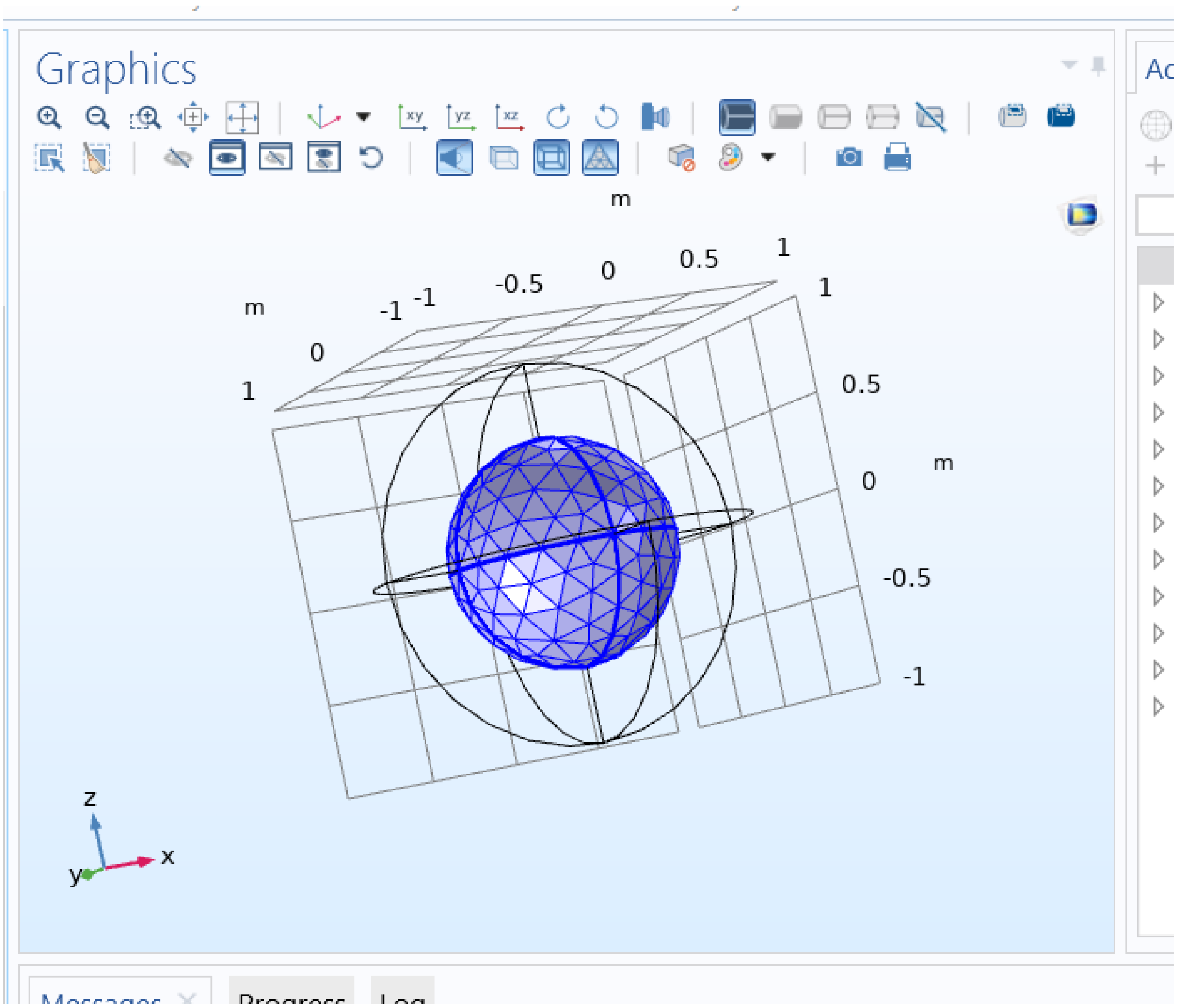
Electromagnetic Theory Home Work 10

Rita Abani 19244

10.05.2022

The outputs in PNG format and the corresponding COMSOL file can also be found on my GitHub repository : https://github.com/DRA-chaos/COMSOL-for-coursework

