Spintronics and Nanomagnetics

ECS 521/641

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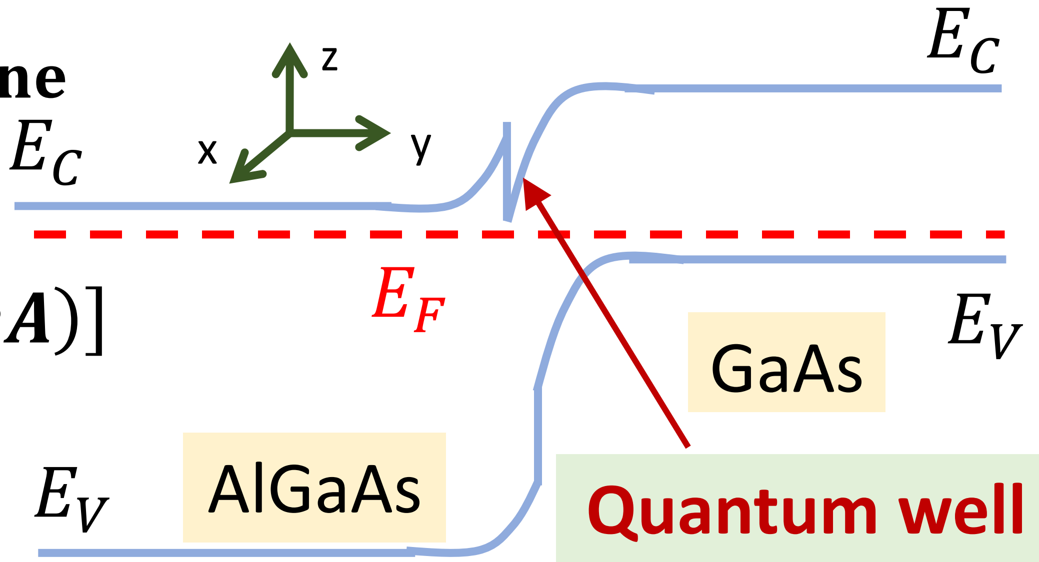
Subbands in the presence of spin-orbit interaction

Subbands in the presence of spin-orbit interaction 2-Dimensional Electron Gas (DEG)

Rashba and Dresselhaus SOI

2D electron gas in $x-z$ plane

$\boldsymbol{E}$ **in y-direction**



$$H_R = \boldsymbol{\eta}_R(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})]$$

$\boldsymbol{B}$ **in $x-z$ plane**

$$\boldsymbol{B} = B_x\,\hat{\boldsymbol{x}} + B_z\,\hat{\boldsymbol{z}}$$

$$\boldsymbol{A} = -B_z y\,\hat{\boldsymbol{x}} + B_x y\,\hat{\boldsymbol{z}}$$

$$H_D = \gamma_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa}$$

γ_D is material constant

$$\kappa_x = \frac{1}{2\hbar^3}\left[(p_x + eA_x)\left\{\left(p_y + eA_y\right)^2 - (p_z + eA_z)^2\right\} + \left\{\left(p_y + eA_y\right)^2 - (p_z + eA_z)^2\right\}(p_x + eA_x)\right]$$

$$\kappa_y = \frac{1}{2\hbar^3}\left[\left(p_y + eA_y\right)\{(p_z + eA_z)^2 - (p_x + eA_x)^2\} + \{(p_z + eA_z)^2 - (p_x + eA_x)^2\}\left(p_y + eA_y\right)\right]$$

$$\kappa_z = \frac{1}{2\hbar^3}\left[(p_z + eA_z)\left\{(p_x + eA_x)^2 - \left(p_y + eA_y\right)^2\right\} + \left\{(p_x + eA_x)^2 - \left(p_y + eA_y\right)^2\right\}(p_z + eA_z)\right]$$

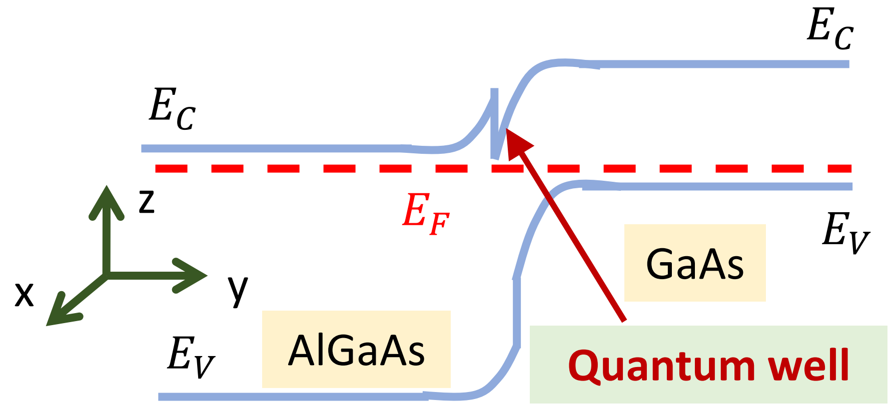
2-DEG in the presence of SOI

2D electron gas in $x - z$ plane

$\boldsymbol{E}$ **in y-direction**

$\boldsymbol{B}$ **in $x - z$ plane** $\quad \boldsymbol{B} = B_x\,\hat{\boldsymbol{x}} + B_z\,\hat{\boldsymbol{z}}$

$$\boldsymbol{A} = -B_z y\,\hat{\boldsymbol{x}} + B_x y\,\hat{\boldsymbol{z}}$$



$$H_R = \boldsymbol{\eta_R}(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + \boldsymbol{eA})]$$

$$H\Psi = E\Psi$$

$$H_D = \gamma_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa}$$

$$H_Z = -\frac{g}{2}\mu_B \boldsymbol{\sigma} \cdot \boldsymbol{B}$$

$$[\Psi(x,y,z)] = e^{ik_x x} e^{ik_z z}[\lambda(y)]$$

$V(y)$: Confining potential

$$\langle p_y^2 \rangle \gg \langle p_x^2 \rangle, \langle p_z^2 \rangle$$

$$H = \frac{|\boldsymbol{p} + e\boldsymbol{A}|^2}{2m^*}[I] + V(y)[I] + H_Z + H_R + H_D$$

Dresselhaus SOI H_D

2D electron gas in $x - z$ plane

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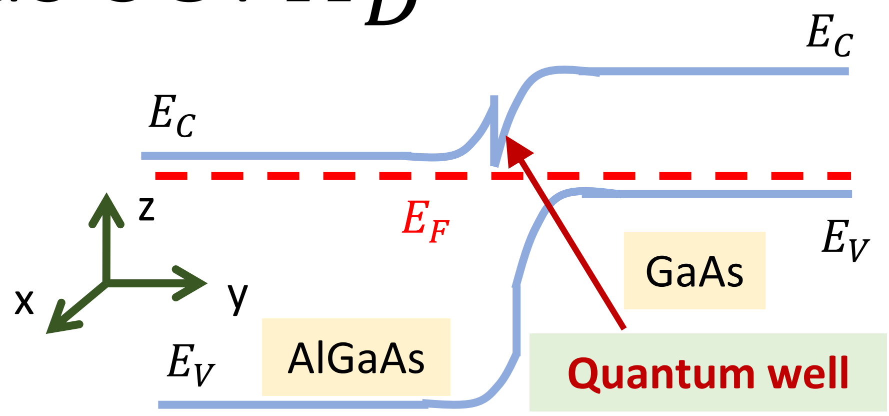

$$H_R = \boldsymbol{\eta_R}(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})] \qquad H\Psi = E\Psi$$

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$$\langle p_y^2 \rangle \gg \langle p_x^2 \rangle, \langle p_z^2 \rangle \qquad H_D = \frac{\gamma_D}{\hbar}\frac{p_y^2}{\hbar^2}[(p_x - eB_z y)\sigma_x - (p_z + eB_x y)\sigma_z]$$

$$\kappa_x = \frac{1}{2\hbar^3}\left[(p_x + eA_x)\left\{\left(p_y + eA_y\right)^2 - (p_z + eA_z)^2\right\} + \left\{\left(p_y + eA_y\right)^2 - (p_z + eA_z)^2\right\}(p_x + eA_x)\right]$$

$$\kappa_y = \frac{1}{2\hbar^3}\left[(p_y + eA_y)\{(p_z + eA_z)^2 - (p_x + eA_x)^2\} + \{(p_z + eA_z)^2 - (p_x + eA_x)^2\}(p_y + eA_y)\right]$$

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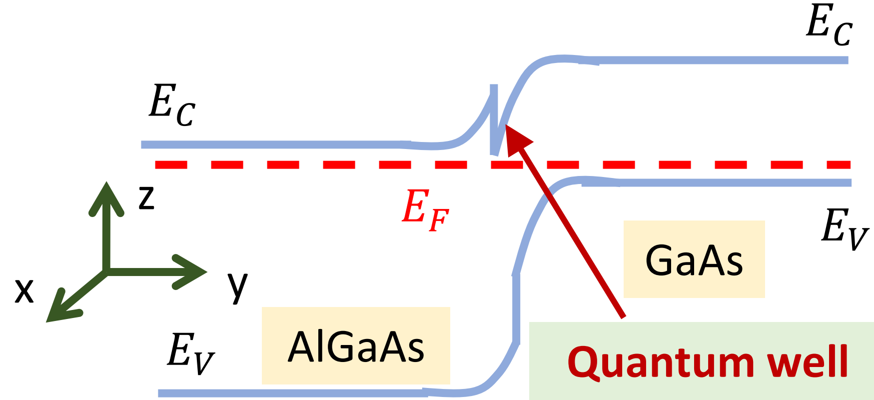
2-DEG Hamiltonian



