Spintronics and Nanomagnetics

ECS 521/641

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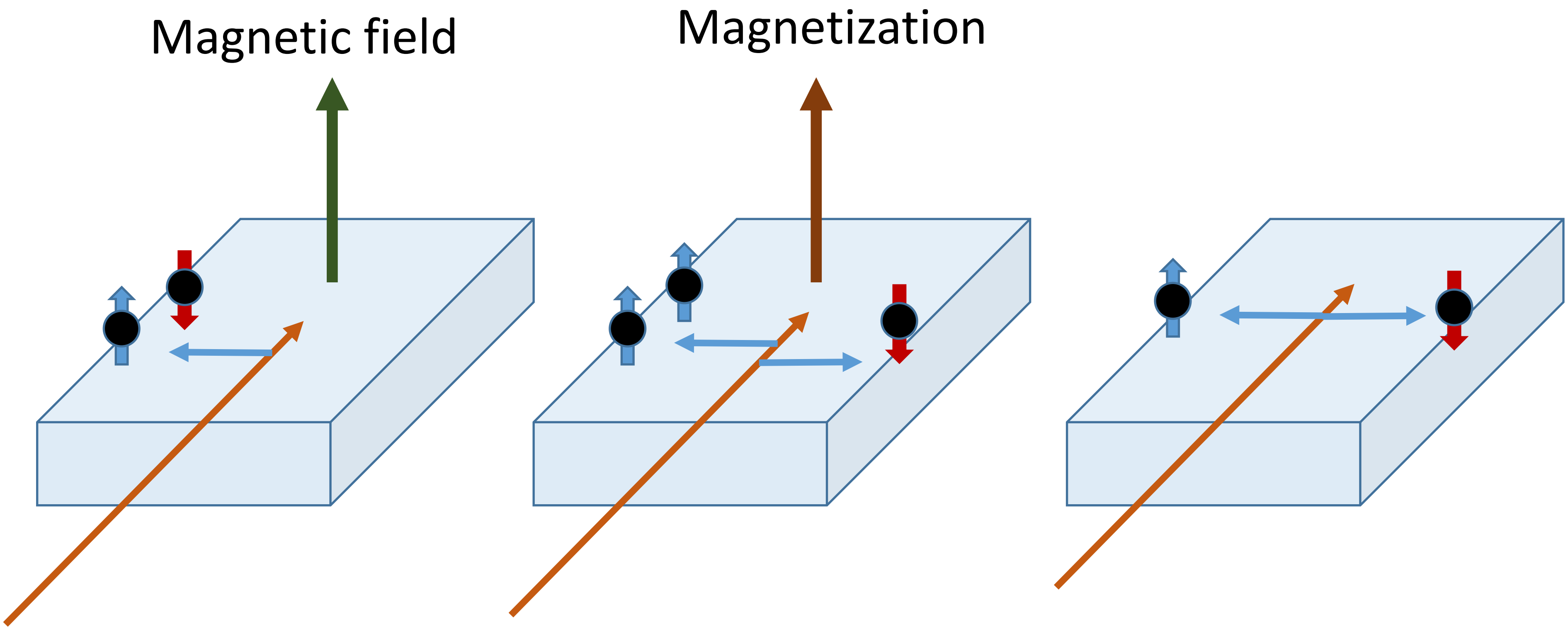
Spin Hall effect

Spin Hall effect



Ordinary Hall effect

Hall voltage
NO
spin accumulation

Anomalous Hall effect

Hall voltage
AND
spin accumulation

(Pure) Spin Hall effect

NO Hall voltage
BUT
spin accumulation

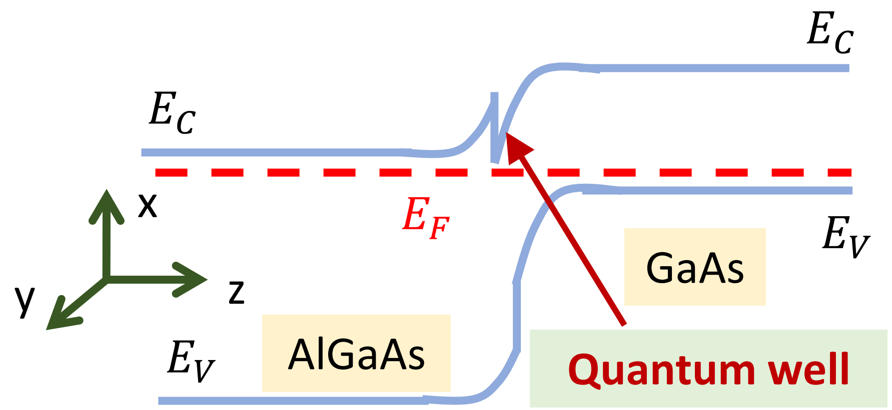
2-DEG in the presence of Rashba SOI

2D electron gas in $x - y$ plane

***E* in z-direction**

$$H_R = \boldsymbol{\eta_R}(\boldsymbol{E}) \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}]$$

