Spintronics and Nanomagnetics

ECS 521/641

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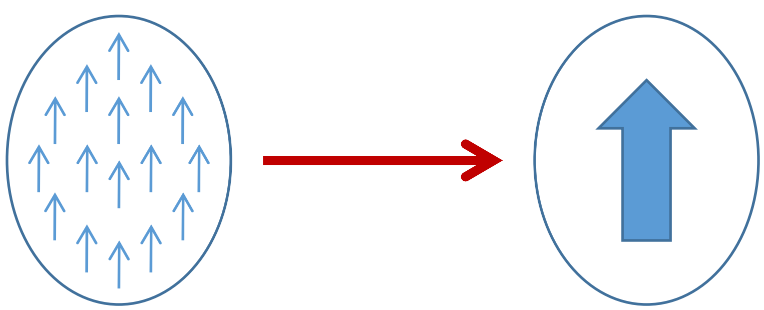
Magnetization dynamics

Single-domain nanomagnets

- Exchange interaction
  - ✓ Pauli's exclusion principle
  - ✓ Coulomb repulsion
- Each electron → small magnet
  - ✓ Ferromagnet
  - ✓ Ferrimagnet
  - ✓ Antiferromagnet

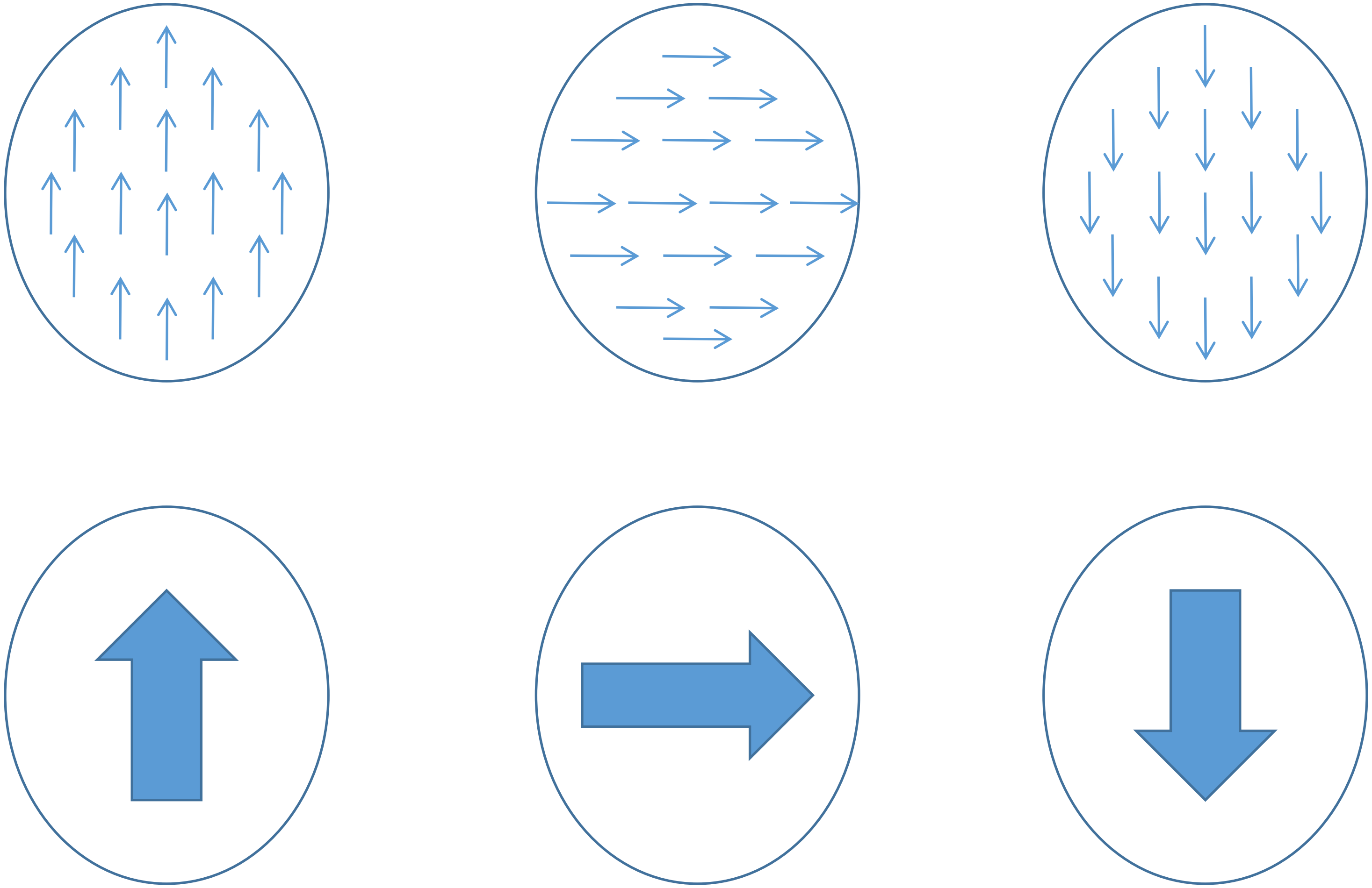
W. F. Brown Jr.,
The fundamental theorem of the ferromagnetic particle theory

Magnetic domain formation should be limited to **very small dimensions (100 nm)** because of the competition between the magnetostatic energy and the quantum-mechanical exchange energy, causing nanomagnets to behave like **single giant spins**



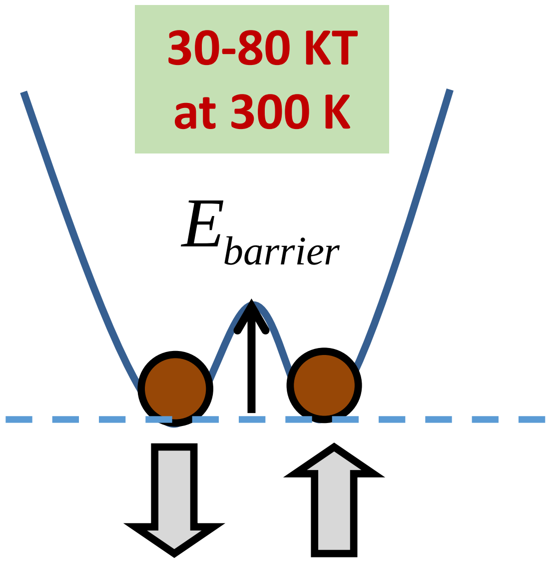
Electron beam lithography (EBL)