2D electron gas in $\boldsymbol{x-z}$ plane

E **in y-direction**

B **in $\boldsymbol{x-z}$ plane** $\quad \boldsymbol{B} = B_x\,\widehat{\boldsymbol{x}} + B_z\,\widehat{\boldsymbol{z}}$

$$\boldsymbol{A} = -B_z y\,\widehat{\boldsymbol{x}} + B_x y\,\widehat{\boldsymbol{z}}$$

$$H_R = \boldsymbol{\eta_R}(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})] \qquad H\Psi = E\Psi$$

$$H_D = \gamma_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa} \qquad H = \frac{|\boldsymbol{p} + e\boldsymbol{A}|^2}{2m^*}[I] + V(y)[I] + H_Z + H_R + H_D$$

$$H = \frac{1}{2m^*}\left[(p_x - eB_z y)^2 + p_y^2 + (p_z + eB_x y)^2\right][I] + V(y)[I]$$

$$-\frac{g}{2}\mu_B\,[B_x\sigma_x + B_z\sigma_z] \qquad [\Psi(x,y,z)] = e^{ik_x x}e^{ik_z z}[\lambda(y)]$$

$$-\frac{\eta_R}{\hbar}[(p_x - eB_z y)\sigma_z - (p_z + eB_x y)\sigma_x]$$

$$+\frac{\gamma_D}{\hbar}\frac{p_y^2}{\hbar^2}[(p_x - eB_z y)\sigma_x - (p_z + eB_x y)\sigma_z]$$

2-DEG Hamiltonian

2D electron gas in $x - z$ plane

$\boldsymbol{E}$ in y-direction

$\boldsymbol{B}$ in $x - z$ plane $\boldsymbol{B} = B_x\,\hat{\boldsymbol{x}} + B_z\,\hat{\boldsymbol{z}}$

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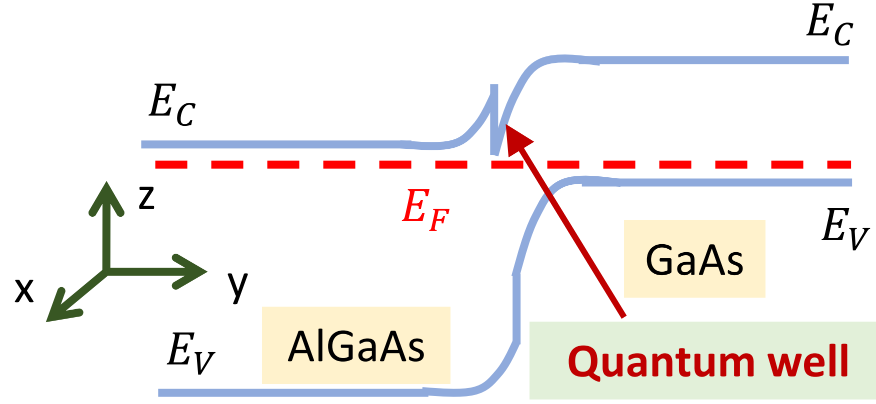

$$H_R = \boldsymbol{\eta_R}(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})] \qquad H\Psi = E\Psi$$

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$$[\Psi(x,y,z)] = e^{ik_x x} e^{ik_z z}[\lambda(y)]$$

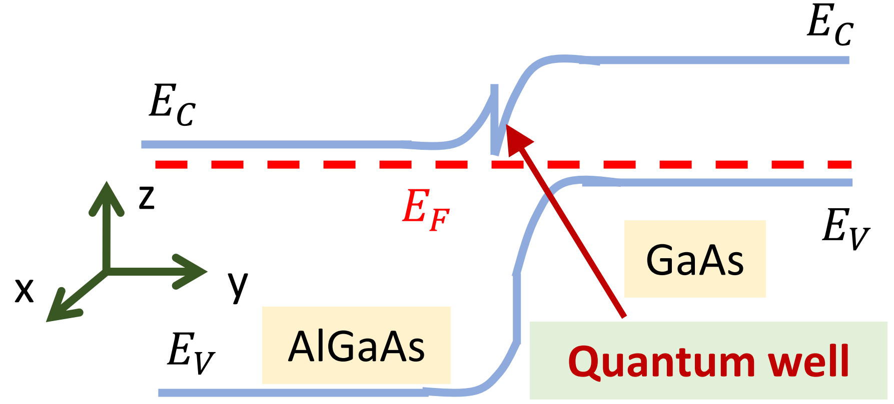
$$\left\{\left[\frac{\hbar^2 k_x^2}{2m^*} + \frac{\hbar^2 k_z^2}{2m^*} - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2} + V(y) - eB_z y\frac{\hbar k_x}{m^*} + eB_x y\frac{\hbar k_z}{m^*} + \frac{e^2 y^2 (B_x^2 + B_z^2)}{2m^*}\right][I] - \frac{g}{2}\mu_B\,[B_x\sigma_x + B_z\sigma_z] - \frac{\eta_R}{\hbar}[(\hbar k_x - eB_z y)\sigma_z - (\hbar k_z + eB_x y)\sigma_x] + \frac{\gamma_D}{\hbar}\frac{p_y^2}{\hbar^2}[(\hbar k_x - eB_z y)\sigma_x - (\hbar k_z + eB_x y)\sigma_z]\right\}[\lambda(y)] = E\,[\lambda(y)]$$

Boundary conditions: $[\lambda(y)](y = d) = [\lambda(y)](y = -d) = [0]$

Actually, triangular potential well

Dispersion relation

$$H\Psi = E\Psi$$



$$[\Psi(x,y,z)] = e^{ik_x x} e^{ik_z z}[\lambda(y)]$$

$$H = \frac{|\boldsymbol{p} + e\boldsymbol{A}|^2}{2m^*}[I] + V(y)[I] + H_Z + H_R + H_D$$

$$\left\{\left[\frac{\hbar^2 k_x^2}{2m^*} + \frac{\hbar^2 k_z^2}{2m^*} - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2} + V(y) - eB_z y\frac{\hbar k_x}{m^*} + eB_x y\frac{\hbar k_z}{m^*} + \frac{e^2 y^2 (B_x^2 + B_z^2)}{2m^*}\right][I] - \frac{g}{2}\mu_B\,[B_x\sigma_x + B_z\sigma_z] - \frac{\eta_R}{\hbar}[(\hbar k_x - eB_z y)\sigma_z - (\hbar k_z + eB_x y)\sigma_x] + \frac{\gamma_D}{\hbar}\frac{p_y^2}{\hbar^2}[(\hbar k_x - eB_z y)\sigma_x - (\hbar k_z + eB_x y)\sigma_z]\right\}[\lambda(y)] = E\,[\lambda(y)]$$

Boundary conditions: $[\lambda(y)](y = d) = [\lambda(y)](y = -d) = [0]$

$$\boldsymbol{k_t} = k_x\,\hat{\boldsymbol{x}} + k_z\,\hat{\boldsymbol{z}}$$

Non-linear in k_x and k_z

$$[\varsigma(y)] = k_x[\lambda(y)]$$

$$\{[B]k_x + [C]\}[\lambda(y)] = [A]k_x^2[\lambda(y)]$$

Dispersion relation

$$\left\{\left[\frac{\hbar^2 k_x^2}{2m^*} + \frac{\hbar^2 k_z^2}{2m^*} - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2} + V(y) - eB_z y\frac{\hbar k_x}{m^*} + eB_x y\frac{\hbar k_z}{m^*} + \frac{e^2y^2(B_x^2+B_z^2)}{2m^*}\right][I] - \frac{g}{2}\mu_B\,[B_x\sigma_x + B_z\sigma_z] - \frac{\eta_R}{\hbar}[(\hbar k_x - eB_z y)\sigma_z - (\hbar k_z + eB_x y)\sigma_x] + \frac{\gamma_D}{\hbar}\frac{p_y^2}{\hbar^2}[(\hbar k_x - eB_z y)\sigma_x - (\hbar k_z + eB_x y)\sigma_z]\right\}[\lambda(y)] = E\,[\lambda(y)]$$

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Non-linear in k_x and k_z

$$[\varsigma(y)] = k_x[\lambda(y)]$$

$$\{[B]k_x + [C]\}[\lambda(y)] = [A]k_x^2[\lambda(y)]$$

$$[A] = -\frac{\hbar^2}{2m^*}[I]$$

$$[B] = -eB_z y\frac{\hbar}{m^*}[I] - \eta[\sigma_z] - \nu_D\frac{\partial^2}{\partial y^2}[\sigma_x]$$

$$\eta = \eta_R \qquad \nu_D = -\frac{\gamma_D}{\hbar^2}$$

$$[C] = \frac{\hbar^2 k_z^2}{2m^*}[I] - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2}[I] + eB_x y\frac{\hbar k_z}{m^*}[I] + V(y)[I] + \frac{e^2y^2(B_x^2+B_z^2)}{2m^*}[I] - \frac{g}{2}\mu_B\{B_x[\sigma_x] + B_z[\sigma_z]\} - E[I] + \frac{eB_z y}{\hbar}\left\{\eta[\sigma_z] + \nu_D\frac{\partial^2}{\partial y^2}[\sigma_x]\right\} + \left(k_z + \frac{eB_x y}{\hbar}\right)\left\{\eta[\sigma_x] + \nu_D\frac{\partial^2}{\partial y^2}[\sigma_z]\right\}$$

Dispersion relation

Boundary conditions: $[\lambda(y)](y=d) = [\lambda(y)](y=-d) = [0]$

$$\boldsymbol{k_t} = k_x\,\hat{\boldsymbol{x}} + k_z\,\hat{\boldsymbol{z}}$$

Non-linear in k_x and k_z

$$[\varsigma(y)] = k_x[\lambda(y)]$$

$$\{[B]k_x + [C]\}[\lambda(y)] = [A]k_x^2[\lambda(y)]$$

$$[A] = -\frac{\hbar^2}{2m^*}[I]$$

$$[B] = -eB_z y\frac{\hbar}{m^*}[I] - \eta[\sigma_z] - \nu_D\frac{\partial^2}{\partial y^2}[\sigma_x]$$

$$\eta = \eta_R$$

$$\nu_D = -\frac{\gamma_D}{\hbar^2}$$

$$[C] = \frac{\hbar^2 k_z^2}{2m^*}[I] - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2}[I] + eB_x y\frac{\hbar k_z}{m^*}[I] + V(y)[I] + \frac{e^2y^2(B_x^2+B_z^2)}{2m^*}[I] - \frac{g}{2}\mu_B\{B_x[\sigma_x] + B_z[\sigma_z]\} - E[I] + \frac{eB_z y}{\hbar}\left\{\eta[\sigma_z] + \nu_D\frac{\partial^2}{\partial y^2}[\sigma_x]\right\} + \left(k_z + \frac{eB_x y}{\hbar}\right)\left\{\eta[\sigma_x] + \nu_D\frac{\partial^2}{\partial y^2}[\sigma_z]\right\}$$

