HW# 6

Problem 3

Velocity along a direction $v_q = \frac{\partial H}{\partial p_q}$

$$H = \frac{1}{2m^*}\left[(p_x - eB_z y)^2 + p_y^2 + (p_z + eB_x y)^2\right] + V(y)$$
$$- \frac{g}{2}\mu_B\left[B_x\sigma_x + B_z\sigma_z\right] - \frac{\eta}{\hbar}\left[(p_x - eB_z y)\sigma_z - (p_z + eB_x y)\sigma_x\right]$$
$$- \frac{\nu}{\hbar}\left[(p_x - eB_z y)\sigma_x - (p_z + eB_x y)\sigma_z\right]$$

$$V_x = \frac{\partial H}{\partial p_x} = \frac{1}{2m^*}\left[(1 - 2eB_z y)\right] - \frac{\eta}{\hbar}(1 - 2eB_z y)\sigma_z$$
$$- \frac{\nu}{\hbar}\left[(1 - 2eB_z y)\sigma_z\right]$$

$$\langle V_x\rangle = \frac{\langle p_x\rangle}{m^*} - \frac{\eta}{\hbar}\langle\sigma_z\rangle - \frac{\nu}{\hbar}\langle\sigma_x\rangle$$

$$\langle V_x^+\rangle = \langle\psi^+|V_x|\psi^+\rangle$$

$$= \frac{\langle\psi^+|p_x|\psi^+\rangle}{m^*} - \frac{\eta}{\hbar}\langle\psi^+|\sigma_z|\psi^+\rangle - \frac{\nu}{\hbar}\langle\psi^+|\sigma_x|\psi^+\rangle$$

$$\psi^+ = \begin{bmatrix}-\sin(\theta_k)\\ \cos(\theta_k)\end{bmatrix} \quad \theta_k = \frac{1}{2}\arctan\left[\frac{(g/2)\mu_B B_z - \eta k_z + \nu k_x}{(g/2)\mu_B B_x + \eta k_x - \nu k_z}\right]$$

$\cancel{\langle\psi^+|p_x|\psi^+\rangle = [-\sin(\theta_k)\ \cos\theta_k]}$ $\quad \langle y\rangle = 0.$

$$\langle\psi^+|\sigma_z|\psi^+\rangle = \underset{1\times 2}{\left[\cancel{-\sin\theta_k\ \cos\theta_k}\right]}\underset{2\times 2}{\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}}\underset{2\times 1}{\begin{bmatrix}-\sin\theta_k\\ \cos\theta_k\end{bmatrix}} = \begin{bmatrix}-\sin\theta_k\\ -\cos\theta_k\end{bmatrix}$$

$$[-\sin\theta_k\ \ \cos\theta_k]\begin{bmatrix}-\sin\theta_k\\ -\cos\theta_k\end{bmatrix} = \left[\sin^2\theta - \cos^2\theta\right] = -\cos 2\theta_k$$

$$\langle \psi^+ | \sigma_x | \psi^+ \rangle = [-\sin\theta_k \ \cos\theta_k] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -\sin\theta_k \\ \cos\theta_k \end{bmatrix}$$

$$= [-\sin\theta_k \ \cos\theta_k] \begin{bmatrix} \cos\theta_k \\ -\sin\theta_k \end{bmatrix} = [-2\sin\theta\cos\theta]$$

$$= [-\sin 2\theta]$$

~~$\langle p_x \rangle = \hbar k_x$~~

$$\therefore v_x^{\pm} = \frac{\hbar k_x}{m^*} \pm \frac{\eta}{\hbar}\cos(2\theta_k) \pm \frac{\nu}{\hbar}\sin(2\theta_k)$$

for the + case $v_x^+ = \frac{\hbar k_x}{m^*} + \frac{\eta}{\hbar}\cos 2\theta_k + \frac{\nu}{\hbar}\sin(2\theta_k)$

for v_z, we have

$$\langle v_z \rangle = \frac{\langle p_z \rangle}{2m^*} + \frac{eB\langle y \rangle}{m^*} + \frac{\eta}{\hbar}\langle \sigma_x \rangle + \frac{\nu}{\hbar}\langle \sigma_z \rangle$$

from the above calculations $\langle \sigma_x \rangle = -\sin 2\theta$

$\langle p_z \rangle = \langle \frac{\partial}{\partial z} \rangle$ and $\langle \sigma_z \rangle = -\cos 2\theta$.

$$\therefore v_z^{\pm} = \frac{\hbar k_z}{m^*} \mp \frac{\nu}{\hbar}\cos(2\theta_k) \mp \frac{\eta}{\hbar}\sin(2\theta_k).$$