Macrospin



All the spins rotate in unison

Magnetic Anisotropy



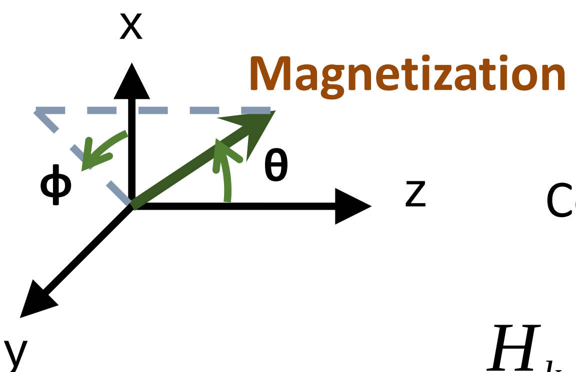
$\theta = 180^o$ $\theta = 0^o$

$$E = \frac{1}{2}\mu_0 M_s H_k \Omega \ \sin^2\theta$$

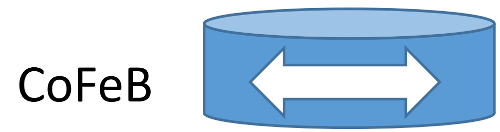
H_k : Coercive field

Ω : Volume

M_s *in* A/m

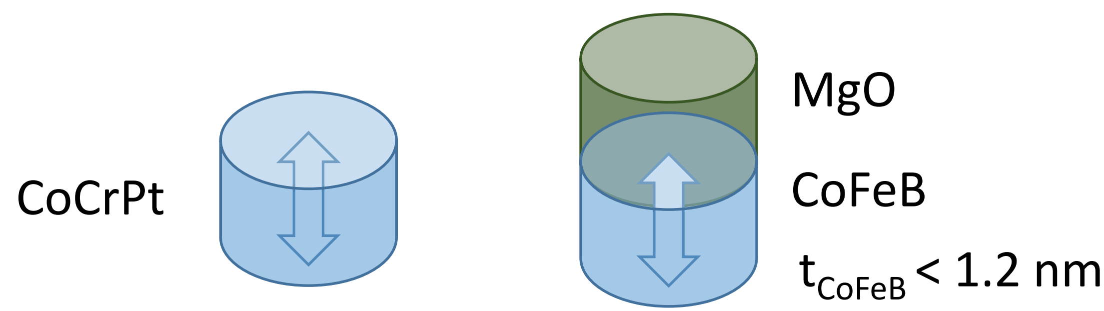


In-plane Shape anisotropy



$$H_k = \left(N_{d-yy} - N_{d-zz}\right) M_s$$

Perpendicular anisotropy

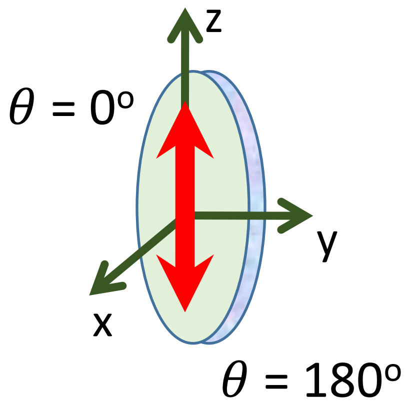


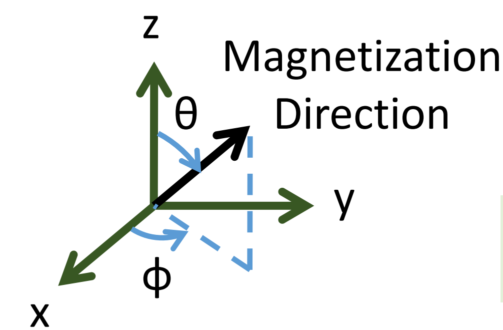
Bulk **Interface**

3D potential landscape of a nanomagnet

Easy axis:
θ = 180°, 0°

Hard axis:
θ = 90°





Magnet's plane: ϕ = ± 90°

$M_s \; in \; A/m$

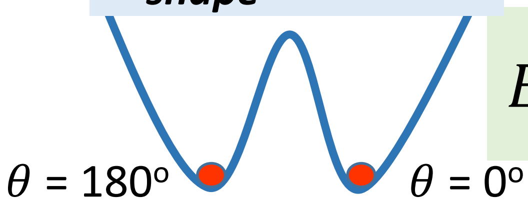
Potential energy

$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s^2 \; \Omega \; N_d(\theta,\phi)$$

$$N_d(\theta,\phi) = N_{d-xx}\sin^2\theta\cos^2\phi + N_{d-yy}\sin^2\theta\sin^2\phi + N_{d-zz}\cos^2\theta$$

$$N_{d-xx} + N_{d-yy} + N_{d-zz} = 1$$

***E**$_{shape}$* **(ϕ = ± 90°)**



$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s[H_k \; + \; H_d \, cos^2\phi]\Omega\sin^2\theta$$

$$H_k = (N_{d-yy} - N_{d-zz})M_s$$

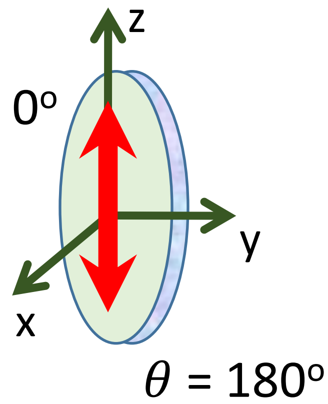
$$H_d = (N_{d-xx} - N_{d-yy})M_s$$

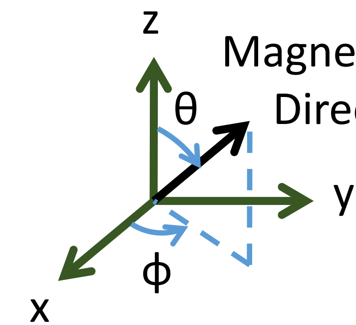
In-plane (**ϕ = ± 90°**) energy barrier: 30 - 80 kT (T=300 K)

3D potential landscape of a nanomagnet

Easy axis:
θ = 180°, 0°

Hard axis:
θ = 90°





$M_s \; in \; A/m$

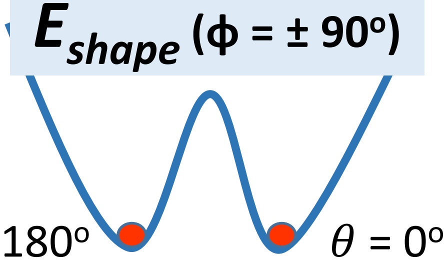
Magnet's plane: ϕ = ± 90°

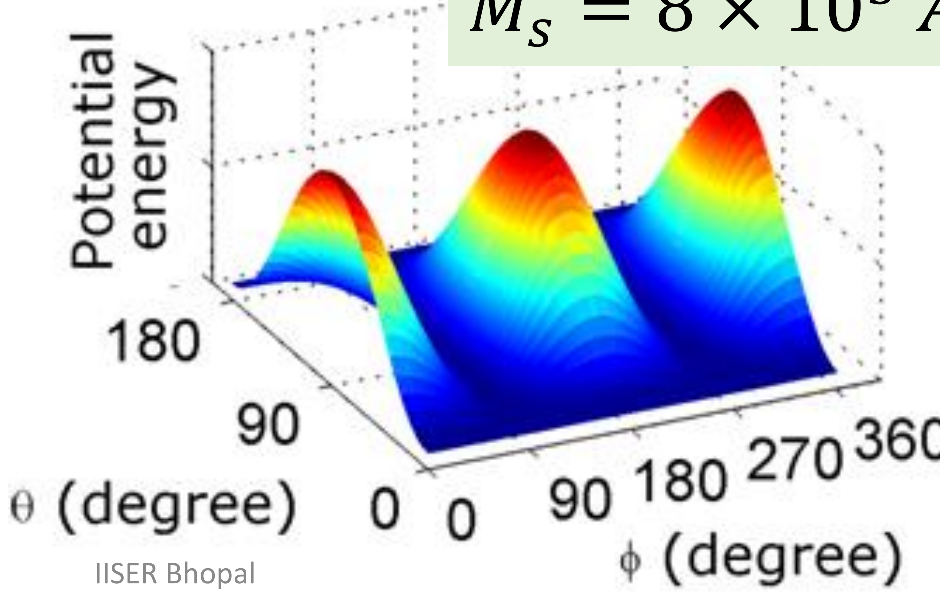
Potential energy

$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s [H_k \; + \; H_d \; cos^2\phi]\,\Omega \sin^2\theta$$

$$H_k = (N_{d-yy} - N_{d-zz})M_s$$
$$H_d = (N_{d-xx} - N_{d-yy})M_s$$

$$M_s = 8 \times 10^5 \; A/m$$



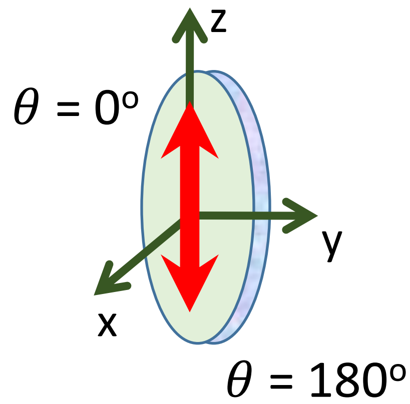


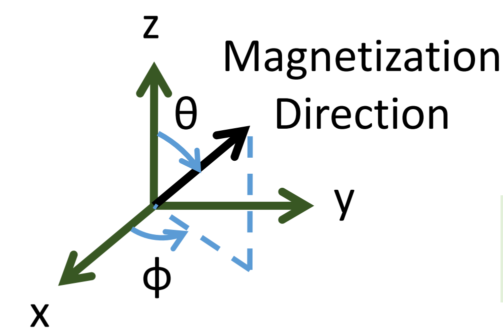
In-plane (**ϕ = ± 90°**) energy barrier: 30 - 80 kT (T=300 K)

3D potential landscape of a nanomagnet

Easy axis:
θ = 180°, 0°

Hard axis:
θ = 90°





Magnet's plane: $\phi = \pm 90°$

M_s *in* A/m

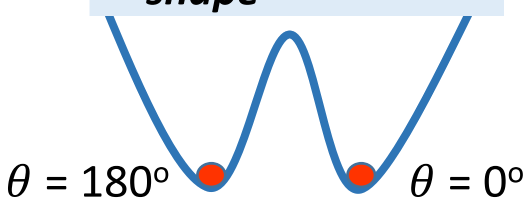
Potential energy

$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s [H_k \; + \; H_d \, cos^2\phi] \, \Omega \sin^2\theta$$