Solve numerically

$$\begin{bmatrix} [0] & [I] \\ [A]^{-1}[C] & [A]^{-1}[B] \end{bmatrix}\begin{bmatrix} \lambda(y) \\ \varsigma(y) \end{bmatrix} = k_x\begin{bmatrix} \lambda(y) \\ \varsigma(y) \end{bmatrix}$$

n-th subband

$$[\lambda(y)] = \begin{bmatrix} \lambda_{+,n,k_x,k_z}(y) \\ \lambda_{-,n,k_x,k_z}(y) \end{bmatrix}$$

E_n versus k_x at any given k_z

Dispersion relation: Variable separation

$$\left\{\frac{\hbar^2 k_x^2}{2m^*} + \frac{\hbar^2 k_z^2}{2m^*} - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2} + V(y) - eB_z y\frac{\hbar k_x}{m^*} + eB_x y\frac{\hbar k_z}{m^*} + \frac{e^2 y^2 (B_x^2 + B_z^2)}{2m^*} - \frac{g}{2}\mu_B\left[B_x\sigma_x + B_z\sigma_z\right] - \frac{\eta_R}{\hbar}\left[(\hbar k_x - eB_z y)\sigma_z - (\hbar k_z + eB_x y)\sigma_x\right] + \frac{\gamma_D}{\hbar}\frac{p_y^2}{\hbar^2}\left[(\hbar k_x - eB_z y)\sigma_x - (\hbar k_z + eB_x y)\sigma_z\right]\right\}[\lambda(y)] = E\,[\lambda(y)]$$

Boundary conditions: $[\lambda(y)](y = d) = [\lambda(y)](y = -d) = [0]$

$$[\lambda(y)] = \begin{bmatrix}\lambda_1(y)\\ \lambda_2(y)\end{bmatrix}$$

$$[\Psi(x,y,z)] = e^{ik_x x}e^{ik_z z}\lambda_0(y)\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

$$[\lambda(y)] = \lambda_0(y)\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

Assumptions

$\nu = -\frac{\gamma_D}{\hbar^2}\langle p_y^2\rangle = \gamma_D\langle\partial^2/\partial y^2\rangle$

Spin orientation of an electron does not depend on the *y*-direction

Narrow 2DEG width: Subbands are well separated

$$\langle y\rangle = \int_0^\infty \lambda_0^*(y)\, y\, \lambda_0(y)dy = y_0 = 0$$

$$E_n = \epsilon_n + \frac{\hbar^2(k_x^2 + k_z^2)}{2m^*} - \frac{g}{2}\mu_B\left[B_x\sigma_x + B_z\sigma_z\right] - \eta\left[k_x\sigma_z - k_z\sigma_x\right] - \nu\left[k_x\sigma_x - k_z\sigma_z\right]$$

$$E_n = \langle H\rangle$$

$$\left[-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2} + \frac{e^2 y^2(B_x^2 + B_z^2)}{2m^*}\right][\lambda_0^n(y)] = \epsilon_n\,[\lambda_0^n(y)]$$

Dispersion relation: Eigenvalues

$$\left[-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2}+\frac{e^2y^2(B_x^2+B_z^2)}{2m^*}\right][\lambda_0^n(y)]=\epsilon_n\,[\lambda_0^n(y)]$$

$$E_n=\epsilon_n+\frac{\hbar^2(k_x^2+k_z^2)}{2m^*}-\frac{g}{2}\mu_B\,[B_x\sigma_x+B_z\sigma_z]-\eta[k_x\sigma_z-k_z\sigma_x]-\nu\,[k_x\sigma_x-k_z\sigma_z]$$

$$E_n[I]=\epsilon_n[I]+\frac{\hbar^2(k_x^2+k_z^2)}{2m^*}[I]-\frac{g}{2}\mu_B\begin{bmatrix}B_z & B_x\\ B_x & -B_z\end{bmatrix}-\eta\begin{bmatrix}k_x & -k_z\\ -k_z & -k_x\end{bmatrix}-\nu\begin{bmatrix}-k_z & k_x\\ k_x & k_z\end{bmatrix}$$

$$E_n[I]=\begin{bmatrix}E_n'-\frac{g\mu_BB_z}{2}-\eta k_x+\nu k_z & -\frac{g\mu_BB_x}{2}+\eta k_z-\nu k_x\\ -\frac{g\mu_BB_x}{2}+\eta k_z-\nu k_x & E_n'+\frac{g\mu_BB_z}{2}+\eta k_x-\nu k_z\end{bmatrix}$$

$$E_n'=\epsilon_n+\frac{\hbar^2(k_x^2+k_z^2)}{2m^*}$$

$$E_\pm=\epsilon_n+\frac{\hbar^2(k_x^2+k_z^2)}{2m^*}\pm\sqrt{\left(\frac{g\mu_BB_z}{2}+\eta k_x-\nu k_z\right)^2+\left(\frac{g\mu_BB_x}{2}-\eta k_z+\nu k_x\right)^2}$$

Dispersion relation: Eigenspinors

$$E_n[I] = \begin{bmatrix} E_n' - \frac{g\mu_B B_z}{2} - \eta k_x + \nu k_z & -\frac{g\mu_B B_x}{2} + \eta k_z - \nu k_x \\ -\frac{g\mu_B B_x}{2} + \eta k_z - \nu k_x & E_n' + \frac{g\mu_B B_z}{2} + \eta k_x - \nu k_z \end{bmatrix}$$

$$E_n' = \epsilon_n + \frac{\hbar^2(k_x^2 + k_z^2)}{2m^*}$$

$$E_\pm = \epsilon_n + \frac{\hbar^2(k_x^2 + k_z^2)}{2m^*} \pm \sqrt{\left(\frac{g\mu_B B_z}{2} + \eta k_x - \nu k_z\right)^2 + \left(\frac{g\mu_B B_x}{2} - \eta k_z + \nu k_x\right)^2}$$

$$\Psi_+(B_x, B_z, k_x, k_z) = \begin{bmatrix} -\sin(\theta_k) \\ \cos(\theta_k) \end{bmatrix}$$

$$\Psi_-(B_x, B_z, k_x, k_z) = \begin{bmatrix} cos(\theta_k) \\ \sin(\theta_k) \end{bmatrix}$$

$$\theta_k = \frac{1}{2} tan^{-1} \left[\frac{\frac{g\mu_B B_x}{2} - \eta k_z + \nu k_x}{\frac{g\mu_B B_z}{2} + \eta k_x - \nu k_z} \right]$$

Velocity versus wavevector

$$H = \frac{1}{2m^*}\left[(p_x - eB_z y)^2 + p_y^2 + (p_z + eB_x y)^2\right] + V(y)$$
$$- \frac{g}{2}\mu_B\,[B_x\sigma_x + B_z\sigma_z]$$
$$- \frac{\eta}{\hbar}[(p_x - eB_z y)\sigma_z - (p_z + eB_x y)\sigma_x]$$
$$- \frac{\nu}{\hbar}[(p_x - eB_z y)\sigma_x - (p_z + eB_x y)\sigma_z]$$

$$v_q = \frac{\partial H}{\partial p_q}$$

$$\langle y \rangle = 0$$

$$\langle v_x \rangle = \frac{\langle p_x \rangle}{m^*} - \frac{\eta}{\hbar}\langle \sigma_z \rangle - \frac{\nu}{\hbar}\langle \sigma_x \rangle$$

$$\langle v_z \rangle = \frac{\langle p_z \rangle}{m^*} + \frac{\eta}{\hbar}\langle \sigma_x \rangle + \frac{\nu}{\hbar}\langle \sigma_z \rangle$$

$$v_x^{\pm} = \frac{\hbar k_x}{m^*} \pm \frac{\eta}{\hbar}\cos(2\theta_k) \pm \frac{\nu}{\hbar}\sin(2\theta_k)$$

$$v_z^{\pm} = \frac{\hbar k_z}{m^*} \mp \frac{\nu}{\hbar}\cos(2\theta_k) \mp \frac{\eta}{\hbar}\sin(2\theta_k)$$

