HW # 10.

Problem 1

We start with the LL equation,

$$\frac{dM}{dt} = -\gamma M \times H_{eff} - \frac{\alpha|\gamma|}{M} M \times M \times H_{eff} \rightarrow ③$$

to linearize the equation for small rotations, we assume that the magnetization vector can be expressed as some

$M = M_0 + \delta M \rightarrow ①$ where M_0 is a constant equilibrium value and δm is a small deviation from this value.

likewise, $H_{eff} = H_{eq} + \delta H \rightarrow ②$

where we assume that the effective magnetic field can be expressed as a sum of constant equilibrium field and a small oscillating field δH.

Substituting ①, ② in ③,

$$\frac{d(M_0+\delta M)}{dt} = -\gamma(M_0+\delta M) \times H_{eff} - \frac{\alpha|\gamma|}{M_0+\delta M}(M_0+\delta M) \times (M_0+\delta M)(H_{eq}+\delta H)$$

expanding the cross products above we get.

$$\frac{d(M_0+\delta M)}{dt} = -\gamma M_0 H_{eq} - \gamma M_0 \times \delta H - \gamma \delta M H_{eq} - \gamma \delta M \times \delta H$$

$$-\frac{\alpha\gamma}{M}(M_0 \times M_0 \times H_{eq}) + M_0 \times \delta M \times H_{eq} + \delta M \times M_0 \times H_{eq}$$

$$+ \delta M \times \delta M \times H_{eq} + M_0 \times M_0 \times \delta H + M_0 \times \delta M \times \delta H$$

$$+ \delta M \times M_0 \times \delta H + \delta M \times \delta M \times \delta H$$

terms involving $\delta H \times \delta H$ and $\delta M \times \delta M \times H_{eq}$ are of higher order than δM and δH, hence they can be neglected (because we are interested in studying small deviations from the equilibrium state, small angles of rotation)

this leaves us with.

$$\frac{d(M_0+\delta M)}{dt} = -\gamma M_0 \times \delta H - \gamma\, \delta M \times H_{eq} - \frac{\alpha\gamma}{M}(M_0 \times \delta M) \times H_{eq} \quad (4)$$

The last term $-\frac{\alpha\gamma}{M}(M_0 \times \delta M) \times H_{eq}$ can be re-written as. (expanding the cross product)

$$\frac{\alpha\gamma}{M}(M_0 \times \delta M) \times H_{eq} = \frac{\alpha\gamma}{M}(M_0 \times \delta M \times H_{eq} + \delta M \times M_0 \times H_{eq})$$

cross product is anticommutative, hence $\delta M \times M_0 \times H_{eq} = -M_0 \times \delta M \times H_{eq}$

$$\frac{\alpha|\gamma|}{M}(M_0 \times \delta M) \times H_{eq} = \frac{\alpha|\gamma|}{M}(2M_0 \times \delta M \times H_{eq})$$

(4) can now be written as

$$\frac{d\delta M}{dt} = -\gamma M_0 \times \delta H - \gamma\, \delta M \times H_{eq} - \frac{\alpha|\gamma|}{M}(2M_0 \times \delta M \times H_{eq})$$

we can ignore $\delta M \times M_0 \times H_{eq}$ because $\gg \delta M$.

$$\therefore \text{ we get } \frac{d\delta M}{dt} = -\gamma M_0 \times \delta H - \gamma\, \delta M \times H_{eq} - \frac{\alpha\gamma}{M}(M_0 \times \delta M) \times H_{eq} \quad (6)$$

$\delta H = H_{y0}\, e^{i\omega t} \Rightarrow \delta M = \delta m_x + i\,\delta m_y$ (5) where δm_x and δm_y are components of deviation in x and y directions.

