2024 International Mathematical Modeling Challenge

Team Control Number: US-13849

Summary

The escalating problem of pet abandonment and inadequate care highlights the necessity for evaluating potential pet owners' readiness. This study introduces a mathematical model that aims to assess household readiness for pet ownership, focusing on balancing the well-being of both humans (HWB) and pets (PWB). The model, designated as the Household Readiness Model - Cats (HRM-C), integrates multifaceted criteria including financial stability, living conditions, and personal commitment to provide a holistic assessment framework. Using a detailed questionnaire linked to indexed pet data, the model then uses vector comparison techniques to quantify the compatibility between household environments and pet characteristics. The outputs, HWB and PWB indices, are derived through a modified cosine similarity function, and further refined into a vector projection model to calculate the suitability of a household for pet ownership.

The flexibility of the HRM-C allows for adaptations to various pet species beyond cats, including dogs, birds, guinea pigs, and fish. The application of this model as shown in simulations across different U.S. states demonstrates its robustness and adaptability, suggesting significant potential for deployment in animal shelters and pet adoption agencies.

Additionally, the model also offers predictive capabilities by analyzing the potential long-term viability of pet ownership. By simulating various economic scenarios and demographic changes, the model can project shifts in household readiness over time, giving insights into population-wise preparedness status and individual pet retention rates.

Ultimately, by providing a quantifiable measure of household readiness, this model serves not only as a tool for assessing potential pet owners but also as a framework for improving the conditions under which pets are adopted, thereby fostering sustainable and mutually beneficial relationships between humans and pets.

Dear *International Mission for the Maintenance and Care of Animals (IMMC-A)*,

In many households, pets are treated as a member of the family. However, not all households retain their pets. Sometimes they are unable to continue caring for their pet, and sometimes they no longer want to care for their pet. The situation saddens us. We appreciate your mission and would like to collaborate with you to create a mathematical model that can benefit both animals and humans.

Through our research and analysis, we determined that the root of this problem is an imbalance of relationships between human well being (HWB) and pet well being (PWB). Therefore, the ultimate objective of our model is to strike a balance between HWB and PWB.

You just read a summary of our model that contained some pretty specific mathematical language. We would like to summarize using terms you may be more familiar with.

Assessment: We created a detailed questionnaire that addresses the financial aspects and lifestyle aspects of owning a pet. We recognize that sufficient money should not guarantee pet ownership. For example, if someone makes a lot of money, but they are unable to dedicate time for maintenance and companionship, our model responds by lowering the PWB value. This helps ensure that only those who are genuinely capable and willing to provide a safe, loving, and stable environment for pets are considered.

Analysis: Our top priority is to limit the possibility of abandonment and/or animal abuse, and our intent is not to increase barriers to pet ownership. We considered creating yes or no questions with "no"s resulting in a recommendation against ownership, but we felt like this approach does not encapsulate the diversity of humans and pets needs. For example, a household may accept a financial burden that is higher than our recommended values in exchange for the emotional benefits of pet ownership. Therefore, our model balances a variety of inputs to quantify the feasibility of a long-lasting bond between pets and humans.

Action: The resulting model could be implemented in shelters and facilities around the United States of America, and is easily adaptable for more pet species, breeds, regions, and countries. We truly believe that this approach will lead to more sustainable, mutually beneficial relationships between pets and their owners, ultimately reducing the number of pets that are returned to shelters.

We are excited about the potential impact of our collaboration and look forward to our journey to bring companionship, happiness, and a stable home.

Sincerely,
Team US-13849

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1 Introduction

2 Preliminary Information

2.1 Definitions

1. **Pet** Any domesticated or tamed animal that is kept as a companion and cared for affectionately[40]. Pets are intended for companionship rather than financial gain by definition. Therefore, all pets are expected to be spayed or neutered.

2. **Household**: A group of individuals living together and sharing common spaces, resources, and responsibilities within a dwelling. A household may consist of family members, roommates, or other individuals who cohabitate and contribute to the functioning and maintenance of the living environment.

3. **Household Readiness**: Refers to the preparedness of a household to responsibly and effectively care for a pet, taking into account factors such as space, resources, and lifestyle compatibility.

4. **Human Well-being**: Encompasses the physical, mental, and emotional health of the individuals within a household. In the context of pet ownership, human well-being directly impacts the owner's desire to own a pet and his or her ability to provide adequate care and companionship to them.

5. **Pet Well-being**: Refers to the health and quality of life of the pet, including their physical, mental, and emotional needs. Pet well-being is influenced by a variety of factors, but many stem from the behavior and companionship provided by the owner(s).

2.2 Assumptions

1. **Pets with the same species can get along:** Animals of the same species often have a better understanding of each other's communication and behavior, leading to harmony within the household.

2. **Number of beds determines how big the residence is:** The number of beds can serve as a proxy for the size of the residence, suggesting whether it has sufficient space to accommodate pets comfortably.

3. **Renting implies proximity to people; noise affects more:** Renting often means living in closer proximity to neighbors or shared spaces, increasing the impact of noise on both the household and potential pets.

4. **A fenced yard indicates outdoor use and is satisfactory:** A fenced yard suggests a safe outdoor space for pets to exercise and explore. This improves the pet's well being.

5. **Dogs should sleep outside only in suitable temperatures:** Referring to guidelines such as those provided by[11], ensuring dogs sleep outdoors only when temperatures are above 45 degrees Fahrenheit prioritizes their health and comfort, contributing to better pet well being.

6. **Urban and rural settings have negligible impact:** In the context of this problem, the distinction between urban and rural settings does not significantly influence household readiness for pet ownership, focusing instead on factors directly related to the household's environment and resources.

7. **Households have sufficient financial resources for initial pet purchase:** Often when households are looking to adopt they will have already saved up for the initial costs. The real cost is the annual cost associated with the pet, so we focus on the annual cost over the initial cost and create this assumption.

8. **Fish are kept in a safe environment:** Ensuring fish are in a safe and secure environment minimizes the risk of harm from other pets or environmental factors, promoting their well-being within the household setup.

3 Household Cat Ownership Readiness Model

3.1 Section Goal

A cat-ready household depends on multiple factors including monetary, emotional, psychological aspects. Some households may not be able to afford premium organic foods or luxurious multi-level cat towers, but they are able to provide substantial affection and companionship. On the other hand, some households are able to afford the "best" cat products and services, but may be emotionally detached from their cat. These two scenarios encapsulate some possibilities, but household situations are incredibly diverse and can vary significantly from each other. In this section, we create a quantitative model based on a number of factors that will determine which households are ready to welcome a cat into their home.

3.2 HRM-C Model Introduction

The HRM-C (Household Readiness Model - Cats) combines user inputs with researched cat data and determines the compatibility of a cat in that specified household using vector comparison techniques. A detailed outline is provided in figure 1. HRM-C is sectioned into three main parts: (1) Input and Data Preparation, (2) Household-Pet Compatibility Diagnosis, and (3) Decision Determiner. See figure 1.

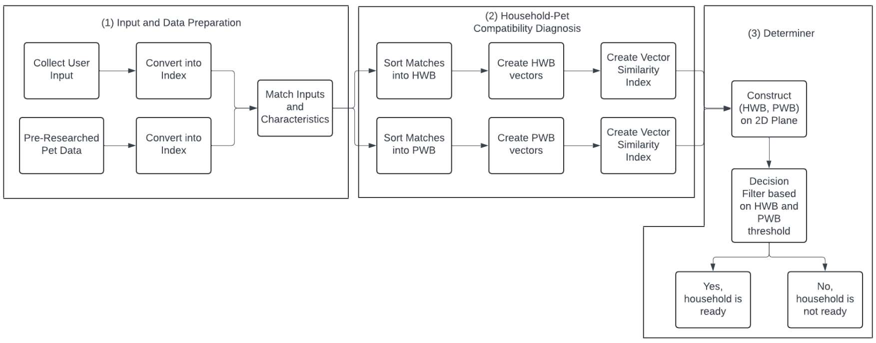

Figure 1: Model Outline

3.3 (1) Input and Data Preparation

Collect User Input and Index Conversion

According to Four Paws International[47], there are nine main reasons that pets, including cats, are given up. Each question in our survey is designed to address one or more reasons why pets are given up. Please note, unwanted litters and profitability are two reasons that are not addressed in the survey because they are addressed in the assumptions.

