HW # 9

Problem 1

for the case of 2-DEG with no external magnetic field and no Dresselhaus interaction.

the expression for $v_x^{\pm}$ is going to be modified

$$\tan 2\theta_n = \frac{g/2\,\mu_B B_z - \eta k_z + \nu k_x}{g/2\,\mu_B B_x + \eta k_x - \nu k_z}$$

In the absence of ext B and Dresselhaus interaction,

$$\tan 2\theta_n = \frac{-k_z}{k_x}, \quad \sin(2\theta_n) = -\frac{k_z}{k} \quad \text{and} \quad \cos(2\theta_n) = \frac{k_x}{k}$$

In that case $v_x^{\pm} = \frac{\hbar k_x}{m^*} \pm \frac{\eta}{\hbar}\frac{k_x}{k}$

and $v_z^{\pm} = \frac{\hbar k_z}{m^*} \pm \frac{\eta}{\hbar}\frac{k_z}{k}$

$$(v^{\pm})^2 = (v_x^{\pm})^2 + (v_z^{\pm})^2 = \left(\frac{\hbar k}{m^*} \pm \frac{\eta}{\hbar}\right)^2.$$

$$v^{\pm} = \pm\left(\frac{\hbar k}{m^*} + \frac{\eta}{\hbar}\right)$$

$$\frac{v_x^{\pm}}{v^{\pm}} = \frac{\frac{\hbar k_x}{m^*} + \frac{\eta}{\hbar}\frac{k_x}{k}}{\pm\left(\frac{\hbar k}{m^*} \pm \frac{\eta}{\hbar}\right)} = \frac{k_x\left(\frac{\hbar}{m^*} + \frac{\eta}{\cancel{\hbar k}}\right)}{\pm k\left(\frac{\hbar}{m^*} + \frac{\eta}{\cancel{\hbar}}\right)}$$

$$\therefore \quad \frac{v_x^{\pm}}{v^{\pm}} = \frac{k_x}{k}$$

Problem 2

We need to show that $\frac{d(\hbar k_x)}{dt} = -eE_x$ is correct even in the presence of spin-orbit interaction.

According to the Ehrenfest theorem,

$$\frac{d\langle A\rangle}{dt} = \frac{1}{i\hbar}\langle [A, H]\rangle$$

The derivitative of the expectation of the position operator along x axis is given by

$$\frac{d\langle x\rangle}{dt} = \frac{1}{i\hbar}\langle x, H\rangle = \frac{1}{i\hbar}\langle [x, \frac{|p|^2}{2m}]\rangle + \frac{1}{i\hbar}\langle [x, V(r)]\rangle + \frac{1}{i\hbar}\langle [x, -\frac{a}{\hbar}E_z.2[\sigma\times p]]\rangle.$$

$$[x, |p|^2] = 2i\hbar p$$

here the first commutator $\frac{1}{i\hbar}\langle [x, \frac{|P|^2}{2m}]\rangle = \frac{1}{i\hbar 2m^*}\langle [x, |p|^2]\rangle$

$$= \frac{1}{i\hbar}\frac{\langle 2i\hbar p\rangle}{2m^*} = \frac{\langle p\rangle}{m^*}.$$

The second commutator $\langle [x, V(r)]\rangle = 0$ because $V(r)$ does not depend on x.

The third commutator is $[x, -\frac{a}{\hbar}E_z.2[\sigma\times p]$

$$= -\frac{a}{\hbar}E_z.2\langle x, [\sigma\times p]\rangle = \frac{-a}{\hbar}E_z.2\langle x, i(\sigma_y p_z - \sigma_z p_y)\rangle$$

$$= \frac{a}{\hbar}E_z.2i\left(\langle x, \sigma_y p_z\rangle - \langle x, \sigma_z p_y\rangle\right)$$

$$= \frac{ia}{\hbar}E_z.2\left(\langle x, \sigma_y\rangle\langle p_z\rangle - \langle x, \sigma_z\rangle\langle p_y\rangle\right)$$

for a spin unpolarized system, $\langle x, \sigma_y\rangle = 0, = \langle x, \sigma_z\rangle = 0$

because $\langle x, \sigma_y\rangle = \langle \psi | x\sigma_y | \psi\rangle = Tr(\rho x \sigma_y) = 0$

$\therefore \frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m^*}$.

hence $\frac{d}{dt}\langle \hbar k_x\rangle = \frac{dp_x}{dt} = \frac{1}{i\hbar}\langle[p_x, H]\rangle$

$= \frac{1}{i\hbar}\langle[p_x, V(r)]\rangle + \frac{1}{i\hbar}\langle[p_x, -\frac{\alpha}{\hbar}E_z z[\sigma\times p]]\rangle$

where

$[p_x, V(r)] = -\frac{\partial V(r)}{\partial x}$

So we can find $\langle[p_x, V(r)]\rangle = -\left\langle\frac{\partial V(r)}{\partial x}\right\rangle = -eE_x$ ↳ ①

For the next part

$v_x = \frac{p_x}{m^*}$.

$\therefore \frac{d\langle v_x\rangle}{dt} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{p_x}{m^*}\right) = \frac{1}{i\hbar}\langle[\frac{p_x}{m^*}, H]\rangle$

$= \frac{1}{m^*}\frac{1}{i\hbar}\langle[p_x, H]\rangle = \frac{1}{m^*} - eE_x$ from ①