Substituting (5) in (6), we get

Considering only the ~~y compo~~ x component after ~~substituting~~ solving,

$$\frac{d\delta m_x}{dt} = -\gamma(M_{0x}\delta H_y + M_{0y}\delta H_x) - \gamma(\delta m_y H_z - \delta m_z H_y),$$

$$\delta M = \delta m_x x + \delta m_y y + \delta m_z z \rightarrow (3).$$

Putting (3) in (2),

$$\frac{d\delta m_x}{dt} = \gamma(M_{0x}\delta H_y + M_{0y}\delta H_x) - \gamma(\delta m_y H_z - \delta m_z H_y)$$

$$-\frac{\alpha\gamma}{M}[(M_{0x}\delta m_y - M_{0y}\delta m_x)H_z - (M_{0x}\delta m_z - M_{0z}\delta m_x)H_y$$

$$+(\delta m_x M_z - \delta m_z M_x)H_z - (\delta m_x M_y - \delta m_y M_x)H_y].$$

grouping ~~the~~ terms on both sides and taking the time derivative of the above, we get

$$\frac{d^2\delta m_x}{dt^2} + \alpha|\gamma|\frac{d\delta m_x}{dt} + \omega_0^2\delta m_x = -\gamma^2 M_0 \delta H_y e^{i\omega t}$$

if we consider $\phi = \delta m_x$.

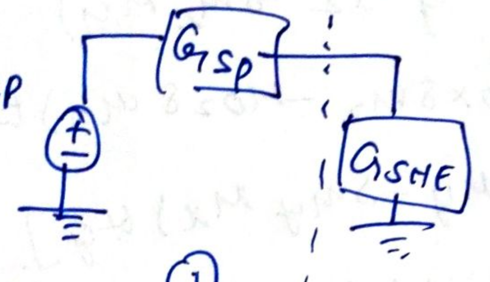
$$\frac{d^2\phi}{dt^2} + \alpha|\gamma|M\frac{d\phi}{dt} + \omega_0^2\phi = |\gamma|^2 M H_{y0} e^{i\omega t}$$

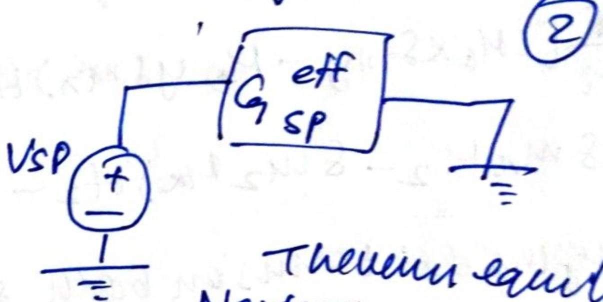
where $\omega_0 = \gamma|H_{eq}|$ is the Larmor frequency.

Problem 2

We need to derive the expression of eff. spin mixing $g^{\uparrow\downarrow}_{eff}$ conductance and the ISHE Voltage due to spin pumping

The instantaneous 3 component ~~ckt~~ spin circuit with voltage source Vsp acts as a spin battery. Gsp is the interfacial spin mixing conductance between the magnetic & SHE layer.





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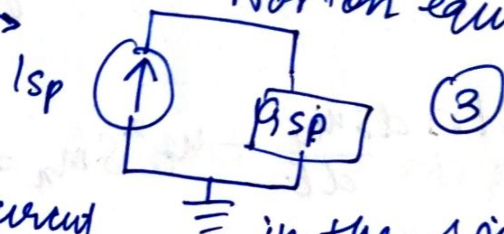
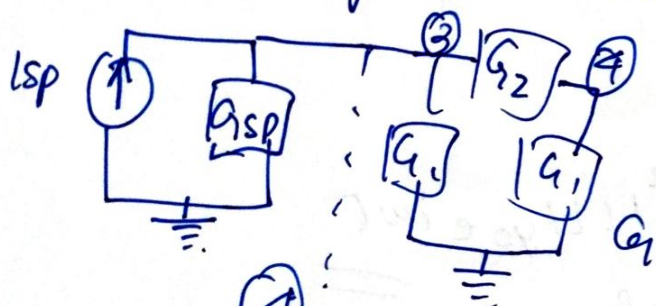


Incorporating the Norton circuit in the spin pumping sys,



In this figure, $V_{sp} = \frac{S\hbar\omega}{2e}\sin^2\theta$

$G_{sp} = lw\frac{2e^2}{h}g^{\uparrow\downarrow}$

$G_{NM} = G_1 + \frac{G_1G_2}{G_1+G_2} = \frac{1}{\frac{\lambda}{\sigma l w}\coth\left(\frac{t}{\lambda}\right)}$