These questions are in a multiple choice format to increase front-end usability and back-end data preparation. This allows us to strategically convert their inputs into indices to create quantifiable metrics. The answers to questions are typically assigned with index from 1 to 5, with some having unique indices. An index system enables us to create concise quantifiable answers without sacrificing the predictive value of HRM-C. The answer choices and accompanying index values are available in the appendix (ref appendix num)

The questions we ask are:

Table 1: Questions for HRM-C

Question	Reason Addressed
1. Why are you interested in adopting a cat?	Boredom/Disinterest
2. How much time do you plan to spend actively caring for your cat per week? (Companionship)	Time Factor
3. How much time and effort are you willing to dedicate to your cat's daily care and maintenance? (Labor)	Animal Health and Age
4. Do you own or rent your home, and if renting, does your lease allow cats?	Change Life Situation, Behavioral Issues
5. How many people live in your household, and are there any children under 3 years old?	Changed Life Situation
6. Is anyone in your household allergic to cats?	Changed Life Situation
7. Does anyone in your household have experience with cats?	Lack of Experience, Behavioural Issues
8. What arrangements will you make for your cat during extended vacations or absences?	Financial Factor
9. How much do you plan to budget annually for your cat's care?	Animal Health and Age

Pre-Researched Pet Data and Index Conversion

We identified eight key characteristics that influence HWB and PWB, and collected data on these characteristics for ten cat breeds. We modified the order of some of the indices to always make the least desirable trait 1 and the most desirable trait 5.

1. Annual Cost: This is the only non-index characteristic. The original dataset provided a range of cost; to increase access to cat ownership, we choose to use the minimum cost for each breed.
2. Shedding Amount: Scored on a scale from 1 (most shedding) to 5 (least shedding).
3. Vocality: Scored from 1 (most noisy) to 5 (least noisy).
4. Friendliness to Children: Scored from 1 (least friendly) to 5 (most friendly).
5. Grooming Requirements (maintenance): Scored from 1 (extensive grooming needed) to 5 (minimal grooming needed).
6. Attention Requirements (companionship): Scored from 1 (most attention needed) to 5 (least attention needed).
7. Docility: Scored from 1 (least obedient) to 5 (most obedient).
8. Activity Level: Scored from 1 (most active) to 5 (least active).

Match Inputs and Pet Characteristics

To address whether a household is cat-ready, we connect input indices with relevant pet characteristics and categorize them into influence components for HWB and PWB. These influence components will be used to compare goodness of fit between household and pet. Note that some user inputs are used in multiple influence components; however, each influence component either impacts HWB or PWB.

The matches are listed in table 8 and its reasoning is provided in the table in appendix.

For each of the eight influence factors: a) If the user input index is greater than or equal to the pet characteristic index, then the influence component is satisfactory b) If the user input index is less than the pet characteristic index, then the influence component is unsatisfactory

3.4 (2) Household-Pet Compatibility Diagnosis

This part of the model building process will use vector comparison techniques to determine the suitability of each of the influence components between the household and the cat. As stated in the table in section 3.3, each index has either a satisfactory and unsatisfactory result. The table also split the influence components into HWB and PWB.

To generate values for HWB and PWB, we use vector comparison techniques. We create four vectors: two based on household criteria ($\vec{H}_{HWB}$ and $\vec{H}_{PWB}$) and two based on pet criteria ($\vec{P}_{HWB}$ and $\vec{P}_{PWB}$).

Human Well Being Index (HWB)

The six influence components relevant to measure HWB are listed in table 2, where $\vec{H}_{HWB}, \vec{P}_{HWB} \in \mathbb{R}^6$.

Table 2: Influence Component

Influence Component	**Household Vector Component** (from user input indices)	**Pet Vector Component** (from cat characteristic indices)
Burden of Annual Expense	H_e	P_e
Burden of Maintenance	H_m	P_m
Burden of Noise	H_n	P_n
Burden of Allergies	H_a	P_a
Risk of Safety	H_s	P_s
Activity Level Compatibility	H_{act}	P_{act}

We utilize cosine similarity to compare two vectors based on the direction each is pointing.

$$cos(\theta) = \frac{\vec{H}_{HWB} \cdot \vec{P}_{HWB}}{|\vec{H_{HWB}}||\vec{P}_{HWB}|} = \frac{H_e P_e + H_m P_m + H_n P_n + H_a P_a + H_s P_s + H_{act} P_{act}}{\sqrt{H_e^2 + H_m^2 + H_n^2 + H_a^2 + H_s^2 + H_{act}^2}\sqrt{P_e^2 + P_m^2 + P_n^2 + P_a^2 + P_s^2 + P_{act}^2}} \tag{1}$$

When the $\vec{H}_{HWB}$ and $\vec{P}_{HWB}$ are pointing in non-similar directions, then the output from HRM-C is unsatisfactory. Although the range of the cosine function is typically [-1,1], in our case, the range is between [0,1] because none of the indices are negative. A cosine value of 0 indicates least compatibility and a value of 1 indicates best compatibility.

To visualize this, we first reduce the number of dimensions into three by taking three example influence components, and plot sample household vectors and pet vectors. In the following diagram (figure 2), it is noticeable that the vectors are somewhat similar. Subjectively, we assume that cosine similarity values greater than 0.75 are similar. Therefore, the cosine similarity value of 0.87 is an acceptable deviation from the maximum of 1, whereas the cosine similarity value of 0.56 is significantly dissimilar.

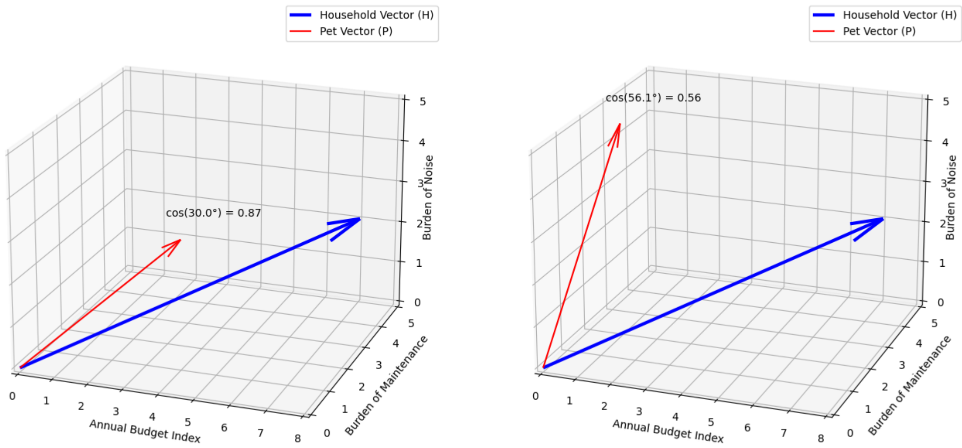

Figure 2: Visualization of Cosine Similarity

Although this sub-model is only three dimensions, cosine similarity can be applied to any number of dimensions. For the purpose of this section, we will remain in $\mathbb{R}^3$ in order to provide supplemental visualizations to the model building.