$$H_R = -\frac{a_R}{\hbar} E_z \hat{\boldsymbol{z}} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}]$$



$$H\Psi = E\Psi$$

$$[\Psi(x,y,z)] = e^{ik_x x} e^{ik_y y} [\lambda(z)]$$

$V(z)$: Confining potential

$$H = \frac{|\boldsymbol{p}|^2}{2m^*}[I] + V(z)[I] + H_R$$

$$H = \begin{bmatrix} \frac{|\boldsymbol{p}|^2}{2m^*} + V(z) & -\frac{a_R}{\hbar} E_z (p_y + ip_x) \\ -\frac{a_R}{\hbar} E_z (p_y - ip_x) & \frac{|\boldsymbol{p}|^2}{2m^*} + V(z) \end{bmatrix}$$

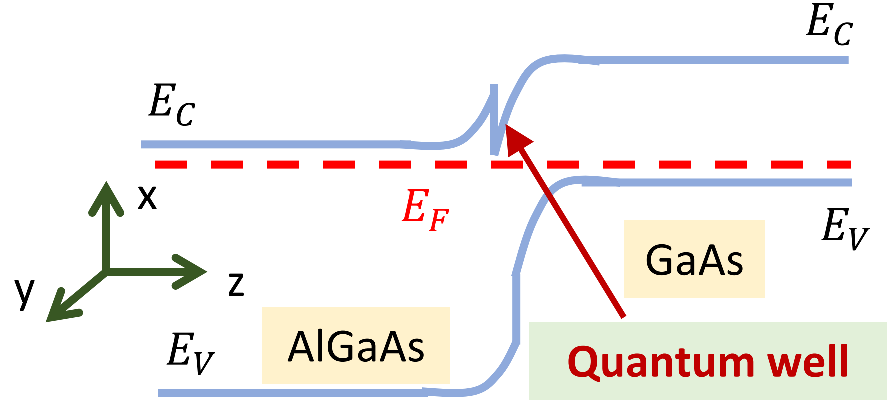
$$\boldsymbol{p} = p_x \hat{x} + p_y \hat{y}$$

$$p = \sqrt{p_x^2 + p_y^2}$$

Velocity versus wavevector

2D electron gas in $x - y$ plane

$\boldsymbol{E}$ in z-direction



$$H_R = -\frac{a_R}{\hbar} E_z \hat{\boldsymbol{z}} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}]$$

$$H = \frac{|\boldsymbol{p}|^2}{2m^*}[I] + V(z)[I] + H_R$$

$$H = \begin{bmatrix} \frac{|\boldsymbol{p}|^2}{2m^*} + V(z) & -\frac{a_R}{\hbar} E_z (p_y + ip_x) \\ -\frac{a_R}{\hbar} E_z (p_y - ip_x) & \frac{|\boldsymbol{p}|^2}{2m^*} + V(z) \end{bmatrix}$$

$$v_q = \frac{\partial H}{\partial p_q}$$

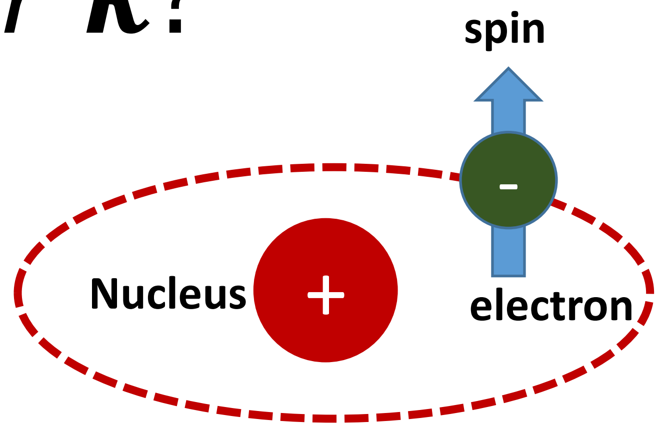
Exercise Determine v_x and v_y

$$v_x = \frac{\hbar k_x}{m^*}[I] + \frac{a_R}{\hbar} E_z \sigma_y \qquad v_y = \frac{\hbar k_y}{m^*}[I] - \frac{a_R}{\hbar} E_z \sigma_x$$

$$\boldsymbol{v} = A\boldsymbol{k} + B$$

$\boldsymbol{B}_{\boldsymbol{Rashba}} \propto \boldsymbol{v}$ *or* $\boldsymbol{k}$?

