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Problem 3

$$H_{shape} = -\frac{1}{M} \nabla E_{shape}$$

$$E_{shape} = \frac{1}{2} M H_K \sin^2\theta$$

$$H_{shape} = -H_K \sin\theta\cos\theta\ \hat{e}_\theta$$

H_M (+z direction) would have $\theta_m = 0$.

$$E_M = -MH\cos\theta$$
$$H_M = -H\sin\theta\ \hat{e}_\theta$$
$$H_{eff} = H_{shape} + H_M$$

Putting this in the LLG equation

$$\frac{dm}{dt} = -|\gamma| m \times H_{eff} + \alpha \left(m \times \frac{dm}{dt}\right)$$

$$\frac{d\theta}{dt}\hat{e}_\theta + \sin\theta \frac{d\phi}{dt}\hat{e}_\phi = |\gamma| \hat{e}_r \left(H_K \sin\theta\cos\theta\ \hat{e}_\theta + H\sin\theta\ \hat{e}_\theta \right) + \alpha \frac{d\theta}{dt}\hat{e}_\phi - \alpha \sin\theta \frac{d\phi}{dt}\hat{e}_\theta$$

$$\frac{d\theta}{dt} = -\alpha \sin\theta \frac{d\phi}{dt} \quad \text{if we compare the terms on both sides above}$$

$\hookrightarrow$ (1)

$$\sin\theta \frac{d\phi}{dt} = \alpha \frac{d\theta}{dt} + |\gamma| \left(H_K \sin\theta\cos\theta + H\sin\theta \right)$$

$$\int_{180}^{0} d\theta \quad \frac{d\theta}{dt} = -\alpha \sin\theta \frac{d\phi}{dt}$$

$$= -\alpha^2 \frac{d\theta}{dt} - \alpha|\gamma|(H_K \sin\theta\cos\theta + H\sin\theta)$$

$$(1+\alpha^2)\frac{d\theta}{dt} = -\alpha|\gamma|(H_K \sin\theta\cos\theta + H\sin\theta)$$

$$\int_{180}^{0} \frac{d\theta}{H_K \sin\theta\cos\theta + H\sin\theta} = \frac{-\alpha|\gamma|}{(1+\alpha^2)} \int_0^{\tau} dt$$

$$m = \frac{H}{H_K}$$

$$\int_{180}^{0} \frac{d\theta}{\sin\theta(\cos\theta + m)} = -\frac{H_K \alpha|\gamma|}{1+\alpha^2} \cdot \int dt$$

assume $\cos\theta = p$

$$\int \frac{-dp}{(1-p^2)(p+m)} = \int \frac{A}{1+p} + \int \frac{B}{1-p} + \int \frac{C}{p+m}$$

$$-1 = A(1-p)(p+m) + B(1+p)(p+m) + C(1+p)(1-p)$$

$$-1 = A(p+m-p^2-pm) + B(p+m+p^2+pm) + C(1-p^2)$$

$$A - m + B + Bm = 0.$$

$$Am + Bm + C = -1$$

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$A = \frac{m - m^3 + 1}{1 - m^2} = \frac{1}{2(m-1)}$

$B = \frac{1}{2(m+1)}$ $\quad C = \frac{1}{1-m^2}$

$\therefore \int \frac{dp}{(1-p^2)(p+m)} = \frac{1}{2(m-1)} \log(1+p) + \frac{1}{2(m+1)} \log(1-p)$

$+ \frac{1}{1-m^2} \log(p+m)$

substituting $p = \cos\theta$.

$$\left[\frac{1}{2(m-1)} \log(1+\cos\theta) + \frac{1}{2(m+1)} \log(1-\cos\theta) + \frac{1}{1-m^2} \log(\cos\theta + m)\right]_{180}^{0}$$

$\frac{1}{2(m-1)} \log(1 + \cos 180) \rightarrow$ this will be α,

hence at $180°$, Hs and HM are both 0. Torque = 0
we need to change the limit, let's assume 0 9