While cosine similarity is a good metric for part of the problem, it is unable to distinguish between some potential cases which logically should result in different HWB indices. For example, the figure below (figure 3) shows two cases where the cosine similarity are identical. However, in context, these two cases are far from equivalent. If $\vec{H}_{HWB}$ is "larger" than $\vec{P}_{HWB}$ as shown on the left side of figure 3, the household's burden tolerance is greater than the burden of the pet. On the other hand, if $\vec{H}_{HWB}$ is "smaller" than $\vec{P}_{HWB}$ as shown on the right side of figure 3, in context, the burden of the pet exceeds the household's burden tolerance, which is unsatisfactory. From this case study, we conclude that cosine similarity cannot be the only metric to determine compatibility between a household and a pet.

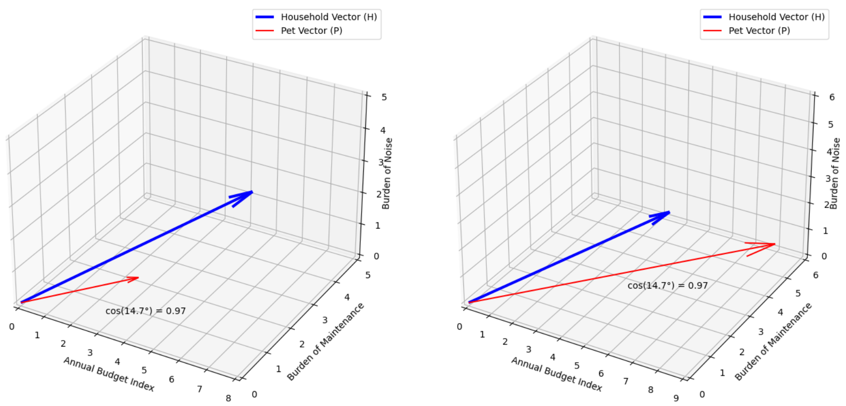

Figure 3: Example of Cosine Similarity Limitation

Upon further scrutiny, we created an additional metric to distinguish cases, like the one previously mentioned. This metric considers the relative magnitudes of both vectors, to find whether household tolerance outweighs the burden of the cat. The following is our final equation for the HWB value. This combines cosine similarity with their relative magnitudes.

$$\begin{aligned} HWB &= \left(\frac{\vec{H}_{HWB}\cdot\vec{P}_{HWB}}{|\vec{H}_{HWB}||\vec{P}_{HWB}|}\right)\left(\frac{|\vec{H}_{HWB}|}{|\vec{P}_{HWB}|}\right) = \frac{\vec{H}_{HWB}\cdot\vec{P}_{HWB}}{|\vec{P}_{HWB}|^2} \\ &= \frac{H_eP_e + H_mP_m + H_nP_n + H_aP_a + H_sP_s + H_{act}P_{act}}{P_e^2 + P_m^2 + P_n^2 + P_a^2 + P_s^2 + P_{act}^2} \end{aligned} \tag{2}$$

If $\left|\vec{H}_{HWB}\right| > \left|\vec{P}_{HWB}\right|$ then the cosine similarity will be scaled up, while if $\left|\vec{H}_{HWB}\right| < \left|\vec{H}_{HWB}\right|$ the cosine similarity will be scaled down. Notice that this is one directional, meaning that $\left|\vec{H}_{HWB}\right|$ must be greater than $\left|\vec{P}_{HWB}\right|$ in order to obtain a more "satisfactory" match, and thus a higher HWB. This effectively takes care of any magnitude discrepancy.

Coincidentally, we noticed that we happened to derive the vector projection formula with the directional vector component removed. Upon further inspection, vector projection makes sense in this situation. In the equation 2, it states a vector projection of the household vector onto the pet vector. This allows for the value to be penalized if the $\vec{H}_{HWB}$ is "smaller" than the $\vec{P}_{HWB}$. This also lets there be great benefits if the $\vec{H}_{HWB}$ is larger than the $\vec{P}_{HWB}$, as shown in the diagram below. Therefore, a projection value (disregarding the direction of the projection vector) is used as the metric for HWB.

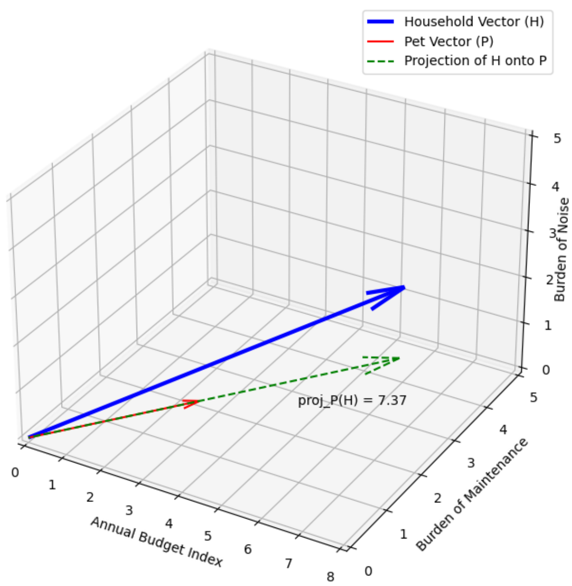

Figure 4: Vector Projection

Pet Well Being Index (PWB)

For the PWB index calculation, we conduct the same process of modified vector projection. There are different influence components that impact PWB instead of HWB, listed in table 3 Note that, $\vec{H}_{HWB}$ and $\vec{P}_{HWB} \in \mathbb{R}^4$.

Table 3: Influence Component

Influence Component	**Household Vector Component** (from user input indices)	**Pet Vector** (cat characteristic indices)
Engagement Compatibility	H_{eng}	P_{eng}
Suitability of plan of vacation extended	H_v	P_v
Burden of Noise	H_n	P_n
Burden of Training	H_t	P_t
Adoption Intention Impacts on Cat	H_{int}	P_{int}

Thus, PWB is calculated with the following equation,

$$PWB = \frac{\vec{H}_{PWB} \cdot \vec{P}_{PWB}}{|\vec{P}_{PWB}|^2} = \frac{H_{eng}P_{eng} + H_vP_v + H_nP_n + H_tP_t + H_{int}P_{int}}{P_{eng}^2 + P_v^2 + P_n^2 + P_t^2 + P_{int}^2} \tag{3}$$

3.5 (3) Decision Determiner

The motivation to create values for HWB and PWB is to ensure that the well being of both the household and the pet are satisfactory. In mathematical terms, we are looking for points that fall in the region generated by the system of inequalities:

$$\begin{cases} \text{HWB} \geq \text{threshold} \\ \text{PWB} \geq \text{threshold} \end{cases}$$

For example, point A indicates household readiness because both HWB and PWB are satisfactory. However, points B, C, and D do not meet both thresholds, so they are not ready for cat ownership.

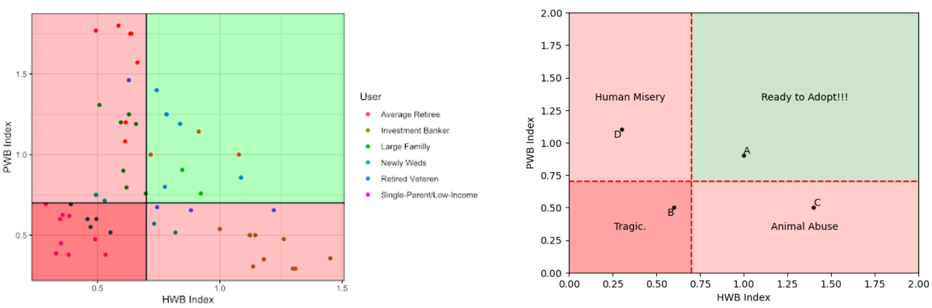

Figure 5: HWB vs PWB Regions and Determiner

Testing our six household conditions with diverse answer pallets, we subjectively determined that a threshold value of 0.7 was appropriate. We intentionally built three profiles we believed to be cat-ready and three profiles we believed were not cat-ready. Then, we classified them based on value and determined that a threshold of 0.7 accurately classified all six households.