A few screenshots from the COMSOL designed simulation are as follows :



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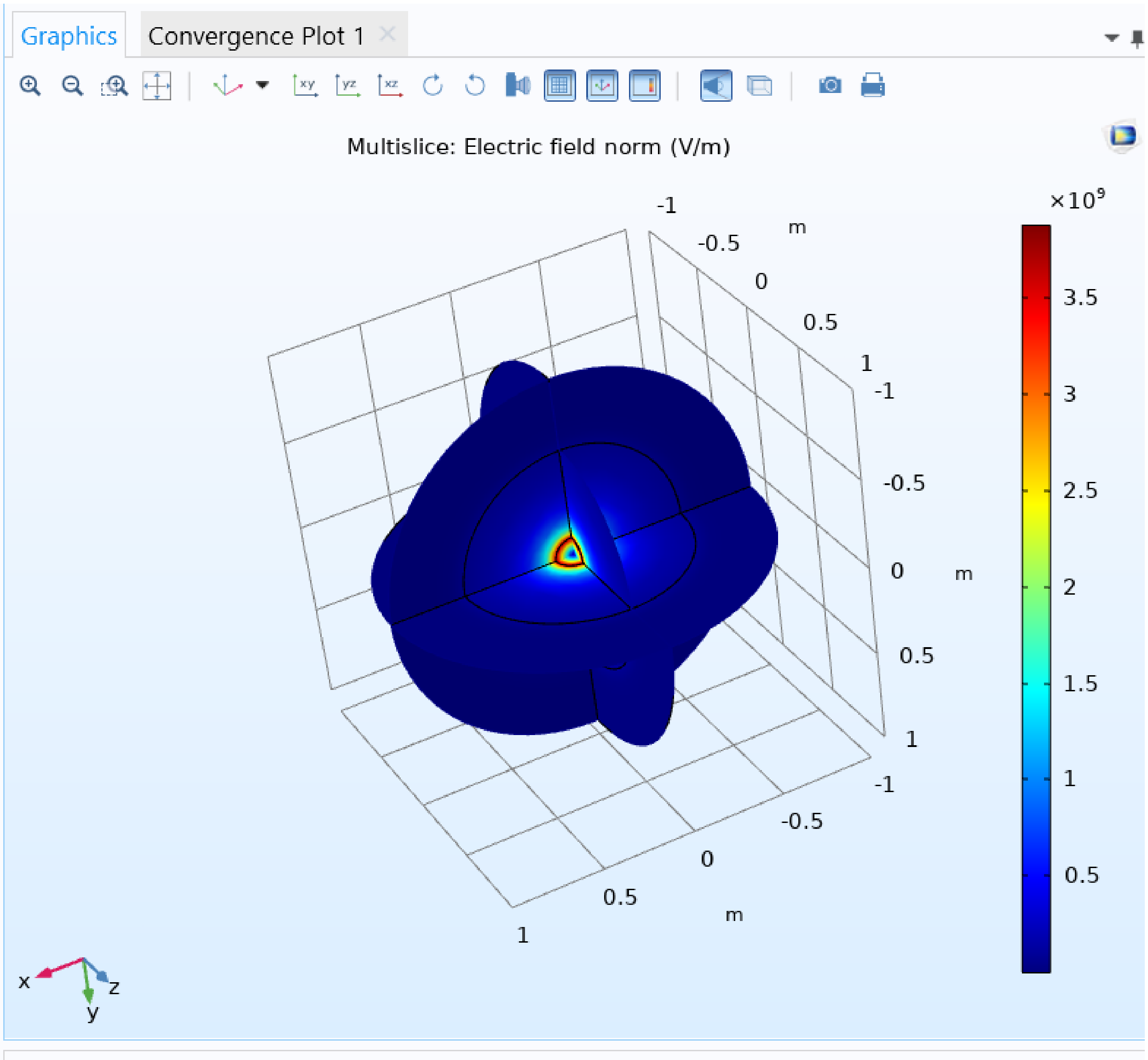

