Spintronics and Nanomagnetics

ECS 521/641

Instructor: Dr. Kuntal Roy

Electrical Engineering and Computer Science (EECS) Dept.

Indian Institute of Science Education and Research (IISER) Bhopal

Email: kuntal@iiserb.ac.in

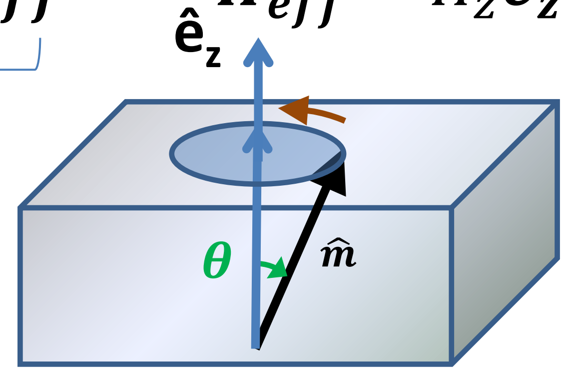
Spin pumping

Ferromagnetic resonance (FMR)

LL Equation

$$\frac{d\boldsymbol{M}}{dt} = \underbrace{-|\gamma|\boldsymbol{M}\times\boldsymbol{H_{eff}}}_{\text{precession}} - \frac{\alpha|\gamma|}{M}\underbrace{\boldsymbol{M}\times\boldsymbol{M}\times\boldsymbol{H_{eff}}}_{\text{damping}}$$

$$\boldsymbol{H_{eff}} = H_z\hat{\boldsymbol{e}}_z$$



Precessing Magnetization

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M}\times\boldsymbol{H_{eff}}$$

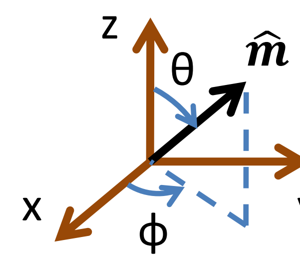
Consider α later

$$H_x^i = H_x - N_{xx}M_x$$

$$H_y^i = H_y - N_{yy}M_y$$

$$M_z = M$$

$$H_z^i = H_z - N_{zz}M_z$$



$$\frac{dM_x}{dt} = -|\gamma|\left(M_yH_z^i - M_zH_y^i\right) = -|\gamma|\left(H_z + \left(N_{yy} - N_{zz}\right)M\right)M_y$$

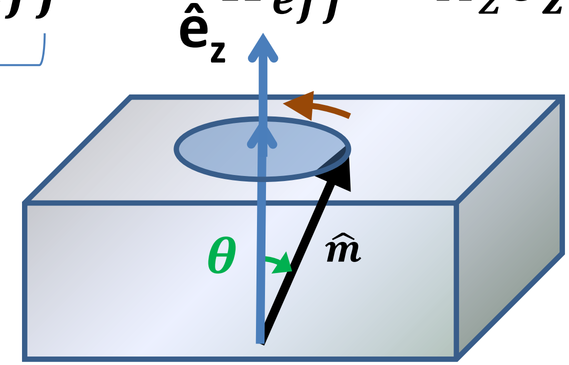
$$\frac{dM_y}{dt} = -|\gamma|\left(M_zH_x^i - M_xH_z^i\right) = |\gamma|(H_z + (N_{xx} - N_{zz})M)M_x$$

Ferromagnetic resonance (FMR)

LL Equation

$$\frac{d\boldsymbol{M}}{dt} = \underbrace{-|\gamma|\boldsymbol{M}\times\boldsymbol{H}_{\boldsymbol{eff}}}_{\text{precession}} - \underbrace{\frac{\alpha|\gamma|}{M}\boldsymbol{M}\times\boldsymbol{M}\times\boldsymbol{H}_{\boldsymbol{eff}}}_{\text{damping}}$$

$$\boldsymbol{H}_{\boldsymbol{eff}} = H_z\hat{\boldsymbol{e}}_{\boldsymbol{z}}$$



Precessing Magnetization

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M}\times\boldsymbol{H}_{\boldsymbol{eff}}$$

Consider α later

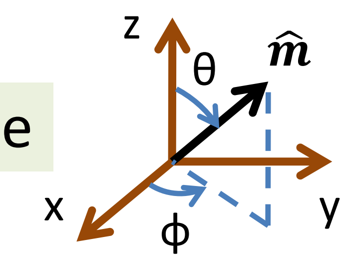
$$M_z = M$$

$$\frac{dM_x}{dt} = -|\gamma|\left(H_z + \left(N_{yy} - N_{zz}\right)M\right)M_y$$

$$\frac{dM_y}{dt} = |\gamma|\left(H_z + \left(N_{xx} - N_{zz}\right)M\right)M_x$$

M_x, M_y

$e^{-i\omega t}$ dependence



$$\begin{vmatrix} i\omega & -|\gamma|\left(H_z + \left(N_{yy} - N_{zz}\right)M\right) \\ |\gamma|\left(H_z + \left(N_{xx} - N_{zz}\right)M\right) & i\omega \end{vmatrix} = 0$$

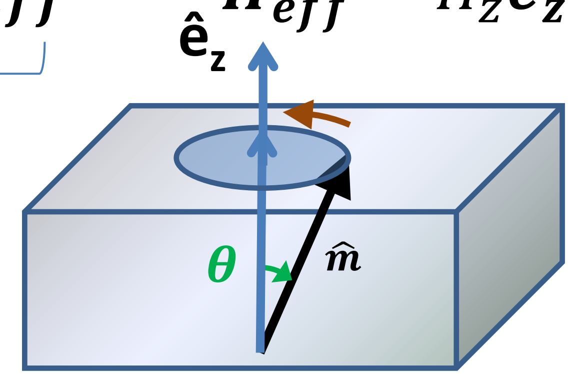
$$\omega^2 = |\gamma|^2\left(H_z + \left(N_{yy} - N_{zz}\right)M\right)\left(H_z + \left(N_{xx} - N_{zz}\right)M\right)$$

Ferromagnetic resonance (FMR)

LL Equation

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\underbrace{\boldsymbol{M}\times\boldsymbol{H_{eff}}}_{\textbf{precession}} - \frac{\alpha|\gamma|}{M}\underbrace{\boldsymbol{M}\times\boldsymbol{M}\times\boldsymbol{H_{eff}}}_{\textbf{damping}}$$

$$\boldsymbol{H_{eff}} = H_z\hat{\boldsymbol{e}}_z$$



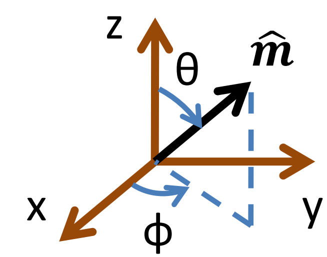
Precessing Magnetization

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M}\times\boldsymbol{H_{eff}}$$

Consider α later

$$M_z = M$$

$$\omega^2 = |\gamma|^2\big(H_z + \big(N_{yy} - N_{zz}\big)M\big)\big(H_z + (N_{xx} - N_{zz})M\big)$$



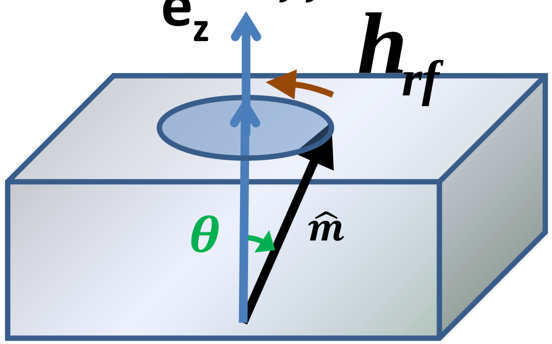
$\omega = \vert\gamma\vert\, H_z$	$N_{xx} = N_{yy} = N_{zz}$	**Sphere**
$\omega = \vert\gamma\vert\sqrt{H_z(H_z + M)}$	$N_{xx} = N_{zz} = 0, N_{yy} = 1$	**In-plane FMR**
$\omega = \vert\gamma\vert(H_z - M)$	$N_{xx} = N_{yy} = 0, N_{zz} = 1$	**Perpendicular FMR**

Ferromagnetic resonance with damping

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M}\times\boldsymbol{H_{eff}} - \frac{\alpha|\gamma|}{M}\boldsymbol{M}\times\boldsymbol{M}\times\boldsymbol{H_{eff}}$$

LL Equation

$$\boldsymbol{H_{eff}} = H_z\hat{\boldsymbol{e}}_z$$



Precessing Magnetization

Linearized for small rotation

Prove

$$\frac{d^2\phi}{dt^2} + \alpha|\gamma|M\frac{d\phi}{dt} + \omega_0^2\phi = 0$$

$$\omega_0 = |\gamma|\sqrt{H_z(H_z + M)}$$

In-plane FMR

Transverse AC field $H_y(t) = H_{y0}e^{i\omega t}$

Negative damping to keep the magnetization rotating

$$\frac{d^2\phi}{dt^2} + \alpha|\gamma|M\frac{d\phi}{dt} + \omega_0^2\phi = |\gamma|^2 M H_{y0}e^{i\omega t}$$

Prove

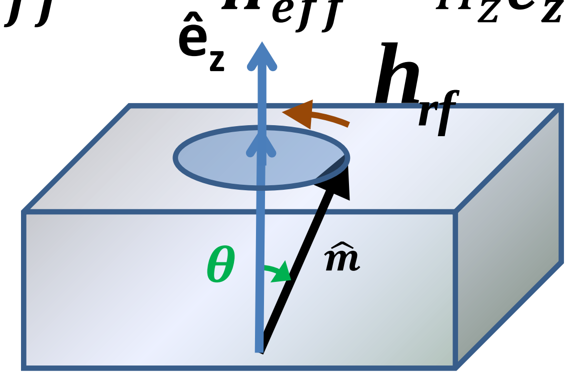
$$\phi(t) = \phi_0 e^{i\omega t} = |\phi_0|e^{i(\omega t+\delta)}$$

Ferromagnetic resonance with damping

LL Equation

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M} \times \boldsymbol{H_{eff}} - \frac{\alpha|\gamma|}{M}\boldsymbol{M} \times \boldsymbol{M} \times \boldsymbol{H_{eff}}$$

$\boldsymbol{H_{eff}} = H_z\hat{\boldsymbol{e}}_z$



Precessing Magnetization

Transverse AC field $H_y(t) = H_{y0}e^{i\omega t}$

$$\frac{d^2\phi}{dt^2} + \alpha|\gamma|M\frac{d\phi}{dt} + \omega_0^2\phi = |\gamma|^2 M H_{y0}e^{i\omega t}$$

$$\phi(t) = \phi_0 e^{i\omega t} = |\phi_0|e^{i(\omega t+\delta)}$$

$$\phi_0 = \frac{|\gamma|^2 M H_{y0}}{(\omega_0^2-\omega^2)^2 + (\alpha|\gamma|M\omega)^2}\left[(\omega_0^2-\omega^2) - i\alpha|\gamma|M\omega\right]$$

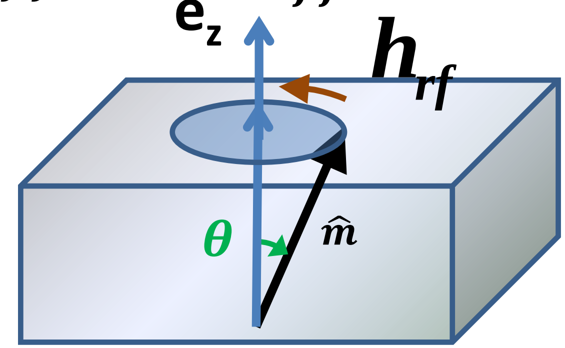
$$|\phi_0| = \frac{|\gamma|^2 M H_{y0}}{\sqrt{(\omega_0^2-\omega^2)^2 + (\alpha|\gamma|M\omega)^2}} \qquad \tan\delta = \frac{-\alpha|\gamma|M\omega}{(\omega_0^2-\omega^2)}$$

Ferromagnetic resonance with damping

LL Equation

$$\frac{d\boldsymbol{M}}{dt} = -|\gamma|\boldsymbol{M}\times\boldsymbol{H}_{\boldsymbol{eff}} - \frac{\alpha|\gamma|}{M}\boldsymbol{M}\times\boldsymbol{M}\times\boldsymbol{H}_{\boldsymbol{eff}}$$

$$\boldsymbol{H}_{\boldsymbol{eff}} = H_z\hat{\boldsymbol{e}}_{\boldsymbol{z}}$$



Precessing Magnetization

Lorentzian

$$Imag(\phi_0) = \frac{-|\gamma|^2 M H_{y0}\alpha|\gamma|M\omega}{(\omega_0^2-\omega^2)^2 + (\alpha|\gamma|M\omega)^2}$$

