ECS 521: Spintronics & Nanomagnetics.

HW # 4

Rita Abani 19244

Problem 1

We know that the Potential energy of a nanomagnet due to shape anisotropy is given as

$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s\,[H_k + H_d\cos^2\phi]\,\Omega\sin^2\theta.$$

$$\therefore H_{shape} = -\frac{1}{M}\nabla E_{shape}.$$

$$\nabla E_{shape} = \nabla E_S = \frac{\partial E_S}{\partial\theta}\hat{e}_\theta + \frac{1}{\sin\theta}\frac{\partial E_S}{\partial\phi}\hat{e}_\phi$$

$$\nabla E_S = \frac{\partial}{\partial\theta}\left(\frac{1}{2}\mu_0 M_s[H_k + H_d\cos^2\phi]\,\Omega\sin^2\theta\right)\hat{e}_\theta + \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}E_S\,\hat{e}_\phi$$

$$= \left(\frac{1}{2}\mu_0 M_s[H_k + H_d\cos^2\phi]\,\Omega\, 2\sin\theta\cos\theta\right)\hat{e}_\theta +$$

$$\frac{1}{\sin\theta}\left(\frac{1}{2}\mu_0 M_s[H_k - 2H_d\cos\phi\sin\phi]\,\Omega\sin^2\theta\right)\hat{e}_\phi$$

$$M = \mu_0 M_s\Omega$$

$$\therefore H_{shape} = \frac{-1}{\mu_0 M_s\Omega}\left(\nabla E_S\right)$$

$$H_{shape} = (H_k + H_d\cos^2\phi)\sin\theta\cos\theta\,\hat{e}_\theta + \frac{1}{2}(H_k - H_d\sin 2\phi)\sin\theta\,\hat{e}_\phi$$

$$\frac{dm}{dt} = \frac{d\theta}{dt}\hat{e}_\theta + \sin\theta \frac{d\phi}{dt}\hat{e}_\phi$$

$$\frac{d\theta}{dt} = \frac{\partial E_s}{\partial \theta} \quad \text{while} \quad \frac{d\phi}{dt} = \frac{1}{\sin^2\theta}\frac{\partial E_s}{\partial \phi}.$$

$$\frac{d\theta}{dt} = (H_k + H_d\cos^2\phi)\sin\theta\cos\theta \times \frac{1}{2}\mu_0 M_s \Omega$$

$$\frac{d\phi}{dt} = \frac{1}{\sin^2\theta}\left(\frac{1}{2}\mu_0 M_s (H_k \phi - 2H_d\cos\phi\sin\phi)\Omega\sin^2\theta\right).$$

Problem 2

a) $(a, b, t) = (150\,nm, 100\,nm, 2\,nm)$

$$\Omega = 150 \times 100 \times 2 \times 10^{-27}\,m^3$$

$$= 30000 \times 10^{-27}\,m^3 = 3 \times 10^{-22}\,m^3$$

$$M_s = 8 \times 10^5\,A/m.$$

$$(N_{d-xx}, N_{d-yy}, N_{d-zz}) = (0.9468, 0.0339, 0.0193).$$

$$E(\theta,\phi) = \frac{1}{2}\mu_0 M_s (H_k + H_d\cos^2\phi)\Omega\sin^2\theta$$

$$H_k = (N_{d-yy} - N_{dzz})M_s = 11680$$

$$H_d = (N_{d-xx} - N_{dyy})M_s = 730320.$$

$$E(\theta,\phi) = \frac{1}{2}\mu_0 \times 8\times10^5(11680 + 730320\cos^2\phi)\times 3\times10^{-22}\sin^2\theta$$

$$= K \times 12 \times 10^{-17}(11680 + 730320\cos^2\phi)\sin^2\theta.$$

The above equation was plotted in matlab

Problem 3:-

$(a, b, t) = (20nm, 20nm, 5nm)$

$M_S = 4.8 \times 10^5$ A/m. $H_{PMA} = M_S$.

The switching delay can be estimated using the macrospin approximation and LLG equation.

$H_{PMA} = H_k = (N_{d-yy} - N_{d-zz}) M_S = M_S$ (given).