The time taken for solving the equations in 3-D space takes into account the degrees of freedom, the local GPU/ processor feedbacks and the runtime. The details of the run are enlisted below :

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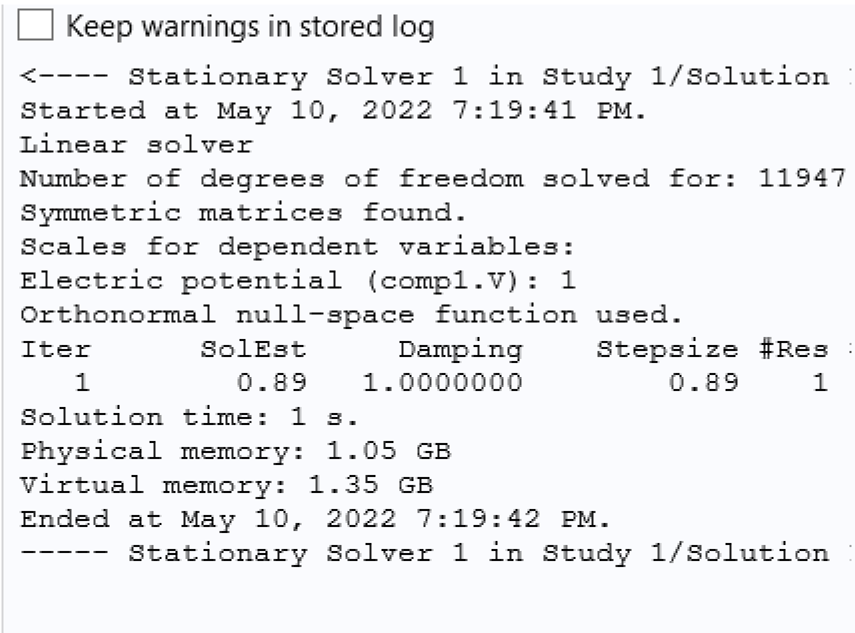
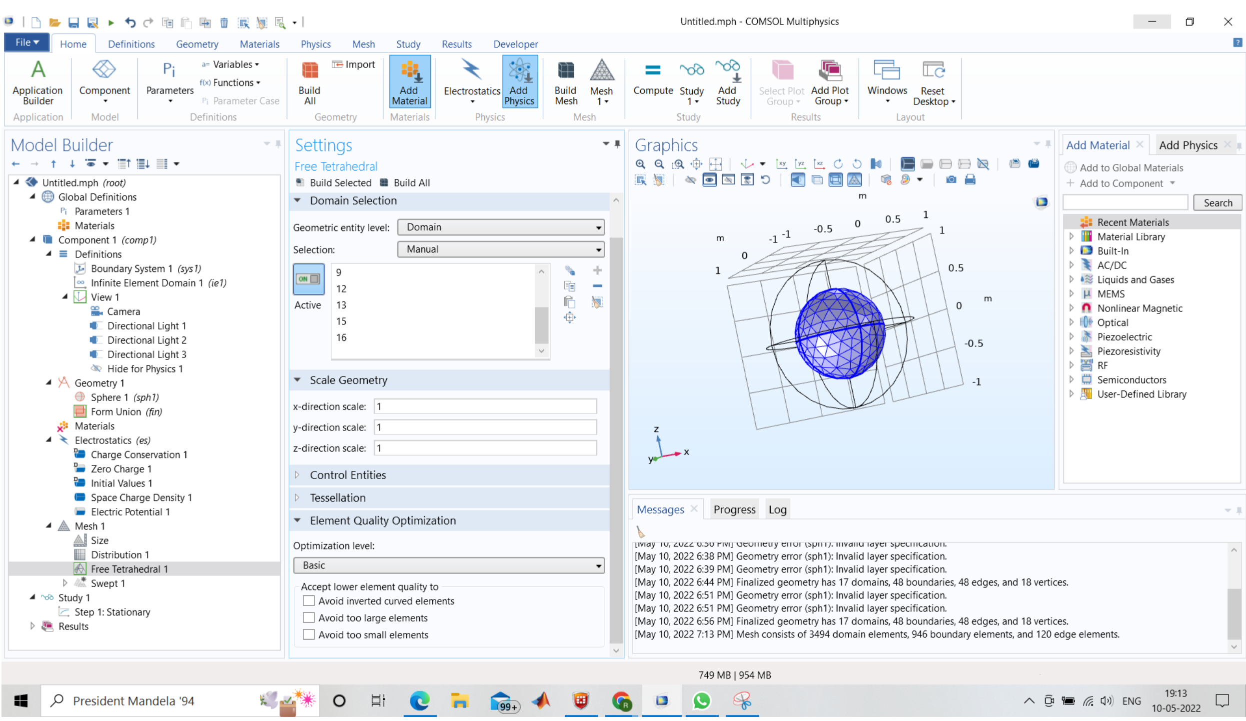


```
Keep warnings in stored log
<---- Stationary Solver 1 in Study 1/Solution
Started at May 10, 2022 7:19:41 PM.
Linear solver
Number of degrees of freedom solved for: 11947
Symmetric matrices found.
Scales for dependent variables:
Electric potential (comp1.V): 1
Orthonormal null-space function used.
Iter      SolEst     Damping    Stepsize #Res
   1        0.89   1.0000000        0.89    1
Solution time: 1 s.
Physical memory: 1.05 GB
Virtual memory: 1.35 GB
Ended at May 10, 2022 7:19:42 PM.
----- Stationary Solver 1 in Study 1/Solution
```



