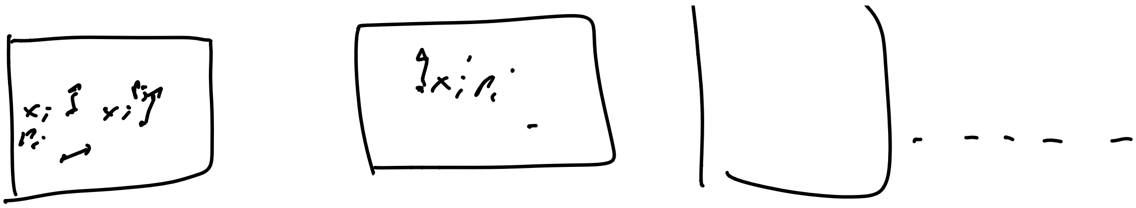
$\Omega(N,V,E)$

$$\langle x \rangle_t = \langle x \rangle_e$$

Ergodicity

N, V, E - miccrocanonical

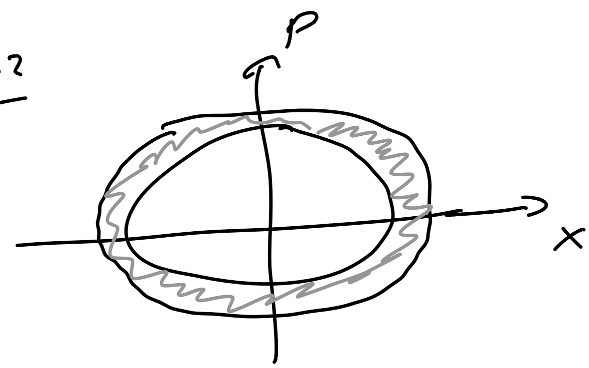
N, V, T - canonical



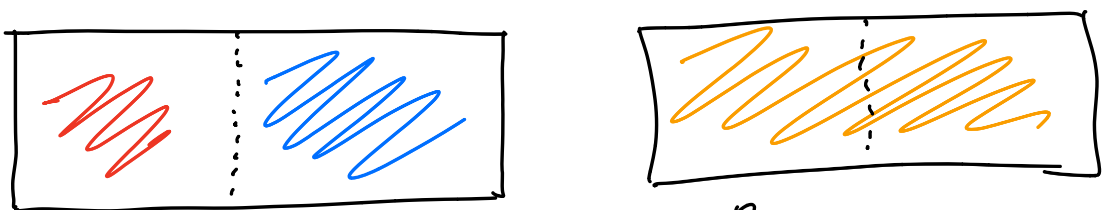
NVE ensemble

$$P(E) = \frac{1}{\Omega(N,E,V)} \begin{cases} E, E+\Delta E \\ 0 \quad \text{otherwise} \end{cases}$$

$$H(p,x) = \frac{p^2}{2m} + \frac{kx^2}{2}$$



$\Omega(N,V,E)$ $\quad 2^{10^{23}}$



A $\quad E = E_1 + E_2$ $\quad$ B

$$\Omega_1(E_1)\,\Omega_2(E_2) = \Omega_A(E_1|E)$$

$$\Omega_1(E_1)\,\Omega_2(E-E_1) = \Omega_B(E_1|E)$$

$$\frac{\partial \Omega_B(E_1|E)}{\partial E_1} = \Omega_2 \frac{\partial \Omega_1}{\partial E_1} - \frac{\partial \Omega_2}{\partial E_2}\Omega_1 = 0$$

$$\frac{\partial \ln \Omega_1(E_1)}{\partial E_1} = \frac{\partial \ln \Omega_2(E_2)}{\partial E_2}$$

$$\frac{\partial S_1}{\partial E_1} = \frac{\partial S_2}{\partial E_2} \qquad T_1 = T_2$$

$$\Delta S = \int_{T_1}^{T_2} \frac{dq}{T} = \int_{T_1}^{T_2} \frac{C_V(T)\,dT}{T} \qquad C_V \ln \frac{T_2}{T_1}$$

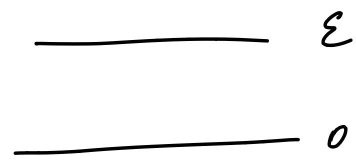
$$\left(\frac{\partial S}{\partial E}\right)_{N,V} = \frac{1}{T} \qquad \left(\frac{\partial^2 S}{\partial E^2}\right) = -\frac{1}{T^2}\left(\frac{\partial E}{\partial T}\right)_{N,V}$$