FMR absorption is given by the imaginary part

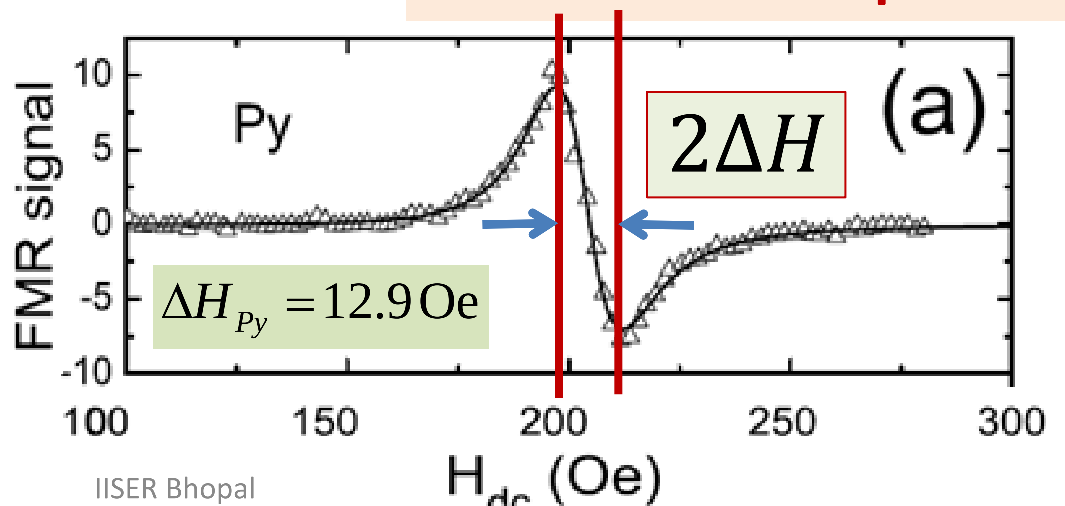
$$Imag(\phi_0) = \frac{H_{y0}|\gamma|}{\alpha\omega_0}$$ at $\omega = \omega_0$

FMR linewidth ΔH

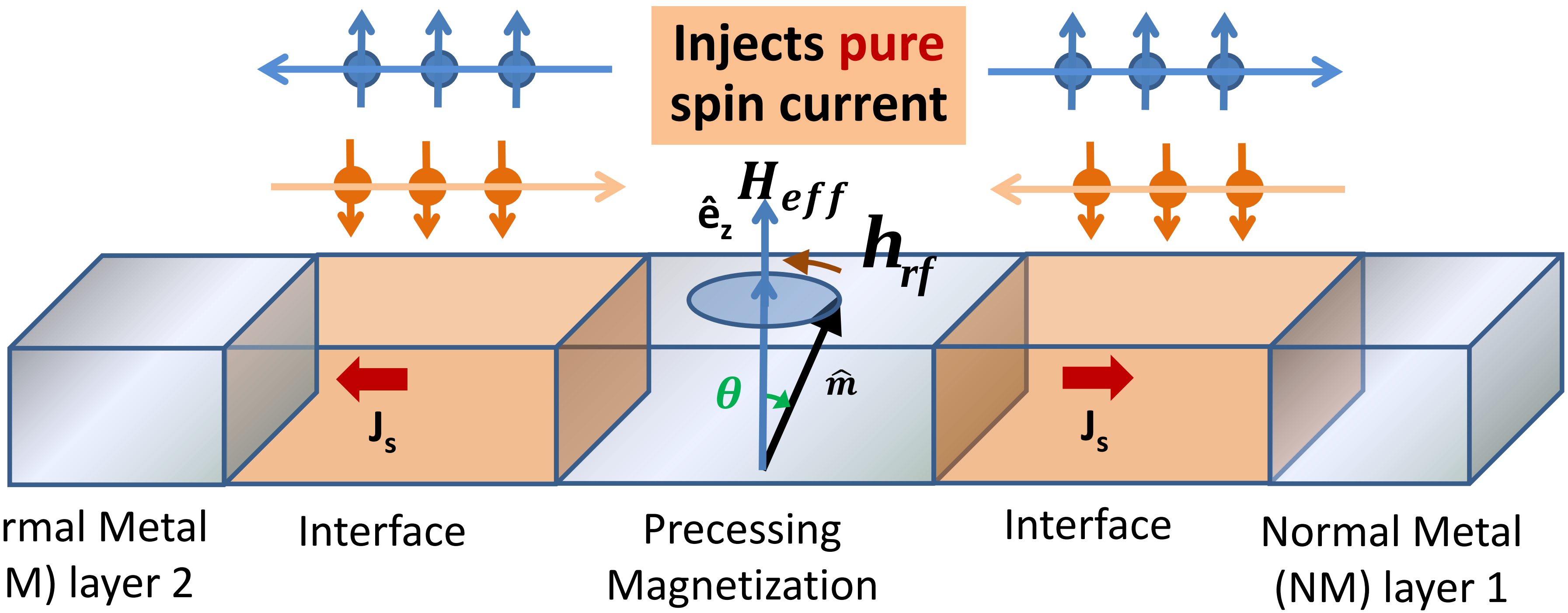
Half width at half maximum (HWHM)

$$\Delta H = \frac{\alpha\omega_0}{|\gamma|}$$

Derivative FMR spectra



Spin pumping



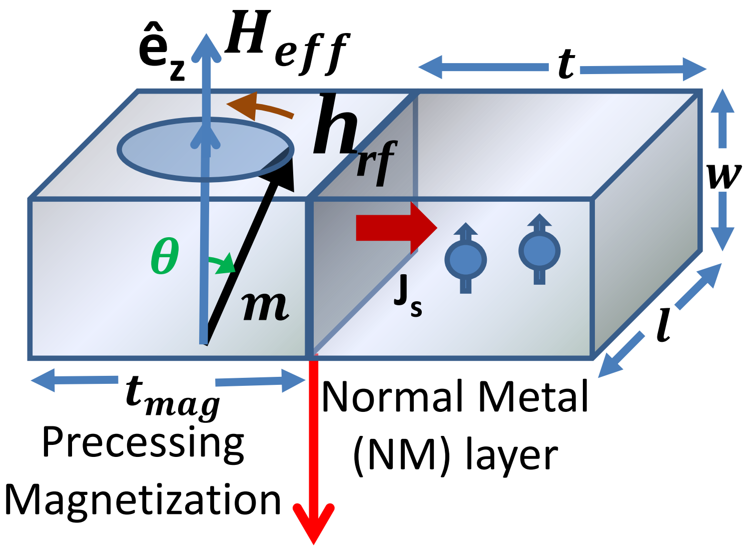
An excited ferromagnet ejects spins into the adjacent materials

Spin battery

Increase in magnetization damping

Tserkovnyak et *al., Phys. Rev. Lett.* **88**, 117601 (2002); *Rev. Mod. Phys.* **77**, 1375 (2005)

Spin pumping: Modification of LLG



LLG Equation

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\mathbf{eff}} + \alpha\, \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

$$J_s\, \mathbf{s} = \frac{\hbar}{4\pi}\left(g^{\uparrow\downarrow}_{r,eff}\, \mathbf{m} \times \frac{d\mathbf{m}}{dt} + g^{\uparrow\downarrow}_{i,eff}\, \frac{d\mathbf{m}}{dt} \right)$$

$$g^{\uparrow\downarrow}_{eff} = g^{\uparrow\downarrow} \,\|\, g_{NM} = g^{\uparrow\downarrow}_{r,eff} + i g^{\uparrow\downarrow}_{i,eff}$$

$g^{\uparrow\downarrow}$: Interfacial spin-mixing conductance, Complex number $(g_r^{\uparrow\downarrow}, g_i^{\uparrow\downarrow})$

$$\frac{d\mathbf{m}}{dt} = -\gamma_{eff}\, \mathbf{m} \times \mathbf{H}_{\mathbf{eff}} + \alpha_{eff}\, \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

$$\alpha_{eff} \frac{\gamma}{\gamma_{eff}} = \alpha + \alpha_{sp}$$

$$\frac{\gamma}{\gamma_{eff}} = 1 - \frac{\hbar\gamma}{(4\pi M_s)\, t_{mag}}\, g^{\uparrow\downarrow}_{i,eff}$$

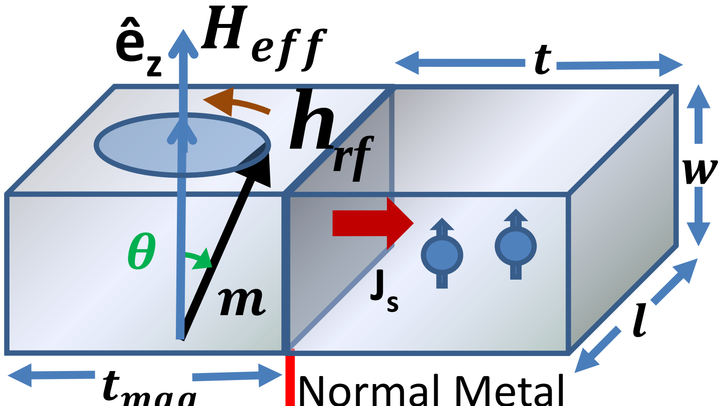
$g^{\uparrow\downarrow}_{r,eff}, g^{\uparrow\downarrow}_{i,eff}$ in $\frac{1}{m^2} = \frac{1}{\mathrm{lw}}$

$$\alpha_{sp} = \frac{\hbar\gamma}{(4\pi M_s)\, t_{mag}}\, g^{\uparrow\downarrow}_{r,eff}$$

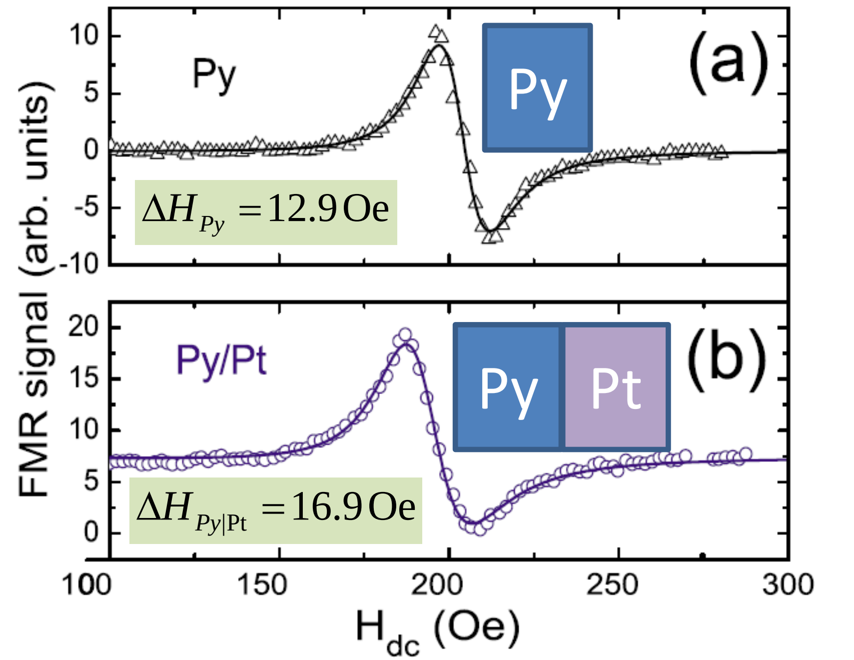
Shift in resonance

Increase in linewidth

Spin pumping
Increase in linewidth and shift in resonance

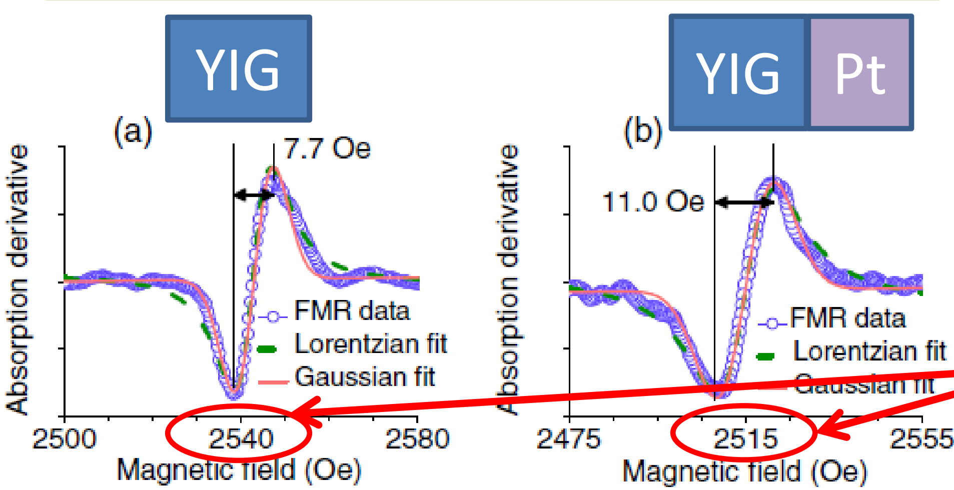


$$g_{eff}^{\uparrow\downarrow} = g^{\uparrow\downarrow} \parallel g_{NM} = g_{r,eff}^{\uparrow\downarrow} + i g_{i,eff}^{\uparrow\downarrow}$$