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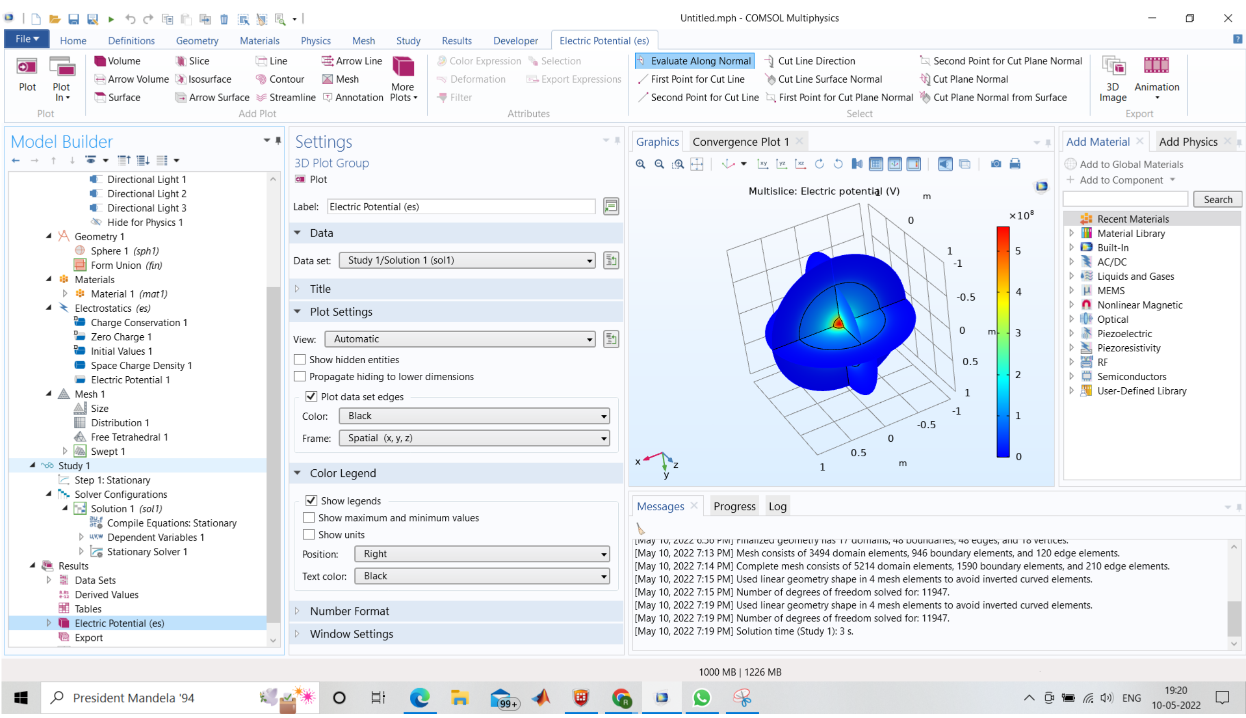