$$= -\frac{1}{T^2 C_V} < 0$$

$$C_V > 0$$

$$S' = k_B \ln \Omega(N, V, E)$$

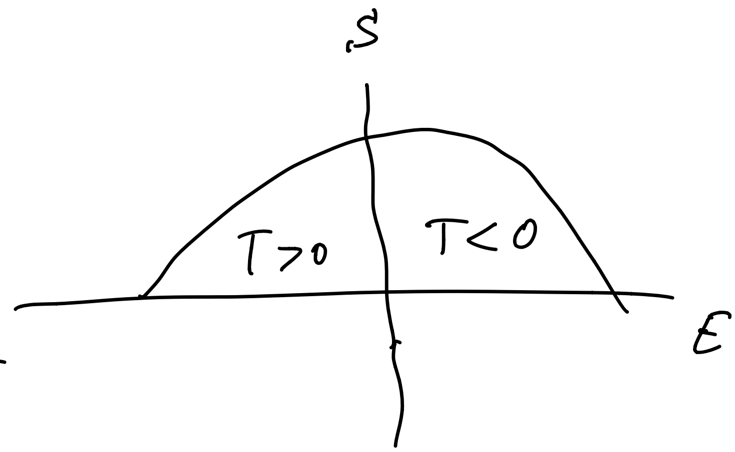
$$\nearrow$$

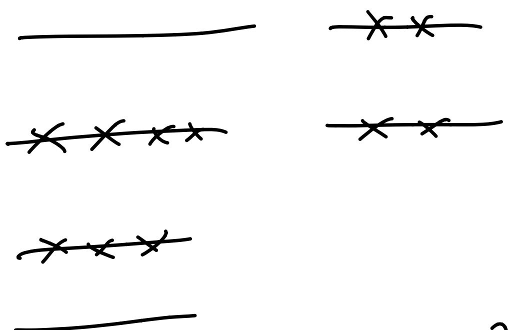


$$\Omega(N, V, E) = \frac{N!}{n!\,(N-n)!}$$

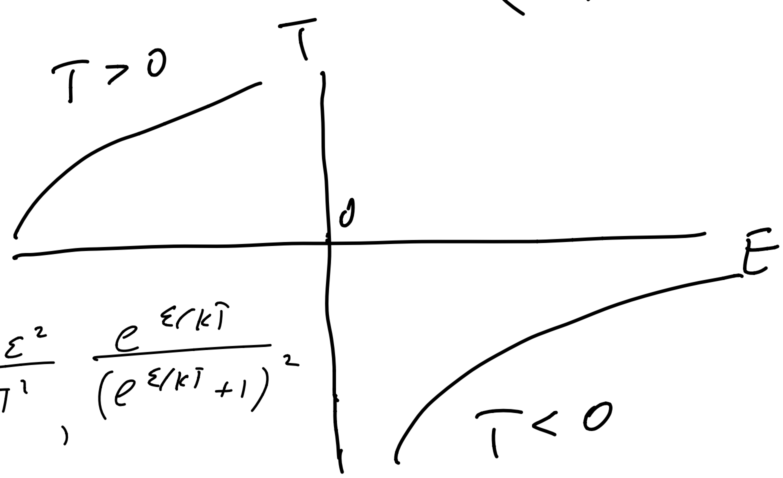
$$\Omega(N, V, E) = \frac{N!}{(E/\varepsilon)!\,(N - E/\varepsilon)!} \qquad n = E/\varepsilon$$

$$S = k_B \ln \Omega(N,V,E) = -N\left[\left(1-\frac{E}{N\varepsilon}\right)\ln\left(1-\frac{E}{N\varepsilon}\right) - \frac{E}{N\varepsilon}\ln\frac{E}{N\varepsilon}\right]$$

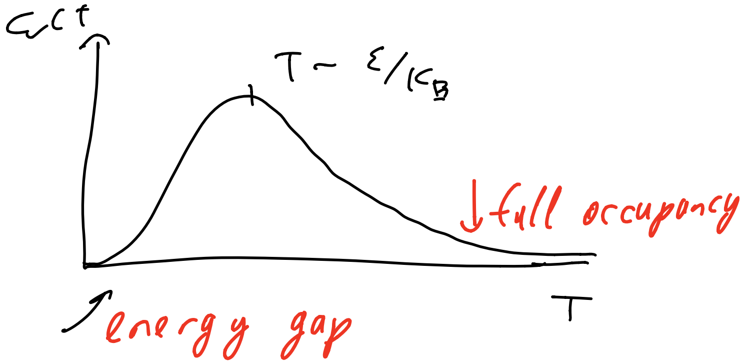




$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)$$



$$C_v = \left(\frac{dE}{dT}\right)_V = \frac{N\varepsilon^2}{kT^2}\,\frac{e^{\varepsilon/kT}}{(e^{\varepsilon/kT}+1)^2}$$



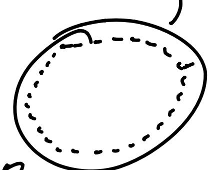
$$\Omega(N,V,E) = \int dp^{3N} dx^{3N} = V^N \int dp^{3N}$$

$$H = \sum_{i=1}^{N} p_i^2 \qquad E \le \sum_i p_i^2 \le E + \Delta E \qquad E \le p_i^2 \le E + \Delta E$$

$$p = \sqrt{2mE}$$

CR^d d-dimension

CR^d



$\cdot\cdot\ (R-\delta R)^d$

$$\frac{C(R-\delta R)^d}{CR^d} = \left(1 - \frac{\delta R}{R}\right)^d \to 0$$

$d \to \infty$

$$\Omega(N,V,E) \sim C_N V^N E^{3N/2-1} \Delta E$$

$$d\left(\frac{4}{3}\pi R^3\right) = 4\pi R^2\, dR$$

$p\, dp^N$

$$\Omega(N,V,E) \sim V^N E^{3N/2} \Delta E \qquad p^{N-1} dp$$

$$S = k_B \ln \Omega = N \log V + \frac{3Nk_B}{2} \log E + \ldots$$

$$\frac{1}{N!} \quad S = k_B N \left[\log\left(\frac{V}{N}\right) + \frac{3Nk_B}{2} \log\left(\frac{E}{N}\right) \right]$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V} = \frac{3}{2}\frac{Nk_B}{E} \Rightarrow E = \frac{3}{2} N k_B T$$

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right) = \frac{N k_B}{V}$$

$$S' = k_B \log \Omega(N, V, E)$$