From the above we have

$\cancel{G_{SHE} = \frac{G_1^2 + G_1^2G_2^2}{G_1^2}}$

$G_{SHE} = \frac{G_1 + G_1G_2}{G_1+G_2} = \frac{G_\lambda}{\coth(t/\lambda)} \rightarrow (i)$

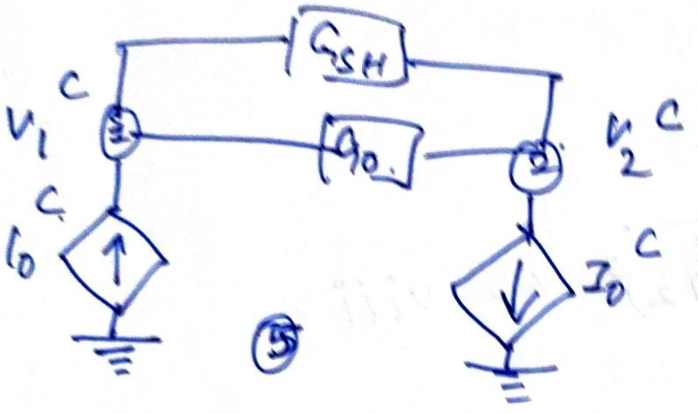
and from figure (1), we have $G_{sp,eff} = \frac{G_{sp}G_{SHE}}{G_{sp}+G_{SHE}} \rightarrow (ii)$

$= lw\left(\frac{2e^2}{h}\right)g^{\uparrow\downarrow}_{eff}$

Combining equations i and ii

$$g_{eff}^{\uparrow\downarrow} = \frac{g^{\uparrow\downarrow}}{1 + \frac{\lambda}{\sigma} \frac{2e^2}{h} g^{\uparrow\downarrow} \coth\left(\frac{t}{\lambda}\right)}$$

To calculate the ISHE volage,



The $V_{ISHE} = V_2^c - V_1^c$

$G_{SH} = \sigma_{mag} t_{mag} \frac{w}{l}$

In circuit ④ from the previous page, charge current depends on spin potential difference between node 3 and 4 as $I_0^c = \beta G_0 (V_3^z - V_4^z) \to iii$ and $G_0 = \frac{\sigma t w}{l}$, $\beta = \frac{\theta_{SH} l}{t}$

where θ_{SH} is the spin hall angle.

In circuit ⑤, we apply KCL at node ① to get.

$$I_0^c = (V_1^c - V_2^c)(G_0 + G_{SH}) \to v$$

and $V_{ISHE} = V_2^c - V_1^c \to iv$

Substituting iii and iv in v,

$$V_{ISH} = -\beta \left(\frac{G_0}{G_0 + G_{SH}}\right)(V_3 - V_4) \to (x)$$

To calculate $(V_3^z - V_4^z)$, apply KCL at nodes 3, 4 in circuit ④ from previous page.

to get

$$(V_3 - V_{sp})G_{sp} + V_3 G_1 + (V_3 - V_4) G_2 = 0 \quad \rightarrow vi$$

and

$$V_4 G_1 + (V_4 - V_3) G_2 = 0 \quad \rightarrow vii$$

Solving vi and vii

$$V_3 = \frac{G_1 + G_2}{(G_1+G_2)(G_{sp}+G_1+G_2) - (G_2)^2} \quad \rightarrow viii$$

and $V_4 = \frac{G_2}{D} V_{sp} G_{sp}$. $\rightarrow$ ix

Substituting (viii) and (ix) in (x)

$$V_{ISHE} = \frac{-\theta_{SH}\, l \lambda e \tilde{S} \omega g^{\uparrow\downarrow} \sin^2\theta \tanh\left(\frac{t}{2\lambda}\right)}{2\pi(\sigma t + \sigma_{mag} t_{mag})\left[1 + \frac{\lambda}{\sigma} \frac{2e^2}{h} g^{\uparrow\downarrow} \coth\left(\frac{t}{\lambda}\right)\right]}$$