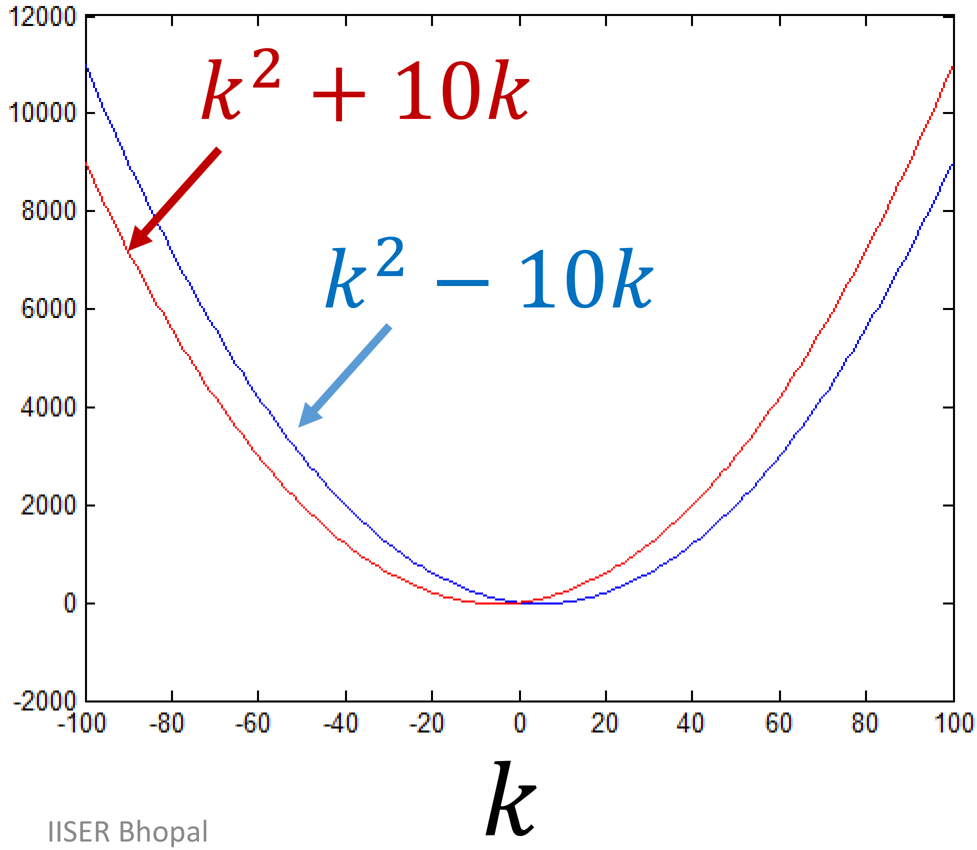
Dispersion relation: $B = 0, \nu = 0$

$$E_\pm = \epsilon_n + \frac{\hbar^2(k_x^2 + k_z^2)}{2m^*} \pm \sqrt{\left(\frac{g\mu_B B_z}{2} + \eta k_x - \nu k_z\right)^2 + \left(\frac{g\mu_B B_x}{2} - \eta k_z + \nu k_x\right)^2}$$

$$E_\pm = \epsilon_n + \frac{\hbar^2 k^2}{2m^*} \pm \eta k$$

$$k^2 \pm 10k$$

Sample example



Subbands in the presence of spin-orbit interaction 1-DEG

1-DEG in the presence of SOI

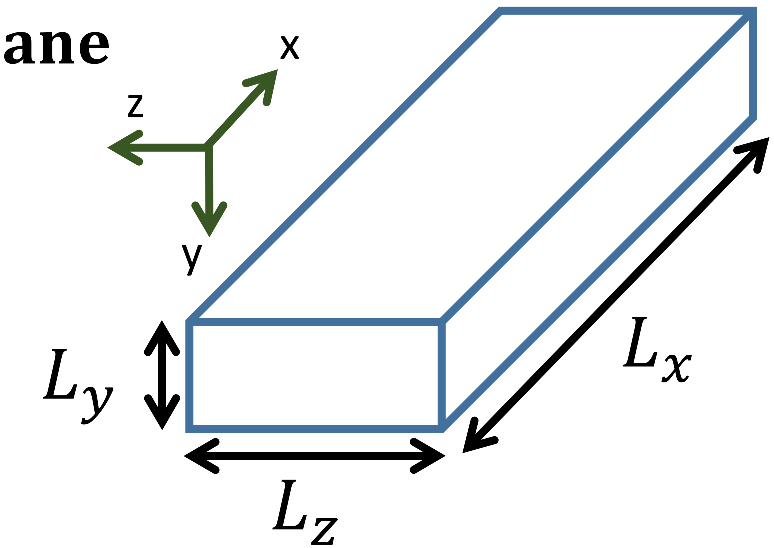
1D electron gas confined in y$-z$ plane

$\boldsymbol{E}$ **in y-direction**

$\boldsymbol{B}$ **in $\boldsymbol{x}$ direction**

$$\boldsymbol{B} = B\,\widehat{\boldsymbol{x}}$$

$$\boldsymbol{A} = -Bz\,\widehat{\boldsymbol{y}}$$



$$H_R = \boldsymbol{\eta_R}(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})]$$

$$H_D = \gamma_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa}$$

$$H_Z = -\frac{g}{2}\mu_B \boldsymbol{\sigma} \cdot \boldsymbol{B}$$

$$H\Psi = E\Psi$$

$$L_y \ll L_z \ll L_x$$

$$\langle p_y^2 \rangle \gg \langle p_x^2 \rangle, \langle p_z^2 \rangle$$

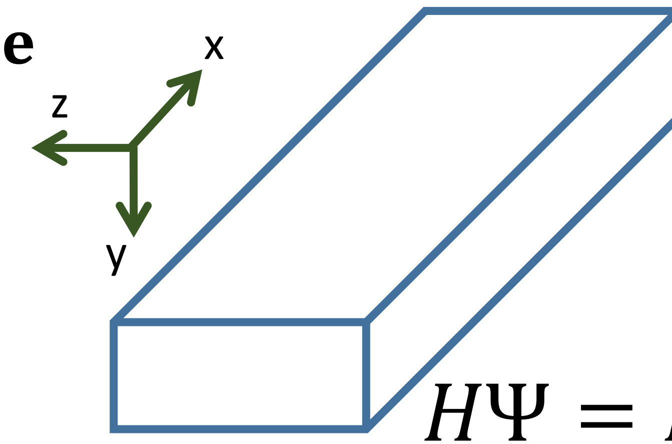
$V(y)\ and\ V(z)$: Confining potentials

$$\langle p_y \rangle = 0 \quad \langle p_z \rangle = 0$$

$$H = \frac{|\boldsymbol{p} + e\boldsymbol{A}|^2}{2m^*}[I] + V(y)[I] + V(z)[I] + H_Z + H_R + H_D$$

Dresselhaus SOI H_D

1D electron gas confined in y−z plane



$\boldsymbol{E}$ **in y-direction** $\quad \boldsymbol{B} = B\,\hat{\boldsymbol{x}}$

$\boldsymbol{B}$ **in x direction** $\quad \boldsymbol{A} = -Bz\,\hat{\boldsymbol{y}}$

$$H_R = \boldsymbol{\eta}_R(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})]$$

$$H_D = \gamma_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa}$$

$$H_Z = -\frac{g}{2}\mu_B \boldsymbol{\sigma} \cdot \boldsymbol{B}$$

$$H\Psi = E\Psi$$

$$H = \frac{|\boldsymbol{p} + e\boldsymbol{A}|^2}{2m^*}[I] + V(y)[I] + V(z)[I] + H_Z + H_R + H_D$$

$$\nu = -\frac{\gamma_D}{\hbar^2}\langle p_y^2 \rangle = \gamma_D \langle \partial^2/\partial y^2 \rangle$$

$$\langle p_y^2 \rangle \gg \langle p_x^2 \rangle, \langle p_z^2 \rangle \qquad H_D = \frac{\gamma_D}{\hbar}\frac{p_y^2}{\hbar^2}[p_x\sigma_x - p_z\sigma_z] \qquad p_z = -i\hbar\partial/\partial z$$

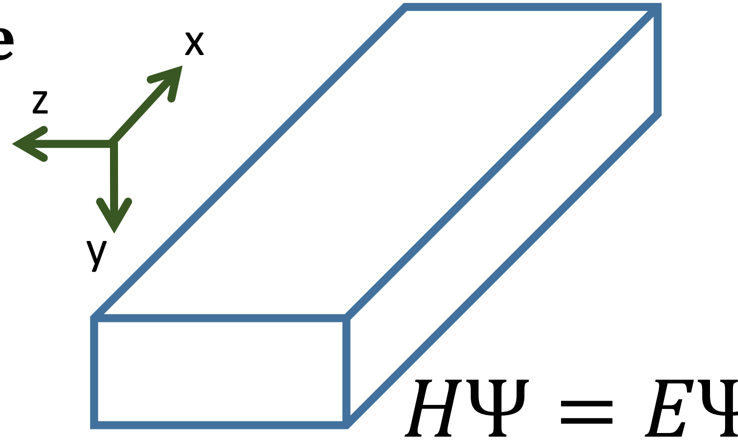
$$\kappa_x = \frac{1}{2\hbar^3}\left[(p_x + eA_x)\left\{(p_y + eA_y)^2 - (p_z + eA_z)^2\right\} + \left\{(p_y + eA_y)^2 - (p_z + eA_z)^2\right\}(p_x + eA_x)\right]$$

$$\kappa_y = \frac{1}{2\hbar^3}\left[(p_y + eA_y)\{(p_z + eA_z)^2 - (p_x + eA_x)^2\} + \{(p_z + eA_z)^2 - (p_x + eA_x)^2\}(p_y + eA_y)\right]$$

$$\kappa_z = \frac{1}{2\hbar^3}\left[(p_z + eA_z)\left\{(p_x + eA_x)^2 - (p_y + eA_y)^2\right\} + \left\{(p_x + eA_x)^2 - (p_y + eA_y)^2\right\}(p_z + eA_z)\right]$$