$$H_k = (N_{d-yy} - N_{d-zz}) M_s$$

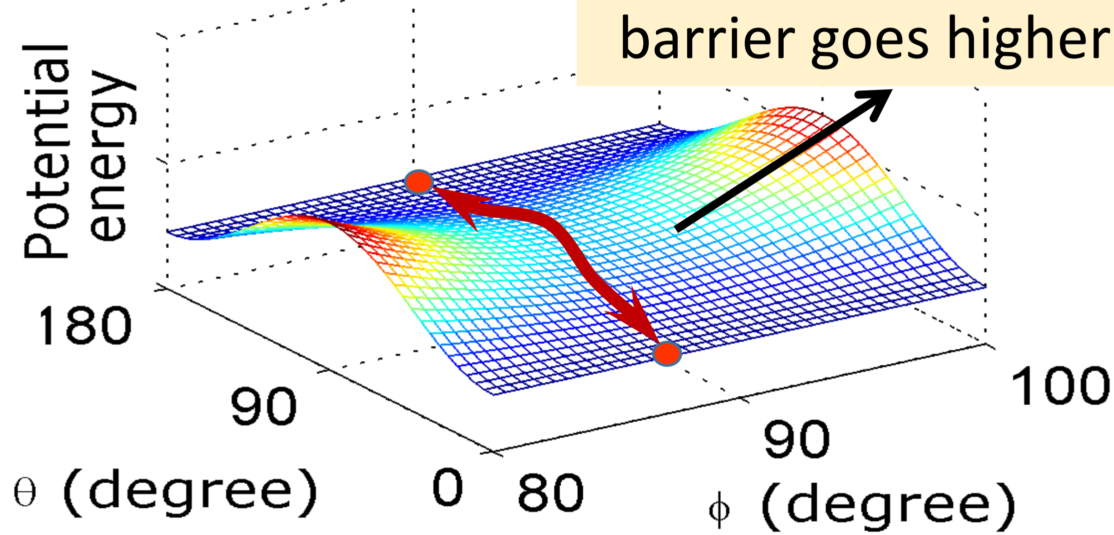
$$H_d = (N_{d-xx} - N_{d-yy}) M_s$$

***E**$_{shape}$* **($\phi = \pm 90°$)**



In-plane (**$\phi = \pm 90°$**) energy barrier: 30 - 80 kT (T=300 K)

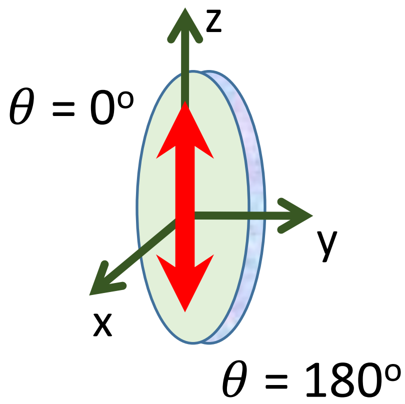
Out-of-plane energy barrier goes higher

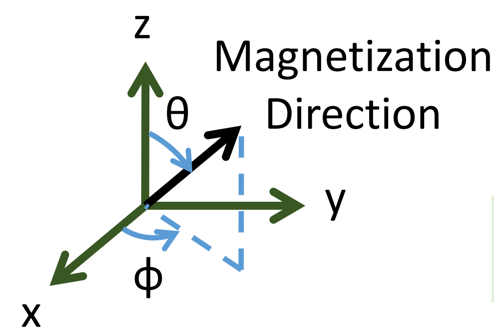


3D potential landscape: Typical parameters

Easy axis:
θ = 180°, 0°

Hard axis:
θ = 90°





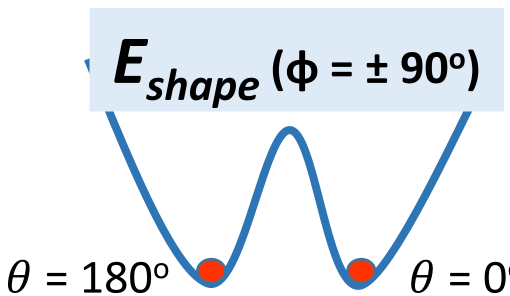
Magnet's plane: ϕ = ± 90°

$M_s \; in \; A/m$

Potential energy

$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s [H_k \; + \; H_d \, cos^2\phi]\Omega \sin^2\theta$$

$$H_k = (N_{d-yy} - N_{d-zz})M_s$$
$$H_d = (N_{d-xx} - N_{d-yy})M_s$$



In-plane (**ϕ = ± 90°**) energy barrier: 30 - 80 kT (T=300 K)

$$M_s = 8 \times 10^5 \frac{A}{m}$$

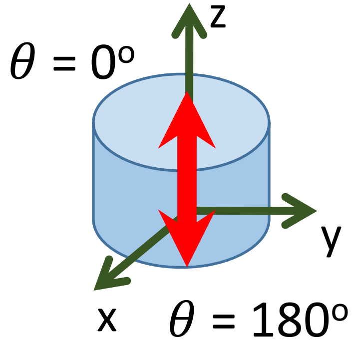
$$(a,b,t) = (100 \text{ nm}, 90 \text{ nm}, 6 \, nm)$$

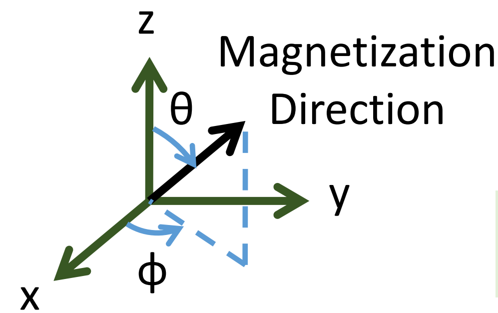
$$\mathrm{N_d} = (0.8529, 0.0788, 0.0683)$$

Perpendicular anisotropy

Easy axis:
θ = 180°, 0°

Hard axis:
θ = 90°





Magnet's plane: ϕ = ± 90°

M_s *in* A/m

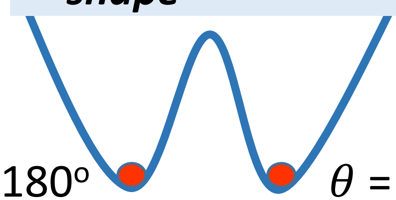
Potential energy

$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s^2\,\Omega\,N_d(\theta,\phi)$$

$$N_d(\theta,\phi) = N_{d-xx}\sin^2\theta\cos^2\phi + N_{d-yy}\sin^2\theta\sin^2\phi + N_{d-zz}\cos^2\theta$$

$$N_{d-xx} + N_{d-yy} + N_{d-zz} = 1$$

E $\boldsymbol{E_{shape}}$ **(ϕ = ± 90°)**



$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s[H_k + H_d\,cos^2\phi]\Omega\sin^2\theta$$

$$H_k = (N_{d-yy} - N_{d-zz})M_s$$

Circular cross-section

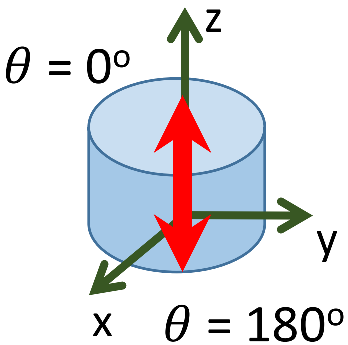
$$H_d = (N_{d-xx} - N_{d-yy})M_s = 0$$

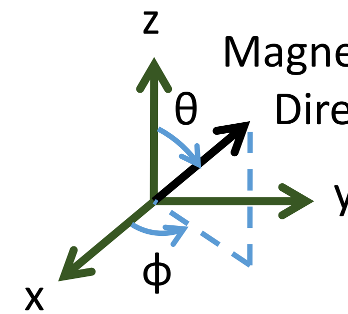
In-plane (**ϕ = ± 90°**) energy barrier: 30 - 80 kT (T=300 K)

Perpendicular anisotropy

Easy axis:
θ = 180º, 0º

Hard axis:
θ = 90º





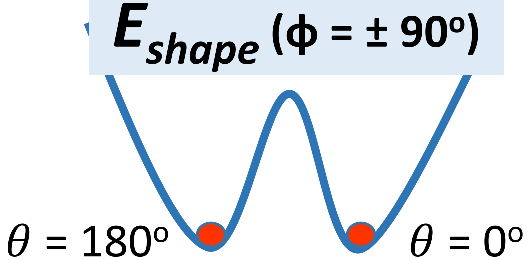
Magnet's plane: ϕ = ± 90º

$M_s \ in \ A/m$

Potential energy
$$E_{shape}(\theta,\phi) = \frac{1}{2}\mu_0 M_s^2 \, \Omega \, N_d(\theta,\phi)$$

$$N_d(\theta,\phi) = N_{d-xx}\sin^2\theta\cos^2\phi + N_{d-yy}\sin^2\theta\sin^2\phi + N_{d-zz}\cos^2\theta$$

E_{shape} (ϕ = ± 90º)