Mosendz *et al. Phys. Rev. B* **82**, 1214403 (2010)

Increase in linewidth



Magnetic Insulators, YIG

ALSO: Shift in resonance

Sun *et al. Phys. Rev. Lett.* **111**, 106601 (2013)

Spin pumping
Increase in magnetization damping

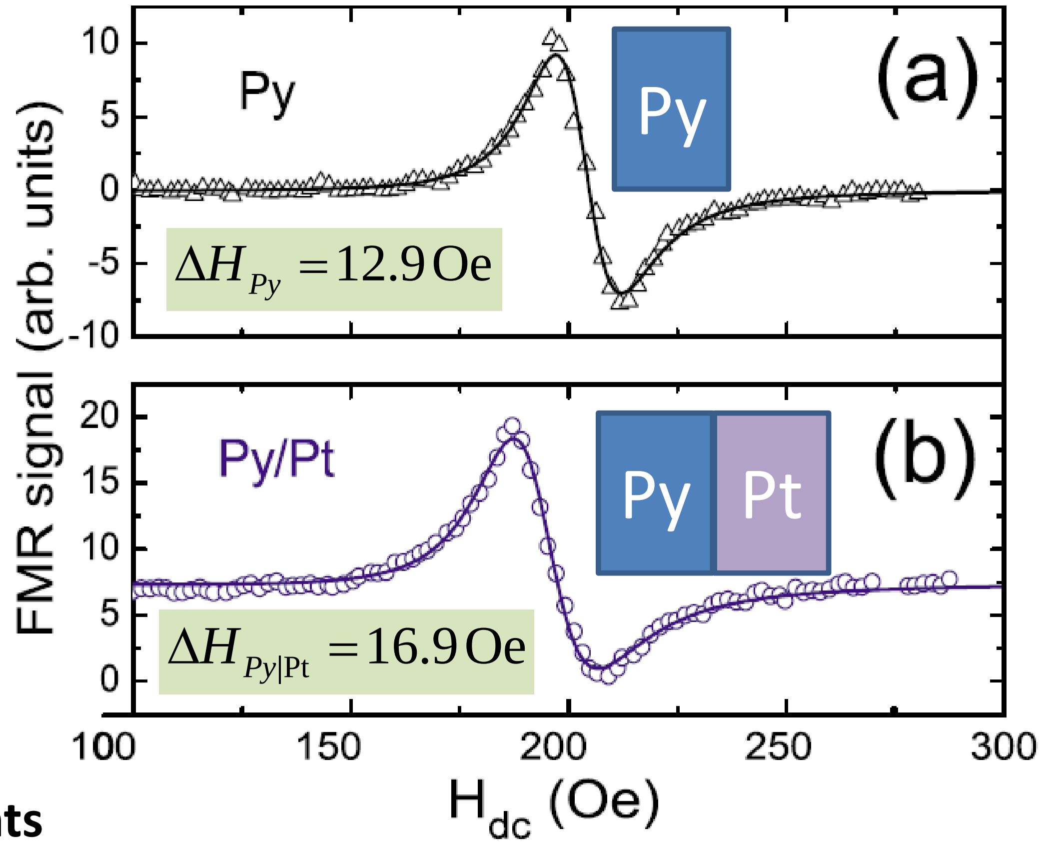
FMR linewidth ΔH

Half width at half maximum (HWHM)

$$\Delta H_{Py} = \alpha\ \omega / \gamma$$

$$\Delta H_{Py|\mathrm{Pt}} = \left(\alpha + \alpha_{sp}\right)\omega / \gamma$$

$$\alpha_{sp} = \frac{g\mu_B}{\left(4\pi M_s\right)t_{Py}} g_{r,eff}^{\uparrow\downarrow}$$



Calculating $g_{eff}^{\uparrow\downarrow}$ from FMR experiments

$$\boxed{g_{r,eff}^{\uparrow\downarrow} = \frac{\left(4\pi M_s\right)\gamma t_{Py}}{g\mu_B\omega}\left(\Delta H_{Py|\mathrm{Pt}} - \Delta H_{Py}\right) = 1.93\times 10^{18}\ \mathrm{m}^{-2}}$$

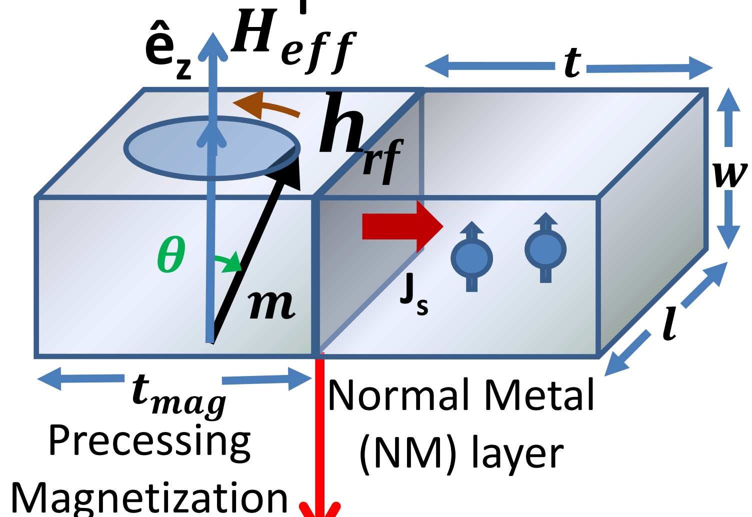
$$4\pi M_s = 8.52e5\ A/m$$

$$t_{Py} = 15\ nm$$

$$f = 4\ GHz$$

Spin pumping

Spin current and spin polarization

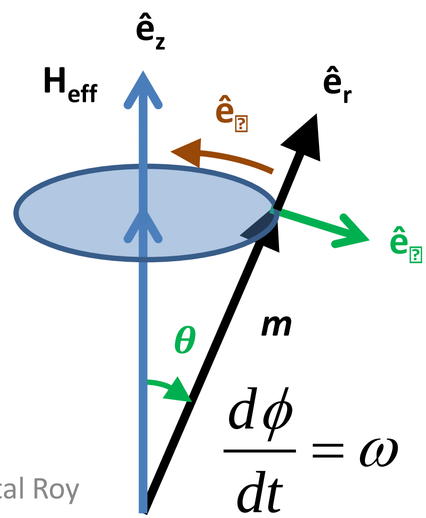


$$\frac{d\boldsymbol{m}}{dt} = -\gamma_{eff}\,\boldsymbol{m}\times\boldsymbol{H}_{eff} + \alpha_{eff}\,\boldsymbol{m}\times\frac{d\boldsymbol{m}}{dt}$$

$$J_s = \frac{\hbar}{4\pi}\left(g^{\uparrow\downarrow}_{r,eff}\,\mathrm{m}\times\frac{d\mathrm{m}}{dt} + g^{\uparrow\downarrow}_{i,eff}\,\frac{d\mathrm{m}}{dt}\right)$$

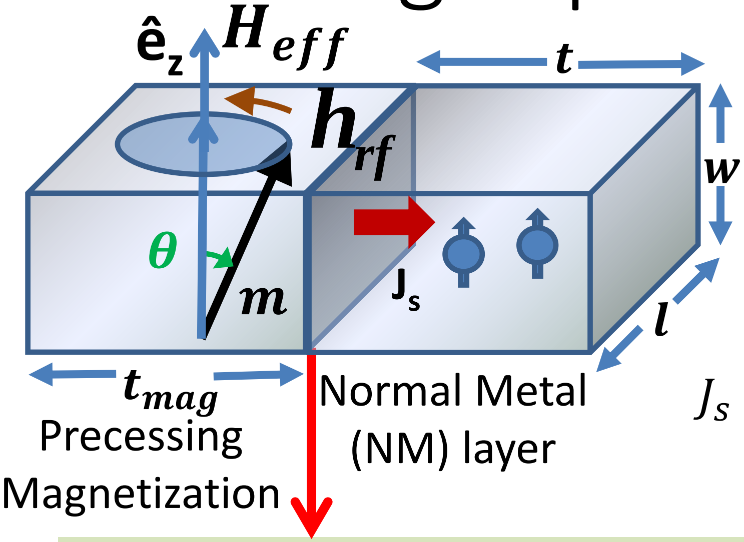
$$g^{\uparrow\downarrow}_{eff} = g^{\uparrow\downarrow} \,||\, g_{NM} = g^{\uparrow\downarrow}_{r,eff} + i g^{\uparrow\downarrow}_{i,eff}$$

$$\frac{d\boldsymbol{m}}{dt} = \sin\theta\,\frac{d\phi}{dt}\,\hat{e}_\phi$$



$$J_s\,\boldsymbol{s} = \frac{\hbar\omega}{4\pi}\,g^{\uparrow\downarrow}_{r,eff}\left[\left(1-m_z^2\right)\hat{\boldsymbol{e}}_z - m_x m_z\hat{\boldsymbol{e}}_x - m_y m_z\hat{\boldsymbol{e}}_y\right] + \frac{\hbar\omega}{4\pi}\,g^{\uparrow\downarrow}_{i,eff}\left[-m_y\hat{\boldsymbol{e}}_x + m_x\hat{\boldsymbol{e}}_y\right]$$

Spin pumping
Average spin current and polarization



Instantaneous

$$J_s = \left(\frac{2e}{\hbar}\right)\frac{\hbar}{4\pi}\left(g_{r,eff}^{\uparrow\downarrow}\,\mathrm{m}\times\frac{d\mathrm{m}}{dt} + g_{i,eff}^{\uparrow\downarrow}\frac{d\mathrm{m}}{dt}\right)$$

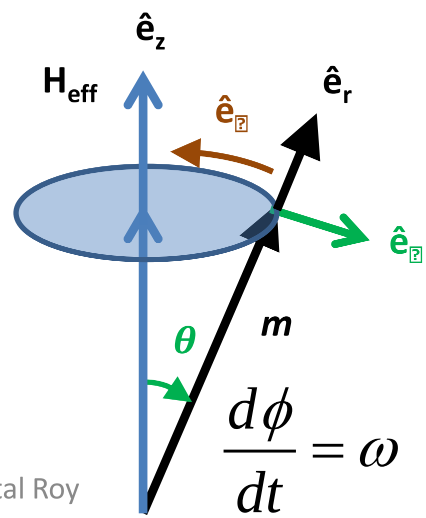
$$J_s = \left(\frac{2e}{\hbar}\right)\frac{\hbar}{4\pi}\left(g_{r,eff}^{\uparrow\downarrow}\,\hat{\mathrm{e}}_{\mathrm{r}}\times\sin\theta\frac{d\phi}{dt}\hat{\mathrm{e}}_{\phi} + g_{r,eff}^{\uparrow\downarrow}\sin\theta\frac{d\phi}{dt}\hat{\mathrm{e}}_{\phi}\right)$$

$$g_{eff}^{\uparrow\downarrow} = g^{\uparrow\downarrow}\,\|\,g_{NM} = g_{r,eff}^{\uparrow\downarrow} + i g_{i,eff}^{\uparrow\downarrow}$$

Average

$$J_s\langle\mathbf{s}\rangle = \left(\frac{2e}{\hbar}\right)\frac{\hbar}{4\pi}g_{r,eff}^{\uparrow\downarrow}\left[\sin\theta\,\omega\left(-\hat{\boldsymbol{e}}_{\theta}\right)\right]$$

$$\hat{\boldsymbol{e}}_{\theta} = \cos\theta\cos\phi\,\hat{\boldsymbol{e}}_x + \cos\theta\sin\phi\,\hat{\boldsymbol{e}}_y - \sin\theta\,\hat{\boldsymbol{e}}_z$$



