2023 High School Mathematical Contest in Modeling

Team Control Number: 13849
Problem B: Charging Ahead with E-buses

Nov. 14, 2023

Summary

According to the EPA[1], the transportation sector produces 28% of the greenhouse gas emissions in the US. Even though public transportation like buses is a more fuel-efficient means of travel, most public buses have diesel engines and therefore pollute the air. One solution to this problem is using BEBs or Battery Electric Buses which do not burn gasoline and do not cause noise pollution like conventional diesel buses. Many metropolitan cities like Shenzhen, China have documented transitioning from being completely dependent on diesel buses to having an all-electric fleet of buses. Acknowledging all the ecological benefits along with long-term economic sustainable growth associated with electric transportation, it is a trend for cities across the world to start adopting such a system, therefore the question lies in "how" this transformation might happen in an economically feasible manner.

To tackle the problem of converting a partially or entirely diesel-powered fleet of buses into BEBs, we used an energy constraint and a price optimization model to determine what number of BEBs and chargers would be needed to supplant the diesel buses in a metropolitan city. We used Boston, Massachusetts—a relatively large metropolitan city in the US—as our first city. Then we applied our model to two other cities, Detroit, Michigan, and Philadelphia Pennsylvania. With our model complete, we developed a detailed 10-year plan for Ebus transformation that lays out the regional-specific costs and charging port placements.

To minimize the cost on cities, we used the Lagrange Multiplier Method to optimize the number of e-buses and chargers needed to completely replace fleets of diesel buses. This model considers factors like the distances covered by buses, the associated continuous cost of electricity, and the one-time costs of purchase and installation to give an accurate prediction of the number of buses and chargers necessary to meet the demand for public bus transportation.

Contents

1 Introduction

1.1 Background

In the status quo, the market for electric buses along with charging ports and batteries has been continuously growing with an increasing number of models and innovations available for local governments and companies to select. However despite all these great efforts, as of this moment, the main problem with electric buses is their limited driving range. On average, a regular-sized diesel bus can travel around 350 to 500 miles on a full tank of gas. On the contrary, an electric bus, could only on average travel 125 miles, at least half of the mileage in comparison to diesel.

1.2 Problem Restatement

Through this paper, our goal was to develop a model that reasonably predicts the number of e-buses and chargers required to accommodate the public bus transportation needs of any metropolitan city. We also used this information to model the cost for cities to make a change from diesel to electric buses. We separated the problem into three fundamental questions:

1. What factors affect the number of BEB's necessary and therefore the number of chargers?
2. How do we optimize the number of e-buses and chargers to limit the cost for cities?
3. How do we apply a model to show the ecological effects of transitioning a city to an all-electric bus fleet?

Answering the first question will intuitively help determine the required number of BEBs and chargers. The second question adds complexity to the model by aiming to make it more cost-efficient. The third question will quantify the predicted emissions abated through the conversion of a city's bus fleet to a fleet of BEBs.

2 Preliminary Information

2.1 Definitions

1. **Battery Electric Bus (BEB)**
   An Electric Bus is any bus whose propulsion and accessory systems are powered exclusively by a zero-emissions electricity source[2].
2. **Active Charger/Charging**
   In this model, an active charger is a charger that has high power ratings relative to depot chargers and is a charger that is placed on a bus stop/terminal.
3. **Depot Charger/Charging**
   Depot charging is the process of charging using plug in/depot chargers when at a bus depot, often during downtime (e.g. late night).

4. **Bus Stop**
   A place where a bus regularly stops.

5. **Bus Terminal**
   A point where a bus route stop and ends.

6. **Time of Use (TOU) Policy** Time of Use rates are a kind of electricity billing arrangement in which the price of electricity changes based on the time of day.[3]

7. **MBTA** The Massachusetts Bay Transportation Authority (MBTA) is the public agency responsible for operating most public transportation services in Greater Boston, Massachusetts.

8. **DDOT** The Detroit Department of Transportation (DDOT) is the primary public transportation operator serving Detroit, Michigan.

9. **SEPTA** The Southeastern Pennsylvania Transportation Authority (SEPTA) is a regional public transportation authority that operates bus, rapid transit, commuter rail, light rail, and electric trolleybus services for nearly four million people in five counties in and around Philadelphia, Pennsylvania.

2.2 Assumptions

1. **Assumption:** The bus schedule will not be impacted by the time spent at an active charger
   Justification: Despite buses having a predefined schedule with times down to the minute, in reality, bus arrival and departures from stops are highly unpredictable. Therefore, trying to fit active charging time into the equation of delaying buses from arriving on time is too complex and on such a minor scale that data for this kind of work may not be available.

2. **Assumption:** Bus bunching, overcrowding, or queuing at bus stops or terminals is neglected
   Justification: Similarly to the first assumption, although bus bunching, overcrowding, and queuing are all common phenomena in every metropolitan transit system, the underlying factors that govern them, such as traffic disruptions, are very unpredictable and would require a model of it in itself. Therefore, for model succinctness, those factors are not considered.

3. **Assumption:** The impact of weather and climate on bus performance, charging efficiency, or battery life is negligible.
   Justification: While it is proven that weather and climate do play a role in the aforementioned traits, the cost of replacing a few batteries or a little more maintenance over the cost of purchasing a whole new fleet of BEBs and installing hundreds of depot and active chargers trump the significance of replacing a few batteries. Therefore, our model focuses on the big-picture costs.

4. **Assumption:** There is a linear relationship between a BEB's state of charge (battery percentage) and voltage.
   Justification: Almost all electric batteries do not have a linear relationship between BEB's state of charge and voltage. Within an electric transit system context, this would mean that a BEB's battery does not train at a constant rate, and may drain more towards the beginning

and the end. Assuming a linear relationship enables us to easily correlate battery percentage with mileage in our model, something a typical SOC-Voltage system would make complex.

5. **Assumption:** The number of BEBs on the road at a given time is always equivalent to the number of buses on the road previously.
Justification: It is a reasonable assumption that the current bus schedule for each metropolitan area is already optimal based on a slew of factors, so modifying the schedule would be ineffective and time-constraining. Therefore, in order to maintain the schedule in our model the number of BEBs on the road must be the same as the number of buses previously on the road. In other words, the additional BEBs the model produces will be idle and will only serve as a substitute bus for when a BEB runs out of electricity.

6. **Assumption:** Every time a BEB passes an active charger, the BEB is assumed to charge, and each charge is the same time, thus the same amount of energy gain.
Justification: To determine where and when a BEB should use an active charger will need the use of individual bus data and schedule, which none of the three metropolitan areas we are exploring have.