In-plane (**ϕ = ± 90º**) energy barrier: 30 - 80 kT (T=300 K)

$$N_{d-xx} + N_{d-yy} + N_{d-zz} = 1$$

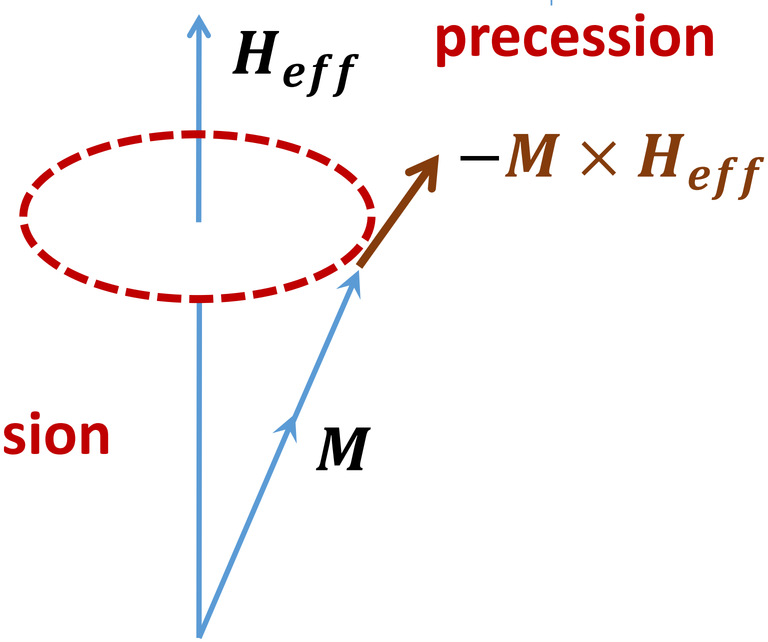
$$E_{PMA}(\theta) = \frac{1}{2}\mu_0 M_s H_{PMA} \Omega \sin^2\theta$$

$$H_k = (N_{d-yy} - N_{d-zz}) M_s$$

$$H_{PMA} = H_K + H_{crystalline} + H_{interface}$$

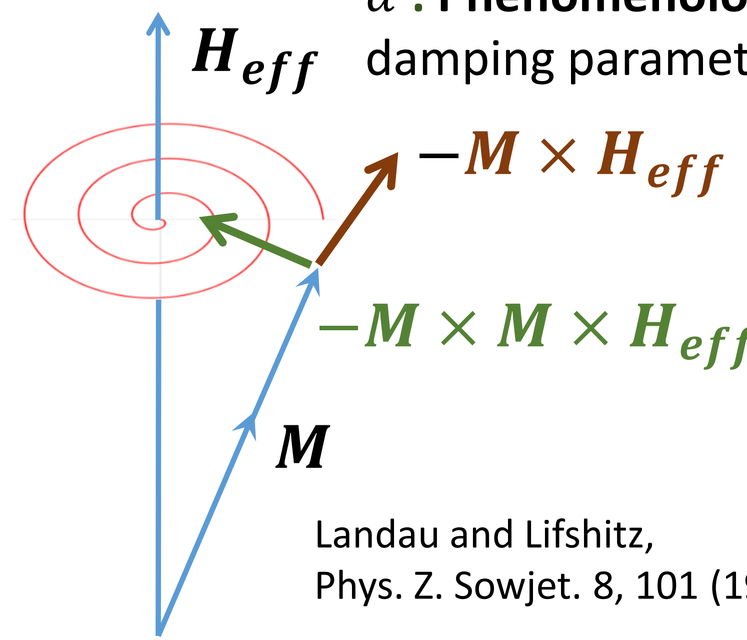
Magnetization dynamics
Landau-Lifshitz (LL) equation

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\underbrace{\boldsymbol{M}\times\boldsymbol{H}_{eff}}_{\textbf{precession}} - \frac{\alpha|\gamma|}{M}\overbrace{\boldsymbol{M}\times\boldsymbol{M}\times\boldsymbol{H}_{eff}}^{\textbf{damping}}$$



precession

α : **Phenomenological** damping parameter



Landau and Lifshitz,
Phys. Z. Sowjet. 8, 101 (1935)

- Damping causes a transfer of energy from **macroscopic** motion to **microscopic** thermal motion, which results in **internal energy losses**
- A damping parameter takes into account the rate of energy transfer

Magnetization dynamics
Landau-Lifshitz (LL) equation

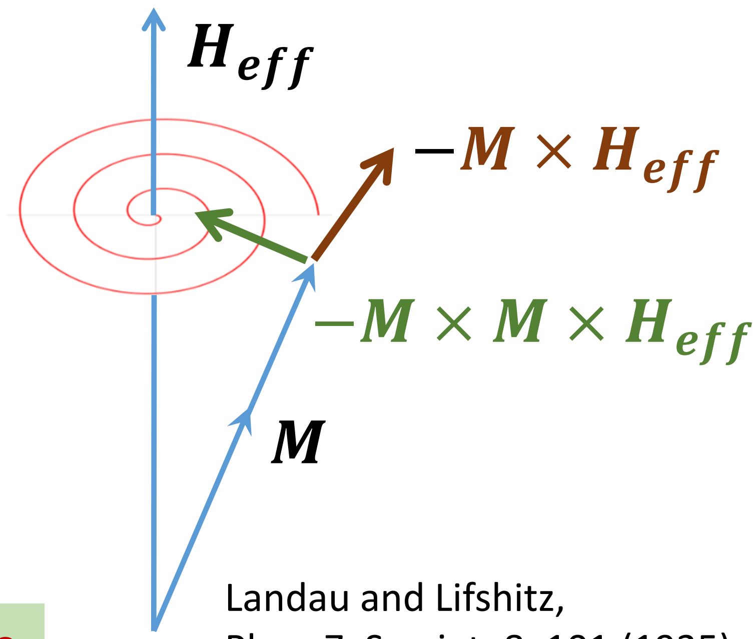
$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\underbrace{\boldsymbol{M} \times \boldsymbol{H_{eff}}}_{\text{precession}} - \frac{\alpha|\gamma|}{M}\underbrace{\boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{H_{eff}}}_{\text{damping}}$$

α : **Phenomenological** damping parameter

$$\boldsymbol{H_{eff}} = -\frac{1}{M}\nabla E$$

$$M = \mu_0 M_s \Omega$$

E : Potential energy
M : Magnetization
Ω : Volume



Landau and Lifshitz,
Phys. Z. Sowjet. 8, 101 (1935)

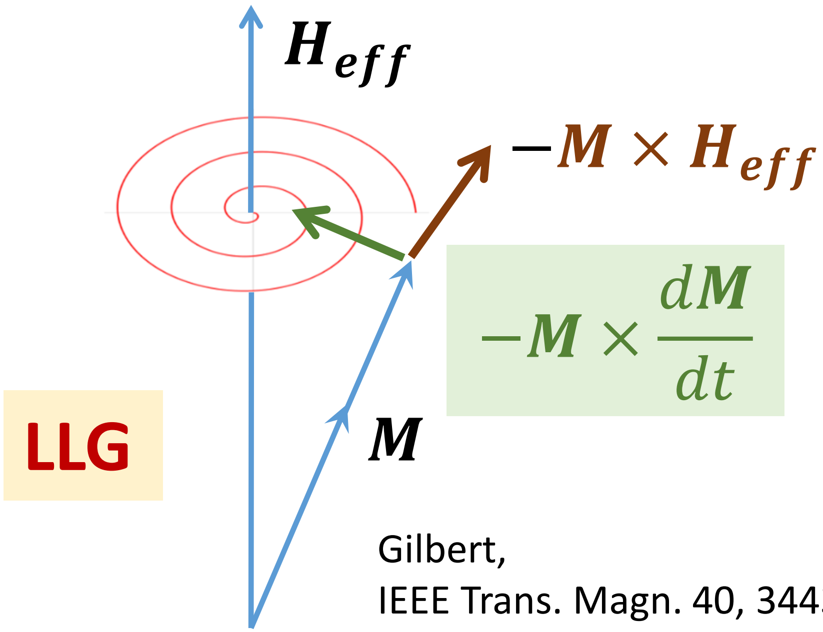
Performance metrics

Switching delay and energy dissipation

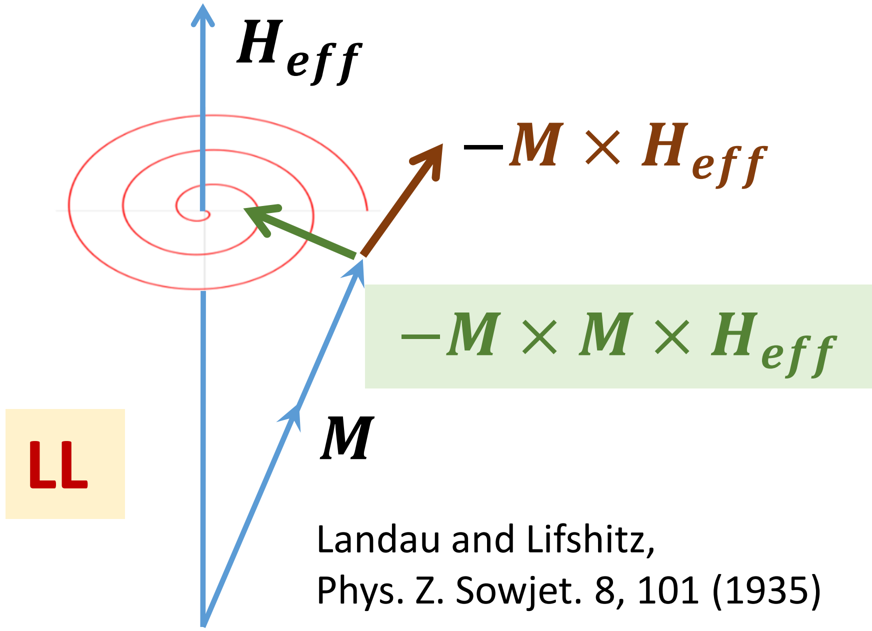
Magnetization dynamics
Landau-Lifshitz-Gilbert (LLG) equation