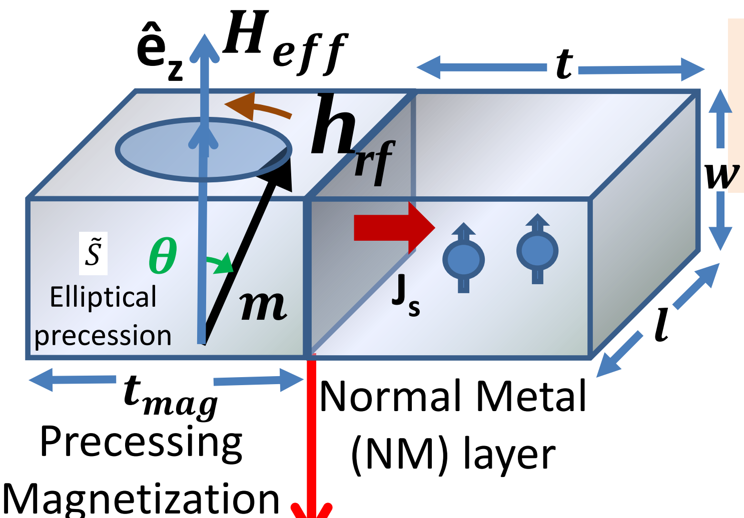
$$I_{SP}^{dc} = G_{SP}^{eff}V_{SP}^{dc}$$

$$\boxed{J_s\langle\mathbf{s}\rangle = \frac{e\omega}{2\pi}g_{r,eff}^{\uparrow\downarrow}\sin^2\theta\,\hat{\boldsymbol{e}}_z}$$

$$V_{SP}^{dc} = \frac{\hbar\omega}{2e}\sin^2\theta$$

$$G_{SP}^{eff} = lw\frac{2e^2}{h}g_{r,eff}^{\uparrow\downarrow}$$

Spin pumping: Elliptical precession factor



Magnetization precession in thin films is elliptic due to strong demagnetization field

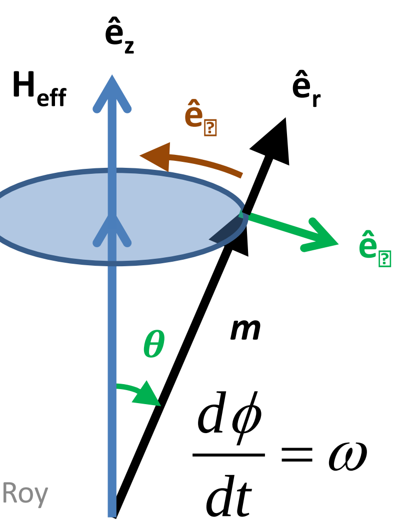
$$g_{eff}^{\uparrow\downarrow} = g^{\uparrow\downarrow} \,||\, g_{NM} = g_{r,eff}^{\uparrow\downarrow} + i g_{i,eff}^{\uparrow\downarrow}$$

Ellipticity correction factor $\tilde{S}$

$$J_{s,dc}^{elliptical} = \tilde{S} J_{s,dc}^{circ}$$

$$\boxed{\tilde{S} = \frac{2\omega\left((4\pi M_s)\gamma + \sqrt{(4\pi M_s)^2\gamma^2 + 4\omega^2}\right)}{(4\pi M_s)^2\gamma^2 + 4\omega^2}}$$

Ando *et al. Appl. Phys. Lett.* **94**, 152509 (2009)

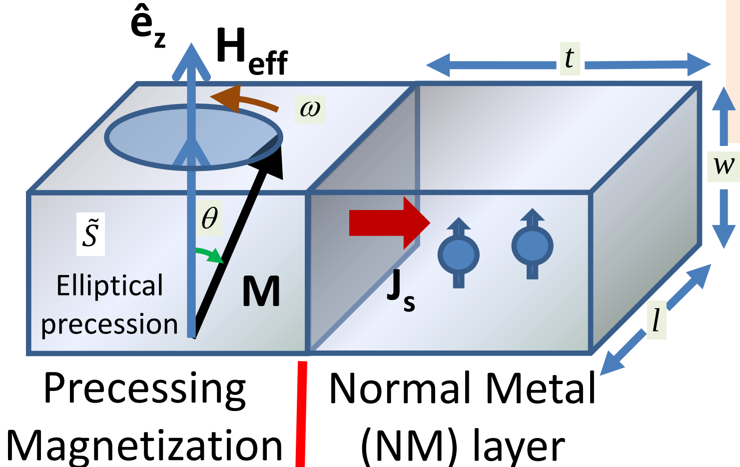


$$I_{SP}^{dc} = G_{SP}^{eff} V_{SP}^{dc} = \tilde{S} \frac{\hbar\omega}{2e} G_{SP}^{eff} \sin^2\theta$$

$$V_{SP}^{dc} = \tilde{S} \frac{\hbar\omega}{2e} \sin^2\theta$$

$$G_{SP}^{eff} = lw \frac{2e^2}{h} g_{r,eff}^{\uparrow\downarrow}$$

Reciprocity: Spin pumping and spin-transfer-torque



Spin-transfer-torque (STT)

$$J_s = -\frac{2e^2}{h}\left(g^{\uparrow\downarrow}_{r,eff}\,\mathrm{m}\times\mathrm{m}\times V_{SP} + g^{\uparrow\downarrow}_{i,eff}\,\mathrm{m}\times V_{SP}\right)$$

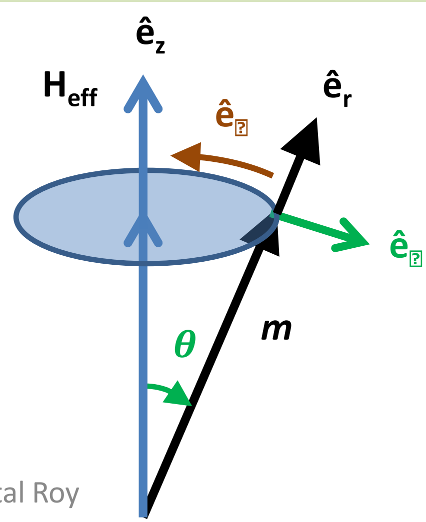
Spin pumping (SP)

$$J_s s = \tilde{S}\,\frac{e}{2\pi}\left(g^{\uparrow\downarrow}_{r,eff}\,\mathrm{m}\times\frac{d\mathrm{m}}{dt} + g^{\uparrow\downarrow}_{i,eff}\,\frac{d\mathrm{m}}{dt}\right)$$

$$g^{\uparrow\downarrow}_{eff} = g^{\uparrow\downarrow} \,\|\, g_{NM} = g^{\uparrow\downarrow}_{r,eff} + i g^{\uparrow\downarrow}_{i,eff}$$

$$\tilde{S}\,\frac{\hbar}{2e}\,\frac{d\mathrm{m}}{dt} = -\mathrm{m}\times V_{SP}$$

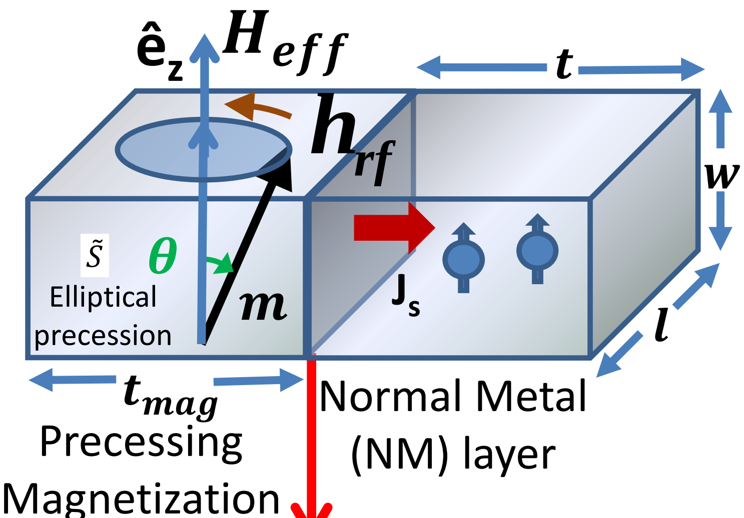
$$I^{dc}_{SP} = G^{eff}_{SP} V^{dc}_{SP} = \tilde{S}\,\frac{\hbar\omega}{2e}\,G^{eff}_{SP}\sin^2\theta$$



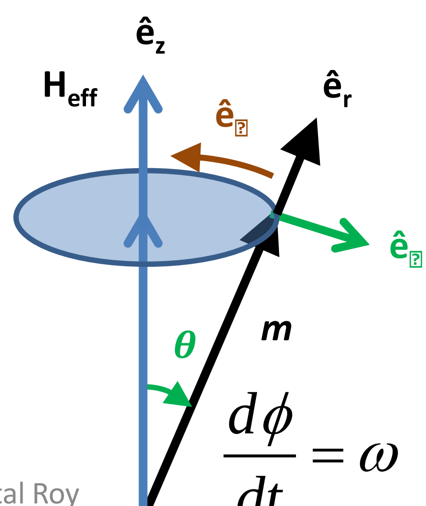
$$V^{dc}_{SP} = \tilde{S}\,\frac{\hbar\omega}{2e}\sin^2\theta$$

$$G^{eff}_{SP} = lw\,\frac{2e^2}{h}\,g^{\uparrow\downarrow}_{r,eff}$$

Spin relaxation

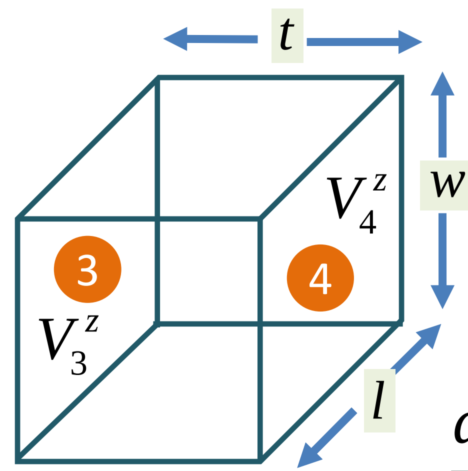


$$g_{eff}^{\uparrow\downarrow} = g^{\uparrow\downarrow} \,||\, g_{NM} = g_{r,eff}^{\uparrow\downarrow} + i g_{i,eff}^{\uparrow\downarrow}$$



$$\frac{d\phi}{dt} = \omega$$

Spin relaxes due to spin diffusion



V_3^z, V_4^z

λ: Spin diffusion length

$$\frac{d^2 V^z(y)}{dy^2} = \frac{V^z(y)}{\lambda^2}$$

$$I_{SP}^{dc} = G_{SP}^{eff} V_{SP}^{dc} = \tilde{S} \frac{\hbar\omega}{2e} G_{SP}^{eff} \sin^2\theta$$

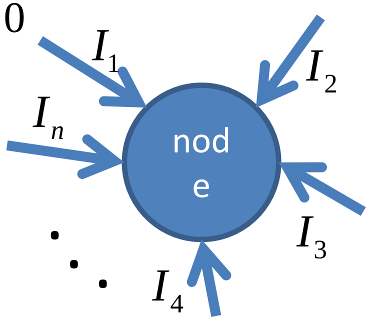
$$V_{SP}^{dc} = \tilde{S} \frac{\hbar\omega}{2e} \sin^2\theta$$

$$G_{SP}^{eff} = lw \frac{2e^2}{h} g_{r,eff}^{\uparrow\downarrow}$$

Spin circuits for spintronic devices

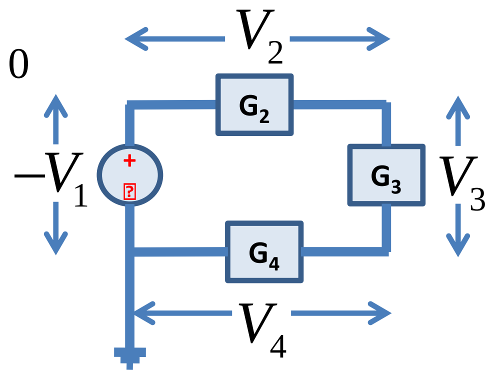
Current Law (KCL)
Conservation of Charge

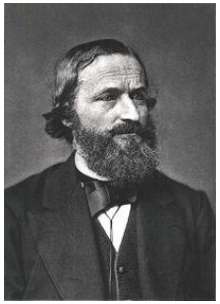
$$\sum_{k=1}^{n} I_k = 0$$



Voltage Law (KVL)
Conservation of Energy

$$\sum_{k=1}^{n} V_k = 0$$





G. R. Kirchoff
(1824-1887)

Development of SPICE

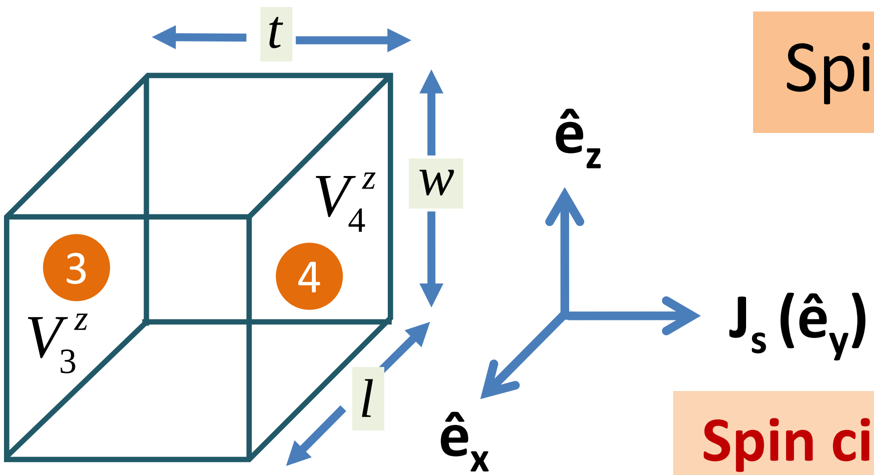
Commercial tool for transistors, e.g., HSPICE, Synopsys Inc.

