ECS 521/641: Spintronics and Nanomagnetics

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HW #3

Problem 1

Do all the derivations in the Lectures 3 and 4.

Problem 2

Locate the points on the Bloch sphere which represent the eigenvectors of the Pauli matrices $(\sigma_x, \sigma_y, \sigma_z)$.

Three other matrices frequently used in the design of quantum logic gates are the S matrix

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix},$$

which is a special case of the phase shift matrix $P(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$,

the $\pi/8$ or T matrix

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix},$$

and the Hadamard matrix

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Calculate the eigenspinors and eigenvalues of the S, T, and Hadamard matrices. Locate the corresponding eigenvectors on the Bloch sphere.

Problem 3

Starting with the general expression for the spinor, show that the probability for the spinor to be aligned along the $+x$-axis at time t is given by

$$\left|\langle \xi_{+x} | \xi_n^+ \rangle\right|^2 = \frac{1 + n_x(t)}{2}$$

Calculate the explicit time dependence of that probability when the original spinor is located at the north pole at $t = 0$.

Repeat for the case when the original spinor is the eigenstate of the Hadamard matrix with eigenvalue $+1$. Find under what condition that probability is maximized.