2.3 Variables, Parameters, Constants

Table 1: Variables Table

Variable	Definition
n	Number of Active Chargers
b	Number of Additional Buses

Table 2: Parameters Table

Parameter	Definition	Value
T	Amount of time spent at each active charger	20 seconds
O_c	Ownership cost duration span	10 years
DOD	Depth of discharge (between 0 and 1)	0.8 [1]
F	External funding transition coverage in percentage	0-50%

Table 3: Location Specific Constants Table

Constant	Definition	Value
b_0	Base number of BEBs required to meet transit system schedule (equivalent to the total number of Diesel Buses)	· Boston MBTA: 980 · Detroit DDOT: 291[5] · Philadelphia SEPTA: 1400[6]
p	Number of routes (major)	· Boston MBTA: 151 · Detroit DDOT: 48 · Philadelphia SEPTA: 152
q	Number of stops (major)	· Boston MBTA: 1001 · Detroit DDOT: 3691 · Philadelphia SEPTA: 12998
D_0	Total distance covered by all pre-electrification buses (generally just diesel buses) per day.	· Boston MBTA: 120,220 miles · Detroit DDOT: 45,132 miles · Philadelphia SEPTA: 239,656 miles

Table 4: Constants Table

Constant	Constant Name	Value
B_{BL}	BEB Battery Life	12 years[7]
B_C	BEB Capacity	351 kWh[7]
B_R	BEB Range	125 miles[8]
B_P	Average Bus Price of Purchase	$887,308[7]
$B_{P,M}$	BEB Price of Maintenance	$0.64/mile[7]
$A_{P,P}$	Active Charger Price of Purchase	$495,636[7]
$A_{P,I}$	Active Charger Price of Installation	$202,811[7]
$A_{P,OM}$	Active Charger Price of Operations and Maintenance (O&M)	$18000/year[7]
A_{kW}	Active Charger Power	150 kW[9]
$D_{P,P}$	Depot Charger Price of Purchase	$50,000[7]
$D_{P,I}$	Depot Charger Price of Installation	$17,050[7]
$D_{P,M}$	Depot Charger Price of Operations and Maintenance (O&M)	$0[7]
D_{kW}	Depot Charger Power	70 kW[7]

2.4 Data Collection

Across all three metropolitan areas, we accessed and used the public GTFS (General Transit Feed Specification) database for routes, stops, and timetables. We will go into specifics for each of the three areas in the following sections.

1. Boston MBTA

We used the "MBTA Bus Arrival Departure Times 2023" [10] data set provided by the MBTA Office of Performance Management & Innovation to access the timetable of Buses in the Boston Metropolitan area. This dataset contains 2,192,992 observations and 13 columns. Some notable columns that we extracted for modeling purposes include the date (year-month-day), bus route, the stops' ID number, the scheduled and actual arrival or departure time, and whether the bus stop is a start point, midpoint, or endpoint. The other dataset that we utilized is the "MBTA Bus Routes and Stops" [11] provided by the Massachusetts Department of Transportation. This public data source provided the specific longitude and latitude coordinates of the stops along with the stop ID number and name. We combine these two data frames together through the matching IP of the Stop Identification number. We then used such a merged dataset for further modeling and analysis.

2. Detroit DDOT

We used the "DDOT Bus Stops" [12] and "DDOT Bus Routes" [13] datasets provided by the City of Detroit as a baseline for analysis, these two datasets together provide information such as the longitude and latitude of bus stops, bus routes associated with a particular stop, direction of the bus, (Eastbound/Westbound/Northbound/Southbound) as well as the distance of a particular route. In addition, due to the lack of an organized dataset regarding the specific timetable and records of individual buses, we instead directly extracted the bus timetable for every route by examining its schedule published by the Detroit Department of Transportation [14].

3. Philadelphia SEPTA

Similar to the situation in Detroit, to model for the Philadelphia metropolitan region, we first extract the bus routes and stop information from the SEPTA GIS Data platform [15], providing us the information on bus routes, stops, directions, and the length in miles. Then due to the lack of an organized timetable dataset, we again refer to the bus schedule and its frequency throughout a typical day [16].

3 Ecological Consequence Model

The obvious benefit of a city switching from a fleet of diesel buses to one with e-buses is that there is an environmental consideration. E-buses do not produce any greenhouse gas emissions and do not cause noise pollution like diesel buses. We used emissions and noise pollution data and applied it to the number of diesel buses in the Boston bus fleet.

3.1 CO2 Emission

According to the Congressional Budget Office, the average carbon dioxide emission from one diesel bus per mile is 0.39 pounds[17]. Then we converted the units from pounds to miles.

$$0.39 \text{ lb} \cdot 453.6 = 176.9 \text{ g}$$

The equation below calculates the amount of carbon dioxide emitted from one fleet of diesel buses over the course of 10 years.

$$\text{10-year Emission(tons)} = 176.9 \cdot D_0 \cdot \left(\frac{1}{1000}\right)^2 \cdot 365 \cdot 10 \tag{1}$$

Where 176.9 is the amount of emission for one per mile in a day, D_0 is the total distance of all buses travel in a day. $\frac{1}{1000}$ is multiplied two times to convert the unit from grams to tons, and 365 days and 10 years are multiplied to figure out the 10-year emission.

Finally, we used the equation 1 to calculate the emission in Boston. We substituted 120220 miles to D_0 since that is the total distance of all buses traveling in one day in Boston.

$$\begin{aligned}\text{Boston Emission(tons)} &= 176.9 \cdot 120220 \cdot \left(\frac{1}{1000}\right)^2 \cdot 365 \cdot 10 \\ &= 77624.25\end{aligned}$$

In conclusion, the carbon dioxide emission will be reduced by 77624.25 tons if Boston changed to full electric bus system for 10 years, since BEBs don't emit any carbon dioxide.

3.2 Noise Pollution

The equation below converts decibels, which measure sound intensity, to sound intensity per square meter. This allows us to combine the sound intensities of all the e-buses into one variable and the sound intensities of the diesel buses into another and compare them. The sound amplitudes (DB) for e-buses is 65 decibels and 82.5 decibels for diesel buses[18].

$$\text{I} = 10^{\frac{DB}{10}} \cdot 10^{-12} \tag{2}$$

Equation 3, below, takes the combined sound intensity of a fleet of e-buses (N_e) and the sound intensity of a fleet of diesel buses (N_d) for the city of Boston and compares them by determining the percent drop of noise pollution by going electric.

$$\text{\% of noise pollution abated} = (1 - \frac{N_e}{N_d}) \cdot 100 = 98\% \tag{3}$$

As expected, the noise pollution produced by a fleet of e-buses is 98% lower than that of diesel buses for the city of Boston.