Can we apply
Kirchoff's circuit laws for **spintronic circuits?**

The circuit elements are 4-component matrices
1 charge, 3 for spin vector

- ✓ **Complex functional devices can be analyzed and proposed using spin circuits**
- ✓ **Simple to conceive**

Spin circuits for a Normal Metal (NM)

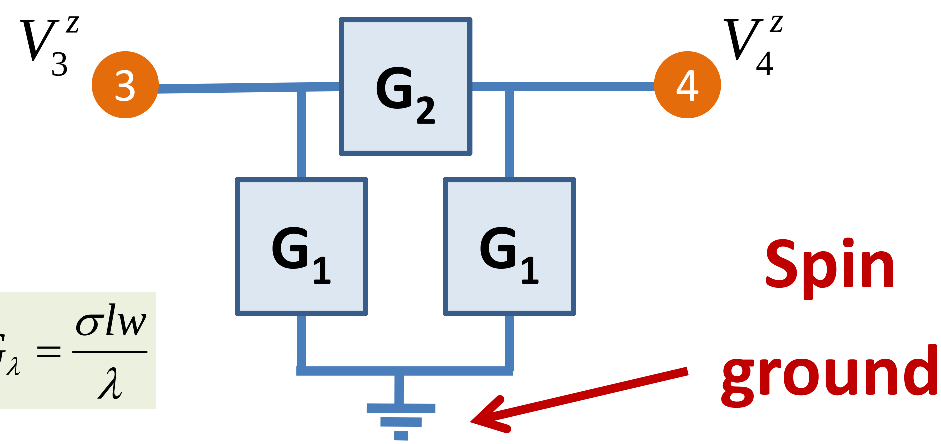


Spin relaxes due to spin diffusion

$$\frac{d^2V^z(y)}{dy^2} = \frac{V^z(y)}{\lambda^2}$$

λ: Spin diffusion length

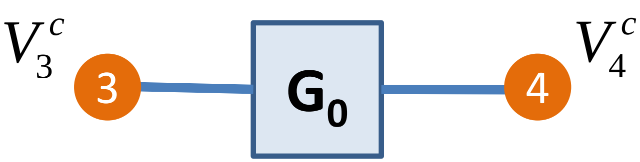
Spin circuit



$$G_\lambda = \frac{\sigma l w}{\lambda}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right) \qquad G_2 = G_\lambda \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

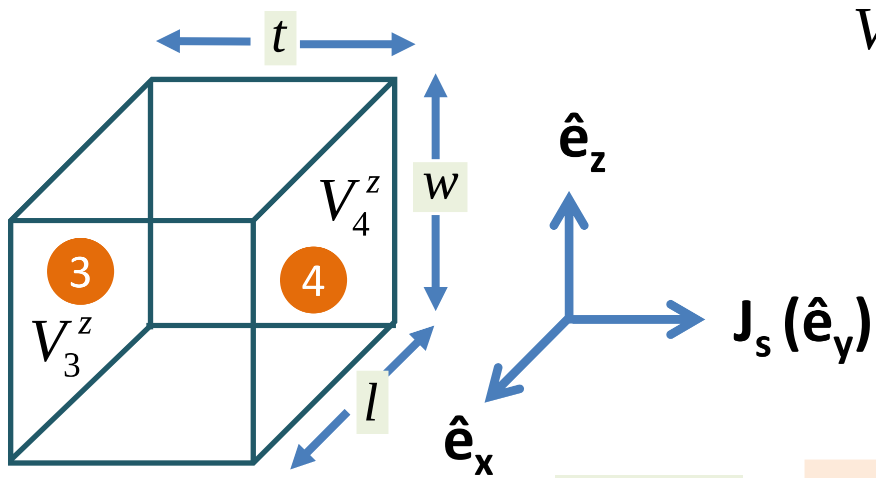
Charge circuit

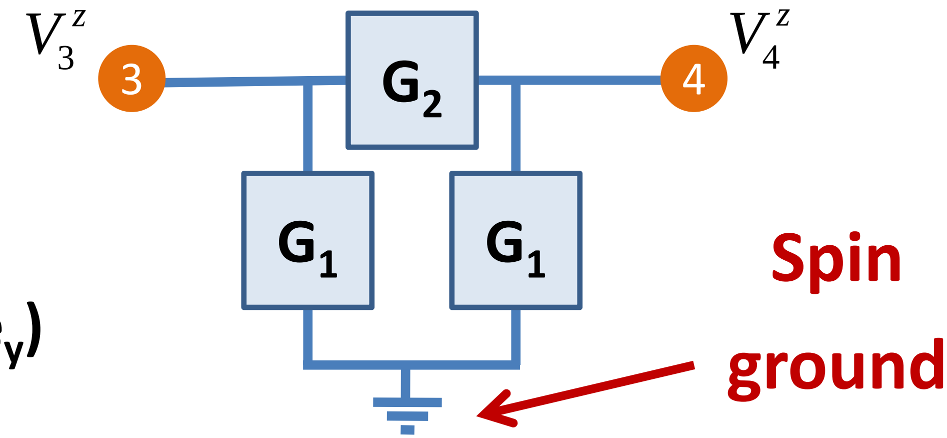


$$G_0 = \frac{\sigma l w}{t}$$

No shunt elements

Spin circuits for a Normal Metal (NM): Proof





$$G_\lambda = \frac{\sigma l w}{\lambda}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

$$G_2 = G_\lambda \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

$$\frac{d^2V^z(y)}{dy^2} = \frac{V^z(y)}{\lambda^2}$$

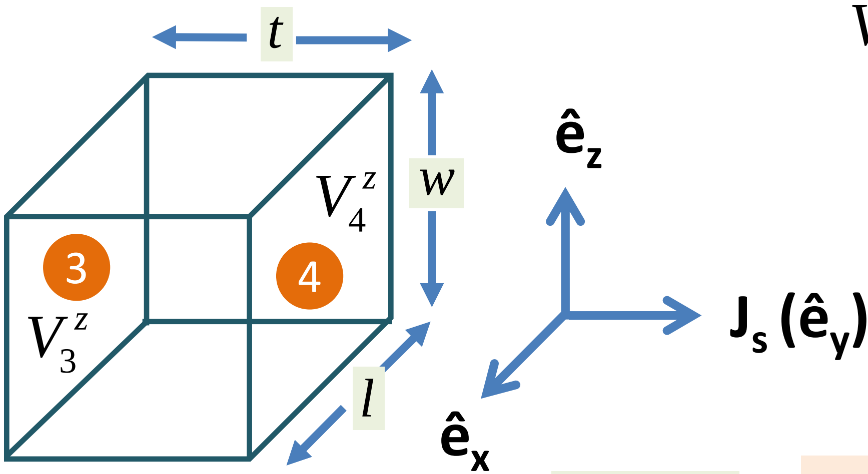
$V^z(y)$?

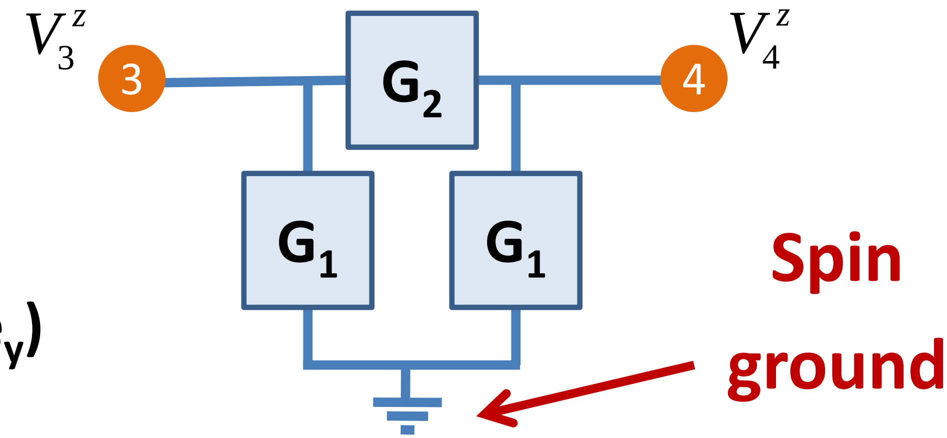
Satisfies

$$V^z(y) = \frac{V_4^z \sinh\frac{y}{\lambda} + V_3^z \sinh\frac{t-y}{\lambda}}{\sinh\frac{t}{\lambda}}$$

$$J(y) = -\sigma\frac{dV^z(y)}{dy} = -\frac{\sigma}{\lambda}\frac{V_4^z \cosh\frac{y}{\lambda} - V_3^z \cosh\frac{t-y}{\lambda}}{\sinh\frac{t}{\lambda}}$$

Spin circuits for a Normal Metal (NM): Proof





$$G_\lambda = \frac{\sigma l w}{\lambda}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

$$G_2 = G_\lambda \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

$$\frac{d^2 V^z(y)}{dy^2} = \frac{V^z(y)}{\lambda^2}$$

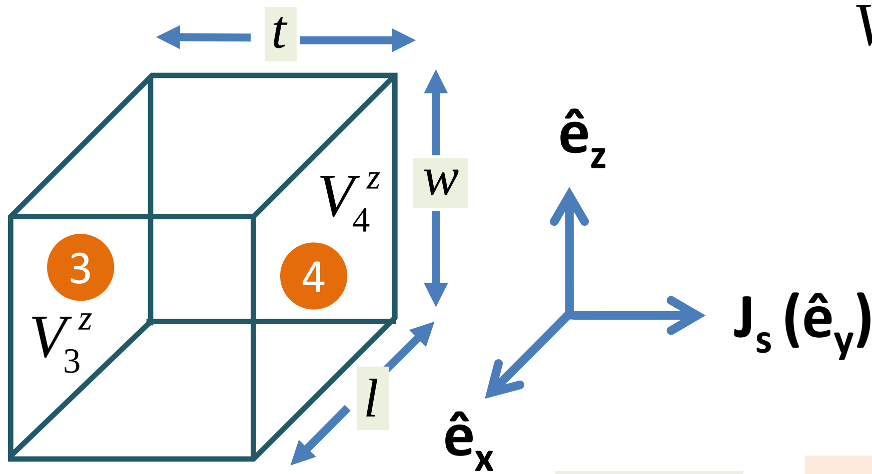
$J(0), J(t)$?

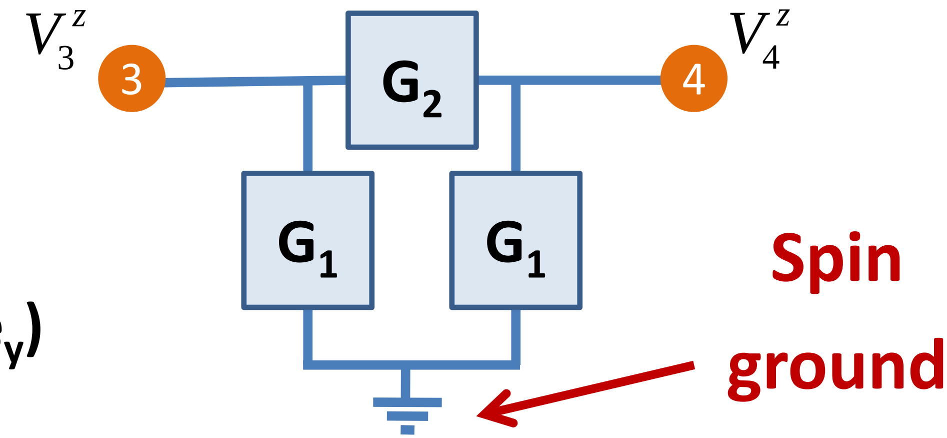
$$J(y) = -\sigma \frac{dV^z(y)}{dy} = -\frac{\sigma}{\lambda} \frac{V_4^z \cosh\frac{y}{\lambda} - V_3^z \cosh\frac{t-y}{\lambda}}{\sinh\frac{t}{\lambda}}$$

$$J(0) = -\frac{\sigma}{\lambda}\left[V_4^z \operatorname{csch}\frac{t}{\lambda} - V_3^z \coth\frac{t}{\lambda}\right]$$

$$J(t) = -\frac{\sigma}{\lambda}\left[V_4^z \coth\frac{t}{\lambda} - V_3^z \operatorname{csch}\frac{t}{\lambda}\right]$$