1-DEG Hamiltonian

1D electron gas confined in $y-z$ plane



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$$H = \frac{|\boldsymbol{p} + e\boldsymbol{A}|^2}{2m^*}[I] + V(y)[I] + V(z)[I] + H_Z + H_R + H_D$$

$$H = \frac{1}{2m^*}\left[p_x^2 + \left(p_y - eBz\right)^2 + p_z^2\right][I] + V(y)[I] + V(z)[I] \qquad \eta = \eta_R$$

$$-\frac{g}{2}\mu_B B \sigma_x$$

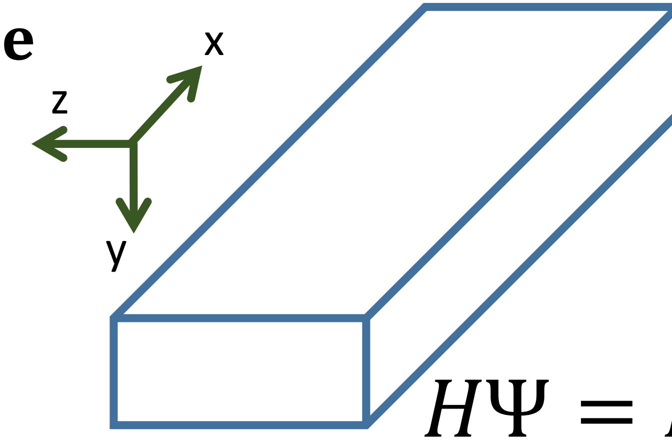
$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$

$$-\eta[k_x\sigma_z + i(\partial/\partial z)\sigma_x]$$

$$-\nu[k_x\sigma_x + i(\partial/\partial z)\sigma_z]$$

1-DEG Hamiltonian

1D electron gas confined in y$-z$ plane



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$$H\Psi = E\Psi$$

$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$

$$\left[\left\{\frac{\hbar^2 k_x^2}{2m^*} + E_m - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2} + \frac{e^2B^2z^2}{2m^*} + V(z)\right\}[I] - \left(\nu k_x + \left(\frac{g}{2}\right)\mu_B B + i\eta(\partial/\partial z)\right)\sigma_x - \left(\eta k_x + i\nu(\partial/\partial z)\right)\sigma_z\right][\lambda(z)] = E[\lambda(z)]$$

$$\left[\frac{p_y^2}{2m^*} + V(y)\right]\phi_m(y) = E_m \phi_m(y)$$

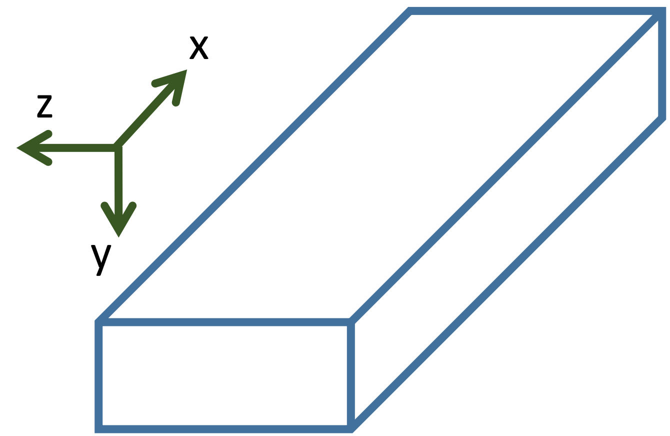
Boundary conditions: $\phi_m(y = d) = \phi_m(y = -d) = 0 \qquad \langle p_y \rangle = 0$

Dispersion relation

$$H\Psi = E\Psi$$

$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$

$$\left[\frac{p_y^2}{2m^*} + V(y)\right]\phi_m(y) = E_m\phi_m(y)$$



$$\left[\left\{\frac{\hbar^2 k_x^2}{2m^*} + E_m - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2} + \frac{e^2B^2z^2}{2m^*} + V(z)\right\}[I] - \left(\nu k_x + \left(\frac{g}{2}\right)\mu_B B + i\eta(\partial/\partial z)\right)\sigma_x - \left(\eta k_x + i\nu(\partial/\partial z)\right)\sigma_z\right][\lambda(z)] = E[\lambda(z)]$$

Boundary conditions: $\phi_m(y=d) = \phi_m(y=-d) = 0$

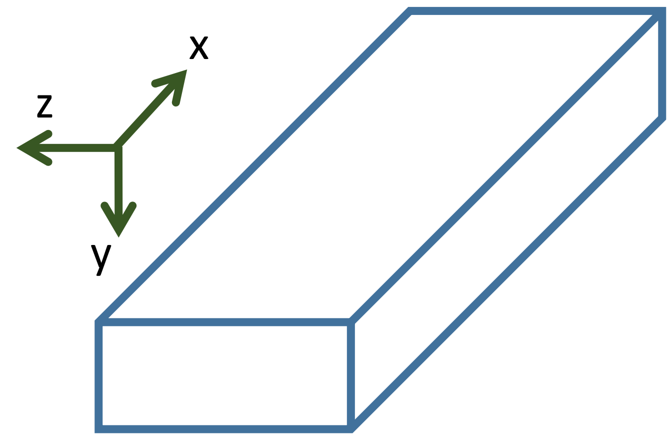
Non-linear in k_x

Obtain an eigenequation in k_x

Solve numerically

Similar to 2-DEG case

Dispersion relation: Variable separation



$$H\Psi = E\Psi$$

$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$

$$\left[\frac{p_y^2}{2m^*} + V(y)\right]\phi_m(y) = E_m\phi_m(y)$$

$$\left[\left\{\frac{\hbar^2 k_x^2}{2m^*} + E_m - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2} + \frac{e^2B^2z^2}{2m^*} + V(z)\right\}[I] - \left(\nu k_x + \left(\frac{g}{2}\right)\mu_B B + i\eta(\partial/\partial z)\right)\sigma_x - \left(\eta k_x + i\nu(\partial/\partial z)\right)\sigma_z\right][\lambda(z)] = E[\lambda(z)]$$

Boundary conditions: $\phi_m(y=d) = \phi_m(y=-d) = 0$

$$[\lambda(z)] = \begin{bmatrix}\lambda_1(z)\\ \lambda_2(z)\end{bmatrix} = \lambda_0(z)\begin{bmatrix}\alpha\\ \beta\end{bmatrix}$$

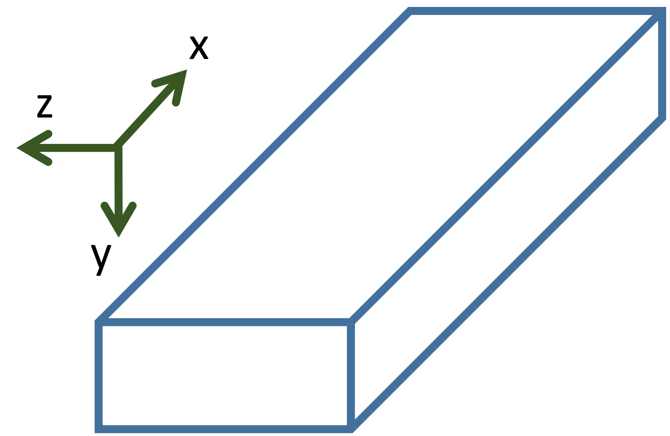
Assumptions

Spin orientation of an electron does not depend on the *z*-direction

Narrow 1-DEG width: Subbands are well separated

Dispersion relation: Variable separation

$$H\Psi = E\Psi$$



$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$

$$\left[\frac{p_y^2}{2m^*} + V(y)\right]\phi_m(y) = E_m\phi_m(y)$$

$$\left[\left\{\frac{\hbar^2 k_x^2}{2m^*} + E_m - \frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2} + \frac{e^2B^2z^2}{2m^*} + V(z)\right\}[I] - \left(\nu k_x + \left(\frac{g}{2}\right)\mu_B B + i\eta(\partial/\partial z)\right)\sigma_x - \left(\eta k_x + i\nu(\partial/\partial z)\right)\sigma_z\right][\lambda(z)] = E[\lambda(z)]$$

$$p_z - i\hbar(\partial/\partial z)$$

$$\langle p_z \rangle = 0$$

$$V(z) = \frac{1}{2}m^*\omega_0^2 z^2 \quad \left[\frac{p_z^2}{2m^*} + \frac{1}{2}m^*\omega^2 z^2\right][\lambda_0(z)] = E_{HO}[\lambda_0(z)]$$

$$\omega^2 = \omega_0^2 + \omega_c^2$$

$$E_{HO} = \left(n + \frac{1}{2}\right)\hbar\omega \qquad \omega_c = \frac{eB}{m^*}$$

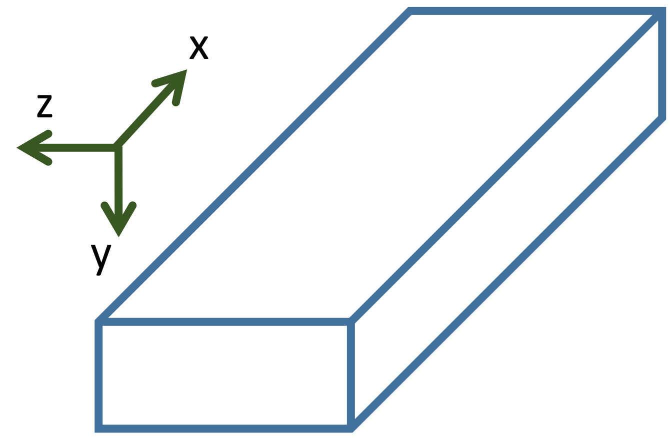
$$E[I] = \left[\left\{\frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO}\right\}[I] - (\nu k_x + (g/2)\mu_B B)\sigma_x - (\eta k_x)\sigma_z\right]$$

Dispersion relation: Eigenvalues

$$H\Psi = E\Psi$$

$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$

$$E_0 = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO}$$



$$E[I] = [E_0[I] - (\nu k_x + (g/2)\mu_B B)\sigma_x - (\eta k_x)\sigma_z]$$

$$E[I] = \begin{bmatrix} E_0 - \eta k_x & -(\nu k_x + (g/2)\mu_B B) \\ -(\nu k_x + (g/2)\mu_B B) & E_0 + \eta k_x \end{bmatrix}$$

$$E_\pm = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO} \pm \sqrt{(\eta k_x)^2 + (\nu k_x + (g/2)\mu_B B)^2}$$