4 Financial Implications Model

4.1 Introduction

In spite of falling battery prices and lower operational and maintenance costs, transitioning to a fully electric fleet still comes with a hefty cost that is determined by a myriad of factors, including the number of chargers and their locations and the size of the BEB fleet. However, It is critical to understand that some of those factors changes with each metropolitan area. In other words, the transit electrification plan for one metropolitan area could look vastly different than another. In this financial implications model, we aim to transform location-specific specifications into an optimal electrification plan.

4.2 Model Overview

In order to quantify the aforementioned factors, we look to a constrained optimization model based on two independent variables: the number of active chargers and the number of (additional) BEBs. For this constrained optimization model there are two primary components: a distance constraint and a price optimizer. The distance constraint will be a mathematical expression that will ensure that our BEB fleet will be able to run throughout the day, or in other words, have enough energy for the required mileage for a day's work. The price optimizer will be another mathematical expression that applies both independent variables to obtain a cost over a 10 year period. For this model, we utilized a Lagrange Multiplier Method for Constrained Optimization to connect the distance constraint and price optimizer to produce an optimal combination of number of active chargers and the number of (additional) buses. Furthermore, other numbers besides the two independent variables can be determined by a metropolitan transit system's data.

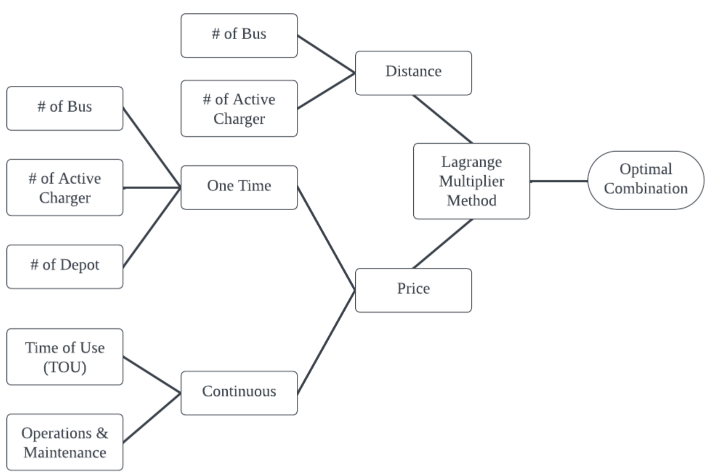


Figure 1: Financial Implications Model Overview Diagram

Figure 1 shows the flow of the model, starting from sub-factors that ultimately flows to the Lagrange Multiplier Method to create an optimal combination. This diagram can be referenced throughout the model for a full picture.

4.3 Optimal Active Charger Location Selector

In any electric transit system, because BEBs might not be able to run all day due of its limited range, active charging is a viable option to charge the BEB at bus stops and/or terminals to increase daily mileage needed to fulfill a day's work. However, transit authorities must take careful consideration of where to place these chargers based on a simple question: where should they place an active charger so that it's the most advantageous?

A succinct method to quantify "most advantageous" is to refer to the "number of visits made to a particular bus stop or terminal," or V_i, where i is the index of stops from $i \in \{1, 2, 3, \ldots q\}$, which can be simply modeled by the following formula:

$$V_i = \sum_{j=1}^{p} (\text{Number of Visits})_j \tag{4}$$

Where, j is the index of routes ranging from $j \in \{1, 2, 3, \ldots q\}$. From there we create a ranking of V_i, rearranging its index i to correspond with its rank by total number of visits with $V_i = 1$ having the greatest number of visits and $V_i = q$ having the lowest.

To illustrate this with an example, refer to Boston's V_i rankings shown in table 5.

Table 5: Boston Metropolitan Area Bus Stop Ranked by Number of Visits Per Day

Longitude, Latitude	Ranking i	Number of Visits, V_i
(-71.08395, 42.32972)	1	1988
(-71.11435, 42.30074)	2	914
(-71.11354, 42.30045)	3	888
(-71.06394, 42.28418)	4	841
(-71.00519, 42.25207)	5	667
⋮	⋮	⋮

From this, to choose optimal locations for chargers, we choose in chronological rank order, choosing $i = 1$ first and $i = q$, so we can add most value into the system.

4.4 Distance Constraint

As stated in the Model Overview section (section 4.2), one of the parts to use for the Lagrange Multiplier Method for constrained optimization is the distance constraint based on the number of active chargers (n) and the number of additional buses (b).

In essence, our aim is to construct a model for the following inequality:

$$\text{Distance covered by full BEB fleet} \geq \text{Distance covered by current fleet of buses} \tag{5}$$

Where "Distance covered by full BEB fleet", $D(n, b)$, can be modeled with subsequent equation (refer to constants in section 2.3):

$$D(n, b) = (b + b_0)\,(B_R)\,DOD + V(n) \cdot A_{kW} \cdot T \cdot \frac{B_R}{B_C} \tag{6}$$

In equation 6, $(b + b_0)(B_R)DOD$ is how many miles can be covered the total number of BEBs in a day from the new fleet $(b + b_0)$ if each BEB uses its allocated range $B_R \cdot DOD$. And $V \cdot A_{kW} \cdot T$ calculates the amount of energy in kWh based on the total number of visits V of all stops combined in a day; finally, $\frac{B_R}{B_C}$ converts that amount of energy into miles.

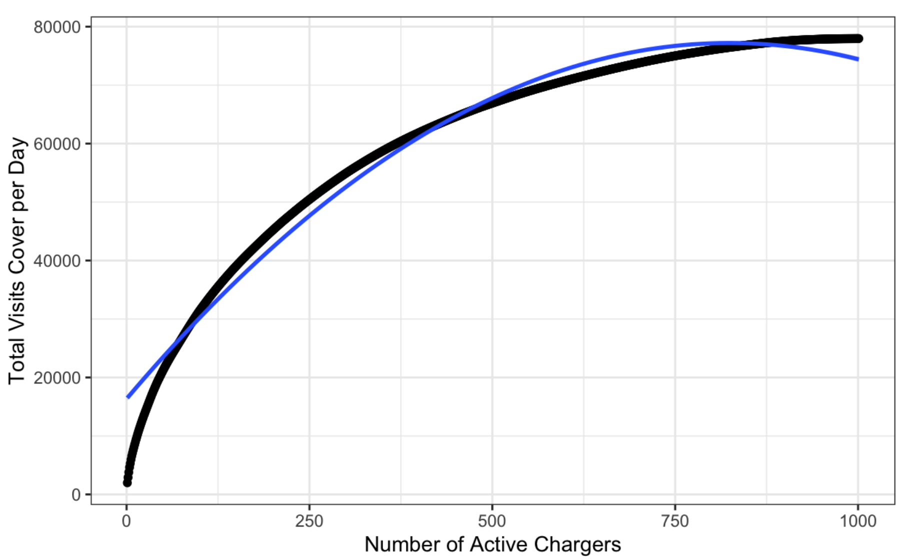
Next, we mutate V to get it in terms of the number of active chargers in the metropolitan area, which is achieved by a quadratic regression.

$$V(n) = An^2 + Bn + C \tag{7}$$

In which $V(n)$ is the sum of visits for the top n stops (based on ranking mentioned in section 4.3, mathematically represented as

$$V(n) = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n}\sum_{j=1}^{p} (\text{Number of Visits})_j \tag{8}$$

where i and j is the index for bus stops and routes respectively. An example regression performed for Boston is shown in figure 4.4. That regression produces the constants A, B, and C for equation 7.