LL $$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M}\times\boldsymbol{H}_{eff} - \underbrace{\frac{\alpha|\gamma|}{M}\boldsymbol{M}\times\boldsymbol{M}\times\boldsymbol{H}_{eff}}$$ **damping**

LLG $$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M}\times\boldsymbol{H}_{eff} + \frac{\alpha}{M}\boldsymbol{M}\times\frac{d\boldsymbol{M}}{dt}$$ **damping**



Gilbert,
IEEE Trans. Magn. 40, 3443 (2004)



Landau and Lifshitz,
Phys. Z. Sowjet. 8, 101 (1935)

Magnetization dynamics
Landau-Lifshitz-Gilbert (LLG) equation

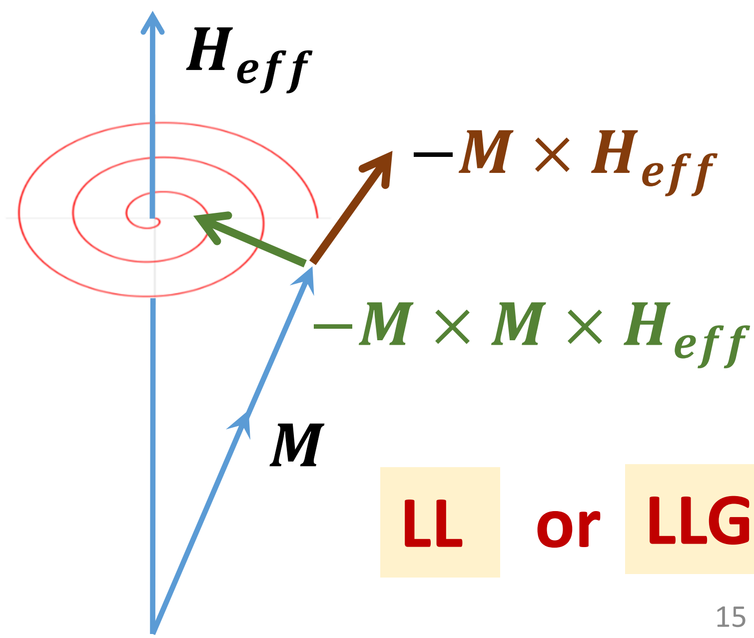
LLG $$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} + \frac{\alpha}{M}\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}$$

In standard form

$$(\boldsymbol{1} + \boldsymbol{\alpha^2})\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} - \frac{\alpha|\gamma|}{M}\boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{H_{eff}}$$

Difference: $(1 + \alpha^2)$ factor

- Landau and Lifshitz formulated the theory of dynamics of magnetization in ferromagnetic bodies
- It cannot account for large noneddy-current damping in thin Permalloy sheets, adjusted by Gilbert



LL or **LLG**

LLG: Deriving the standard form

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} + \frac{\alpha}{M}\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}$$

Exercise

Derive standard form

$$\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{H_{eff}} + \frac{\alpha}{M}\boldsymbol{M} \times \left(\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}\right)$$

$$\boldsymbol{M} \times \left(\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}\right) = \boldsymbol{M}\left(\boldsymbol{M} \cdot \frac{d\boldsymbol{M}}{dt}\right) - \frac{d\boldsymbol{M}}{dt}(\boldsymbol{M} \cdot \boldsymbol{M}) = -M^2\frac{d\boldsymbol{M}}{dt}$$

zero

$$(1+\alpha^2)\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} - \frac{\alpha|\gamma|}{M}\boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{H_{eff}}$$

LLG: Energy dissipation due to damping

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} + \frac{\alpha}{M}\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt} \qquad \mathbf{(1)}$$

In standard form

$$\left(\boldsymbol{1} + \boldsymbol{\alpha}^{\boldsymbol{2}}\right)\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} - \frac{\alpha|\gamma|}{M}\boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{H_{eff}} \qquad \mathbf{(2)}$$

$$E_d = \int_0^\tau P_d(t)dt$$

$$P_d(t) = \boldsymbol{H_{eff}} \cdot \frac{d\boldsymbol{M}}{dt} = \frac{\alpha}{|\gamma|M}\left|\frac{d\boldsymbol{M}}{dt}\right|^2 = \frac{\alpha|\gamma|}{(1+\alpha^2)M}|\boldsymbol{M} \times \boldsymbol{H_{eff}}|^2$$

Use $\mathbf{(1)}$ **Use $\mathbf{(2)}$**

LLG: Including magnetic field

$$\frac{d\boldsymbol{m}}{dt} = -|\gamma|\boldsymbol{m} \times \boldsymbol{H_{eff}} + \alpha\left(\boldsymbol{m} \times \frac{d\boldsymbol{m}}{dt}\right)$$

$$\frac{d\boldsymbol{m}}{dt} = \frac{d\theta}{dt}\hat{e}_\theta + sin\theta\frac{d\phi}{dt}\hat{e}_\phi$$

$$\boldsymbol{m} = \frac{\boldsymbol{M}}{M} = \hat{e}_r$$

$$M = \mu_0 M_s \Omega$$

$$\alpha\left(\boldsymbol{m} \times \frac{d\boldsymbol{m}}{dt}\right) = \alpha\frac{d\theta}{dt}\hat{e}_\phi - \alpha\, sin\theta\frac{d\phi}{dt}\hat{e}_\theta$$

$$\boldsymbol{H_M} = -\frac{1}{M}\nabla E_M$$

$$\boldsymbol{H_{eff}} = \boldsymbol{H_{shape}} + \boldsymbol{H_M}$$

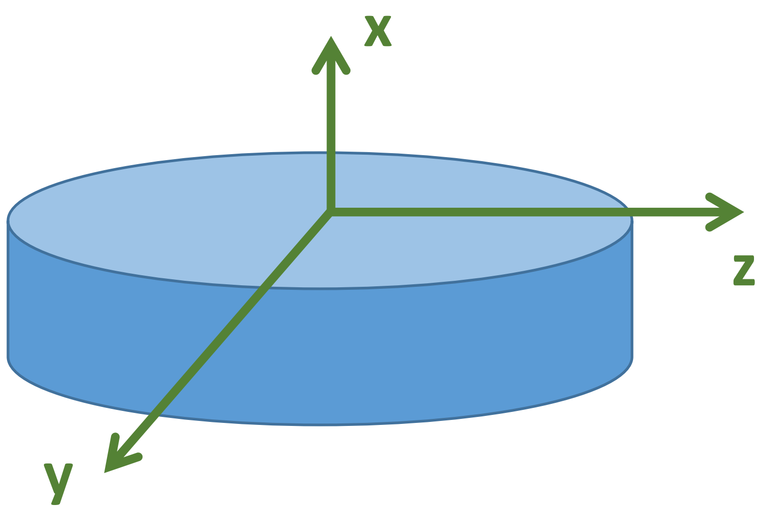
$$\nabla E_M = \frac{\partial E_M}{\partial\theta}\hat{e}_\theta + \frac{1}{sin\theta}\frac{dE_M}{d\phi}\hat{e}_\phi$$

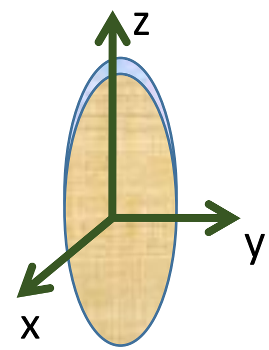
$$E_M = -\boldsymbol{M} \cdot \boldsymbol{H_M}$$
$$= -MH_M(sin\theta\, cos\phi\, sin\theta_m cos\phi_m$$
$$+ sin\theta\, sin\phi\, sin\theta_m\, sin\phi_m$$
$$+ cos\theta\, cos\theta_m)$$