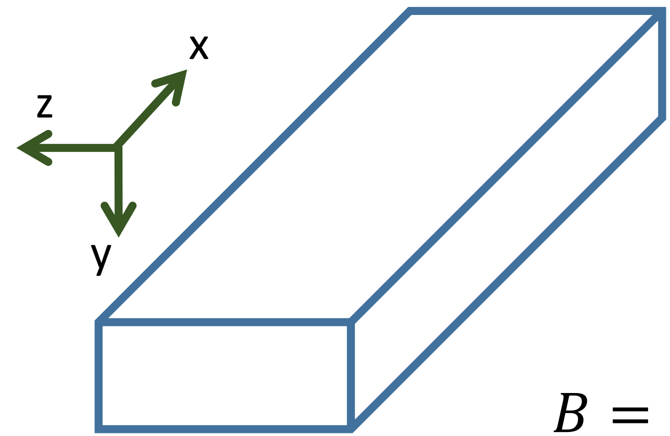
$$\beta = (g/2)\mu_B B$$

$$E_\pm = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO} \pm \sqrt{(\eta^2 + \nu^2)\left(k_x + \frac{\nu\beta}{\eta^2 + \nu^2}\right)^2 + \frac{\eta^2}{\eta^2 + \nu^2}\beta^2}$$

Dispersion relation: Eigenspinors

$$H\Psi = E\Psi$$

$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$



$$B = B_x$$

$$E_0 = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO}$$

$$E_{\pm} = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO} \pm \sqrt{(\eta k_x)^2 + (\nu k_x + (g/2)\mu_B B)^2}$$

$$\beta = (g/2)\mu_B B$$

$$E_{\pm} = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO} \pm \sqrt{(\eta^2+\nu^2)\left(k_x + \frac{\nu\beta}{\eta^2+\nu^2}\right)^2 + \frac{\eta^2}{\eta^2+\nu^2}\beta^2}$$

$$\Psi_+(B_x,k_x,x) = \begin{bmatrix} -\sin(\theta_k) \\ \cos(\theta_k) \end{bmatrix} e^{ik_x x}$$

$$\Psi_-(B_x,k_x,x) = \begin{bmatrix} cos(\theta_k) \\ \sin(\theta_k) \end{bmatrix} e^{ik_x x}$$

$$\theta_{k_x} = \frac{1}{2} tan^{-1}\left[\frac{\nu k_x + (g/2)\mu_B B}{\eta k_x}\right]$$

Velocity versus wavevector

$$H = \frac{1}{2m^*}\left[p_x^2 + \left(p_y - eBz\right)^2 + p_z^2\right][I] + V(y)[I] + V(z)[I]$$
$$-\frac{g}{2}\mu_B B\sigma_x$$
$$-\eta[k_x\sigma_z - i(\partial/\partial z)\sigma_x]$$
$$-\nu[k_x\sigma_x + i(\partial/\partial z)\sigma_z]$$

$$v_x = \frac{\partial H}{\partial p_x}$$

$$p_z - i\hbar(\partial/\partial z)$$

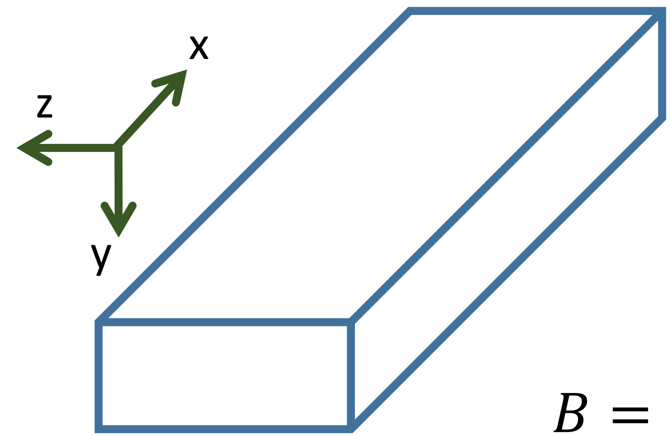
$$\langle p_z\rangle = 0$$

$$\langle v_x\rangle = \frac{\langle p_x\rangle}{m^*} - \frac{\eta}{\hbar}\langle\sigma_z\rangle - \frac{\nu}{\hbar}\langle\sigma_x\rangle$$

$$v_x^{\pm} = \frac{\hbar k_x}{m^*} \pm \frac{\eta}{\hbar}\cos\left(2\theta_{k_x}\right) \pm \frac{\nu}{\hbar}\sin\left(2\theta_{k_x}\right)$$

Dispersion relation: No magnetic field

$$H\Psi = E\Psi$$



$$[\Psi(x,y,z)] = e^{ik_x x}\phi(y)[\lambda(z)]$$

$$E_0 = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO}$$

$$B = B_x$$

$$\theta_{k_x} = \frac{1}{2}tan^{-1}\left[\frac{\nu k_x + (g/2)\mu_B B}{\eta k_x}\right]$$

$$\beta = (g/2)\mu_B B$$

$$E_\pm = \frac{\hbar^2 k_x^2}{2m^*} + E_m + E_{HO} \pm \sqrt{(\eta^2+\nu^2)\left(k_x + \frac{\nu\beta}{\eta^2+\nu^2}\right)^2 + \frac{\eta^2}{\eta^2+\nu^2}\beta^2}$$

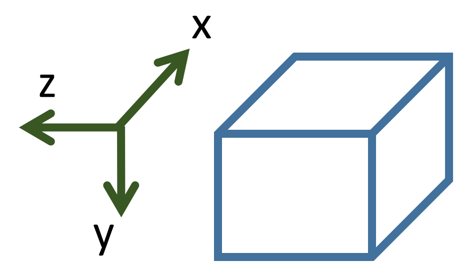
$$E_\pm = \frac{\hbar^2}{2m^*}\left(k_x \pm \frac{m^*\sqrt{\eta^2+\nu^2}}{\hbar^2}\right)^2 + E_m + E_{HO} - \frac{m^*(\eta^2+\nu^2)}{2\hbar^2}$$

$$\theta_{k_x} \neq f(k_x) \qquad v_x^\pm \propto k_x \pm constant$$

Subbands in the presence of spin-orbit interaction 0-DEG

0-DEG in the presence of SOI

0D electron gas confined in all directions



E **in *y*-direction**

B **in *x*-direction** $\boldsymbol{B} = B\,\hat{\boldsymbol{x}}$

$$\boldsymbol{A} = -Bz\,\hat{\boldsymbol{y}}$$

$$H_R = \boldsymbol{\eta_R}(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + \boldsymbol{eA})]$$

$$H_D = \gamma_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa}$$

$$H\Psi = E\Psi$$

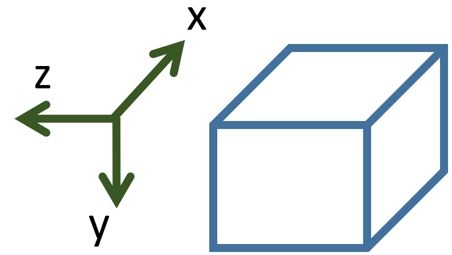
$$H_Z = -\frac{g}{2}\mu_B \boldsymbol{\sigma} \cdot \boldsymbol{B}$$

$V(x), V(y)\ and\ V(z)$: Confining potentials

$$H = \frac{|\boldsymbol{p} + e\boldsymbol{A}|^2}{2m^*}[I] + V(x)[I] + V(y)[I] + V(z)[I] + H_Z + H_R + H_D$$

Dresselhaus SOI H_D

0D electron gas confined in all directions



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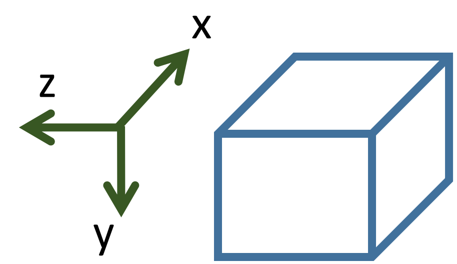
$$\kappa_y = \frac{1}{2\hbar^3}\left[\left(p_y + eA_y\right)\{(p_z + eA_z)^2 - (p_x + eA_x)^2\} + \{(p_z + eA_z)^2 - (p_x + eA_x)^2\}\left(p_y + eA_y\right)\right]$$

$$\kappa_z = \frac{1}{2\hbar^3}\left[(p_z + eA_z)\left\{(p_x + eA_x)^2 - \left(p_y + eA_y\right)^2\right\} + \left\{(p_x + eA_x)^2 - \left(p_y + eA_y\right)^2\right\}(p_z + eA_z)\right]$$

Dresselhaus SOI H_D

0D electron gas confined in all directions



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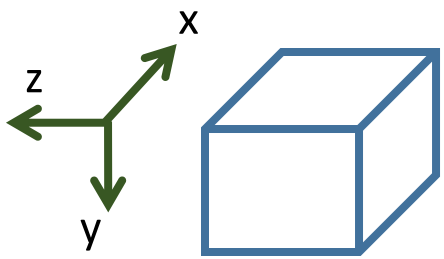
$$\kappa_x = \frac{1}{2\hbar^3}\left[p_x\left\{\left(p_y - eBz\right)^2 - p_z^2\right\} + \left\{\left(p_y - eBz\right)^2 - p_z^2\right\}p_x\right]$$

$$\kappa_y = \frac{1}{2\hbar^3}\left[\left(p_y - eBz\right)\{p_z^2 - p_x^2\} + \{p_z^2 - p_x^2\}\left(p_y - eBz\right)\right]$$

$$\kappa_z = \frac{1}{2\hbar^3}\left[p_z\left\{p_x^2 - \left(p_y - eBz\right)^2\right\} + \left\{p_x^2 - \left(p_y - eBz\right)^2\right\}p_z\right]$$

0-DEG Hamiltonian

0D electron gas confined in all directions



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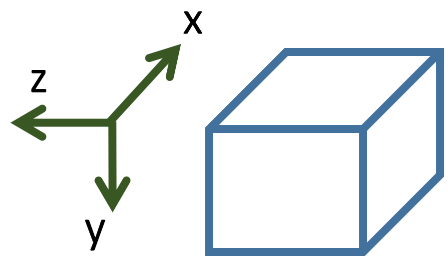
$$H = \frac{1}{2m^*}\left[p_x^2 + \left(p_y - eBz\right)^2 + p_z^2\right][I] + V(x)[I] + V(y)[I] + V(z)[I]$$
$$-\frac{g}{2}\mu_B B\sigma_x$$
$$-\frac{\eta}{\hbar}[p_x\sigma_z - p_z\sigma_x]$$
$$+\gamma_D\left(\sigma_x\kappa_x + \sigma_y\kappa_y + \sigma_z\kappa_z\right)$$