Substituting equation 7 into equation 6, we produce the equation 9 for the distance constraint.

$$D(n,b) = (b+b_0)\,(B_R)\,DOD + \left(An^2 + Bn + C\right) \cdot A_{kW} \cdot T \cdot \frac{B_R}{B_C} \tag{9}$$

4.5 Quantification of Price

In this section, we will build the second part of the Lagrange Multiplier Method set up, the price equation. In this scenario, the multitude of cost factors can be arranged into categories one-time costs and continuous costs listed below:

One-time costs

1. BEB Price of Purchase
2. Active Charger Price of Purchase
3. Active Charger Price of Installation
4. Depot Charger Price of Purchase
5. Depot Charger Price of Installation

Continuous costs

1. BEB Price of Operations and Maintenance
2. Active Charger Price of Operations and Maintenance
3. Depot Charger Price of Operations and Maintenance
4. Time of Use Policy - Active Charger Energy Consumption Costs
5. Depot Charger Energy Consumption Cost

To model the one-time costs we multiply those constants by its corresponding quantitiy/variable, which is illustrated in the equation 10.

$$\text{One-time Costs} = B_P(b+b_0) + (A_{P,P} + A_{P,I})n + (D_{P,P} + D_{P,I})b_0 \tag{10}$$

The first addition factor, second addition factor, and third addition factor correspond to the BEB fleet total purchase price, total active charger purchase and installation cost, and total depot charger purchase and installation cost, respectively.

Specifically for the total cost of depot chargers, we decided that the number of depot chargers should be equivalent to the base number of buses b_0 because most transit authorities conduct it this way[19]. However, in the same source[19], it states that some transit authorities are considering a method to minimize the number of depot chargers through charger sharing. While building a model to optimize the schedule of when buses should be charged in order to minimize the number of depot chargers, given the time complexity and the lack of individual bus data, we determined it was more feasible to stick with a constant equivalent to the base number of buses. Furthermore, additional buses will not need depot chargers of their own because they are expected to charge when the base number of buses are on the road (b_0) as per assumption 5.

Next, we perform a similar methodology to model the continuous costs, which are measured in

\$USD/year. In the continuous cost equation 11, each addition factor corresponds to the continuous cost list above, in the same order.

$$\begin{aligned}\text{Continuous Costs} = &\left(B_{P,OM} \cdot \frac{D_0}{b_0} \cdot 365\right) \cdot (b + b_0) + A_{P,OM} \cdot n + D_{P,OM} \cdot b_0 \\ &+ (\text{TOU} + 0.0357 \cdot (B_C \cdot DOD \cdot (b + b_0)) \cdot 365\end{aligned} \tag{11}$$

Specific constants can be looked up in table 4 and 3. More specifically, $0.0357 \cdot B_C \cdot DOD \cdot (b+b_0)$ is the number of kWh that every BEB (total of $b+b_0$) needs to charge each day, assuming that they use up their allocated DOD space, multiplied by the charging price 0.0357 \$USD/kWh, the cost rate during the nighttime. Next, because the TOU (Time of Use) Policy dictates the energy consumption rate at different times of the day (shown in table 6), we will need to split up energy consumption by different hour groups to calculate daily energy consumption costs used by active chargers.

Table 6: Average National TOU Rates[20]

Hours of Day	Price per Kilowatt Hour
10 PM - 5 AM	\$0.0357/kWh
5 AM - 7 AM & 9 AM - 10 PM	\$0.0468/kWh
7 AM - 9 AM	\$0.1652/kWh

Energy consumption is based on the number of visits to active chargers, and breaking those energy consumption by hourly intervals and applying the time of use constants, we achieve the following relationship,

$$\begin{aligned}\text{Time of Use} = &\ 0.0468 \cdot A_{kW} \cdot T(\sum_{h=5}^{14} V_h + \sum_{18}^{21} V_h) + 0.0357 \cdot A_{kW} \cdot T(\sum_{22}^{23} V_h + \sum_{0}^{4} V_h) \\ &+ 0.1652 \cdot A_{kW} \cdot T \sum_{15}^{17} V_h\end{aligned} \tag{12}$$

Where $B_C \cdot DOD$ is the number of kWh that a bus needs to charge on average, assuming that a bus finishes its day after using 80% Depth of Discharge, or used down to 20% battery life. And where V_h is the number of visits made to all active chargers in a specified hour h where $h \in \{1, 2, 3, \ldots 24\}$. To get V_h in terms of the number of active chargers in a metropolitan area, we perform a quadratic regression model very similar to the regression model described in section 4.4. However, for the time of use, instead of performing one total quadratic model as conducted in equation 13, we execute 24 different quadratic regressions for each of the hourly time intervals, or mathematically written as,

$$V_h = a_h n^2 + b_h n + c_h \tag{13}$$

Substituting equation 13 into equation 12, we produce the equation:

$$\begin{aligned}\textbf{Time of Use} = A_{kW} \cdot T\Big(&0.0468\left(a_{G_1}n^2 + b_{G_1}n + c_{G_1}\right) + 0.0357\left(a_{G_2}n^2 + b_{G_2}n + c_{G_2}\right)\\ &+ 0.1652\left(a_{G_3}n^2 + b_{G_3}n + c_{G_3}\right)\Big)\end{aligned} \tag{14}$$

Where each group, G, represents the hours for each Time of Use hour group;
$G_1 \in \{5, 6, 7, \ldots 14\} \bigcup \{18, 19, 20, 21\}$, $G_2 \in \{22, 23\} \bigcup \{0, 1, 2, 3, 4\}$, and $G_3 \in \{15, 16, 17\}$.

Finally, we amalgamate the one-time and continuous cost equations into one price equation, equation 15, although with one simple add on, the multiplication of O_c on the continuous cost equation to calculate the ownership cost over a 10 year period, allowing us to properly add the one-time and continuous costs where units permit.

$$\begin{aligned}P(n,b) =& \left[\left(B_{P,OM} \cdot \frac{D_0}{b_0} \cdot 365\right) \cdot (b + b_0) + A_{P,OM} \cdot n + D_{P,OM} \cdot b_0\right]\\ &+ \Bigg[(A_{kW} \cdot T\Big(0.0468\left(a_{G_1}n^2 + b_{G_1}n + c_{G_1}\right) + 0.0357\left(a_{G_2}n^2 + b_{G_2}n + c_{G_2}\right)\\ &+ 0.1652\left(a_{G_3}n^2 + b_{G_3}n + c_{G_3}\right)\Big) + 0.0357 \cdot B_C \cdot DOD(b + b_0)\Bigg] \cdot 365 \cdot O_c\end{aligned} \tag{15}$$

For clarity, the constants and variables in the first large bracket pair is the fixed cost the the second large bracket pair for the continuous costs; $P(n, b)$ is in units of \$USD.