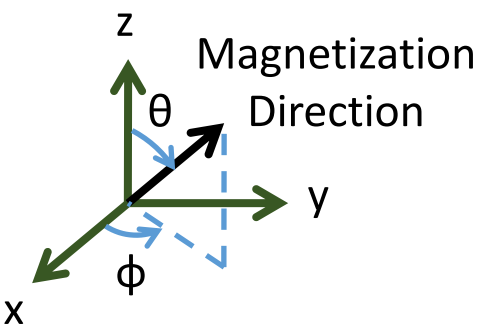
Exercise

Determine $\frac{d\theta}{dt}$ and $\frac{d\phi}{dt}$

Magnetic Field, H: Simplified 2D analysis







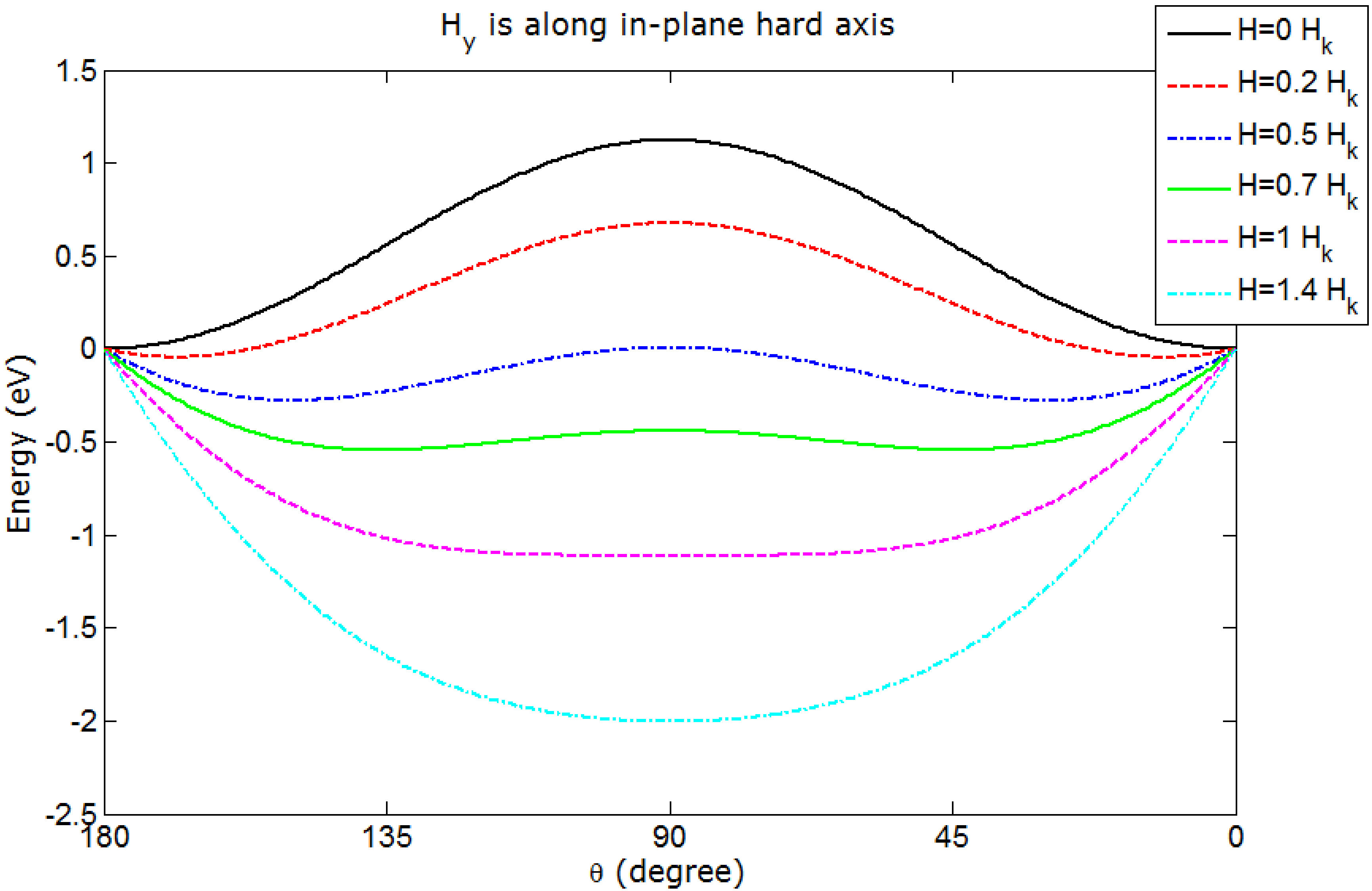
Magnet's plane: $\phi = \pm 90^{\circ}$

$$E = \frac{1}{2}\mu_0 M_s H_k \sin^2\theta - \mu_0 M_s H_y \sin\theta$$

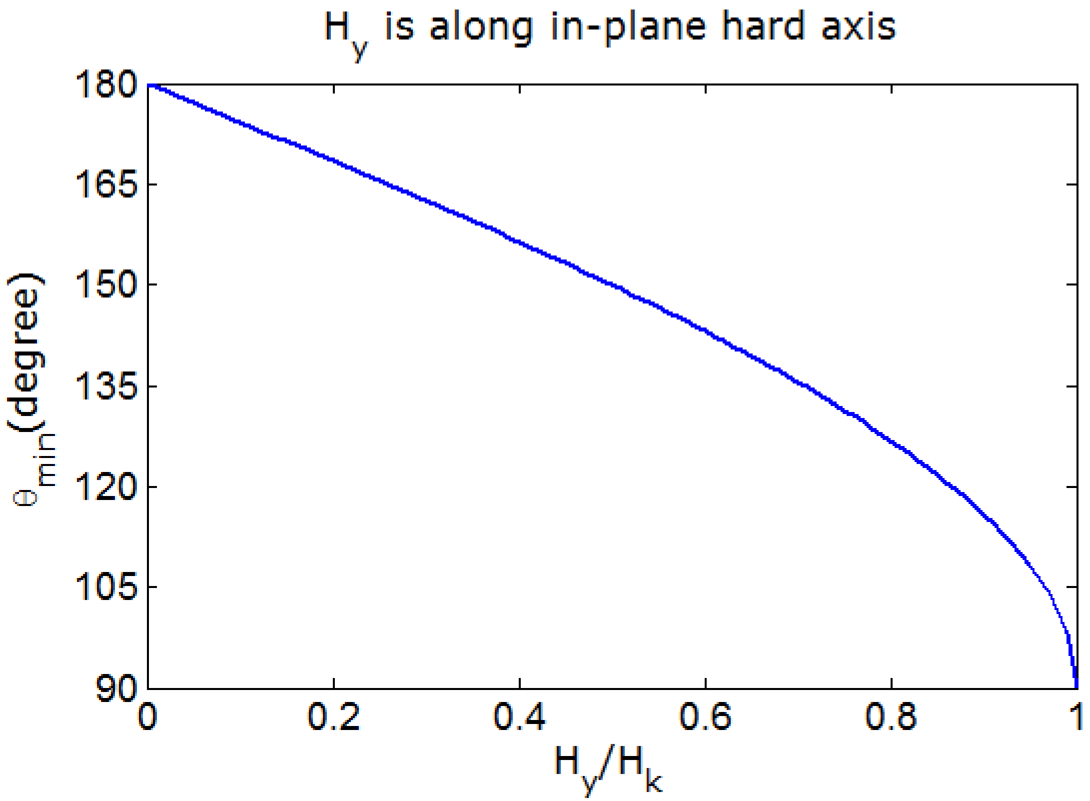
$$\frac{dE}{d\theta} = \mu_0 M_s H_k \sin\theta\cos\theta - \mu_0 M_s H_y \cos\theta = 0$$

$$\sin\theta_{\min} = \frac{H_y}{H_k} \Rightarrow \boxed{\theta_{\min} = \sin^{-1}\left(\frac{H_y}{H_k}\right)}$$