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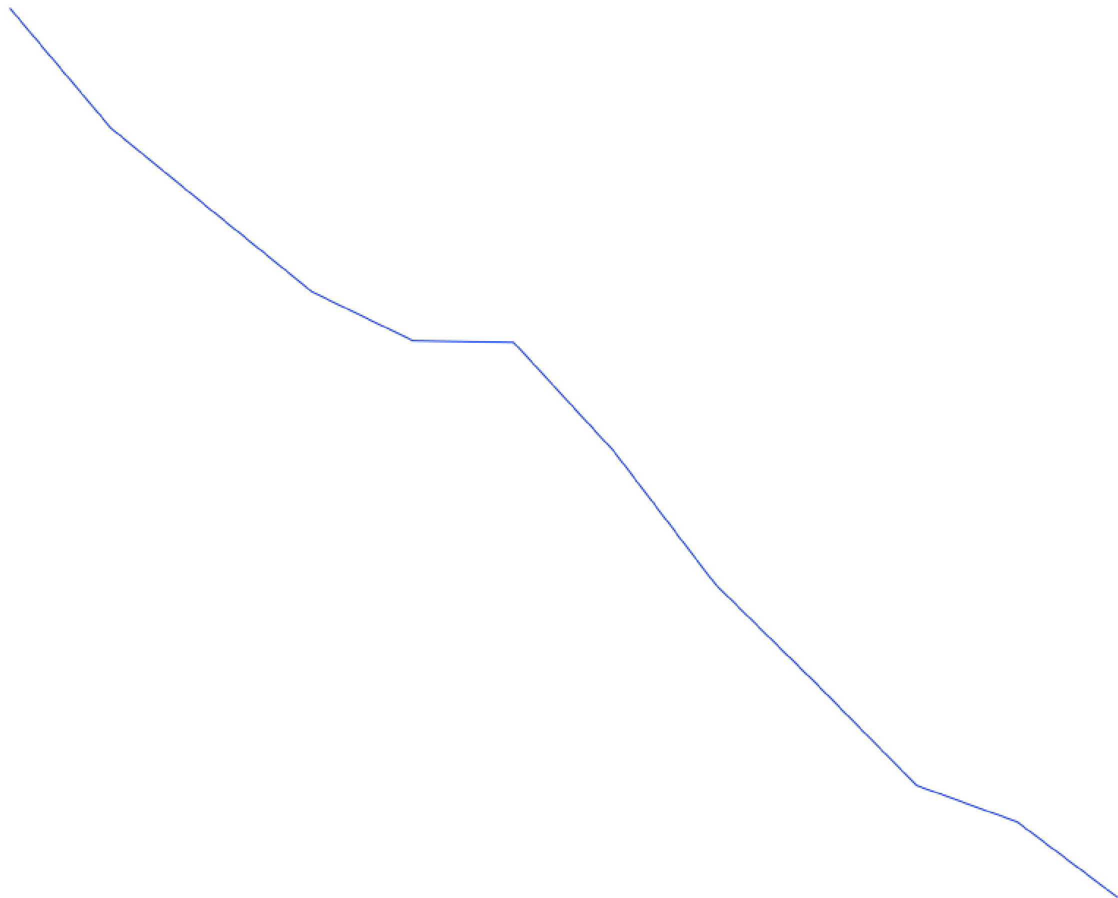