Spin circuits for a Normal Metal (NM): Proof





$$G_\lambda = \frac{\sigma l w}{\lambda}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

$$G_2 = G_\lambda \,\mathrm{csch}\left(\frac{t}{\lambda}\right)$$

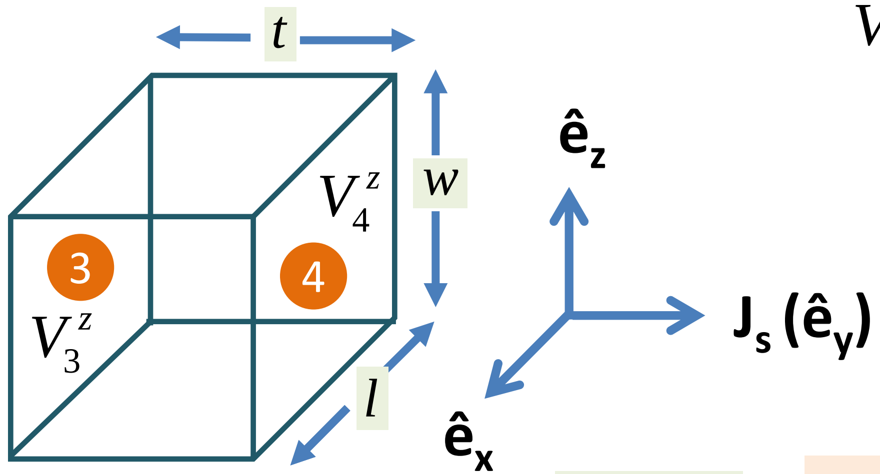
$$\frac{d^2 V^z(y)}{dy^2} = \frac{V^z(y)}{\lambda^2}$$

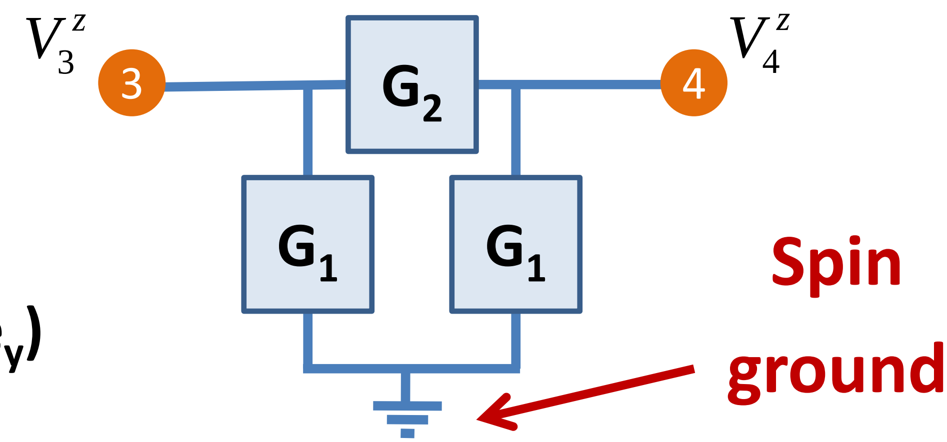
$$J(0) = -\frac{\sigma}{\lambda}\left[V_4^z \,\mathrm{csch}\,\frac{t}{\lambda} - V_3^z \coth\frac{t}{\lambda}\right] \qquad J(t) = -\frac{\sigma}{\lambda}\left[V_4^z \coth\frac{t}{\lambda} - V_3^z \,\mathrm{csch}\,\frac{t}{\lambda}\right]$$

$$\frac{1}{lw}\begin{bmatrix} I_3^z \\ I_4^z \end{bmatrix} = -\frac{\sigma}{\lambda}\begin{bmatrix} -\coth\frac{t}{\lambda} & \mathrm{csch}\,\frac{t}{\lambda} \\ -\mathrm{csch}\,\frac{t}{\lambda} & \coth\frac{t}{\lambda} \end{bmatrix}\begin{bmatrix} V_3^z \\ V_4^z \end{bmatrix}$$

$$I_3^z = \frac{\sigma l w}{\lambda}\left[(V_3^z - V_4^z)\,\mathrm{csch}\,\frac{t}{\lambda} + V_3^z\left(\coth\frac{t}{\lambda} - \mathrm{csch}\,\frac{t}{\lambda}\right)\right]$$

Spin circuits for a Normal Metal (NM): Proof





$$\frac{d^2V^z(y)}{dy^2} = \frac{V^z(y)}{\lambda^2}$$

$$G_\lambda = \frac{\sigma l w}{\lambda}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

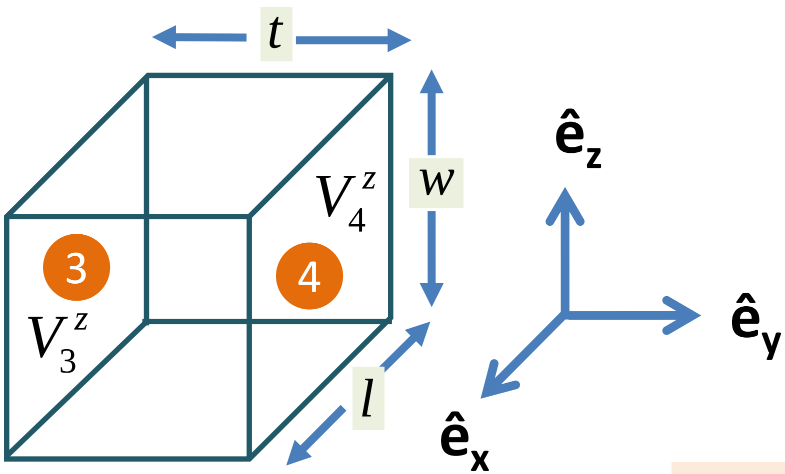
$$G_2 = G_\lambda \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

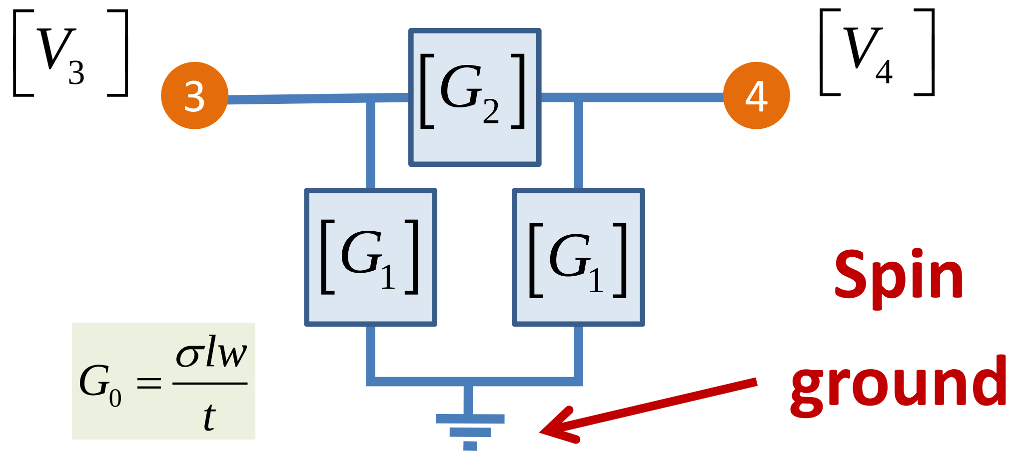
$$I_3^z = \frac{\sigma l w}{\lambda}\left[(V_3^z - V_4^z)\operatorname{csch}\frac{t}{\lambda} + V_3^z\left(\coth\frac{t}{\lambda} - \operatorname{csch}\frac{t}{\lambda}\right)\right]$$

$$G_1 = \frac{\sigma l w}{\lambda}\left[\coth\frac{t}{\lambda} - \operatorname{csch}\frac{t}{\lambda}\right] = G_\lambda \tanh\frac{t}{2\lambda}$$

$$G_2 = G_\lambda \operatorname{csch}\frac{t}{\lambda}$$

Spin circuits: 4-component model for NM



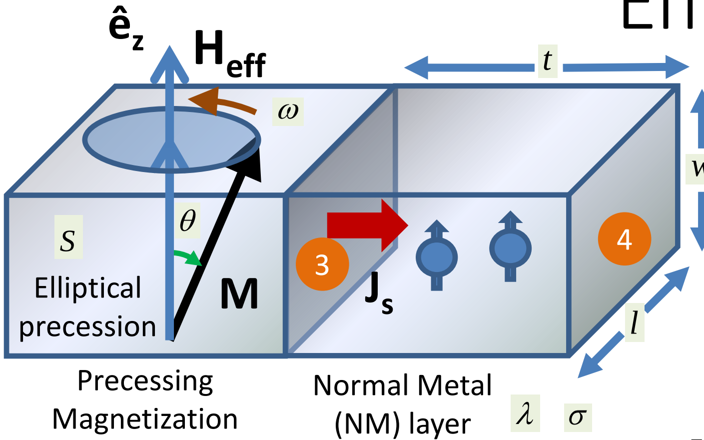


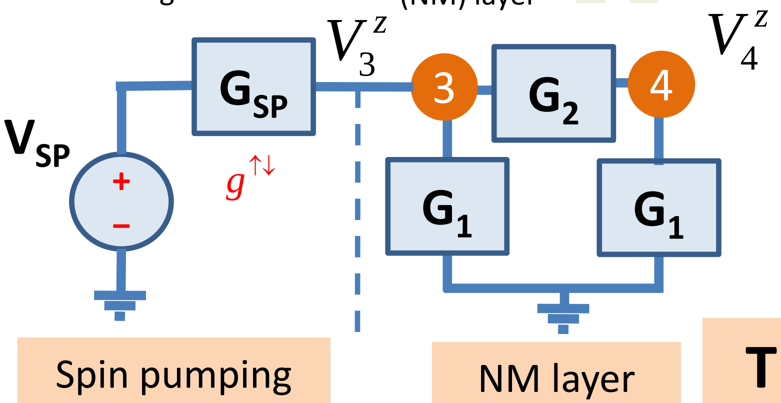
$$G_0 = \frac{\sigma l w}{t}$$

$$G_1 = \frac{\sigma l w}{\lambda} \tanh\left(\frac{t}{2\lambda}\right) \qquad G_2 = \frac{\sigma l w}{\lambda} \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

$$\begin{array}{cc} & \begin{array}{cccc} c & z & x & y \end{array} \\ [G_1] = & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & G_1 & 0 & 0 \\ 0 & 0 & G_1 & 0 \\ 0 & 0 & 0 & G_1 \end{pmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cccc} c & z & x & y \end{array} \\ [G_2] = & \begin{pmatrix} G_0 & 0 & 0 & 0 \\ 0 & G_2 & 0 & 0 \\ 0 & 0 & G_2 & 0 \\ 0 & 0 & 0 & G_2 \end{pmatrix} \end{array}$$

Spin circuit representation of spin pumping
Effective spin mixing conductance





$$V_{SP} = \frac{S\hbar\omega}{2e}\sin^2\theta$$

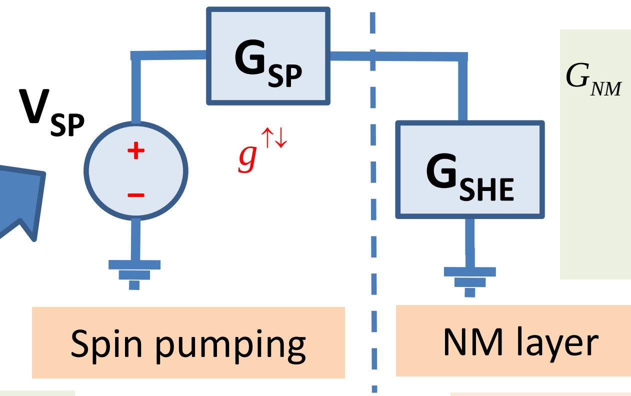
$$G_{SP} = lw\frac{2e^2}{h} g^{\uparrow\downarrow}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

$$G_2 = G_\lambda \,\mathrm{csch}\left(\frac{t}{\lambda}\right)$$

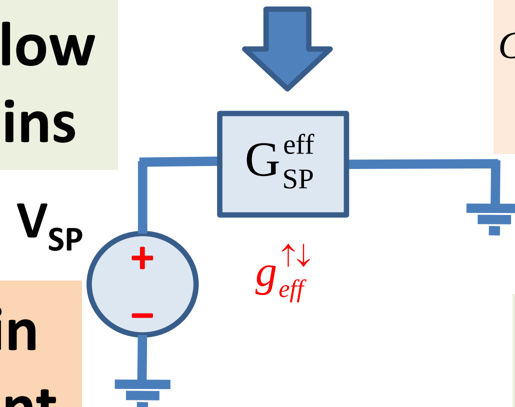
$$G_\lambda = \frac{\sigma l w}{\lambda}$$