Potential Profiles
H is along in-plane hard-axis

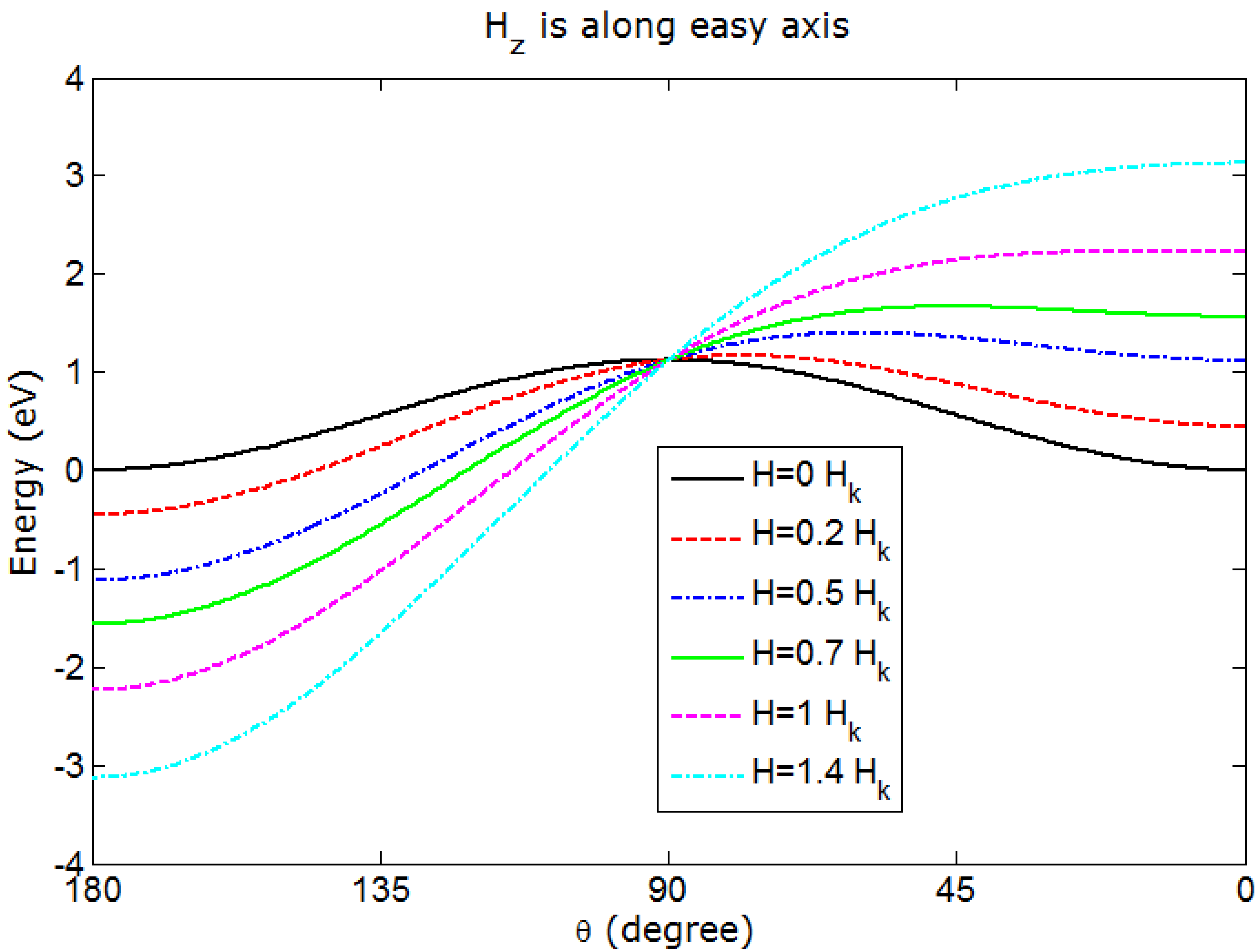


H_y vs θ_{min}

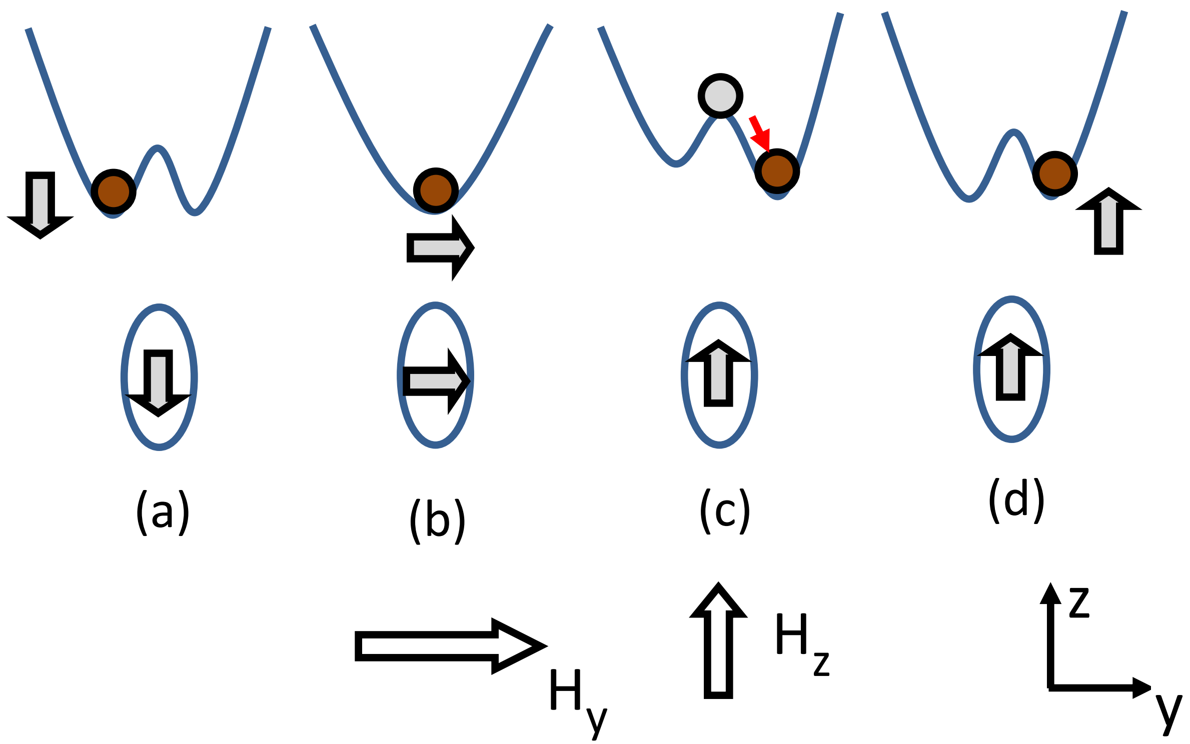


$$\boxed{\theta_{min} = \sin^{-1}\left(\frac{H_y}{H_k}\right)}$$

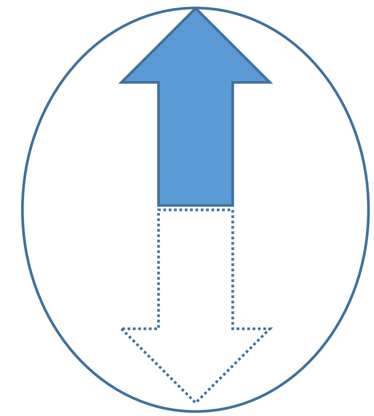
Potential Profiles
H is along easy-axis



Magnetization switching: Magnetic field



DOWN to UP



Magnetic Random Access Memory (MRAM)

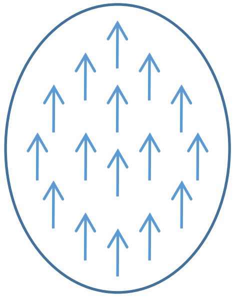
- Huge energy dissipation to generate the magnetic field: 10^7 - 10^8 KT (~**1 pJ**)
- Magnetic field is **difficult to confine** in small space

- **Spin current**
- **Voltage control**

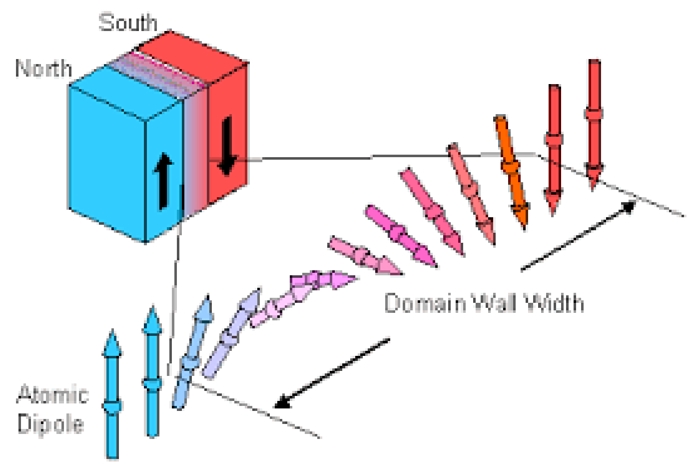
Macrospin versus Multispin analysis

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} + \frac{\alpha}{M}\boldsymbol{M} \times \frac{d\boldsymbol{M}}{dt}$$

$$\frac{\partial \boldsymbol{M}(\boldsymbol{r},t)}{\partial t} = -|\gamma|\boldsymbol{M}(\boldsymbol{r},t) \times \boldsymbol{H_{eff}}(\boldsymbol{r},t) + \frac{\alpha}{M}\boldsymbol{M}(\boldsymbol{r},t) \times \frac{\partial \boldsymbol{M}(\boldsymbol{r},t)}{\partial t}$$



- ➢ Exchange interaction
  - ✓ Pauli's exclusion principle
  - ✓ Coulomb repulsion
- ➢ Dipole interaction



