ECS 521/641: Spintronics and Nanomagnetics

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HW #2

Problem 1

Apply the operators $\sigma_x, \sigma_y, \sigma_z$ to the 2-component wavefunction

$$\psi(x) = \begin{bmatrix} \psi_1(x) \\ \psi_2(x) \end{bmatrix}.$$

Problem 2

The expected value along the n-th coordinate axis at location $\boldsymbol{r} = (x, y, z)$ at an instant of time t is $\langle S_n \rangle(\boldsymbol{r}, t) = [\psi(\boldsymbol{r}, t)]^+ [S_n][\psi(\boldsymbol{r}, t)]$, where $S_n = (\hbar/2)\sigma_n$. Determine $\langle S_n \rangle$ for $n = x, y, z$. Use the 2-component wave function in the problem above.

For the states $|+\rangle_z$ and $|-\rangle_z$, determine $\langle S_n \rangle$ for $n = x, y, z$.

Problem 3

Prove the following equality. With this equation, Dirac was able to derive the Pauli equation without the spin-orbit term. The second part of the right-hand side of the equation is the Zeeman splitting energy term.

$$[\boldsymbol{\sigma}.(\boldsymbol{p} + e\boldsymbol{A})]^2 = (\boldsymbol{p} + e\boldsymbol{A})^2 + 2m_0\,\mu_B \boldsymbol{B} \cdot \boldsymbol{\sigma}$$

Problem 4

If θ is real and if the matrix A is such that $A^2 = I$, prove the following identity.

$$e^{i\theta A} = cos\theta\, I + i\, sin\theta\, A$$

This is the generalization to operators of the well-known Euler relation for complex numbers.

$$e^{iz} = cos\, z + i\ \sin z.$$