0-DEG: 2-component wavefunctions

0D electron gas confined in all directions



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$$H_Z = -\frac{g}{2}\mu_B \boldsymbol{\sigma} \cdot \boldsymbol{B}$$

$$H\Psi = E\Psi$$

$$H = \frac{1}{2m^*}\left[p_x^2 + \left(p_y - eBz\right)^2 + p_z^2\right][I] + V(x)[I] + V(y)[I] + V(z)[I]$$
$$-\frac{g}{2}\mu_B B\sigma_x - \frac{\eta}{\hbar}[p_x\sigma_z - p_z\sigma_x] + \gamma_D\left(\sigma_x\kappa_x + \sigma_y\kappa_y + \sigma_z\kappa_z\right)$$

Quantum Dot: $\langle p_x\rangle, \langle p_y\rangle, \langle p_z\rangle = 0$

No effect of SOI **unless there is a magnetic field** lifting the spin degeneracy via the Zeeman effect

SOI couples to different bands differently and have non-trivial effect

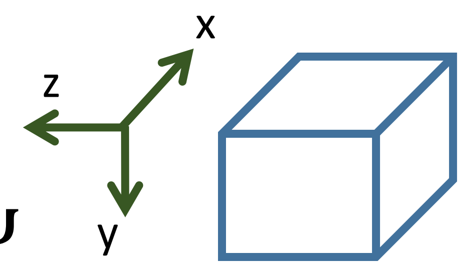
0-DEG: SOI as perturbation

0D electron gas confined in all directions

E **in *y*-direction** $\qquad \boldsymbol{B} = B\,\hat{\boldsymbol{x}}$

B **in *x*-direction** $\qquad \boldsymbol{A} = -Bz\,\hat{\boldsymbol{y}}$

$$H\Psi = E\Psi$$



$$H = \frac{1}{2m^*}\left[p_x^2 + \left(p_y - eBz\right)^2 + p_z^2\right][I] + V(x)[I] + V(y)[I] + V(z)[I]$$
$$- \frac{g}{2}\mu_B B\,\sigma_x - \frac{\eta}{\hbar}[p_x\sigma_z - p_z\sigma_x] + \gamma_D\left(\sigma_x\kappa_x + \sigma_y\kappa_y + \sigma_z\kappa_z\right)$$

$$H = H_0 + H_{SO} \quad \Rightarrow \quad [H_0 + H_{SO}][\Psi] = E[\Psi]$$

$$[\Psi] = a_\uparrow\{\phi^\uparrow\} + a_\downarrow\{\phi^\downarrow\}$$

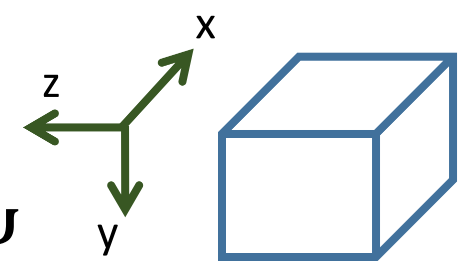
Unperturbed eigenspinors (unperturbed by SOI)?

$+x$ and $-x$
spin-polarized states

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \text{ and } \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

0-DEG: 2-component wavefunction

0D electron gas confined in all directions



E **in *y*-direction** $\boldsymbol{B} = B\,\hat{\boldsymbol{x}}$

B **in *x*-direction** $\boldsymbol{A} = -Bz\,\hat{\boldsymbol{y}}$

$$H\Psi = E\Psi$$

$$H = \frac{1}{2m^*}\left[p_x^2 + \left(p_y - eBz\right)^2 + p_z^2\right][I] + V(x)[I] + V(y)[I] + V(z)[I]$$

$$-\frac{g}{2}\mu_B B\sigma_x - \frac{\eta}{\hbar}[p_x\sigma_z - p_z\sigma_x] + \gamma_D\left(\sigma_x\kappa_x + \sigma_y\kappa_y + \sigma_z\kappa_z\right)$$

$$H = H_0 + H_{SO} \quad \Rightarrow \quad [H_0 + H_{SO}][\Psi] = E[\Psi]$$

$$[\Psi] = a_\uparrow\{\phi^\uparrow\} + a_\downarrow\{\phi^\downarrow\}$$

$$\phi(x) \neq \phi'(x)$$

$$\phi(y) \neq \phi'(y)$$

$$\phi(z) \neq \phi'(z)$$

$$\{\phi^\uparrow(x,y,z)\} = \frac{1}{\sqrt{2}}\phi(x)\phi(y)\,\phi(z)\begin{bmatrix}1\\1\end{bmatrix}$$

$$\{\phi^\downarrow(x,y,z)\} = \frac{1}{\sqrt{2}}\phi'(x)\phi'(y)\,\phi'(z)\begin{bmatrix}1\\-1\end{bmatrix}$$

0-DEG: 2-component wavefunction

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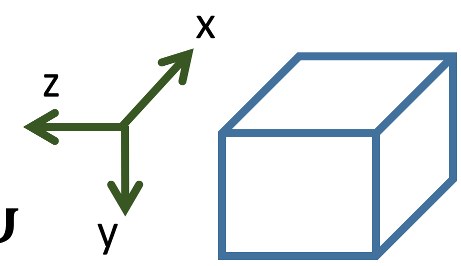

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$$\phi(x) \neq \phi'(x)$$
$$\phi(y) \neq \phi'(y)$$
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In a finite potential well, **upper spin level'**s wavefunction will be **less confined** than the lower spin level's one

0-DEG: Perturbed Hamiltonian

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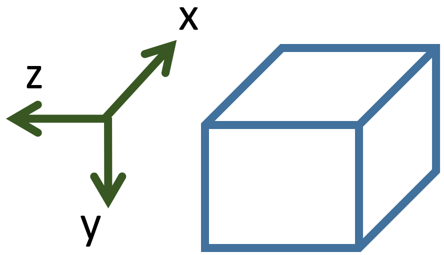

$$[H_0 + H_{SO}][\Psi] = E[\Psi] \qquad [\Psi] = a_\uparrow\{\phi^\uparrow\} + a_\downarrow\{\phi^\downarrow\}$$

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$$\begin{bmatrix}\langle H_1\rangle + \langle H_{SO}\rangle_{11} & \langle H_{SO}\rangle_{12}\\ \langle H_{SO}\rangle_{21} & \langle H_2\rangle + \langle H_{SO}\rangle_{22}\end{bmatrix}\begin{pmatrix}a_\uparrow\\a_\downarrow\end{pmatrix} = E\begin{pmatrix}a_\uparrow\\a_\downarrow\end{pmatrix}$$

$$\phi_s^\uparrow = \phi(x)\phi(y)\,\phi(z) \qquad \langle H_{SO}\rangle_{11} = \langle\phi_s^\uparrow|H_{SO}|\phi_s^\uparrow\rangle = 0$$

$$\phi_s^\downarrow = \phi'(x)\phi'(y)\,\phi'(z) \qquad \langle H_{SO}\rangle_{22} = \langle\phi_s^\downarrow|H_{SO}|\phi_s^\downarrow\rangle = 0$$

$$\langle H_1\rangle = \langle\phi_s^\uparrow|H_0|\phi_s^\uparrow\rangle \qquad \langle H_{SO}\rangle_{12} = \langle\phi_s^\uparrow|H_{SO}|\phi_s^\downarrow\rangle$$

$$\langle H_2\rangle = \langle\phi_s^\downarrow|H_0|\phi_s^\downarrow\rangle \qquad \langle H_{SO}\rangle_{21} = \langle\phi_s^\downarrow|H_{SO}|\phi_s^\uparrow\rangle$$

Dispersion relation: Eigenvalues

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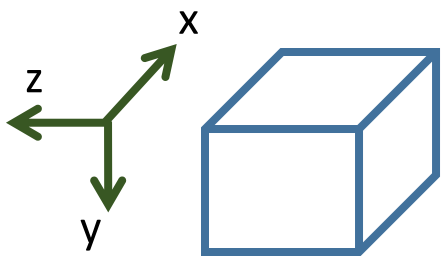

$$[H_0 + H_{SO}][\Psi] = E[\Psi] \qquad [\Psi] = a_\uparrow\{\phi^\uparrow\} + a_\downarrow\{\phi^\downarrow\}$$

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$$\begin{bmatrix}\langle H_1\rangle & \langle H_{SO}\rangle_{12}\\ \langle H_{SO}\rangle_{21} & \langle H_2\rangle\end{bmatrix}\begin{pmatrix}a_\uparrow\\a_\downarrow\end{pmatrix} = E\begin{pmatrix}a_\uparrow\\a_\downarrow\end{pmatrix}$$

$$E_\pm = \frac{\langle H_1\rangle + \langle H_2\rangle}{2} \pm \sqrt{\left(\frac{\langle H_1\rangle - \langle H_2\rangle}{2}\right)^2 + \langle H_{SO}\rangle_{12}\langle H_{SO}\rangle_{21}}$$

$$\langle H_1\rangle - \langle H_2\rangle = g\mu_B B$$

Perturbation: Rashba SOI

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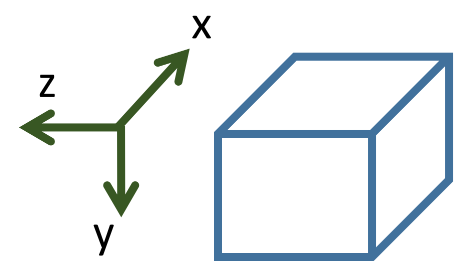

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$$E_\pm = \frac{\langle H_1\rangle + \langle H_2\rangle}{2} \pm \sqrt{\left(\frac{g\mu_B B}{2}\right)^2 + \langle H_{SO}\rangle_{12}\langle H_{SO}\rangle_{21}} \qquad H_{SO}^R = -\frac{\eta}{\hbar}[p_x\sigma_z - p_z\sigma_x]$$