$$\boldsymbol{B}_{\boldsymbol{Rashba}} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}} \simeq \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2}$$



Ehrenfest theorem: Time evolution of the expectation value of a time-dependent observable for a quantum-mechanical system

$$\frac{d\langle \boldsymbol{S}(t)\rangle}{dt} = \frac{1}{i\hbar}\langle[\boldsymbol{S}(t), H(t)]\rangle + \left\langle \frac{d\boldsymbol{S}(t)}{dt} \right\rangle$$

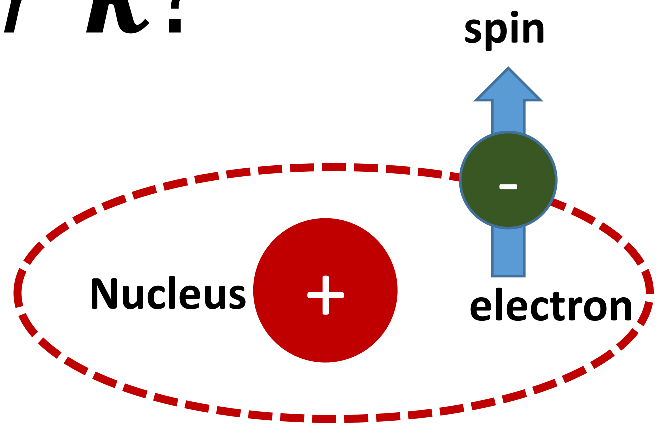
$$\frac{g\mu_B \boldsymbol{B}_{\boldsymbol{Rashba}}}{\hbar} \times \langle \boldsymbol{S}(t)\rangle = \frac{1}{i\hbar}\langle[\boldsymbol{S}(t), H_R(t)]\rangle \qquad H_R = -\frac{a_R}{\hbar} E_z \hat{\boldsymbol{z}} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}]$$

$$\boldsymbol{E} = E_z \hat{\boldsymbol{z}}$$

$$\frac{g\mu_B \boldsymbol{B}_{\boldsymbol{Rashba}}}{\hbar} \times \langle \boldsymbol{S}(t)\rangle = -\frac{1}{i\hbar}\left\langle \left[\boldsymbol{S}(t), \frac{a_R}{\hbar} \boldsymbol{E} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}] \right] \right\rangle$$

$\boldsymbol{B}_{\boldsymbol{Rashba}} \propto \boldsymbol{v}$ or $\boldsymbol{k}$?

$$\boldsymbol{B}_{\boldsymbol{Rashba}} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}} \simeq \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2}$$



$$H_R = -\frac{a_R}{\hbar} E_z \hat{\boldsymbol{z}} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}] \qquad \boldsymbol{E} = E_z \hat{\boldsymbol{z}}$$

$$\frac{g\mu_B \boldsymbol{B}_{\boldsymbol{Rashba}}}{\hbar} \times \langle \boldsymbol{S}(t) \rangle = -\frac{1}{i\hbar} \left\langle \left[\boldsymbol{S}(t), \frac{a_R}{\hbar} \boldsymbol{E} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}] \right] \right\rangle$$

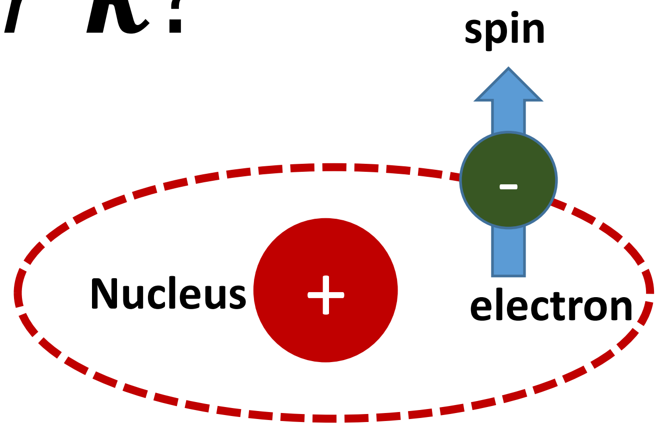
$$[\boldsymbol{\sigma}, \boldsymbol{E} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})] = \boldsymbol{\sigma}\boldsymbol{E} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p}) - \boldsymbol{E} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})\boldsymbol{\sigma}$$

$$= -\boldsymbol{\sigma}\boldsymbol{\sigma} \cdot (\boldsymbol{E} \times \boldsymbol{p}) + \boldsymbol{\sigma} \cdot (\boldsymbol{E} \times \boldsymbol{p})\boldsymbol{\sigma}$$

$$= (\boldsymbol{\sigma} \cdot \boldsymbol{q})\boldsymbol{\sigma} - \boldsymbol{\sigma}(\boldsymbol{\sigma} \cdot \boldsymbol{q}) \qquad \boldsymbol{q} = \boldsymbol{E} \times \boldsymbol{p}$$

