Typing Systems for PL - 2018S - Makeup - Midterm

11 April 2018

Exercise 1

Provide a typing context Γ, type σ and a typing derivation of the typing judgement $\Gamma \vdash M : \sigma$, where M is the following term:

```
let r : Ref (Nat->Nat) = ref (λ x: Nat.x)
in (r:= (λ x:Nat.succ x));
   (!r) 3
```

The typing rule for ";" is

$$\frac{\Gamma \rhd M : \mathbb{U} \quad \Gamma \rhd N : \sigma}{\Gamma \rhd M; N : \sigma}$$

Exercise 2

Reduce the following term to normal form using the call-by-value, small-step operational semantics given in class.

```
let r : Ref (Nat->Nat) = ref (λ x:Nat.x)
in (r:= (λ x:Nat.if zero?(x) then 0 else x + (!r) (x-1)));
   (!r) 3
```

The rules for zero are:

$$\frac{}{zero?(0) \to true} \qquad \frac{}{zero?(\underline{n+1}) \to false} \qquad \frac{M \to M'}{zero?(M) \to zero?(M')}$$

Exercise 3

Infer the type of the expression: $\lambda \mathsf{r}.\mathsf{r} := !\mathsf{r} + 1; !\mathsf{r}$.