$\left\langle H_{SO}^R\right\rangle_{12}$**?**

$$\left\langle H_{SO}^R\right\rangle_{12} = -\frac{\eta}{2\hbar}[1 \quad 1]\begin{bmatrix}\langle p_x\rangle & -\langle p_z\rangle\\ -\langle p_z\rangle & -\langle p_x\rangle\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = -\frac{\eta}{\hbar}\langle p_x\rangle = D$$

$$\left\langle p_{x,z}\right\rangle = \left\langle \phi(x)\phi(y)\,\phi(z)\middle|p_{x,z}\middle|\phi'(x)\phi'(y)\,\phi'(z)\right\rangle$$

Perturbation: Rashba SOI

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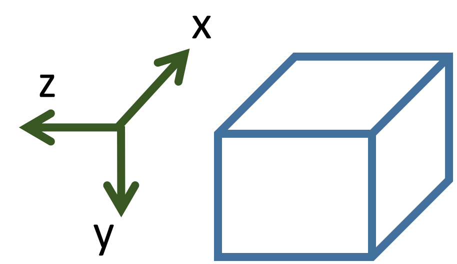

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$$H_{SO}^R = -\frac{\eta}{\hbar}[p_x\sigma_z - p_z\sigma_x]$$

$\langle H_{SO}^R\rangle_{21}$ **?**

$$\langle H_{SO}^R\rangle_{21} = -\frac{\eta}{2\hbar}\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}\langle p_x'\rangle & -\langle p_z'\rangle\\ -\langle p_z'\rangle & -\langle p_x'\rangle\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = -\frac{\eta}{\hbar}\langle p_x'\rangle = D^*$$

$$\langle p_{x,z}'\rangle = \langle \phi'(x)\phi'(y)\,\phi'(z)|p_{x,z}|\phi(x)\phi(y)\,\phi(z)\rangle$$

p_x **is Hermitian** $\quad \langle p_{x,z}'\rangle = \langle p_{x,z}\rangle^*$ $\quad \phi(x)$ **is real**

$$D = i|D|$$

$$D^* = -D$$

Perturbation: Dresselhaus SOI

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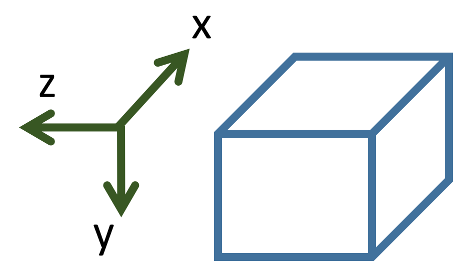

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$$\kappa_x = \frac{1}{2\hbar^3}\left[p_x\left\{(p_y - eBz)^2 - p_z^2\right\} + \left\{(p_y - eBz)^2 - p_z^2\right\}p_x\right]$$

$$\kappa_y = \frac{1}{2\hbar^3}\left[(p_y - eBz)\{p_z^2 - p_x^2\} + \{p_z^2 - p_x^2\}(p_y - eBz)\right]$$

$$\kappa_z = \frac{1}{2\hbar^3}\left[p_z\left\{p_x^2 - (p_y - eBz)^2\right\} + \left\{p_x^2 - (p_y - eBz)^2\right\}p_z\right]$$

Perturbation: Dresselhaus SOI

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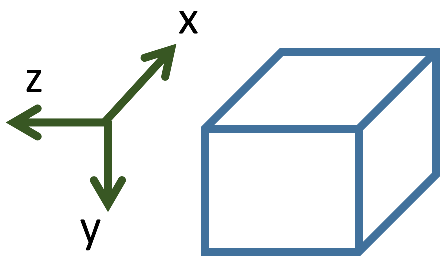

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$$H_D = \gamma_D \boldsymbol{\sigma}\cdot\boldsymbol{\kappa}$$

$\left\langle H_{SO}^D\right\rangle_{12}$**?** $\quad \left\langle H_{SO}^D\right\rangle_{21}$**?**

$$\left\langle H_{SO}^D\right\rangle_{12} = \frac{1}{2}[1 \quad 1]\begin{bmatrix}A & B\\B^* & -A\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = [A - (B - B^*)/2]$$

$$\left\langle H_{SO}^D\right\rangle_{21} = \frac{1}{2}[1 \quad -1]\begin{bmatrix}A' & B'\\B'^* & -A'\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = [A' + (B' - B'^*)/2]$$

$$\left\langle H_{SO}^D\right\rangle_{21}^* = \left\langle H_{SO}^D\right\rangle_{12}$$

$$Rashba: \left\langle H_{SO}^R\right\rangle_{21}^* = \left\langle H_{SO}^R\right\rangle_{12}$$

Perturbation: Rashba and Dresselhaus SOI

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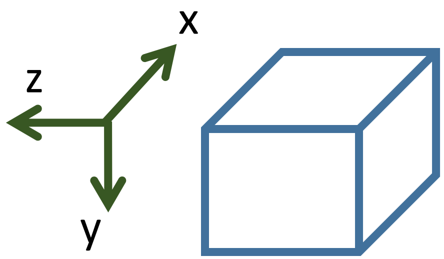

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$$\langle H_{SO}^R\rangle_{21}^* = \langle H_{SO}^R\rangle_{12}$$
$$\langle H_{SO}^D\rangle_{21}^* = \langle H_{SO}^D\rangle_{12}$$

H_{SO} *is Hermitaian*

$$\langle H_{SO}\rangle_{12}\langle H_{SO}\rangle_{21} = |\langle H_{SO}\rangle_{12}|^2$$

$$E_\pm = \frac{\langle H_1\rangle + \langle H_2\rangle}{2} \pm \sqrt{\left(\frac{g\mu_B B}{2}\right)^2 + |\delta|^2}$$

$$\delta = \langle H_{SO}\rangle_{12}$$
$$= D + A - (B - B^*)/2$$

Dispersion relation : Eigenspinors

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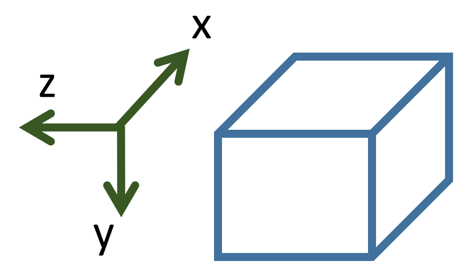

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$$E_\pm = \frac{\langle H_1\rangle + \langle H_2\rangle}{2} \pm \sqrt{\left(\frac{g\mu_B B}{2}\right)^2 + |\delta|^2}$$

$$\phi_s^\uparrow = \phi(x)\phi(y)\,\phi(z)$$

$$\phi_s^\downarrow = \phi'(x)\phi'(y)\,\phi'(z)$$

$$a_\uparrow^+ = -\sin\zeta \qquad a_\downarrow^+ = \cos\zeta\; e^{i\xi}$$

$$a_\uparrow^- = \cos\zeta \qquad a_\downarrow^- = \sin\zeta\; e^{i\xi}$$

$$\delta = |\delta|e^{i\xi}$$

$$\zeta = \frac{1}{2}\tan^{-1}\left[\frac{2|\delta|}{g\mu_B B}\right]$$

$$\Psi_+ = \frac{a_\uparrow^+}{\sqrt{2}}\phi_s^\uparrow\begin{bmatrix}1\\1\end{bmatrix} + \frac{a_\downarrow^+}{\sqrt{2}}\phi_s^\downarrow\begin{bmatrix}1\\-1\end{bmatrix}$$

$$\Psi_- = \frac{a_\uparrow^-}{\sqrt{2}}\phi_s^\uparrow\begin{bmatrix}1\\1\end{bmatrix} + \frac{a_\downarrow^-}{\sqrt{2}}\phi_s^\downarrow\begin{bmatrix}1\\-1\end{bmatrix}$$

Dispersion relation : Eigenspinors

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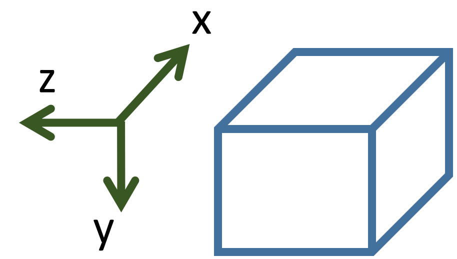

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$$E_\pm = \frac{\langle H_1\rangle + \langle H_2\rangle}{2} \pm \sqrt{\left(\frac{g\mu_B B}{2}\right)^2 + |\delta|^2}$$

$$\delta = |\delta| e^{i\xi} \qquad \zeta = \frac{1}{2}tan^{-1}\left[\frac{2|\delta|}{g\mu_B B}\right]$$

$$\Psi_+ = -\frac{1}{\sqrt{2}}\begin{bmatrix}\phi_s^\uparrow \sin\zeta - \phi_s^\downarrow \cos\zeta\ \ e^{i\xi}\\ \phi_s^\uparrow \sin\zeta + \phi_s^\downarrow \cos\zeta\ \ e^{i\xi}\end{bmatrix} \qquad \Psi_- = \frac{1}{\sqrt{2}}\begin{bmatrix}\phi_s^\uparrow \cos\zeta + \phi_s^\downarrow \sin\zeta\ \ e^{i\xi}\\ \phi_s^\uparrow \cos\zeta - \phi_s^\downarrow \sin\zeta\ \ e^{i\xi}\end{bmatrix}$$

$$\langle S_x^\pm\rangle = \mp\frac{\hbar}{2}\cos 2\zeta \qquad \langle S_y^\pm\rangle = \pm\frac{\hbar}{2}\langle\phi_s^\uparrow\phi_s^\downarrow\rangle \sin 2\zeta \sin\xi \qquad \langle S_z^\pm\rangle = \mp\frac{\hbar}{2}\langle\phi_s^\uparrow\phi_s^\downarrow\rangle \sin 2\zeta \cos\xi$$