Additionally, our model considers 10 different breeds of cat, each with its own pet vector (visualized in figure 6). If at least one of the ten (HWB, PWB) meets both threshold requirements, that household is ready to own a cat. However, if the number of (HWB, PWB) pairs that meet the threshold requirements is less than ten, the household should only be approved for the acceptable breeds.

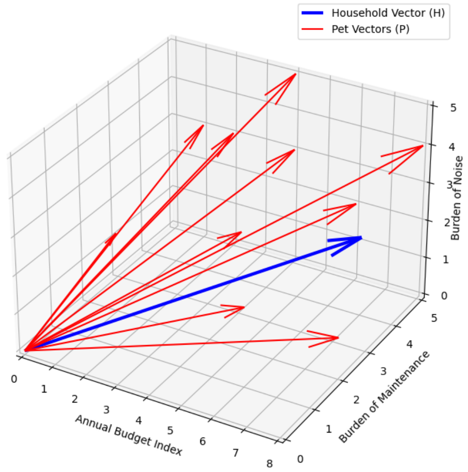

Figure 6: 10 Breeds Represented by Vectors

3.6 HRM-C Application to six households

In this section we apply our model to six households that capture a diversity of households. The table below shows the profile of six households and the inputs entered for each household. Those household inputs are combined with the backend researched cat characteristics and entered through the vector comparison model. The results of each household's (HWB, PWB) values for each of the ten cat breeds are shown in figure 6, and the final results are listed in the table below.

Table 4: Household Readiness Data

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Budget (Annual)	2000	550	450	600	300	760
Maintenance Emphasis	1	5	2	4	1	4
Own/Rent	Own	Own	Rent	Own	Rent	Own
Number of Babies	0	1	0	2	1	0
Allergies Members	0	1	0	3	2	0
Household Size	1	3	2	7	4	1
Hours of Activeness	1	30	3	7	2	25
Years of Experience	0	8	1	1	0	5
Vacation Decision	1	2	0	2	0	2
Intention	0	2	0	0	1	2
Decision	No	Yes (3)	No	Yes (2)	No	Yes (6)

4 Application to Countries/Regions

To assess the current number of households prepared to own a cat in three regions, California, West Virginia, and New York, the existing model needs to be adjusted and scaled to reflect accurate population statistics and household readiness based on broader demographic data. For example, instead of an individualized input of household size, homeownership status, and expected expense on the pet, we will need to use the average number of people in a household for a particular state, their income which is associated with the amount available for pet monetary contributions, and the rate of homeownership which could serve as a probability for "own" vs "rent". On the contrary, some inputting factors, such as number of years of experience, maintenance preference, vacation plan, and purchasing intention are way too specific to find any meaningful probabilistic/demographic generalization. We will discuss the modification and adjustment of every input factor and estimate the number of households in each state that meet the criteria for being considered prepared for cat ownership.

1. Annual Budget: To estimate the financial readiness of households in California, West Virginia, and New York for pet ownership, our model uses the state-wide statistics on average annual take-home pay and along with personal finance expert Forrest McCall's[25] recommendation that pet care expenses should not exceed 5% to 6% of an individual's net income. By randomizing a budget allocation within this range for each household, the model calculates an annual budget for pet expenses based on each household's take-home pay. For example, a household in New York with a take-home pay of $50,000 might be assigned a pet care budget of 5.5%, equating to $2,750 annually. This approach helps in assessing the affordability of pet ownership across the states, factoring in real-life financial constraints and ensuring the model's predictions are both practical and reflective of individual financial situations. The following table references the specific range of generation of take-home income by state.[16]

   Table 5: Income statistics by state

State	California	West Virginia	New York
Median	62,525	40,539	56,897
-90%	6,252.5	4,053.9	5,689.7
+10%	68,777.5	44,592.9	62,586.7

   There are two reasons that we design this "unbalanced" range of take-home income.

   (a) Given the inherent skew of income distribution toward higher values, using only the median income to assess the annual financial burden of maintaining a pet cat can be misleading. Thus, the objective is to evaluate and expand access to those on the verge of financial strain or even below it.

   (b) Since the model is calculated using indices, a higher amount wouldn't necessarily make a difference, yet the model specifies different index values for lower annual expense values.

2. Maintenance Emphasis: The maintenance emphasis factor for potential pet owners is randomly assigned a value between 1 and 5 for each individual in the simulation. This strategy is used due to the highly individualized nature of pet maintenance preferences, which are quite unpredictable and lack clear evidence to suggest that one particular state might have a significantly different emphasis on pet maintenance than other states.

3. Own/Rent: For the Own/Rent factor we are able to find data provided by the FED on the homeownership rate across the three states. Using this information, we can directly translate

that to being the percentage of "Own" vs. "Rent" in our Monte Carlo simulation.[29] [31] [30]

4. Number of Babies: According to 2000 Census data, the average number of children per family of California, West Virginia, and New York are 1.01, 0.72, and 0.90 respectively. Entering the 21st century, birth rates in the United States have been declining consistently, therefore to account for that trend we construct the following table of probability and expected value to reflect that temporal trend but also maintain the general difference between the states.

Table 6: Probability Distribution of Number of Babies by State

Number of Babies	**California**	**West Virginia**	**New York**
0	0.5	0.6	0.55
1	0.25	0.2	0.25
2	0.15	0.15	0.1
3	0.1	0.05	0.1
Expected Value	0.85	0.65	0.75

5. Household Sizes: We randomize and generate household sizes using the following equation:

$$\text{Household Size}_i = \max\left(1 + \text{BabiesInput}_i, \text{Poisson}(v_s)\right)$$

where $\text{Poisson}(v_s)$ represents a Poisson distributed variable with parameter v_s. Therefore, for every iteration of simulation (i), the household sizes input takes the maximum of two options:

- The number of babies at that iteration plus one, to ensure that the household is always greater than the number of babies.
- A value generated using the Poisson distribution where $\lambda = v_s$ corresponds to the average family size of a particular state. These values are as follows: California - 2.94, West Virginia - 2.4, New York - 2.55[2].

6. Allergies Members: According to NIH: "Allergies to dogs and cats affect 10%–20% of the population worldwide and is a growing public health concern as these rates increase". Given this trend, we will use the upper limit of 20% to simulate the number of household members with allergies. Therefore, considering the binary outcome—where having an allergy is a success and not having one is a failure—we can treat each household as a series of fixed trials, equivalent to its size. Then by applying the ceiling function to the average household size, we ensure the number of trials is a whole number. Additionally, we presume independence, meaning that one person's allergy status in a household does not influence another's. Thus, with all the appropriate conditions, we decide to use a binomial distribution to generate the number of household members with allergy issues.

7. Hours of Activeness, Vacation Decision, Intention: Similar to Maintenance (2), due to the individuality and the specificness of these inputs, we will randomly generate using the sample pool of 0 to 35, 1 to 3, and 1 to 3, respectively for each of the inputs.

8. Years of Experience: For the experience of owning a pet, we decided to use a Poisson distribution as a reference for our Monte Carlo simulation with a mean of 2 years of experience because it effectively models events that occur independently and at a constant average rate over a given interval of time. Based on research, the majority of US Households (66% according to Forbes) own a pet, yet 50% of the pet owners surveyed report owning their pets for 1 to 3 years. Therefore using a Poisson distribution with the highest probability at two and restrained to above or equal to 0 as a reference for simulation can better capture this tendency for this particular characteristic.

Using all the modified inputs alongside the cats' characteristics, we run our Monte Carlo simulation with the increasing number of households. As the following figure shows, starting approximately at the Simulation Sample Size of 5000 (5000 generated households per state, 15000 households total), we can see that the Pet Ownership Prepared Percentage (Number of Household Ready for Cat Ownership/ Sample Size) for the three states gradually converges.