Conjugate gradients of the Electric Field obtained in 3D

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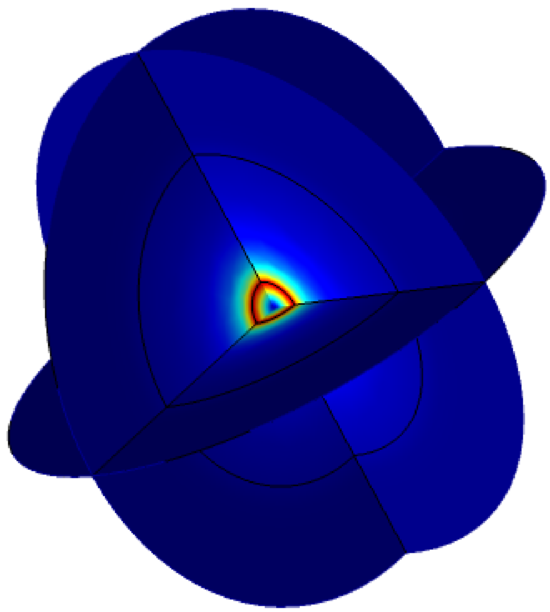

Graphics output of the Electric Field in 3D

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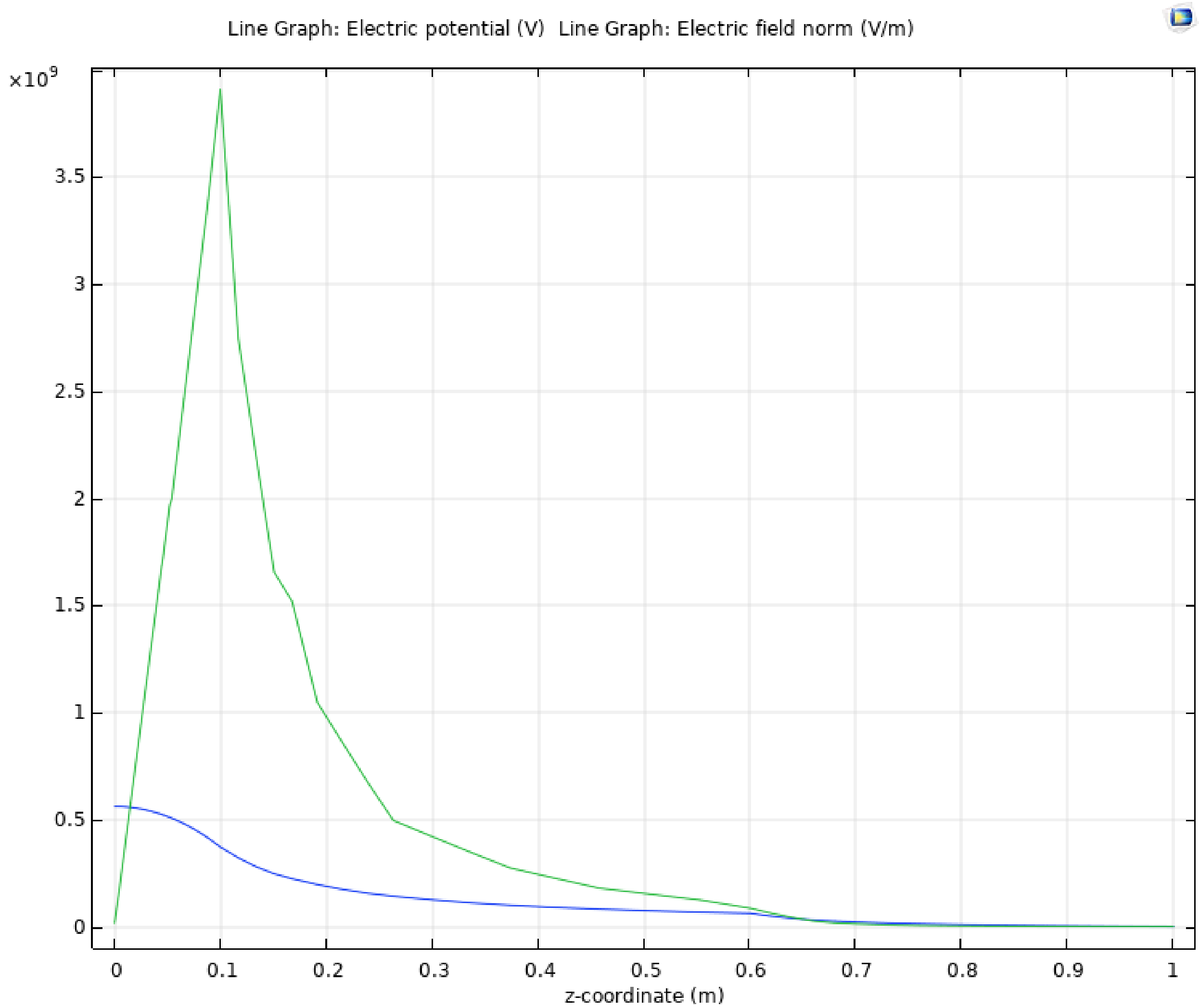
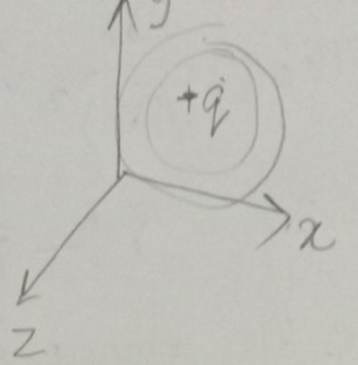