Micromagnetic modelling

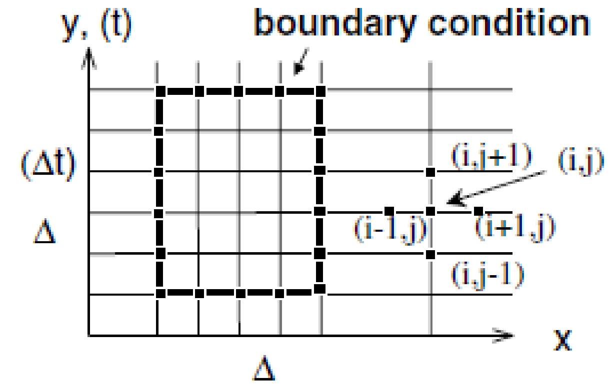
$$\frac{\partial J}{\partial t} = -\frac{|\gamma|}{1+\alpha^2}(J \times H_{eff}) - \frac{\alpha}{J_s(1+\alpha^2)}[J \times (J \times H_{eff})] \tag{5}$$

with

$$H_{eff} = -\frac{\delta E_t}{\delta J} \tag{6}$$

or in the equivalent form given by Gilbert (1955)

$$\frac{\partial J}{\partial t} = -|\gamma|(J \times H_{eff}) + \frac{\alpha}{J_s}\left(J \times \frac{\partial J}{\partial t}\right).$$



$$u(x+\Delta x, y, z, t) = u(x, y, z, t) + \Delta x \frac{\partial u(x, y, z, t)}{\partial x} + \frac{(\Delta x)^2}{2}\frac{\partial^2 u(x, y, z, t)}{\partial x^2} + \cdots. \tag{8}$$

$$H_{ani} = \frac{2K_1}{J_s^2} u_c (J \cdot u_c).$$

$$H_{exch,i} = \frac{2A}{\Delta x^2 \cdot J_s^2} \sum_{i \in NN} J_i$$

$$H_{dip} = -\frac{\Delta x^3}{\mu_0 4\pi} \sum_{j \neq i} \left(\frac{J_j}{R_{ij}^3} - 3\frac{R_{ij}(J_j \cdot R_{ij})}{R_{ij}^5} \right)$$

Fidler, J. and Schrefl, T., Micromagnetic modelling—the current state of the art, J. Phys. D: Appl. Phys. 33, R135–R156 (2000).

OOMMF

https://math.nist.gov/oommf/

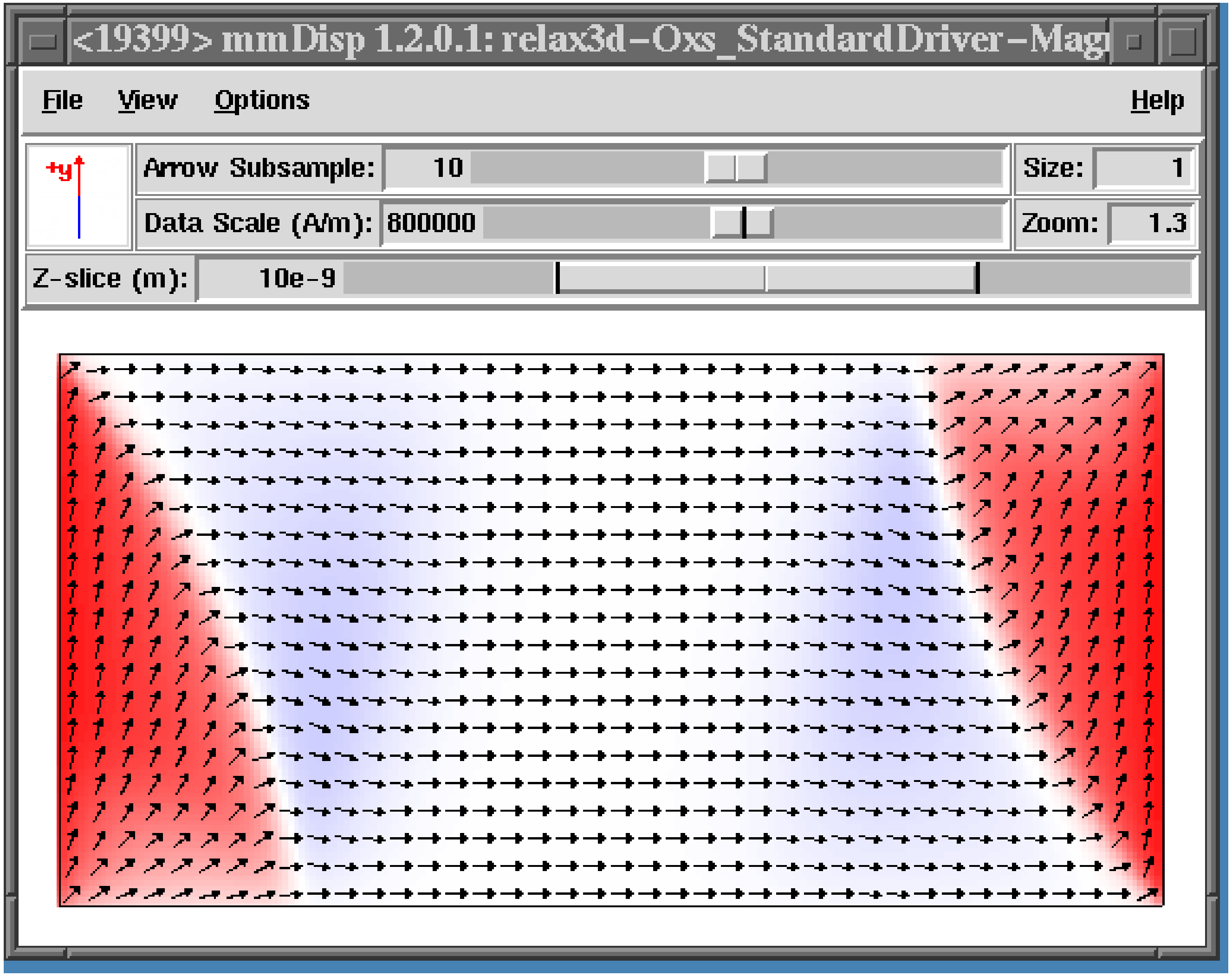
The Object Oriented MicroMagnetic Framework (OOMMF) project at ITL/NIST

Background on the ITL/NIST micromagnetics public code project

OOMMF is a project in the Applied and Computational Mathematics Division (ACMD) of ITL/NIST, in close cooperation with µMAG, aimed at developing portable, extensible public domain programs and tools for micromagnetics. This code forms a completely functional micromagnetics package, with the additional capability to be extended by other programmers so that people developing new code can build on the OOMMF foundation. OOMMF is written in C++, a widely-available, object-oriented language that can produce programs with good performance as well as extensibility. For portable user interfaces, we make use of Tcl/Tk so that OOMMF operates across a wide range of Unix, Windows, and Mac OS X platforms. The main contributors to OOMMF are Mike Donahue, and Don Porter.

OOMMF

https://math.nist.gov/oommf/



Flower and leaf states

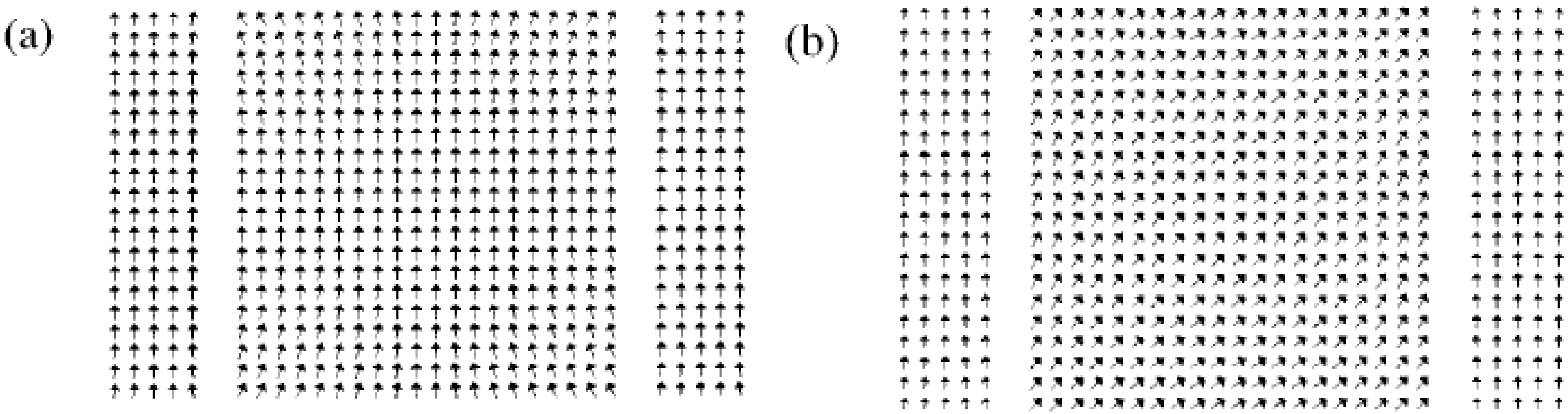


FIG. 1. Examples of the magnetization vectors in (a) the flower and (b) the leaf magnetization distribution in 60×60×15 nm nanostructures. The central part shows a plan view, whereas the side parts show the left and right side surfaces.

Cowburn, R. P. and Welland, M. E., Micromagnetics of the single-domain state of square ferromagnetic nanostructures, Phys. Rev. B 58(14), 9217 (1998).

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- Fidler, J. and Schrefl, T., Micromagnetic modelling—the current state of the art, J. Phys. D: Appl. Phys. 33, R135–R156 (2000).
- Cowburn, R. P. and Welland, M. E., Micromagnetics of the single-domain state of square ferromagnetic nanostructures, Phys. Rev. B 58(14), 9217 (1998).