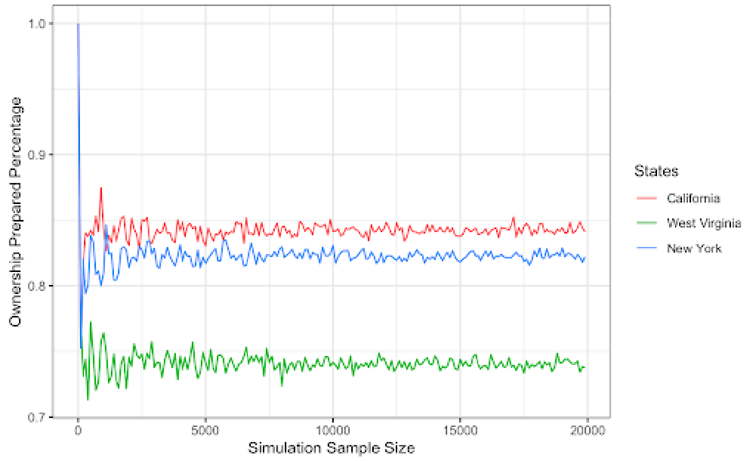

Figure 7: Law of Large Number with Monte Carlo Simulation

5 Model Generalization

In this section, we will expand on the HRM-C model to a more general HRM-P (Pets) model, where it applies to many other pets.

5.1 HRM-P Model

For each species, we define $m, n \in \mathbb{Z}$, where m is the number of Influence Components that impact HWB and n is the number of Influence Components that impact PWB.

Each breed is associated with an $\vec{H}_{HWB} \in \mathbb{R}^m$ and an $\vec{H}_{PWB} \in \mathbb{R}^n$ generated based on user inputs, and a $\vec{P}_{HWB} \in \mathbb{R}^m$ and a $\vec{P}_{PWB} \in \mathbb{R}^n$ generated by prior research. For each breed,

$$\vec{H}_{HWB} = \left\langle (H_{HWB})_1, (H_{HWB})_2, ..., (H_{HWB})_m \right\rangle$$

$$\vec{H}_{PWB} = \left\langle (H_{PWB})_1, (H_{PWB})_2, ..., (H_{PWB})_n \right\rangle$$

$$\vec{P}_{HWB} = \left\langle (P_{HWB})_1, (P_{HWB})_2, ..., (P_{HWB})_m \right\rangle$$

$$\vec{P}_{PWB} = \left\langle (P_{PWB})_1, (P_{PWB})_2, ..., (P_{PWB})_n \right\rangle$$

we calculate $(HWB, PWB) \in \mathbb{R}^2$ by

$$(HWB, PWB) = (proj_{\vec{P}_{HWB}}(\vec{H}_{HWB}), proj_{\vec{P}_{PWB}}(\vec{H}_{HWB})) = \left(\frac{\vec{H}_{HWB} \cdot \vec{P}_{HWB}}{|\vec{P}_{HWB}|^2}, \frac{\vec{H}_{PWB} \cdot \vec{P}_{PWB}}{|\vec{P}_{PWB}|^2} \right)$$

On $\mathbb{R}^2$, each (HWB, PWB) represents a specific breed of the species.

Let $S = \{(HWB, PWB) | HWB \geq 0.7 \text{ and } PWB \geq 0.7\}$. If $S \neq \emptyset$, then the household is ready for a pet of that species. The shelter or pet store should only allow adoption for pets in set S.

Application to Different Species

Then we will expand on the model to four additional pet species: dogs, birds, guinea pigs, and fish. The model flow uses the same utilities as outlined in figure 1, meaning vector comparison techniques will still be used. The only difference is in the first two blocks: collect user input and collect pet data.

We decided to have different input questions depending on the pet species. Since dogs require some questions that cats don't require, for instance, the average temperature and whether there is a fenced-in yard or not, it was reasonable for each pet species to have different questions less than or equal to 10. Note, that we were able to gain multiple information from one user input question. Therefore, our number of variables may go over 10.

5.2 Cat

For input on cats, the only modification that is needed is additional information on the number of cats and dogs the prospective pet owner currently has. The modified inputs for the six households profiles for cat are in appendix table 9.

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Decision	0	Yes(3)	No	Yes(2)	0	Yes(4)

Using the same input as before for cats it can be visualized how the result didn't change as much. Those who didn't qualify for pet ownership before were denied ownership as well, which would make intuitive sense.

5.3 Dog

For the dog input, there were some modifications from the cat input. First, the question about the vacation plan was taken out. This question was significant for cats since they get upset on small changes in their daily life[24] and PWB might be affected negatively when the owner travels. However, this is less applicable for dogs. Thus we decided to take out this question.

Second, in the residence question, we added a sub-question "How many bedrooms does your residence have?". We added a sub-question because it was closely relevant to the residence question. Refer to assumption 2, we assume that a number of bedroom can determine the residence size. This residence size will be compared to the energy level the dog has for each of the breeds since PWB will increase when energetic dogs have a big residence; they can move around as they want.

Third, in the experience question, we modified the answer options, which are shown below:

- 0 years
- 0 - 1 year, currently own
- 0 - 1 years, previously own
- 1 - 3 years, currently own
- 1 - 3 years, previously own
- 3 - 5 years, currently own
- 3 - 5 years, previously own
- 5+ yrs, currently own
- 5+ yrs, previously owned

Then, when a user choose the option with currently own, a sub-question will show up and asks how many dogs do you currently have and how many cats do you have. This information will be used to account for a situation where the household has multiple pets. This situation with multiple pets are further discussed in section 5.7.

Fourth, the question "Do you have a fenced in yard?" is added on to the user input questions. It is beneficial for a dog to have a fenced in yard, especially for a big dog. Having a fenced in yard could be an additional space for dog to live (refer to assumption 4) Therefore, answering this question will affect positively on PWB.

Finally, the last question we added is "Which state do you live in?" From this answer, we can get the information about temperature. This temperature factor is necessary to consider because dogs should only sleep outside when the temperature is above 45 degrees Fahrenheit(refer to assumption 5). All the new inputs are applied to the six households and outcome are listed in appendix table 10.

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Decision	Yes(10)	Yes(7)	No	Yes(7)	No	Yes(10)

The wealthy investment banker was qualified for having a dog. Dogs are the perfect match for the investment banker and the retired veterans since they own a residence, have multiple bedrooms, and have a fenced in yard. The average retiree and the large family are also qualified to have a dog, with having plenty of active hours.

5.4 Bird

For birds, we used very similar input questions such as Budget, Maintenance, Babies, and Allergies. Additionally, we also ask for input on previous/current ownership of birds and the users' previous experience (if any) with birds. The following table represents the possible responses of the 6 identified households, one thing to keep note of is that we only identify other Birds as possible influence actors instead of pets like cats and dogs. This is due to the unique ecological placement of birds and how they occupy and interact significantly differently than terrestrial animals like the two explored above. The six households profiles inputs for bird are in appendix table 11.

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Decision	No	Yes(2)	Yes(4)	No	No	Yes(4)

For those that could receive a bird as a pet option here is the detailed Results based on the specific Breeds:

1. Average Retiree: Cockatiel and Lovebird
2. Newly Weds: Budgie, Cockatiel, Conure, and Lovebird
3. Retired Veterans: African Gray Parrot, Cockatiel, Conure, and Lovebird

5.5 Guinea Pig

As for Guinea Pigs, many of the input factors such as Budget, Allergies, and Households remain the same, yet the input on the current number of pets Guinea Pigs, Cats, and Dogs possess also holds significant weight in decision-making. The six households profiles inputs for Guinea Pig are in appendix table 12.