If we have a point charge $+q$ situated at x, y, z, we want to figure out the



electric field and potential at a point say (p, q, r) both theoreticaly and computationally (using COMSOL)

The error in these two realms is drawn from the fact that we may use different spherical shells with different radii

Ideally $E = \frac{kq}{r^2} = \frac{kq}{(x-p)^2 + (y-q)^2 + (z-r)^2}$

and $V = -\frac{kq}{r}$

Given a stationary charge distribution $\rho(\vec{r})$, we can calculate the electric field,

$$\bar{E}(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\Delta\hat{r}}{(\Delta r)^2} \rho(\bar{r}')\, d\tau$$

where $\Delta\bar{r} = \bar{r}' - \bar{r}$

but since the above expression is tougher to evaluate, we can first evaluate $V(\bar{r})$

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{1}{\Delta r} \rho(\bar{r}')\, d\tau' \quad \text{as } \bar{E} = -\bar{\nabla} V.$$

this can be generalized to 3 Dimensions as.

$$\bar{E} = -\left(\frac{\partial}{\partial x} V_x + \frac{\partial}{\partial y} V_y + \frac{\partial}{\partial z} V_z\right)$$

The Poisson's equaton is often a better approach to determine V

$$\bar{\nabla}^2 V = -\frac{\rho}{\varepsilon_0}$$

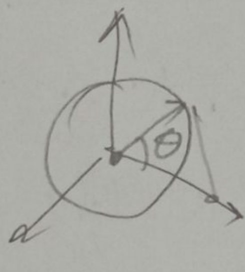
With appropriate boundary conditions,

$$\bar{\nabla}^2 V = 0 \text{ when } \rho = 0$$

We have,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

We can use the properties of solution of the 1D Laplace equation to be valid here as well



$$V(x,y,z) = \frac{1}{4\pi R^2} \oint_S V R^2 \sin\theta \, d\theta \, d\phi$$

$$V_p = \frac{1}{4\pi\varepsilon_0} \frac{q}{d}$$

$$d^2 = r^2 + R^2 - 2rR\cos\theta$$

$$\therefore V_p = \frac{1}{4\pi\varepsilon_0} \frac{q}{\sqrt{r^2 + R^2 - 2rR\cos\theta}}$$

We can obtain the average potential by integrating V_p across the surface of the sphere.

$$V_{average} = \frac{1}{4\pi R^2} \int_S V_p R^2 \sin\theta \, d\theta \, d\phi$$

We have simulated the scenario using COMSOL.

The electrostatic potential V has no local maxima or minima as can be seen from the graphs. All the extremes occur at the boundaries.

We can simulate similar results using different materials and it would indeed be interesting to see the outputs obtained.