$H_{PMA} = 4.8 \times 10^5$ A/m

Let's assume $\alpha = 0.1$

applying a z directed magnetic field 'H' to the nanomagnet, we get $H(t) = H_0 e^{-t/\tau}$

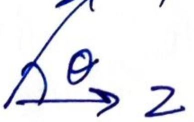
τ = switching time and H_0 = amplitude of magnetic field.

Using the macrospin approximation, we solve the LLG for the 'z' component of the magnetization vector m_z

$$\frac{dm_z}{dt} = -\gamma m_z \times H_{eff} + \alpha \left(m_z \times \frac{dm_z}{dt} \right)$$

Assume that initial magnetization is aligned with +z axis, $m_z(0) = 1$, so switching delay can be time for magnetization to flip and reach -1.

Solving the above using ansätz.



Let $m_z(t) = \cos\theta(t)$.

$$\frac{d}{dt}(\cos\theta(t)) = -\gamma \cos\theta(t) \times (H_0 + H_K) + \alpha\left(\cos\theta(t) \times -\sin\theta(t)\right)$$

$\theta(t) = \sin^{-1}\left(e^{-\frac{\gamma(H_0+H_k)}{\alpha}M_s}\right)\sin\left(\gamma H_0 \frac{t}{\alpha}\right)$

Switching delay is the time taken for $\theta(t)$ to reach $\pi/2$.

$\therefore \quad \tau = \dfrac{\pi \alpha M_s}{\gamma(H_0 + H_k)}$ let $\gamma = 1.76 \times 10^{11}\ T^{-1}s^{-1}$

$H_0 = H_z = 2H_k$.

$$\tau = \frac{3.14 \times 0.1 \times \cancel{4.8 \times 10^5}}{1.76 \times 10^{11} \times 3 \times \cancel{4.8 \times 10^5}}$$

$\tau = 0.0594 \times 10^{-11} = 5.94 \times 10^{-9} = \underline{5.94\ ns}$

Common solution for Problem 3 and 2 d

To numerically solve LLG using finite difference, we need to discretize the equation in space and time.

$$\frac{dm}{dt} = -\gamma(m \times H_{eff}) + \alpha\left(m \times \frac{dm}{dt}\right)$$

On discretizing the above,

$$\frac{\left(m_i^{(n+1)} - m_i^n\right)}{\Delta t} = -\gamma\left(m_i^n \times H_{eff}^n\right) + \alpha\left(m_i^n \times \frac{dm_i}{dt}^n\right)$$

$$m_i^{n+1} = m_i^n - \gamma\left(m_i^n \times H_{eff}^n\right)\Delta t + \alpha\left(m_i^n \times \frac{dm_i}{dt}^n\right)\Delta t.$$

The code illustrates solving the above in python.