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Decision	Yes(5)	Yes(1)	Yes(3)	Yes(3)	No	Yes(5)

It is interesting how the busy and wealthy investment banker is able to qualify and bond with a pet. A Guinea Pig is a perfect match with the profile status of the Investment Banker since it doesn't require as much attention and maintenance effort to take care of. Additionally, the empty household offers the perfect safety and stability for the Guinea pigs to survive and thrive.

5.6 Fish

Since fish is easier to take care of than other pets and it doesn't interact with any of the pets, we only need to consider 3 inputs, which are annual budget, maintenance, and intention inputs.The six households profiles inputs for fish are in appendix table 13.

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Decision	No	Yes	No	No	No	Yes

The average retiree and the retired veterans could only get a fish. We were expecting all of the household to have a fish since it is less expensive and easy to take care of. However, because there were only three inputs, the maintenance emphasis and intention was weighted significantly compared to the other pets, it made the households hard to qualify for having a fish.

5.7 Multiple Pets

For all the pets, we included an additional influence variable that considers the possibility that a household possesses multiple pets. For each pet, there is a unique index chart to consider the relationship between different pets. For example, if a household already owns a dog and is looking to adopt a cat, the index will provide a low index that will decrease the pet well being, given that dogs and cats generally do not mix. On the other hand, if a household already owns a cat and is looking to adopt another cat, then the index will be higher.

This approach is not without its flaws. The relationship between different animals vary significantly between breeds, environment, and other situational factors. This generalization generally holds true, but is subject to many nuanced.

6 Retention Extension of Model

6.1 Single Household Retention Model

This model is versatile in that there is relative finding of new HWB and PWB models based on changing the inputs. In this section, we will expand on the pet-readiness household model and analyze how a household behavior or condition change can positively or negatively influence pet retention.

So why are pets given away? After the research [47], we compiled a list of reasons for abandonment and their corresponding human input.

Table 7: Reasons and index change for retention

Reasons	**Household/Pet** Index Change	**Corresponding** Index Change
Maintenance requirement cannot be tolerated	Household	Decrease in maintenance tolerance index
Time required to companion pet becomes scarce	Household	Decrease in active companionship time index
A loss of job or significant income	Household	Decrease in annual budget index
A move of residence	Household	Decrease in noise index (noise is more burden)
Having a baby	Household	Change in number of children index
Animal's health requires additional maintenance	Pet	Increase in maintenance requirement index
Animal's health requires additional treatment costs	Pet	Increase in annual cost index

Notice that the first five reasons all influence the household's conditions while the last reason influences the pet's condition. Based on this information, for the first three reasons, the household input indices would be adjusted and thus the influence components of the household vectors would change. Mathematically, if $\vec{H}_{HWB}$ and $\vec{H}_{PWB}$ represents the initial state of the household, let $\vec{H'}_{HWB}$ and $\vec{H'}_{PWB}$ represent the adjusted state of the household. Similarly, for the last two reasons, the pet's index data would be adjusted. Mathematically, if $\vec{P}_{HWB}$ and $\vec{P}_{PWB}$ represents the initial state of the household, let $\vec{P'}_{HWB}$ and $\vec{P'}_{PWB}$ represent the adjusted state of the pet.

To visualize the adjusted state method, we limit the visual example to three sample influence components to get it in R^3. The visual can be applied to both the HWB and PWB category. This produces the visual in figure 8.

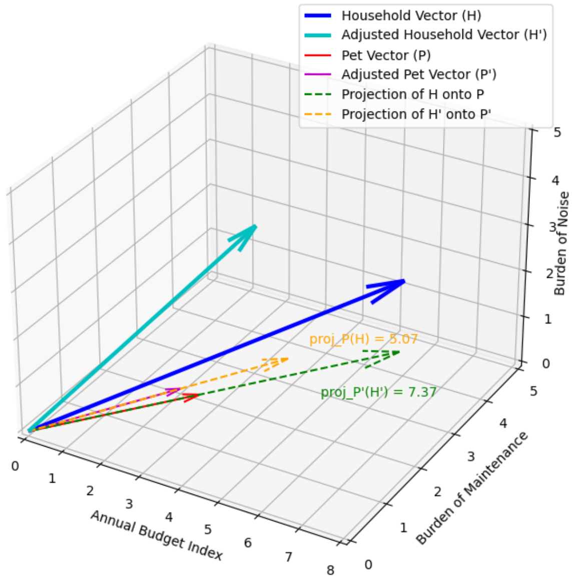

Figure 8: Vector Projection Change Comparisons

The above figure shows how adjusting $\vec{H}$ and $\vec{P}$ changes the projection values, which can either be HWB or PWB. In the illustrated example, it shows the vector projection values decreased. If this decrease was applied for both the HWB and PWB axis, then the transformation could be as follows:

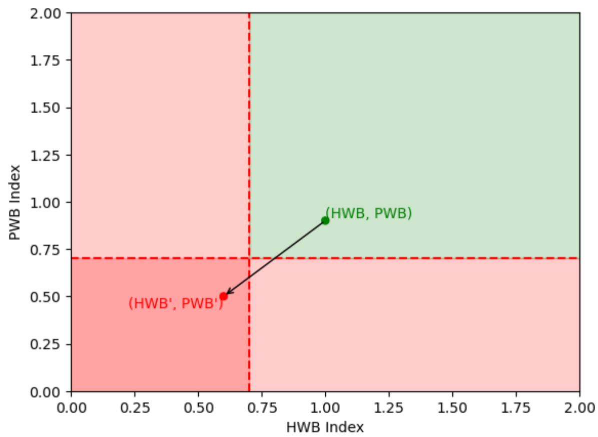

Figure 9: Example of Shifted Household Readiness

Note, the vectors and visualizations in this section do not represent real household or pet values, because (1) they do not consider all the dimensions and (2) they are used for illustration purposes.

The idea of this method is to show how a household is not fixed to its initial inputs and that changes in the household state is frequent and this model is versatile in that it is able to capture that relationship. This is how we can theoretically determine a household's retention if we had access to how likely their inputs were to change.

6.2 Widescale Household Retention Model

In this section we will run through the process of determining statewide or region-wide retention rates by taking the Single Household Retention Model a step further. Instead of looking at a particular case of adjusted initial inputs, we look at many different potential changes. Namely, we aim to grasp the probabilities of different transformations from (HWB, PWB) to (HWB', PWB').

To begin, we create probability distributions for each index in table 7. These distributions will give us estimates on how likely an index is to change and to what value. An example distribution for decrease in maintenance tolerance index could look like the following:

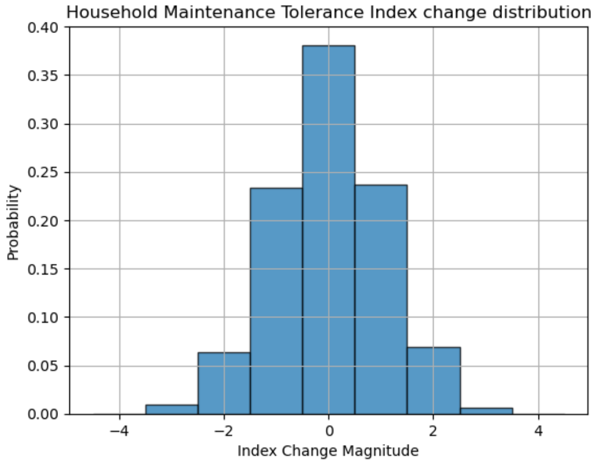

Figure 10: Index Change Distribution for Maintenance

The choice to use a distribution akin to a normal distribution made sense because large deviations in values (like ± 4) are unlikely to happen. For maintenance, it is unlikely for a household's maintenance to go from 5 to 1 or 1 to 5, it is much more likely for the maintenance to shift from 4 to 3 to 3 to 2. Using the distributions of each index, we are able to use a Monte Carlo simulation to generate a large number of (HWB', PWB'). A sample distribution of all these primes are shown in figure 11. This allows us to calculate the proportion of (HWB', PWB') that move from the "ready" side to any of the "not ready" sides.