4.6 Lagrange Multiplier Method for Price Optimization

As mentioned in 4.2 and shown in figure 1, the final step is to obtain a optimization of active chargers, n, and additional buses , b, through the Lagrange Multiplier Method. This method is suitable for this situation for a selection of reasons: (1) its ability to optimize a multivariable function, in our case $P(n, b)$, (2) applying constraints on that multivariable function so we can ensure it meets our distance requirement (3) its relative ease of computation and easy diagnosis.

To begin, the Lagrange Multiplier can be set up with a mathematical representation of our goal:

$$\underset{n,b}{\arg\min}\ P(n,b)$$

$$\begin{aligned}\text{Subject to: } & D(n,b) \geq D_0\\ & n \geq 0\\ & b \geq 0\end{aligned} \tag{16}$$

To begin the optimization, we can model the above optimization-constraint relationship using a Lagrange function that integrates both equation 15 and 9, shown in equation 17.

$$\mathcal{L}(n,b,\lambda) = P(n,b) - \lambda(D_0 - D(n,b)) \tag{17}$$

Then, we set the gradient of the Lagrangian equal to the zero vector in equation 18, setting up a systems of equation type of solution.

$$\nabla\mathcal{L}(n,b,\lambda) = \begin{bmatrix} \frac{\partial\mathcal{L}}{\partial n}(n,b,\lambda) \\ \frac{\partial\mathcal{L}}{\partial b}(n,b,\lambda) \\ \frac{\partial\mathcal{L}}{\partial \lambda}(n,b,\lambda) \end{bmatrix} = \mathbf{0} \tag{18}$$

where, substituting the price and distance equations into equation 17 and taking each partial derivative, we get:

$$\begin{aligned} \frac{\partial\mathcal{L}}{\partial n}(n,b,\lambda) =& A_{P,OM} + 365 \cdot O_c \cdot A_{kW} \cdot T(0.0468a_{G_1} + 0.0375a_{G_2} + 0.1652a_{G_3})(2n+1) \\ &+ \lambda(A_{kW} \cdot T \cdot \frac{B_R}{B_C})(2An + B) = 0 \end{aligned} \tag{19}$$

$$\frac{\partial\mathcal{L}}{\partial b}(n,b,\lambda) = B_{P,OM} \cdot \frac{D_0}{b_0} \cdot 365 + D_{P,OM} + 0.0357 \cdot B_C \cdot DOD \cdot + \lambda(B_R \cdot DOD) = 0 \tag{20}$$

$$\frac{\partial\mathcal{L}}{\partial \lambda}(n,b,\lambda) = -D_0 + DOD \cdot B_R(b + b_0) + A_{kW} \cdot T \cdot \frac{B_R}{B_C}(An^2 + Bn + C) = 0 \tag{21}$$

In each of these equations, refer to variables, parameters, and constants section (section 2.3) for numerical numbers. Ultimately, solving for n and b in this system of three equations will provide the optimal number of active chargers (n^*) and number of additional buses (n^*).However, from a computational perspective, given that there is a myriad of data to compute and the hand-solving complexity of this Lagrangian set up is difficult, we utilized MATLAB's *fmincon* function to compute the Lagrangian to deduce the solution. Note, MATLAB does not produce integers, so a simple ceiling function has been applied.

$$\arg\min_{n,b} P(n,b) = \lceil n^* \rceil, \lceil b^* \rceil \tag{22}$$

To get the corresponding cost:

$$10 \text{ Year Cost} = (1 - F) \cdot P(\lceil n^* \rceil, \lceil b^* \rceil) \tag{23}$$

where F is the percentage of external funding coverage. In our location-specific sections (section 5), in addition to the 10 Year Cost, we can also obtain a cost breakdown.

4.7 Strengths and Weaknesses

4.7.1 Strengths

1. The interconnectedness of the number of active chargers and the number of additional buses. Rather than building two separate models for active chargers and buses, our model takes both into consideration at the same time through the use of Lagrange Multiplier Method for constrained optimization. Furthermore, the use of optimization methodology ensures we are not buying too many active chargers or additional buses, but rather the sweet spot.

2. A location oriented method of finding the optimal active charger locations described in section 4.3. While a big part of this model is "how many active chargers a metropolitan area should have," the location aspect enables us to produce more detailed information. Analyzing real transit data, our model was able to select locations with most visits in a day, and thus locations where multiple buses can share an active charger. Our model being able to select locations based off these criterias is a great selling point to city planners, knowing that the active chargers they would purchase are being used to the fullest extent.

3. Flexibility of parameters and constants. While this paper has not considered a multitude of BEB and charger types with different prices and specifications (a weakness explored in the subsequent section), this model is adaptable to new constants and if time were to permit, it would be possible to apply this model to all sorts of locations and BEB and Charger specifications.

4.7.2 Weaknesses

1. Because our model's constants are for the most part based off of federal averages, our model does not capture the opportunity to explore different BEB and charger model types. If we were to integrate a catalog of different models within the optimization model, we would be able to provide stronger guidance to these metropolitan areas and give more specific information of what types of BEBs and chargers would be optimal to purchase.

2. The number of depot chargers being constant and equivalent to the base number of buses. Our model overlooks the ability to optimize the number and location of depot chargers.

3. As mentioned in model overview, section 4.2, a limitation of our model is not being able to construct a model from the route-specific scope. Being able to access and manipulate route-specific information, if data is provided, will allow our model to ensure that all routes are meeting their needs, optimally. More specifically, if route-specific information is available, our Lagrangian model could be based off of every route and provide optimal solutions for every route, mathematically expressed as follows:

$$\mathcal{L}(n_1, n_2, \dots n_p, b_1, b_2, \dots b_p, \lambda_1, \lambda_2, \dots\lambda_p) = P(n_1, n_2, \dots n_p, b_1, b_2, \dots b_p) - \sum_{i=1}^{p} \lambda_i((D_0)_i - D(n_i, b_i)) \tag{24}$$

5 Application to Metropolitan Areas

5.1 Greater Boston, Massachusetts, United States

According to the US census, the Greater Boston Area is ranked as the 11th largest metropolitan area, with a population of around 4.9 million people[21]. Correspondingly, its transit system is the lifeblood of its bustling city, carrying nearly half a million people a day (metro and bus)[22]. With this in mind, Boston is a great candidate for a transit electrification plan as its city continues to grow and its dependency on buses are increasing.