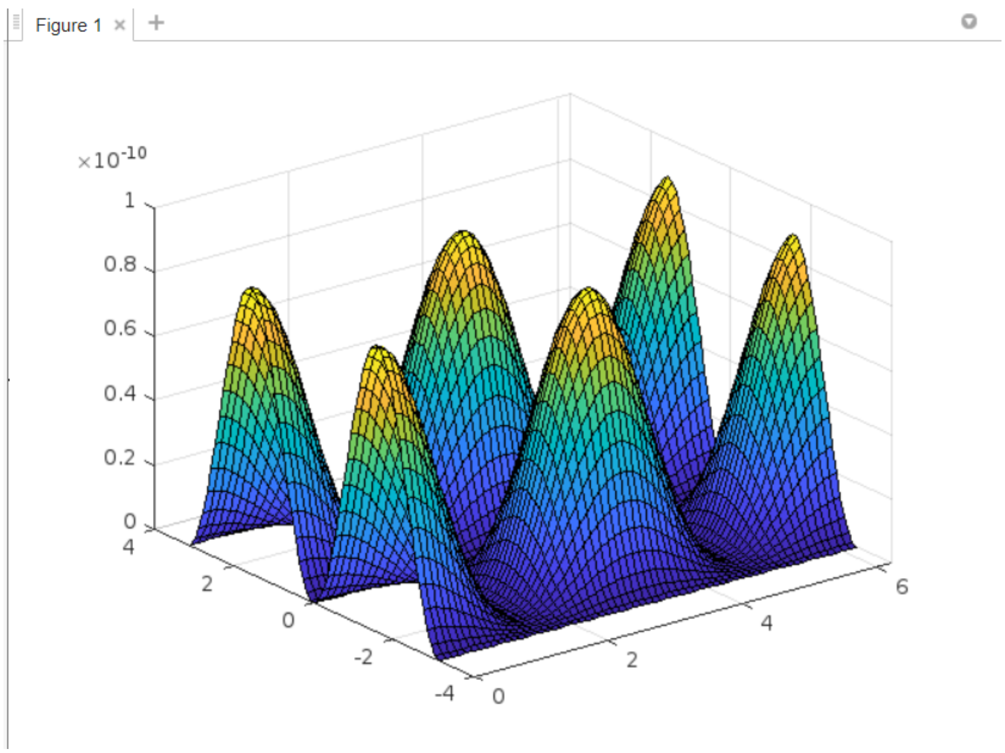
Problem 2

a)

Code

```
% Problem 2 a
[X,Y] = meshgrid(0:0.1:2*pi ,-pi:0.1:pi );
k=1.2*10^(-16);
Z= k*(11680 + 730320*(cos(X)).^2).*(sin(Y)).^2;
colorbar
surf(X,Y,Z)
```

Potential landscape :



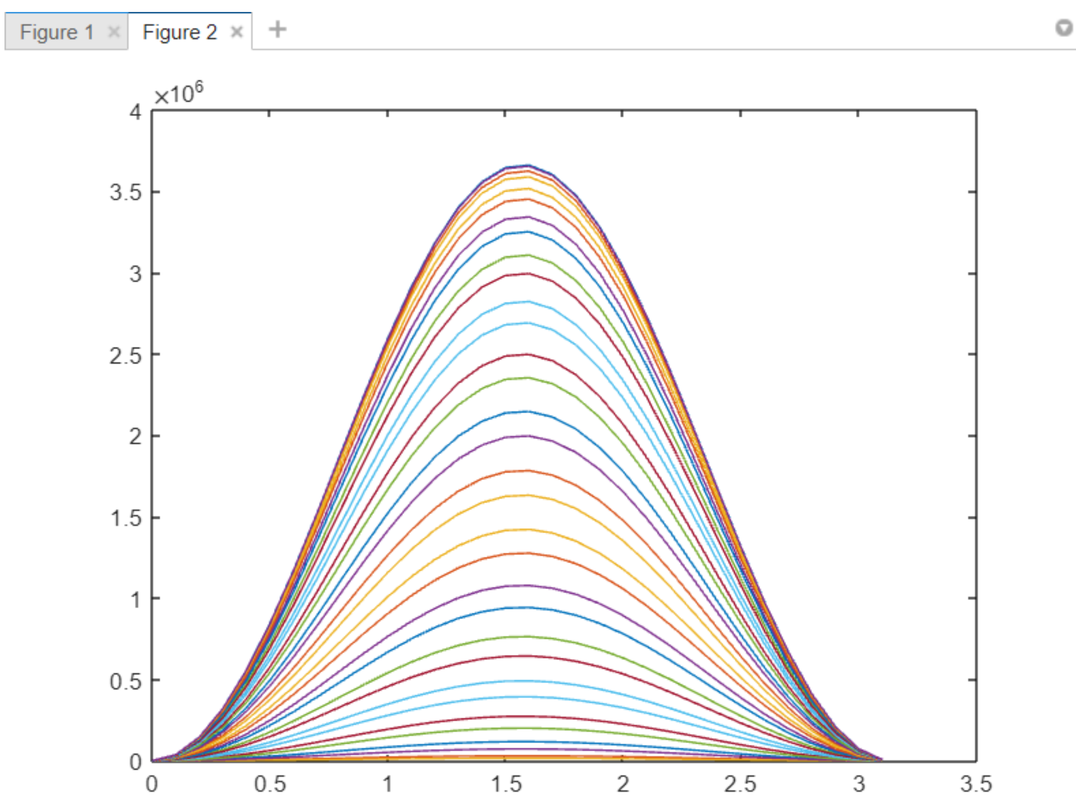
b)
Code

```
[X,Y] = meshgrid(0:0.1:pi ,-0:0.1:pi );
%k=1;

k= [0,0.2,0.5,0.7,1,1.4];
for i = 1:5
    Z= (11680 + 730320.*i.*(cos(X)).^2).*(sin(Y)).^2;
end

figure
plot(Y,Z)
```