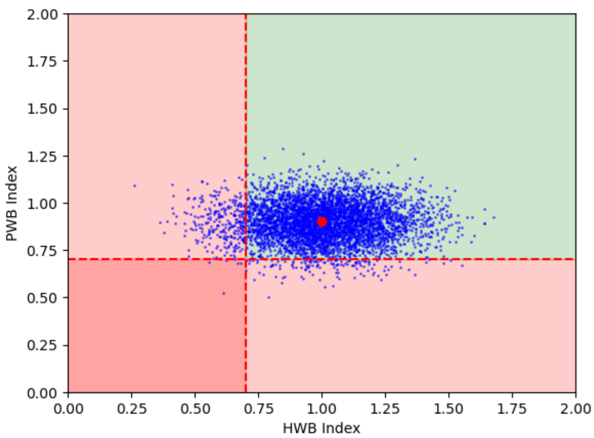

Figure 11: A large number of (HWB', PWB')

The estimated proportion of points that move to the "not ready" regions are around 7%. We notice the deviation from the point is not entirely large, with most points clustered very tightly with the original point. This is expected because big changes in the influence components are unlikely, as seen in one of the influence components in figure 10.

To generalize this model to different households with different starting conditions, we run more and more samples. For each sample, we estimate the proportion of (HWB', PWB') that cross into the red regions and take an average proportion of all the different starting household conditions. Ultimately, we found that on average the probability of becoming a "not ready" household is around 5-6%. For the purpose of future projections in the following section, we assume this number to directly resemble the number of pets abandoned or given away.

7 The Future of Pet Ownership

It is very difficult to project long into the future with the given interval of 5 years, 10 years, and especially 15 years. Therefore, instead, we decide to approach this question using the methods of range, of upper bound vs lower bound, of best case scenario vs worst case scenario.

We will use the cat as the benchmark for this comparison due to its establishment in this paper and the fact that it also has much better documentation in terms of data availability.

In section 4, we've discussed how some factors could be altered based on demographics, yet for others, we decided to fully randomize due to their specificness and level of individuality. Therefore, our estimation and analysis of future pet ownership will mainly rely on changes to the Budget input of a prospective user. We will define two ranges of comparison for each of the 5, 10, and 15-year projections: one with economic optimism of which the take-home salary is consistently on the rise at the rate of 3% per year for the given time projecting interval, the other with economic depression of which the take-home salary consistently decrease for the given interval by 3% per year. The following tables report such results:

Time Interval	California	West Virginia	New York
5 Years (+)	0.8329	0.7528	0.8164
10 Years (+)	0.8604	0.8401	0.7858
15 Years (+)	0.8812	0.8127	0.8656
5 Years (-)	0.7749	0.6534	0.7541
10 Years (-)	0.7334	0.6052	0.7009
15 Years (-)	0.6882	0.5328	0.6561

Based on the results of the extremes, it is evident that an economic downturn is much more impactful and can the damage potential pet ownership population significantly. Moreover, a loss of income would also contribute to the existing pet ownership as it increases the possibility of abandonment due to the lack of adequate pet care and a complete unbalance of HWB and PWB.

8 Strengths and Weaknesses

Our model boasts remarkable adaptability, making it uniquely suited for various pets, even beyond the five defined pets. Its flexible infrastructure effortlessly adjusts dimensions based on the number of household inputs, allowing us to tailor questions precisely for each pet. With the right data and index conversion, the vector projection calculation can go as high of dimensions as possible even past 10. This adaptability extends to real-world applications, empowering pet centers to customize questions and indices as needed.

Additionally, our model excels in handling different breeds. The HRM pet models can evaluate multiple breeds for compatibility, offering insights beyond a simple yes or no assessment. For example, if a household is ready for two breeds but not the eight other breeds, then it might help for the animal shelter or pet store to ask further questions about the household condition. This functionality is invaluable to them, enabling them to assess breed suitability and provide detailed recommendations.

One of the model's standout strengths lies in its holistic approach. While filters provide binary outcomes, our HWB vs PWB graph paints a nuanced picture. With the model, we're able to determine how close a household may be to be ready. Therefore, stores and shelters may provide suggestions to improve household conditions and lead to adoption. After all, our model doesn't serve a purpose to turn away eager pet owners, its a tool that can help proliferate ownership by improving the adoption conditions.

In essence, our model's versatility, breed-specific capabilities, and holistic analysis make it a powerful tool for enhancing pet adoption processes and ensuring long-term household-pet compatibility.

One notable weakness of our model is the uniform influence assigned to influence components. Currently, most indices are rated on a scale of 1 to 5, implying that each influence component carries a similar weight in calculating the HWB and PWB. However, in reality, certain factors may significantly outweigh others in determining a household's readiness for pet ownership. For instance, the burden of noise might be more crucial than the burden of maintenance. To enhance the model's practicality, an essential improvement would involve introducing a weighted system that accurately reflects the varying impact of different influence components on the vector projection model.

This uniformity in influence components can further lead to another weakness: the inequitable balancing of influences. Through observations in household case studies, we've noticed instances where influence components with high indices can offset those with low indices, based on their "geometry" within their respective dimension. For example, a financially well-off household might exhibit a robust HWB due to a strong annual budget, masking weaker influence components. Conversely, a household with lower financial resources could compensate for this by having stronger influences in other areas. This balancing act can create misleading outcomes and should be addressed to ensure a more accurate evaluation of HWB and PWB.

In summary, refining the weighting system for influence components and addressing the imbalanced balancing of influences are crucial steps toward enhancing the model's accuracy and practicality in assessing household readiness for pet ownership.

9 Conclusion

In conclusion, this paper delves into the mathematical process of gauging a household's readiness for pet ownership by gathering user input regarding their living conditions. We crafted user-friendly questions and translated them into indices for easy comparison. These indices were then cross-referenced with pre-researched data on various pet species and breeds using a modified cosine similarity function, later refined into a vector projection model. The model also took into consideration the human and pet well being, to recognize that pet ownership is mutual and companionship must be met both ways.

The flexibility of this model allowed us to tailor it to five distinct pet species (cat, dog, bird, guinea pig, and fish), adapting it to suit the unique needs of different households. We applied this model to six diverse households and extended its scope to three states in America: California, New York, and West Virginia. Our analysis revealed that California exhibits the highest level of preparedness for pet ownership, followed by New York and West Virginia, with ownership readiness percentages ranging from the mid-70s to the mid-80s.

While our model efficiently categorizes households as "ready" or "not ready," its true strength lies in its ability to assess how enhancements to household conditions can significantly boost the likelihood of successful pet adoption. We see our model not merely as an assessment tool for pass or fail but as a guide to improving and promoting responsible pet ownership, benefiting both humans and animals alike.