Problem 3

X spin is not parallel to B_z. Hence, at $t=0$, spin is not eigenstate of the Hamiltonian describing the system.
The spin polarization hence shall change with time

In absence of dissipative forces, ϕ will obey time dependent Pauli's eq^n

$$\left[i\hbar \frac{\partial}{\partial t} + \langle H_0 \rangle + \frac{g}{2} \mu_B \vec{B} \cdot \vec{\sigma} \right] [\phi] = 0$$

The solution can be written in the matrix form with E_-, E_+ as eigenvalues

$$[\phi](t) = e^{[-i(\langle H_0 \rangle + (g/2)\mu_B B \cdot \sigma) t/\hbar]} [\phi](0)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} e^{-iE_+ t/\hbar} & 0 \\ 0 & e^{-iE_- t/\hbar} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [\phi](0). \quad \to (1)$$

$E_\pm = \langle H_0 \rangle \pm \left(\frac{g}{2}\right) \mu_B B$ where $\langle H_0 \rangle = \langle \phi_0 | H_0 | \phi_0 \rangle$.
and corresponding eigenspinors are $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(1) is the eigenspinor at some time t.

$[\phi](0)$ is the x-polarized state $\frac{1}{\sqrt{2}}[1 \ \ -1]^{\dagger}$,

So $[\phi](t) = \begin{bmatrix} e^{-iE_+ t/\hbar} \\ -e^{-iE_- t/\hbar} \end{bmatrix}$

$S_x(t) = [\phi]^+(t)[\sigma_x][\phi]^+(t)$ is the spin compont along x

$$= \left[\frac{e^{iE_+t/\hbar}}{\sqrt{2}} \quad -\frac{e^{iE_-t/\hbar}}{\sqrt{2}}\right]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \frac{e^{-iE_+t/\hbar}}{\sqrt{2}} \\ -\frac{e^{-iE_-t/\hbar}}{\sqrt{2}} \end{bmatrix}$$

$= -\cos\theta$ since $\theta = \frac{(E_+ - E_-)}{\hbar}t = \frac{g\mu_B Bt}{\hbar}$

Likewise,

$S_y(t) = [\phi]^+(t)[\sigma_y][\phi]^+(t)$

$$= \left[\frac{e^{iE_+t/\hbar}}{\sqrt{2}} \quad -\frac{e^{iE_-t/\hbar}}{\sqrt{2}}\right]\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} \frac{e^{-iE_+t/\hbar}}{\sqrt{2}} \\ -\frac{e^{-iE_-t/\hbar}}{\sqrt{2}} \end{bmatrix}$$

$= -\sin\theta$

and

$S_z(t) = [\phi]^+(t)[\sigma_y][\phi]^+(t)$

$$= \left[\frac{e^{iE_+t/\hbar}}{\sqrt{2}} \quad -\frac{e^{iE_-t/\hbar}}{\sqrt{2}}\right]\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} \frac{e^{-iE_+t/\hbar}}{\sqrt{2}} \\ -\frac{e^{-iE_-t/\hbar}}{\sqrt{2}} \end{bmatrix}$$

$= 0.0 = 0.$

$$\frac{dS}{dt} = x\frac{dS_x}{dt} + y\frac{dS_y}{dt} + z\frac{dS_z}{dt}$$

$$= -x\frac{d(\cos\theta)}{dt} - y\frac{d(\sin\theta)}{dt}$$

$$= \frac{d\theta}{dt}\left[\hat{x}\sin\theta - \hat{y}\cos\theta\right]$$

$$= \frac{g}{2}\frac{eB}{m}\left[\sin\theta\,\hat{x} - \cos\theta\,\hat{y}\right]$$

$$= \frac{eB}{m^*}\left[\sin\theta\,\hat{x} - \cos\theta\,\hat{y}\right] \rightarrow (3)$$

$$\Omega \times S = \frac{eB}{m^*} \times S = \frac{eB}{m^*}\left[-\hat{x}\,S_y + \hat{y}\,S_x\right]$$

$$= \frac{eB}{m^*}\left[\sin\theta\,\hat{x} - \cos\theta\,\hat{y}\right] \rightarrow (4)$$

From (3), (4).

$$\frac{dS}{dt} = \Omega \times S$$