5.1.1 Application of Financial Implications Model

Using the data that we described in section 2.4, we determined some necessary numbers to provide into the Lagrange Equation. Ultimately, running the Lagrange with those new constants, it is determined that the optimal number of active chargers is 134, and the optimal number of additional buses is 120, added to the 980 base number of buses. Based on these quantities, the breakdown of costs is calculated in table 7

Table 7: Metropolitan Boston Transit Electrification Cost Breakdown

Cost Category	Cost
BEB Fleet Total Price of Purchase	\$907 Million
BEB Fleet Total Price of Operations and Maintenance	\$17.8 Million/year
Active Charger Total Price of Purchase	\$66 Million
Active Charger Total Price of Installation	\$27 Million
Active Charger Total Price of Operations and Maintenance	\$2.4 Million/year
Depot Charger Total Price of Purchase	\$49 Million/year
Depot Charger Total Price of Installation	\$17 Million/year
Active Charger Total Cost of Electricity Consumption	\$1,998/day
Depot Charger Total Cost of Electricity Consumption	\$11,027/day
TOTAL OVER 10 YEARS RANGE (considering external funding F from 50% to 0%)	\$1.47 to \$2.93 Billion Dollars

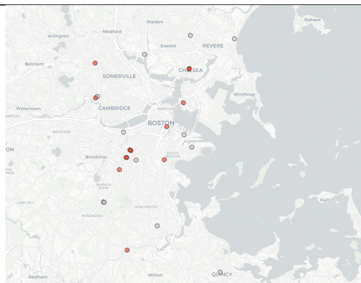
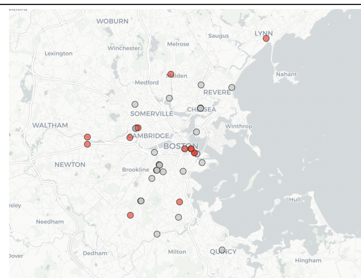
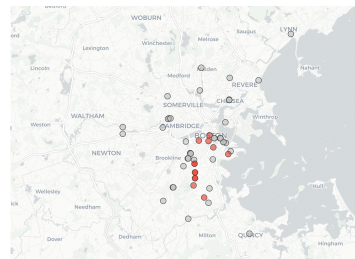
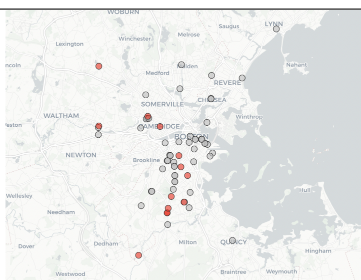
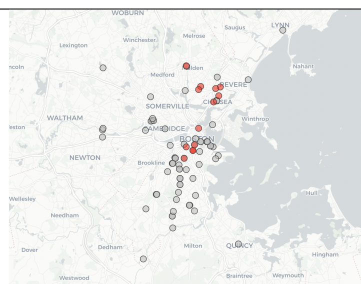
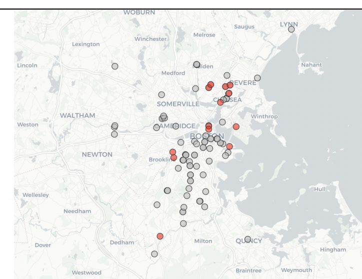
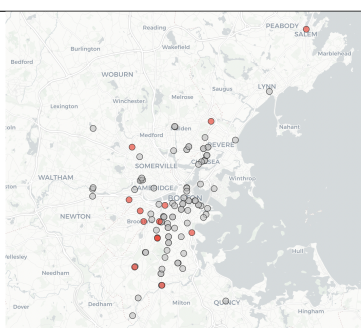
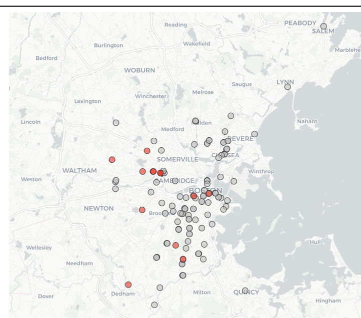
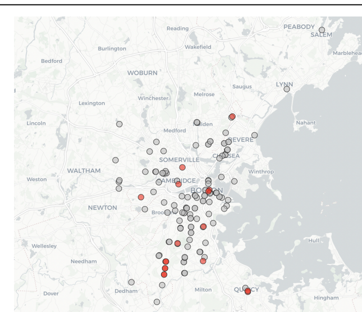
5.1.2 10 year plan

In the lens of a transit authority, to fully transition to an E-Bus transit to be feasible, the costs of implementation should be spread out throughout the 10 years. A simple yet effective plan would be for the MBTA to roll out 110 BEBs, 98 depot chargers, and 13 active chargers each year and adjust the number of diesel vehicles on the road.

Our estimates show that in order to achieve this yearly implementation, the MBTA needs to invest in 90.7-,6.6-, and 9.1-million for BEBs, depot chargers, and active chargers respectively each year, for a total of 106.4 million dollars of annual investment. In fact, this figure is not far from the MBTA's current investment plan of 100 million dollars a year[23].

For the active chargers, we recommend a structured year-by-year plan of where to place active chargers. Recall section 4.3, where we developed a model that ranks active charger location placement based on number of visits; that ranking model is used to make the informed plan in table 8.

Table 8: MBTA Yearly Additions of Active Chargers

2025	2026	2027
		
2028	2029	2030
		
2031	2032	2033
		

5.2 Greater Detroit, Michigan, United States

The Detroit metropolitan area is a prominent urban center, ranking among the top metropolitan areas in the United States. With a population of approximately $4.3 million[24], the greater area of Detroit, like Boston, also has a bus system, DDOT, that keeps its city moving.

5.2.1 Application of Financial Implications Model

Likewise to Greater Boston, substituting location-specific constants (D_0 and b_0) and the regression coefficients for sub-equation **??** and sub-equation 14, the solution to the Lagrange Multiplier 139 additional buses onto the 291 current bus fleet size, for a total of 423 BEBs. It is interesting to note, however, that the optimal combination outputted zero active chargers. While this number may seem as a anomaly at first, when looking over the map of the DDOT, it was comparably much smaller by mile coverage than the MBTA, which is a potential explanation of why buses did not need extra charge along the way because it would be more economical to have other buses standing by. For DDOT's cost breakdown, refer to table 10.