Backflow of spins



$$G_{NM} = G_1 + \frac{G_1 G_2}{G_1 + G_2} = \frac{1}{\dfrac{\lambda}{\sigma l w}\coth\left(\dfrac{t}{\lambda}\right)}$$

$$G_{SP}^{eff} = \frac{G_{SP} G_{NM}}{G_{SP} + G_{NM}} = \frac{G_{SP}}{1 + \dfrac{G_{SP}}{G_{NM}}}$$



$$g_{i,eff}^{\uparrow\downarrow} \ll g_{r,eff}^{\uparrow\downarrow}$$

$$G_{SP}^{eff} = lw\frac{2e^2}{h} g_{eff}^{\uparrow\downarrow}$$

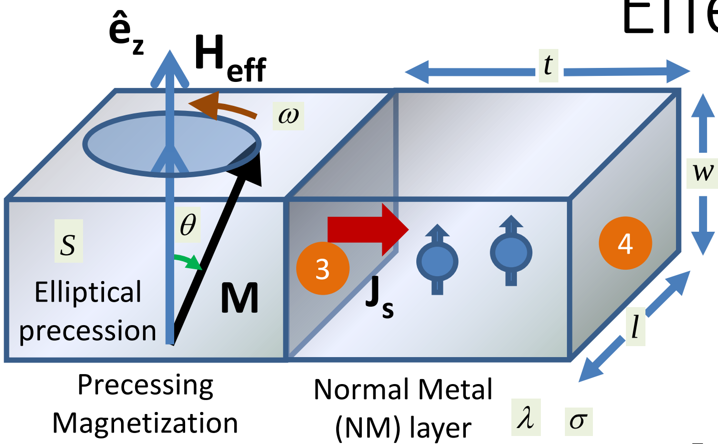
Thevenin equivalent

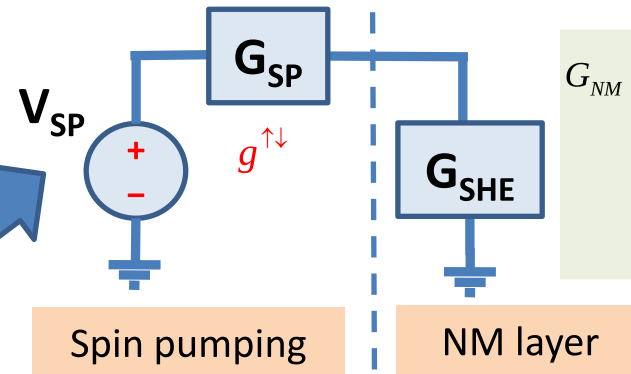
K. Roy, Phys Rev Appl. (Letter) 8, 011001 (2017)

$$\boxed{g_{eff}^{\uparrow\downarrow} = \frac{g^{\uparrow\downarrow}}{1 + \dfrac{\lambda}{\sigma}\dfrac{2e^2}{h} g^{\uparrow\downarrow}\coth\dfrac{t}{\lambda}}}$$

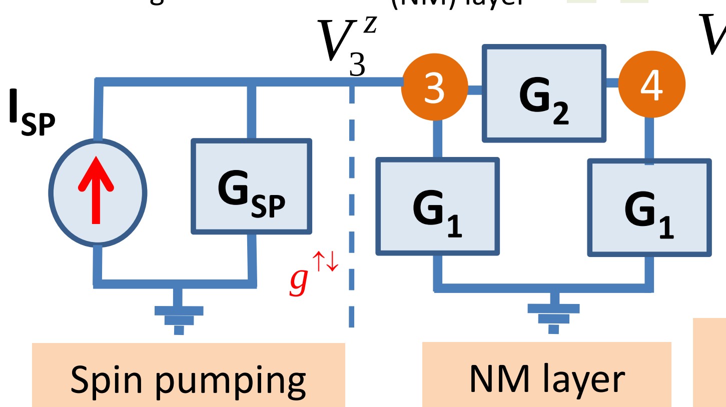
$$g_i^{\uparrow\downarrow} \ll g_r^{\uparrow\downarrow}$$

Spin circuit representation of spin pumping
Effective spin mixing conductance



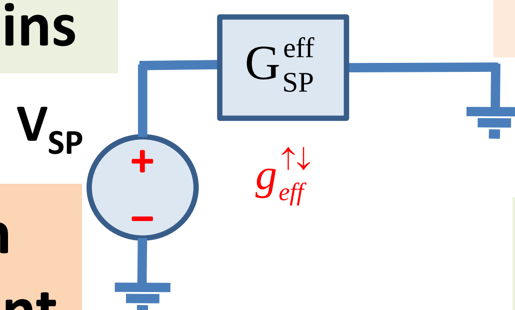


$$G_{NM} = G_1 + \frac{G_1 G_2}{G_1 + G_2} = \frac{1}{\frac{\lambda}{\sigma l w}\coth\left(\frac{t}{\lambda}\right)}$$



Backflow of spins

$$G_{SP}^{eff} = \frac{G_{SP} G_{NM}}{G_{SP} + G_{NM}} = \frac{G_{SP}}{1 + \frac{G_{SP}}{G_{NM}}}$$



$$g_{i,eff}^{\uparrow\downarrow} \ll g_{r,eff}^{\uparrow\downarrow}$$

Norton equivalent

$$G_{SP}^{eff} = lw \frac{2e^2}{h} g_{eff}^{\uparrow\downarrow}$$

K. Roy, Phys Rev Appl. (Letter) 8, 011001 (2017)

$$V_{SP} = \frac{S\hbar\omega}{2e}\sin^2\theta$$

$$G_{SP} = lw\frac{2e^2}{h} g^{\uparrow\downarrow}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

$$G_2 = G_\lambda \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

$$G_\lambda = \frac{\sigma l w}{\lambda}$$

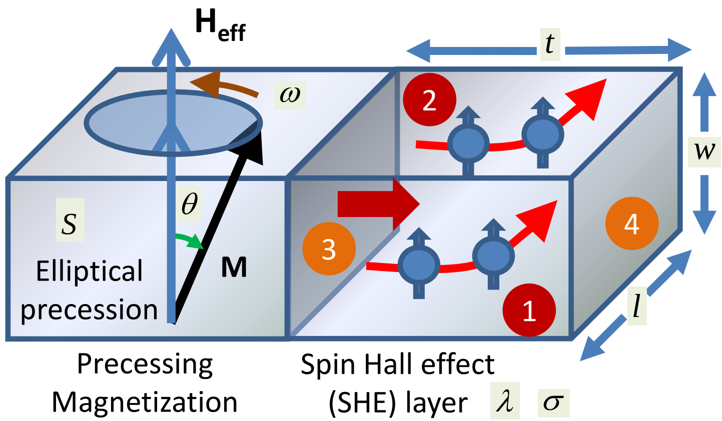
$$I_{SP} = V_{SP} G_{SP}$$

$$\boxed{g_{eff}^{\uparrow\downarrow} = \frac{g^{\uparrow\downarrow}}{1 + \frac{\lambda}{\sigma}\frac{2e^2}{h} g^{\uparrow\downarrow} \coth\frac{t}{\lambda}}}$$

$$g_i^{\uparrow\downarrow} \ll g_r^{\uparrow\downarrow}$$

Spin circuit representation of spin pumping

Inverse SHE voltage V_{ISHE}



$$V_{SP} = \frac{\tilde{S}\hbar\omega}{2e}\sin^2\theta$$

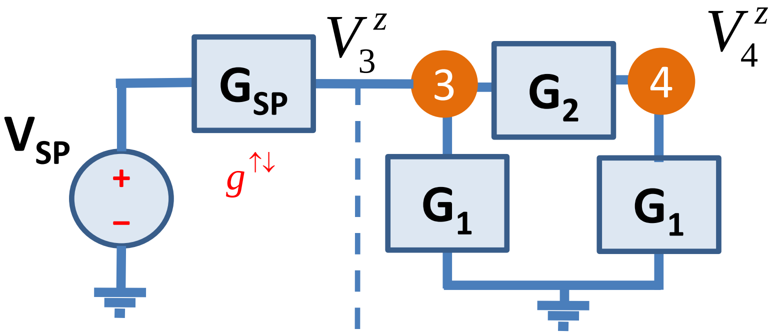
$$G_{SP} = lw\frac{2e^2}{h}g^{\uparrow\downarrow}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

$$G_2 = G_\lambda \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

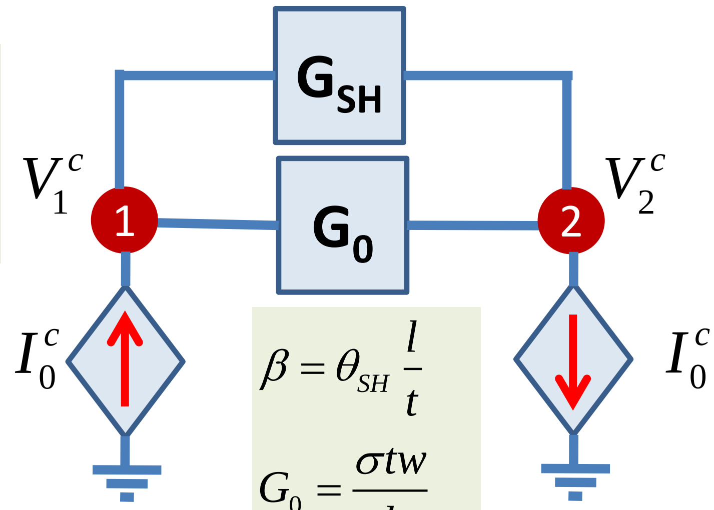
$$G_\lambda = \frac{\sigma l w}{\lambda}$$

$$G_{SH} = \frac{\sigma_{sh} t_{sh} w}{l}$$



Spin pumping

SHE layer



$$\beta = \theta_{SH}\frac{l}{t}$$

$$G_0 = \frac{\sigma t w}{l}$$

$$I_0^c = \beta G_0\left(V_3^z - V_4^z\right)$$

Solving KCL

$$V_{ISHE} = -\beta\left(\frac{G_0}{G_0 + G_{SH}}\right)\left(\frac{G_1}{(G_1+G_2)(G_{SP}+G_1+G_2)-G_2^2}\right)V_{SP}G_{SP}$$

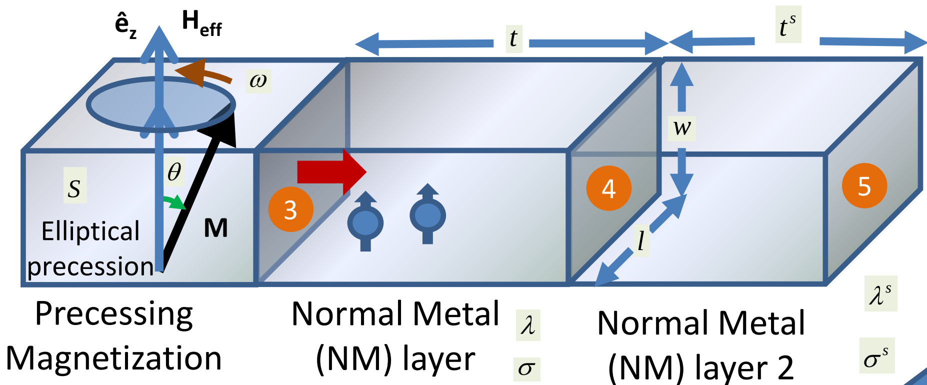
$$V_{ISHE} = -\frac{\theta_{SH} l\lambda e\tilde{S}\omega g^{\uparrow\downarrow}\sin^2\theta\tanh\left(\frac{t}{2\lambda}\right)}{2\pi(\sigma t + \sigma_{sh}t_{sh})\left(1 + \frac{\lambda}{\sigma}\frac{2e^2}{h}g^{\uparrow\downarrow}\coth\left(\frac{t}{\lambda}\right)\right)}$$

