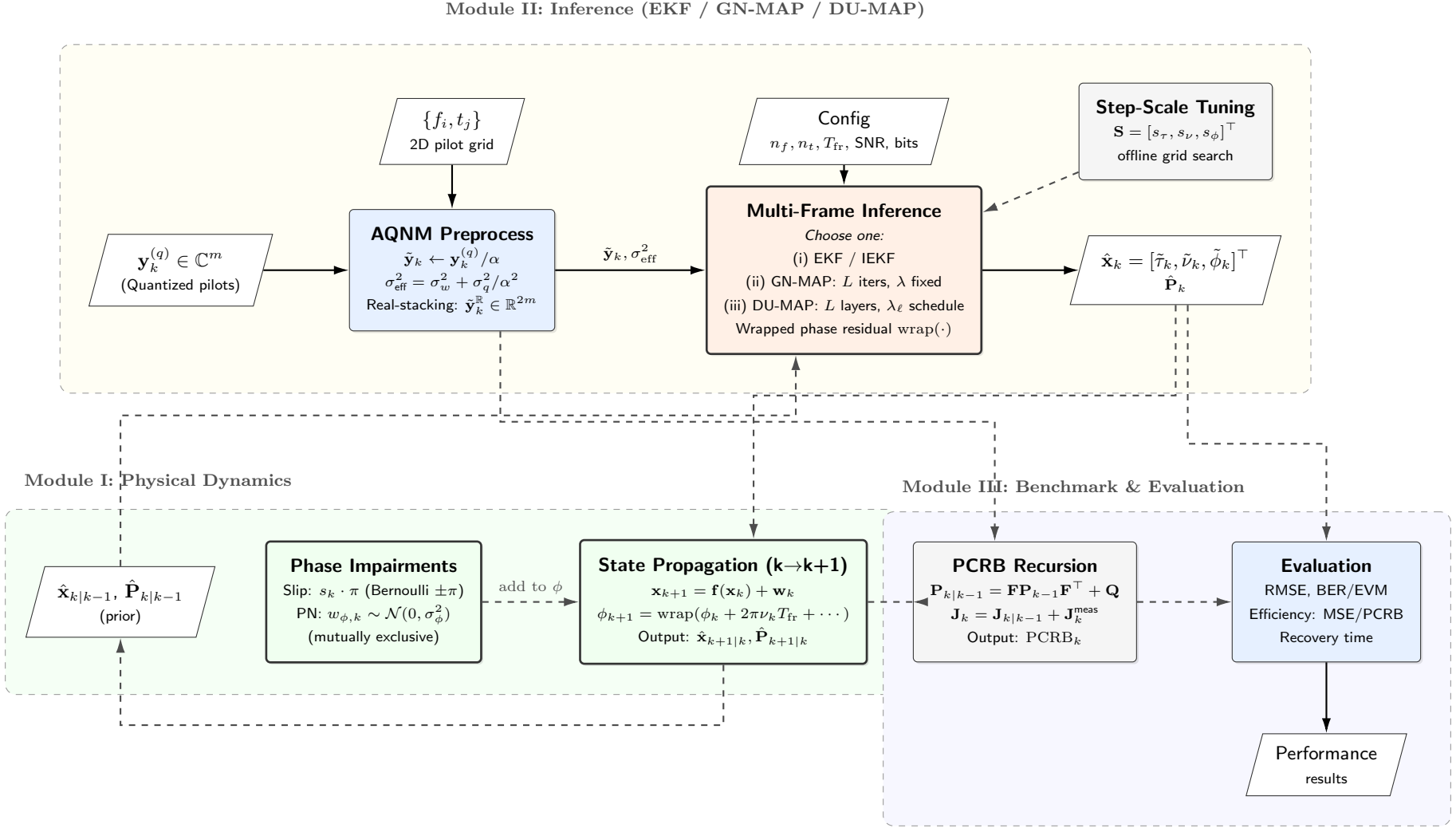
Module II: Inference (EKF / GN-MAP / DU-MAP)
$\{f_i, t_j\}$
2D pilot grid
Config
n_f, n_t, T_{fr}, SNR, bits
Step-Scale Tuning
$\mathbf{S} = [s_\tau, s_\nu, s_\phi]^\top$
offline grid search
$\mathbf{y}_k^{(q)} \in \mathbb{C}^m$
(Quantized pilots)
AQNM Preprocess
$\tilde{\mathbf{y}}_k \leftarrow \mathbf{y}_k^{(q)}/\alpha$
$\sigma_{\text{eff}}^2 = \sigma_w^2 + \sigma_q^2/\alpha^2$
Real-stacking: $\tilde{\mathbf{y}}_k^{\mathbb{R}} \in \mathbb{R}^{2m}$
$\tilde{\mathbf{y}}_k, \sigma_{\text{eff}}^2$
Multi-Frame Inference
Choose one:
(i) EKF / IEKF
(ii) GN-MAP: L iters, λ fixed
(iii) DU-MAP: L layers, λ_ℓ schedule
Wrapped phase residual $\text{wrap}(\cdot)$
$\hat{\mathbf{x}}_k = [\tilde{\tau}_k, \tilde{\nu}_k, \tilde{\phi}_k]^\top$
$\hat{\mathbf{P}}_k$
Module I: Physical Dynamics
Module III: Benchmark & Evaluation
$\hat{\mathbf{x}}_{k|k-1}$, $\hat{\mathbf{P}}_{k|k-1}$
(prior)
Phase Impairments
Slip: $s_k \cdot \pi$ (Bernoulli $\pm\pi$)
PN: $w_{\phi,k} \sim \mathcal{N}(0, \sigma_\phi^2)$
(mutually exclusive)
add to ϕ
State Propagation (k→k+1)
$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k) + \mathbf{w}_k$
$\phi_{k+1} = \text{wrap}(\phi_k + 2\pi\nu_k T_{\text{fr}} + \cdots)$
Output: $\hat{\mathbf{x}}_{k+1|k}, \hat{\mathbf{P}}_{k+1|k}$
PCRB Recursion
$\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1}\mathbf{F}^\top + \mathbf{Q}$
$\mathbf{J}_k = \mathbf{J}_{k|k-1} + \mathbf{J}_k^{\text{meas}}$
Output: PCRB_k
Evaluation
RMSE, BER/EVM
Efficiency: MSE/PCRB
Recovery time
Performance
results