Table 9: Metropolitan Detroit Transit Electrification Cost Breakdown

Cost Category	Cost
BEB Fleet Total Price of Purchase	\$354 Million
BEB Fleet Total Price of Operations and Maintenance	\$7.3 Million/year
Depot Charger Total Price of Purchase	\$15.3 Million
Depot Charger Total Price of Installation	\$5.2 Million
Depot Charger Total Cost of Electricity Consumption	\$4,240/day
TOTAL OVER 10 YEARS RANGE (considering external funding F from 50% to 0%)	\$223 to \$446 Million

5.2.2 10 year plan

Our recommendation for Greater Detroit, through the DDOT, is to purchase and install around 3-4 active chargers, 29 depot chargers, and 42 BEBs every year, which amasses to around \$38.7 Million in yearly investment.

5.3 Greater Philadelphia, Pennsylvania, United States

The largest out of the three metropolitan, the greater Philadelphia region has a population of around 6.5 million people[25] and its SEPTA transit system is larger as well. Ranking 6th in the US by ridership, the SEPTA system carries 142 million people annual during pre-COVID times.

5.3.1 Application of Financial Implication Model

Repeating the same Lagrange Multiplier process that we applied for Greater Boston and Greater Detroit for Greater Philadelphia, our model obtains the numbers: 153 Active Chargers, 1400 Depot

Chargers, and 1820 BEBs. The cost breakdown of Greater Philadelphia's electrification of public bus system is tabulated in table 10.

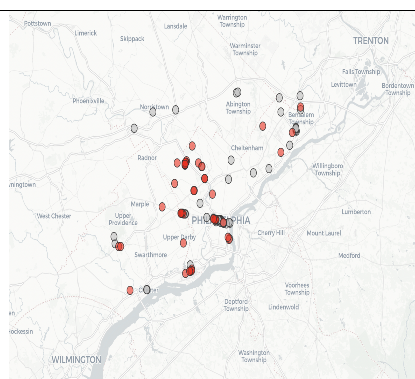
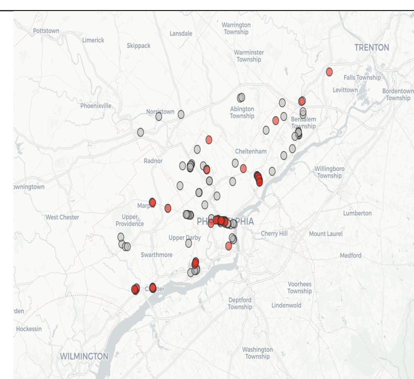
Table 10: Metropolitan Philadelphia Transit Electrification Cost Breakdown

Cost Category	Cost
BEB Fleet Total Price of Purchase	\$1.5 Billion
BEB Fleet Total Price of Operations and Maintenance	\$21.2 Million/year
Active Charger Total Price of Purchase	\$75.8 Million
Active Charger Total Price of Installation	\$31 Million
Active Charger Total Price of Operations and Maintenance	\$2.7 Million/year
Depot Charger Total Price of Purchase	\$21 Million
Depot Charger Total Price of Installation	\$7.2 Million
Active Charger Total Cost of Electricity Consumption	\$2269/day
Depot Charger Total Cost of Electricity Consumption	\$18,244/day
TOTAL OVER 10 YEARS RANGE (considering external funding F from 50% to 0%)	1.96 to \$3.91 Billion

5.3.2 10 year plan

Our recommendation for Greater Philadelphia, is to purchase and install 15 active chargers, 140 depot chargers, and purchase 182 BEBs every year, which amasses to around \$169 Million in yearly investment. The specific arrangement of when and where to place which chargers are shown in table 11.

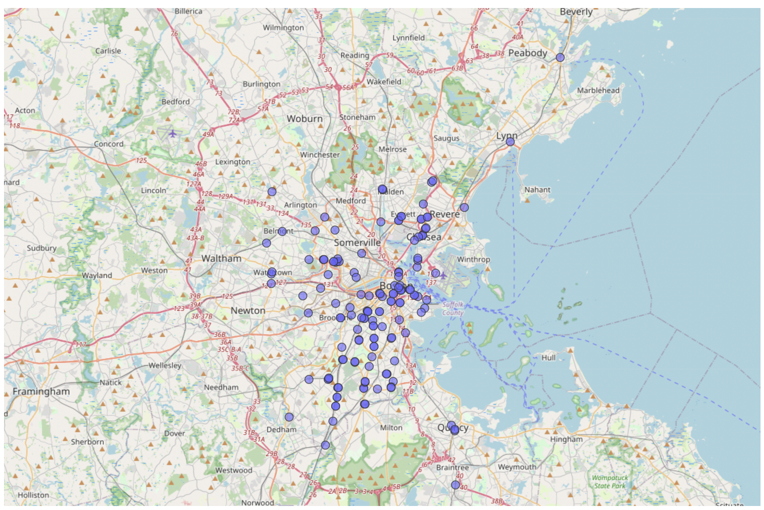
Table 11: SEPTA Yearly Additions of Active Chargers

2024-2026	2027-2029	2030-3033
		

6 One Page Letter

Dear Mr. Phillip Eng, CEO of the MBTA,

As you probably know well, pollution and climate change are pressing issues facing world leaders. As someone in charge of a large, metropolitan city's bus transportation program, you're uniquely positioned to help Boston move away from a dependence on fossil fuels and use cleaner, less-polluting electric buses. I have developed a 10-year plan to transition the MBTA into an all-electric fleet of buses. The plan calls for the purchase of 110 e-buses, 98 depot chargers and 13 active charging stations each year for ten years. This rollout will cost 106.4 million dollars every year. This investment will ensure that there are enough e-buses to replace the MBTA's fleet of diesel buses. This program will also build the infrastructure needed to support an all-electric bus fleet with active and depot chargers. Our recommendation includes that the active chargers be placed at stops where there is the most number of visits by buses. We have determined that the following locations on a map of the greater Boston area should have active charging stations:



Looking at the ecological impact of replacing diesel buses with e-buses, 77,624.25 tons of carbon dioxide emissions will be prevented over a ten-year period. Through this transition, the MBTA will help to address the poor air quality in Boston by moving towards carbon-neutral transportation methods. Also, transitioning to a fleet of e-buses would decrease the amount of noise pollution caused by all public buses collectively by 98 percent.

So, considering the environmental benefit of converting the MBTA's diesel bus fleet to e-buses, I recommend taking decisive action and investing in e-buses through our 10-year plan. It's vital that Boston invests in next-generation bus technology like BEB's to ensure its future prosperity as a forward-thinking, innovative city.