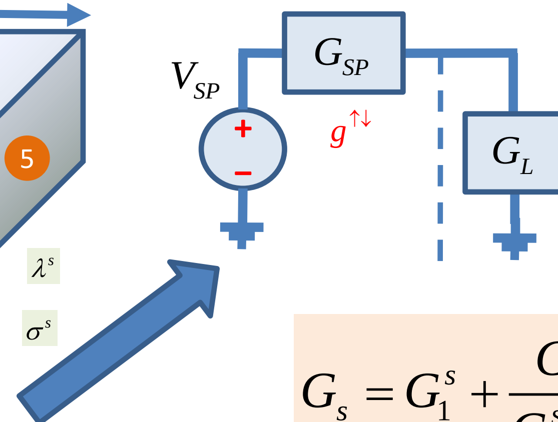
$$g_{eff}^{\uparrow\downarrow} = \frac{g^{\uparrow\downarrow}}{1 + \frac{\lambda}{\sigma}\frac{2e^2}{h}g^{\uparrow\downarrow}\coth\frac{t}{\lambda}}$$

$$V_{ISHE} = V_2^c - V_1^c$$

K. Roy, Phys Rev Appl. (Letter) 8, 011001 (2017)

Spin pumping in multilayered structures

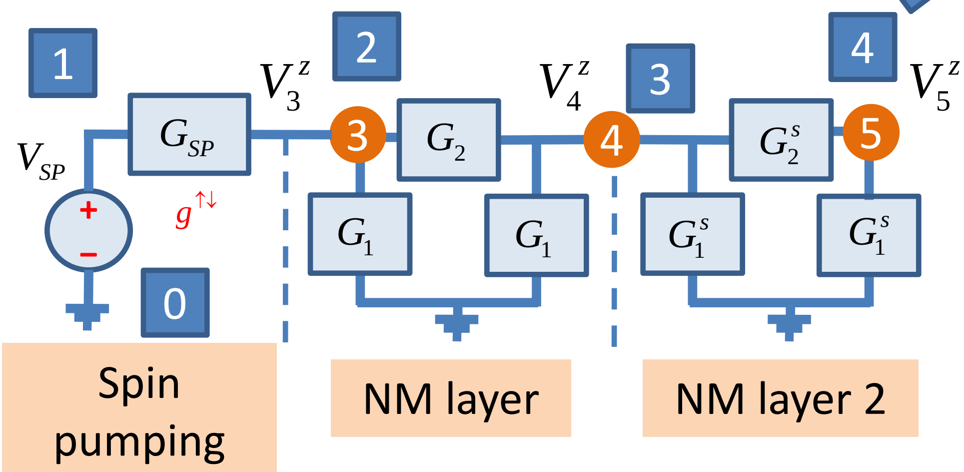




$$G_{SP}^{eff} = \frac{G_{SP} G_L}{G_{SP} + G_L} = \frac{G_{SP}}{1 + \frac{G_{SP}}{G_L}}$$

$$G_L = G_1 + \frac{G_2 (G_1 + G_s)}{G_2 + (G_1 + G_s)}$$

$$G_s = G_1^s + \frac{G_1^s G_2^s}{G_1^s + G_2^s} = G_\lambda^s \tanh\left(\frac{t^s}{\lambda^s}\right)$$



$$\frac{1}{G_L} = \frac{1}{G_\lambda} \frac{G_\lambda \cosh\left(\frac{t}{\lambda}\right) + G_\lambda^s \sinh\left(\frac{t}{\lambda}\right) \tanh\left(\frac{t^s}{\lambda^s}\right)}{G_\lambda \sinh\left(\frac{t}{\lambda}\right) + G_\lambda^s \cosh\left(\frac{t}{\lambda}\right) \tanh\left(\frac{t^s}{\lambda^s}\right)}$$

K. Roy, Phys Rev Appl. (Letter) 8, 011001 (2017)

$$V_{SP} = \frac{S\hbar\omega}{2e} \sin^2\theta$$

$$G_{SP} = lw \frac{2e^2}{h} g^{\uparrow\downarrow}$$

$$G_1 = G_\lambda \tanh\left(\frac{t}{2\lambda}\right)$$

$$G_2 = G_\lambda \operatorname{csch}\left(\frac{t}{\lambda}\right)$$

$$G_\lambda = \frac{\sigma l w}{\lambda}$$

$$G_1^s = G_\lambda^s \tanh\left(\frac{t^s}{2\lambda^s}\right)$$

$$G_1^s = G_\lambda^s \operatorname{csch}\left(\frac{t^s}{\lambda^s}\right)$$

$$G_\lambda^s = \frac{\sigma^s l w}{\lambda^s}$$

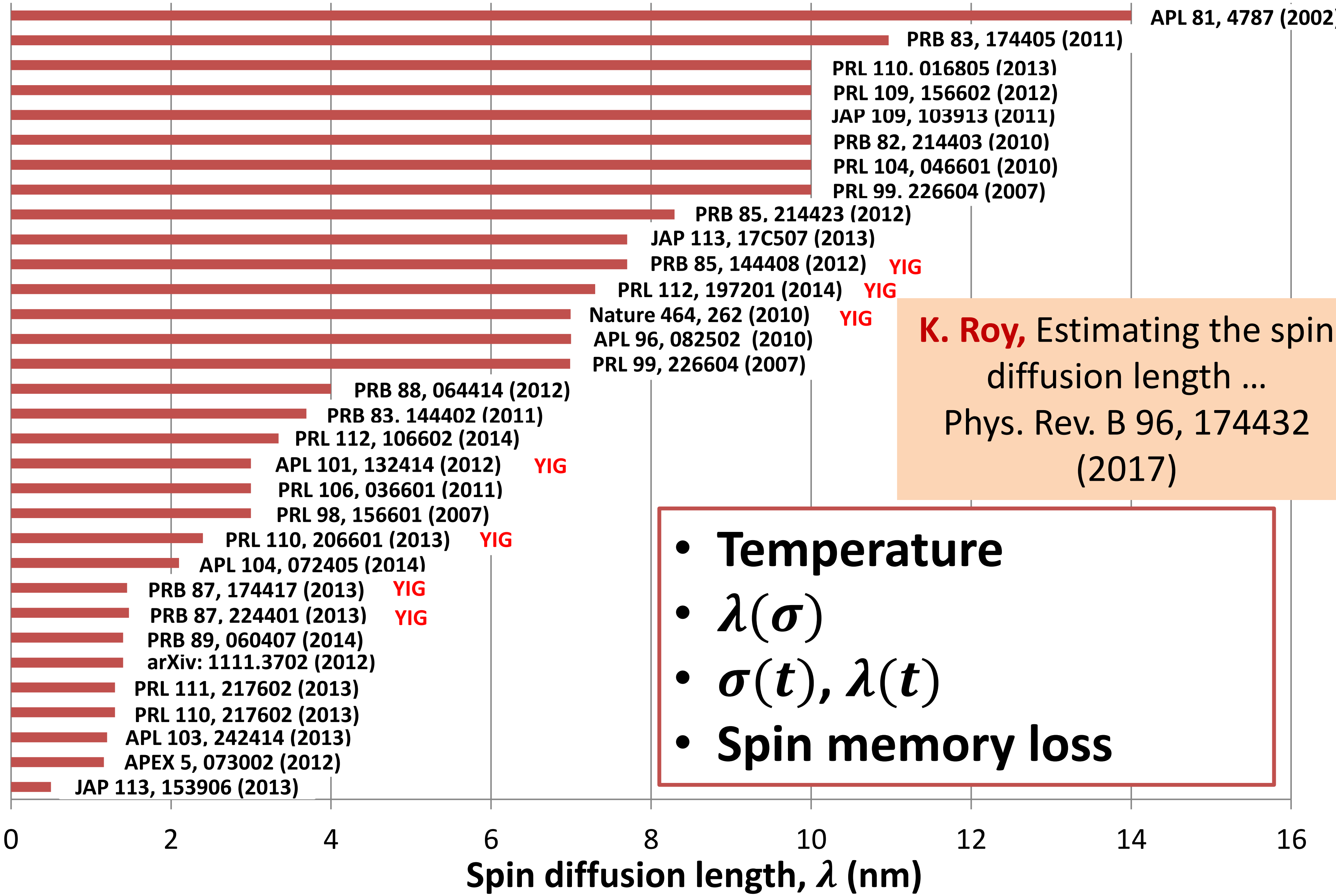
SPICE Netlist

conductances=[1 2 G_{SP} ;
2 0 G_1; 2 3 G_2; 3 0 G_1 ;
3 0 G_1^s ; 3 4 G_2^s ; 4 0 G_1^s]
voltageSources = [1 0 V_{SP}]

References

- Kittel, C., Solid State Physics.
- Tserkovnyak, Y. et al, Nonlocal magnetization dynamics in ferromagnetic heterostructures, Review of Modern Physics 77, 1375 (2005).
- Roy, K., Spin-Circuit Representation of Spin Pumping, Phys. Rev. Appl. (Letter) 8, 011001 (2017).
- Roy, K., Estimating the spin diffusion length and the spin Hall angle from spin pumping induced inverse spin Hall voltages, Phys. Rev. B 96, 174432 (2017).

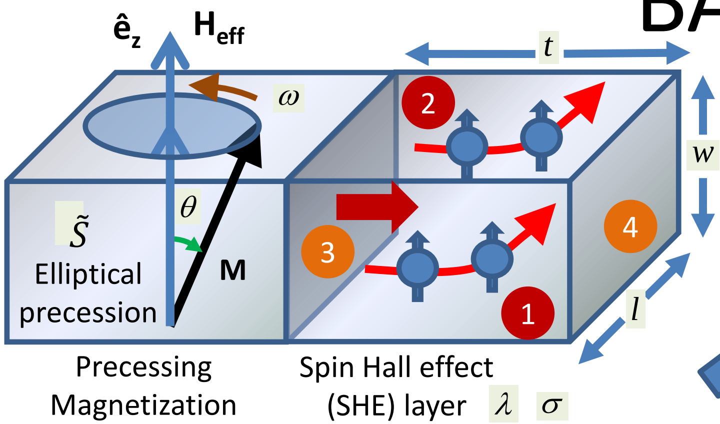
Reported values of Pt's spin diffusion length

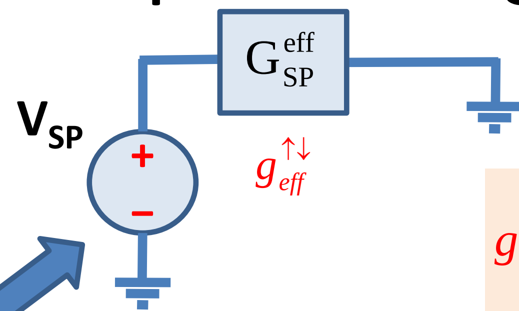


K. Roy, Estimating the spin diffusion length … Phys. Rev. B 96, 174432 (2017)

- **Temperature**
- $\lambda(\sigma)$
- $\sigma(t), \lambda(t)$
- **Spin memory loss**

Spin circuit representation for spin pumping
BARE spin mixing conductance

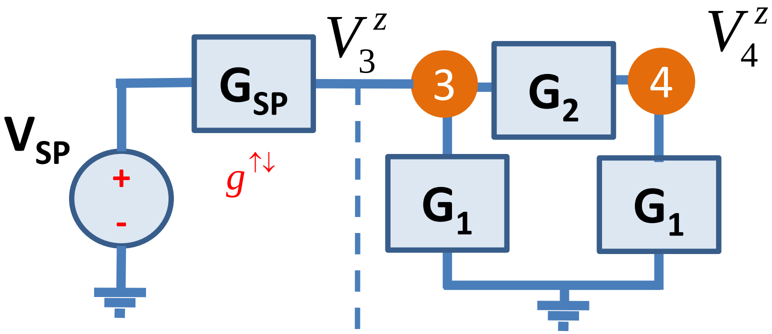




$$G_{SP}^{eff} = lw\frac{2e^2}{h}g_{eff}^{\uparrow\downarrow}$$

$$g_{eff}^{\uparrow\downarrow} = \frac{g^{\uparrow\downarrow}}{1+\frac{\lambda}{\sigma}\frac{2e^2}{h}g^{\uparrow\downarrow}\coth\frac{t}{\lambda}}$$

$$\boxed{g^{\uparrow\downarrow} = \frac{g_{eff}^{\uparrow\downarrow}}{1-\frac{\lambda}{\sigma}\frac{2e^2}{h}g_{eff}^{\uparrow\downarrow}\coth\frac{t}{\lambda}}}$$



Spin pumping

SHE layer

$$G_{SP} = lw\frac{2e^2}{h}g^{\uparrow\downarrow}$$

$$G_{SHE} = \frac{1}{\frac{\lambda}{\sigma lw}\coth\left(\frac{t}{\lambda}\right)}$$

Mosendz et al. PRB **82**, 1214403 (2010)

FMR experiments Py|Pt

$$g_{eff}^{\uparrow\downarrow} = 2.1\times10^{19}\ \mathrm{m}^{-2}$$

$$\sigma = 2.4\times10^{6}\ 1/\Omega\,\mathrm{m}$$

$$t = 15\ \mathrm{nm}$$

$$\frac{\lambda}{\sigma}\frac{2e^2}{h}g_{eff}^{\uparrow\downarrow}\coth\frac{t}{\lambda} < 1$$

$$\lambda < 1.475\,\mathrm{nm}$$

$$\frac{G_{SP}^{eff}}{G_{SHE}} = \frac{G_{SP}}{G_{SP}+G_{SHE}} < 1$$

PRB 82, 1214403 (2010): $\lambda = 10$ nm $\Rightarrow$ $g^{\uparrow\downarrow}$ *negative!*

arXiv:1111.3702v3 (2012): $\lambda = 1.4$ nm $\Rightarrow$ $g^{\uparrow\downarrow} = 4.1\times10^{20}\ \mathrm{m}^{-2}$

APL 103, 242414 (2013): $\lambda = 1.2$ nm $\Rightarrow$ $g^{\uparrow\downarrow} = 1.1\times10^{20}\ \mathrm{m}^{-2}$

Sets the maximum value of λ given other parameters