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11 Appendix

<table>
<tr><td>Question Number</td><td>Question</td><td>Answer choices and corresponding choices</td></tr>
<tr><td>1</td><td>Why are you interested in adopting a cat?</td><td><table>
<tr><td>Intention</td><td>Index</td></tr>
<tr><td>Companionship</td><td>2</td></tr>
<tr><td>Mental Health</td><td>1</td></tr>
<tr><td>Physical Health</td><td>1</td></tr>
<tr><td>To teach responsibility</td><td>0</td></tr>
<tr><td>Fashionable/Cool</td><td>0</td></tr>
<tr><td>Want to try something new</td><td>0</td></tr>
</table></td></tr>
<tr><td>2</td><td>How much time do you plan to spend actively caring for your cat per week? (Companionship)</td><td><table>
<tr><td>Hours per week</td><td>Index</td></tr>
<tr><td>Less than 1 hrs</td><td>1</td></tr>
<tr><td>1 - 2 hrs</td><td>2</td></tr>
<tr><td>2 hrs</td><td>3</td></tr>
<tr><td>3 hrs</td><td>4</td></tr>
<tr><td>4+ hrs</td><td>5</td></tr>
</table></td></tr>
<tr><td>3</td><td>How much time and effort are you willing to dedicate to your cat's daily care and maintenance? (Labor)</td><td><table>
<tr><td>Index</td></tr>
<tr><td>1</td></tr>
<tr><td>2</td></tr>
<tr><td>3</td></tr>
<tr><td>4</td></tr>
<tr><td>5</td></tr>
</table></td></tr>
<tr><td>4</td><td>Do you own or rent your home, and if renting, does your lease allow cats?</td><td>N/A - explained in section [REF SECTION explaining]</td></tr>
<tr><td>5</td><td>How many people live in your household, and are there any children under 3 years old?</td><td><table>
<tr><td># of Children < 3 yrs</td><td>Index</td></tr>
<tr><td>0</td><td>3</td></tr>
<tr><td>1</td><td>2</td></tr>
<tr><td>2+</td><td>1</td></tr>
</table></td></tr>
<tr><td>6</td><td>Is anyone in your household allergic to cats?</td><td><table>
<tr><td># of People Allergic</td><td>Household Allergic Percentage</td></tr>
<tr><td>0</td><td>0 %</td></tr>
<tr><td>1</td><td>1 / # of people</td></tr>
<tr><td>2</td><td>2 / # of people</td></tr>
<tr><td>3+</td><td>3 / # of people</td></tr>
</table>
<table>
<tr><td>Household Allergic Percentage</td><td>Index</td></tr>
<tr><td>0 - 20%</td><td>5</td></tr>
<tr><td>20-40%</td><td>4</td></tr>
<tr><td>40-60%</td><td>3</td></tr>
<tr><td>60-80%</td><td>2</td></tr>
<tr><td>80-90%</td><td>1</td></tr>
</table></td></tr>
<tr><td>7</td><td>Does anyone in your household have experience with cats?</td><td><table>
<tr><td>Years owned</td><td>Index</td></tr>
<tr><td>0 yrs</td><td>1</td></tr>
<tr><td>0 - 1 yrs</td><td>2</td></tr>
<tr><td>1 -3 yrs</td><td>3</td></tr>
<tr><td>3 -5 yrs</td><td>4</td></tr>
<tr><td>5+ yrs</td><td>5</td></tr>
</table></td></tr>
<tr><td>8</td><td>What arrangements will you make for your cat during extended vacations or absences?</td><td><table>
<tr><td>Plan</td><td>Index</td></tr>
<tr><td>Leave it alone</td><td>0</td></tr>
<tr><td>Cat Sitter</td><td>2</td></tr>
<tr><td>Leaving to friends or relatives</td><td>2</td></tr>
<tr><td>Boarding</td><td>1</td></tr>
<tr><td>Bring the pet (if permitted)</td><td>0</td></tr>
</table></td></tr>
<tr><td>9</td><td>How much do you plan to budget annually for your cat's care?</td><td>Open response (in the backend we convert to an index)</td></tr>
</table>

Table 8: Impact Influence Component User Input Indices

Impact	Influence Component	User Input Indices	Cat Characteristic Indices	Justification of Connection
HWB	Burden of Annual Expense	Annual Budget (Q1)	Annual Cost	Trivial
	Burden of Maintenance	Willingness to maintain (Q8)	Grooming	Grooming is essential for cat health, so it is a good metric of maintenance[48]
	Burden of Noise	Rent or Own (Q3), Number of Young Children (Q4)	Vocality	Households with babies under 3 and (by assumption) renters have lower noise tolerance. (see appendix for calculation)
	Burden of Allergies	Percentage of cat-allergic individuals (Q6)	Shedding	Trivial
	Risk of Safety	Number of Young Children (Q4)	Child Friendly	Young children have less impulse control and may unintentionally upset a cat. Cats often retaliate[23].
	Activity Level Compatibility	Hours of active companionship (Q2)	Cat Activity	Trivial
PWB	Engagement Compatibility	Hours of active companionship (Q2)	Attention Requirement	Trivial
	Suitability of plan for extended	Plan of vacation (Q7)	N/A - Compared to a constant index	A constant index is used to create weights of different vacation plans.
	Burden of Training	Ownership Experience (Q5)	Docility	Docility refers to obedience, which by extension applies to trainability, and ownership experience impacts ease of training
	Adoption Intention Impacts on Cat	Intention of Adoption (Q9)	N/A - Compared to a constant index	A constant index is used to create weights of different adoption impacts[47].

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Budget (Annual)	2000	550	450	600	300	760
Maintenance Emphasis	1	5	2	4	1	4
Own/Rent	Own	Own	Rent	Own	Rent	Own
Number of Babies	0	1	0	2	1	0
Allergies Members	0	1	0	3	2	0
Household Size	1	3	2	7	4	1
Hours of Activeness	1	30	3	7	2	25
Vacation Decision	1	2	0	2	0	2
Intention	0	2	0	0	1	2
Multi-pet (Cat)	0	2	0	1	0	0
Multi-pet (Dog)	0	1	1	0	0	2
Decision	0	Yes(3)	No	Yes(2)	0	Yes(4)

Table 9: Six Household Inputs and Outcome for Cat

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Budget (Annual)	2500	1000	800	1300	500	1600
Maintenance Emphasis	1	5	2	3	1	4
Own/Rent	Own	Own	Rent	Own	Rent	Own
Number of Babies	0	1	0	2	1	0
Allergies Members	0	1	0	3	2	0
Household Size	1	3	2	7	4	1
Hours of Activeness	1	30	3	7	2	25
Experience	1	5	1	2	1	3
Vacation Decision	1	2	0	2	0	2
Intention	0	2	0	0	1	2
Bedroom	5	2	2	4	2	3
Yard	5	5	1	5	1	5
State	New York	West Virginia	New York	Los Angeles	Los Angeles	West Virginia
Multi-pet (Cat)	0	2	0	1	0	0
Multi-pet (Dog)	0	1	1	0	0	2
Decision	Yes(10)	Yes(7)	No	Yes(7)	No	Yes(10)

Table 10: Six Household Inputs and Outcome for Dog

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Budget (Annual)	1500	350	450	650	250	700
Maintenance Emphasis	0	1	1	2	0	1
Ownership History	N/A	Curr.	Prev.	N/A	N/A	N/A
Ownership Experience	0	4	3	0	0	1
Number of Babies	0	1	0	2	1	0
Allergies Members	0	1	0	3	2	0
Household Size	1	3	2	7	4	1
Intention	0	2	0	0	1	2
Multi-pet (Bird)	0	2	1	0	0	0
Decision	No	Yes(2)	Yes(4)	No	No	Yes(4)

Table 11: Six Household Inputs and Outcome for Bird

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Budget (Annual)	1200	250	350	550	500	600
Maintenance Emphasis	0	1	1	2	0	1
Allergies Members	0	1	0	3	2	0
Household Size	1	3	2	7	4	1
Intention	0	2	0	1	1	2
Multi-pet (Pigs)	0	2	1	1	0	0
Multi-pet (Cat)	0	0	0	1	0	0
Multi-pet (Dog)	0	0	1	0	0	1
Decision	Yes(5)	Yes(1)	Yes(3)	Yes(3)	No	Yes(5)

Table 12: Six Household Inputs and Outcome for Pig

Category	Investment Banker	Average Retiree	Newly Weds	Large Family	Single-Parent & Low-Income	Retired Veterans
Budget (Annual)	600	300	200	300	150	450
Maintenance Emphasis	1	5	2	4	1	4
Intention	0	2	0	0	1	2
Decision	No	Yes	No	No	No	Yes

Table 13: Six Household Inputs and Outcome for Fish