Sincerely,

Team 13849

7 Conclusion

In this paper, we have proposed a constrained optimization model using the Lagrange multiplier technique in order to evaluate the cost and feasibility of cities to completely transform to an electric bus fleet. The proposed optimization model was then applied to three major metropolitan areas - Boston, Detroit, and Philadelphia - under their unique transit network. The result provides local decision-makers with a detailed ten-year plan as to the cost of a full E-bus transformation, including fixed costs for the purchase of depot chargers, active chargers, and additional buses as well as continuous costs mainly related to the usage of electricity from the grid along with operation and maintenance. All the factors considered are processed in a way that matches reality as closely as possible which includes the simulation of the timetable, examination of routes and stops, and adjusting the electricity price following the local Time of Use policy. A unique feature of the model is the consideration of the individual bus stops and ranks them based on the amount of influence they could have on the operation of the entire transit network. Along with our optimization model, the paper also considers the less-monetary but still significant ecological consequences of transitioning to a fully electric fleet, two of the most significant ecological factors include a reduction in CO2 emission and lessened urban noise pollution.

There are several possible extensions to this study. First, due to the time frame of analysis, the cost of depreciation of electric buses, batteries, and charging stations has been all kept negligible, incorporating these factors into the model could extend the projection of the current ten-year plan into the future and achieve possible sustainable growth. Second, the state of charge/discharge has not been considered in this model. For common battery types like Lithium-ion, a lower state of charger (SOC) could decrease the voltage and hence increase electricity consumption, incorporating this factor will allow the model to better plan the buses' charging schedule. Third, the buses' timetable and route assignments have remained fixed, adjusting and optimizing these two factors could improve the overall efficiency of the electric bus fleet and reduce the overall cost. Of course, other considerations such as traffic, weather, and passenger demands are all significant elements that would add complexities, and improve the overall efficiency of the model.

8 Works Cited

References

[1] *Sources of greenhouse gas emissions*. (2023, October 5). United States Environmental Protection Agency. Retrieved November 14, 2023, from https://www.epa.gov/ghgemissions/sources-greenhouse-gas-emissions

[2] *What is electric bus?* (n.d.). BAE Systems. Retrieved November 14, 2023, from https://www.baesystems.com/en-us/definition/what-is-an-electric-bus

[3] Zientara, B. (2023, January 17). *What are time of use rates - and when is electricity cheapest?* Solar Reviews. Retrieved November 14, 2023, from https://www.solarreviews.com/blog/what-are-time-of-use-rates-and-when-is-electricity-cheapest

[4] *Depth of discharge: what it is and why it's important*. (2022, December 3). Foxtron. Retrieved November 14, 2023, from https://www.foxtronpowersolutions.com/depth-of-discharge/

[5] *Detroit department of transportation*. (n.d.). CPTDB Wiki. Retrieved November 14, 2023, from https://cptdb.ca/wiki/index.php/Detroit_Department_of_Transportation

[6] *Septa bus roster*. (2023, September 28). Philadelphia Transit Vehicles. Retrieved November 14, 2023, from https://philadelphiatransitvehicles.info/septas-bus-roster/

[7] Johnson, Caley, Nobler, Erin, Eudy, Leslie, & Jeffers, Matthew. (2022, June 1). *Financial Analysis of Battery Electric Transit Buses*. United States. Retrieved November 11, 2023, from https://doi.org/10.2172/1659784

[8] Boudreau, C. (2023, November 9). *See what the future of zero-emissions transit looks like in suburban Maryland*. Business Insider. Retrieved November 12, 2023, from https://www.businessinsider.com/clean-zero-emission-bus-transit-montgomery-county-maryland-2023-11

[9] *Electric bus pantograph up*. (n.d.). ABB. Retrieved November 12, 2023, from https://new.abb.com/ev-charging/pantograph-up

[10] *MBTA bus arrival departure times 2023*. (2023, May 9). GeoDOT Portal. Retrieved November 14, 2023, from https://mbta-massdot.opendata.arcgis.com/datasets/MassDOT::mbta-bus-arrival-departure-times-2023/about

[11] *MBTA bus routes and stops*. (2022, December 9). ArcGIS Hub. Retrieved November 14, 2023, from https://hub.arcgis.com/maps/massgis::mbta-bus-routes-and-stops-1/about

[12] *DDOT bus routes*. (2022, November 22). City of Detroit Open Data Portal. Retrieved November 14, 2023, from https://data.detroitmi.gov/datasets/detroitmi::ddot-bus-routes/about

[13] *DDOT BUS STOPS AND ROUTES 2014*. (2017, May 8). ArcGIS Hub. Retrieved November 14, 2023, from https://hub.arcgis.com/maps/259d33193b7044a48fbab295f20e404d/about

[14] *Ddot.info*. (n.d.). DDOT. Retrieved November 14, 2023, from https://ddot.info/

[15] *Bus stops (Fall 2023)*. (n.d.). Southeastern Pennsylvania Transportation Authority. Retrieved November 14, 2023, from https://gis-septa.hub.arcgis.com/search?tags=Bus

[16] LINES & ROUTES. (2023, November 14). SEPTA. Retrieved November 14, 2023, from https://www5.septa.org/travel/routes/

[17] Congressional Budget Office. (2022, December). *Emissions of carbon dioxide in the transportation sector*. https://www.cbo.gov/system/files/2022-12/58566-co2-emissions-transportation.pdf

[18] Stewart, W. H. (n.d.). *Noise pollution*. Edmonton Trolley Coalition. Retrieved November 14, 2023, from http://www.trolleycoalition.org/noise.html

[19] Lepre, N., Burget, S., & McKenzie, L. (2022, June). *Deploying charging infrastructure for electric transit buses*. https://atlaspolicy.com/wp-content/uploads/2022/05/Deploying-Charging-Infrastructure-for-Electric-Transit-Buses.pdf

[20] Electric vehicle time of use. (n.d.). CCEC. Retrieved November 14, 2023, from https://www.ccemc.com/EVTOU

[21] McDonald, D. (2020, December 25). *Greater Boston is no longer one of the country's 10 largest metro areas. How did that happen?* The Boston Globe. Retrieved November 14, 2023, from https://www.bostonglobe.com/2020/12/25/metro/greater-boston-is-no-longer-one-top-10-largest-metro-areas-how-did-that-happen/

[22] *Transportation facts*. (2015, August 14). Massachusetts Department of Transportation. Retrieved November 14, 2023, from https://www.mass.gov/doc/massachusetts-transportation-facts/download

[23] *Bus electrification*. (n.d.). Massachusetts Bay Transportation Authority. Retrieved November 14, 2023, from https://www.mbta.com/projects/bus-electrification

[24] *Detroit-Warren-Dearborn, MI metro area*. (n.d.). Census Reporter. Retrieved November 14, 2023, from https://censusreporter.org/profiles/31000US19820-detroit-warren-dearborn-mi-metro-area/

[25] *Meet greater Philadelphia's communities*. (n.d.). Select Greater Philadelphia. Retrieved November 14, 2023, from https://selectgreaterphl.com/doing-business/demographics/