Plot:

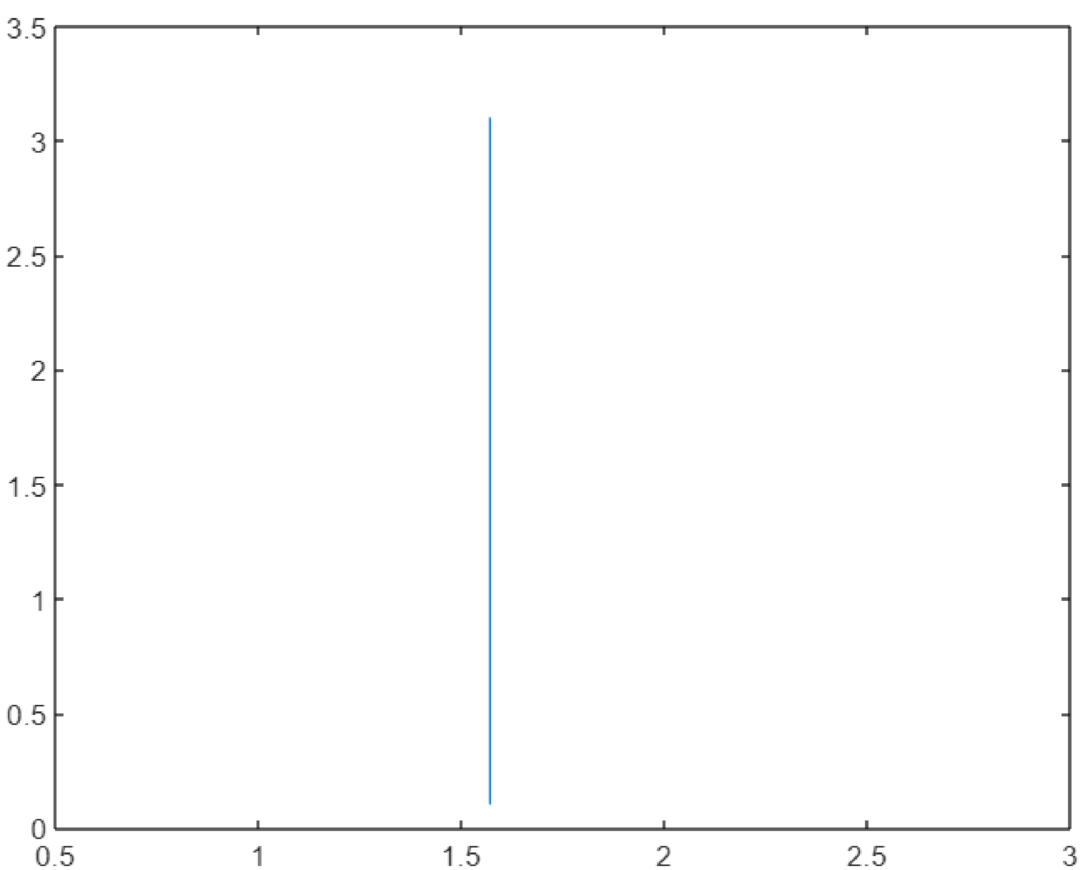


2c)

tled2.m × spintronics2.m × spintronics3.m × problem2b.m × problem2a.m × +

```
[X,Y] = meshgrid(0:0.1:pi ,-0:0.1:pi );
%k=1;
A = 1*(11680 + 730320*(cos(X)).^2).*(sin(Y)).^2;
k= [0,0.2,0.5,0.7,1,1.4];
for i = 1:5
    Z= (11680 + 730320.*i.*(cos(X)).^2).*(sin(Y)).^2;
end

B = Z/A;
C = asin(Z/A);
figure
plot(C,Y)
```



2e)

```
[X,Y] = meshgrid(0:0.1:pi ,-0:0.1:pi );
%k=1;
B = 1*(11680 + 730320*(cos(X)).^2).*(sin(Y)).^2;
k= [0,0.2,0.5,0.7,1,1.4];
for i = 1:5
    Z= (11680 + 730320.*i.*(cos(X)).^2);
end


figure
plot(Y,Z)
```