$$= \left(\sigma_x q_x + \sigma_y q_y + \sigma_z q_z\right)\left(\sigma_x \hat{\boldsymbol{x}} + \sigma_y \hat{\boldsymbol{y}} + \sigma_z \hat{\boldsymbol{z}}\right)$$
$$- \left(\sigma_x \hat{\boldsymbol{x}} + \sigma_y \hat{\boldsymbol{y}} + \sigma_z \hat{\boldsymbol{z}}\right)\left(\sigma_x q_x + \sigma_y q_y + \sigma_z q_z\right)$$

$\boldsymbol{B}_{\boldsymbol{Rashba}} \propto \boldsymbol{v}\ \ or\ \boldsymbol{k}?$



$$\boldsymbol{B}_{\boldsymbol{Rashba}} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}} \simeq \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2}$$

$$H_R = -\frac{a_R}{\hbar} E_z \hat{\boldsymbol{z}} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}] \qquad \boldsymbol{E} = E_z \hat{\boldsymbol{z}}$$

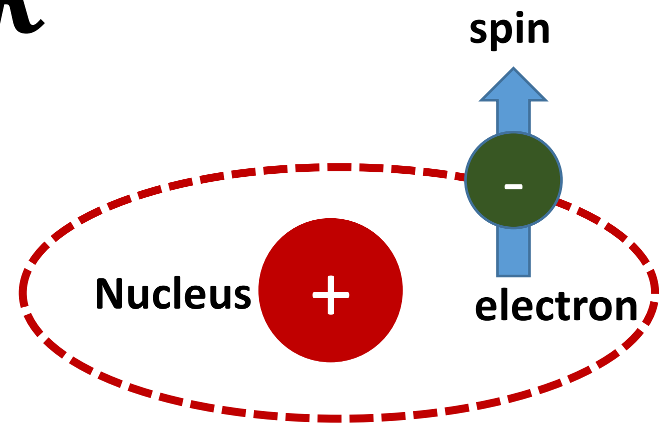
$$\frac{g\mu_B \boldsymbol{B}_{\boldsymbol{Rashba}}}{\hbar} \times \langle \boldsymbol{S}(t) \rangle = -\frac{1}{i\hbar}\left\langle \left[\boldsymbol{S}(t), \frac{a_R}{\hbar} \boldsymbol{E} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}] \right] \right\rangle$$

$$\boldsymbol{q} = \boldsymbol{E} \times \boldsymbol{p}$$

$$\begin{aligned}
[\boldsymbol{\sigma}, \boldsymbol{E} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})] &= \left(\sigma_x q_x + \sigma_y q_y + \sigma_z q_z\right)\left(\sigma_x \hat{\boldsymbol{x}} + \sigma_y \hat{\boldsymbol{y}} + \sigma_z \hat{\boldsymbol{z}}\right) \\
&\quad - \left(\sigma_x \hat{\boldsymbol{x}} + \sigma_y \hat{\boldsymbol{y}} + \sigma_z \hat{\boldsymbol{z}}\right)\left(\sigma_x q_x + \sigma_y q_y + \sigma_z q_z\right) \\
&= \left(2i\sigma_y q_z \hat{\boldsymbol{x}} - 2i\sigma_z q_y \hat{\boldsymbol{x}}\right) \\
&\quad + \left(2i\sigma_z q_x \hat{\boldsymbol{y}} - 2i\sigma_x q_z \hat{\boldsymbol{y}}\right) \quad = 2i\, \boldsymbol{\sigma} \times \boldsymbol{q} \\
&\quad + \left(2i\sigma_x q_y \hat{\boldsymbol{z}} - 2i\sigma_y q_x \hat{\boldsymbol{z}}\right)
\end{aligned}$$

$$\sigma_i \sigma_j = -\sigma_j \sigma_i = i\sigma_k$$

$\boldsymbol{B_{Rashba}} \propto \boldsymbol{k}$



$$\boldsymbol{B_{Rashba}} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}} \simeq \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2}$$

$$H_R = -\frac{a_R}{\hbar} E_z \hat{\boldsymbol{z}} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}] \qquad \boldsymbol{E} = E_z \hat{\boldsymbol{z}}$$

$$\frac{g\mu_B \boldsymbol{B_{Rashba}}}{\hbar} \times \langle \boldsymbol{S}(t) \rangle = -\frac{1}{i\hbar} \left\langle \left[\boldsymbol{S}(t), \frac{a_R}{\hbar} \boldsymbol{E} \cdot [\boldsymbol{\sigma} \times \boldsymbol{p}] \right] \right\rangle$$

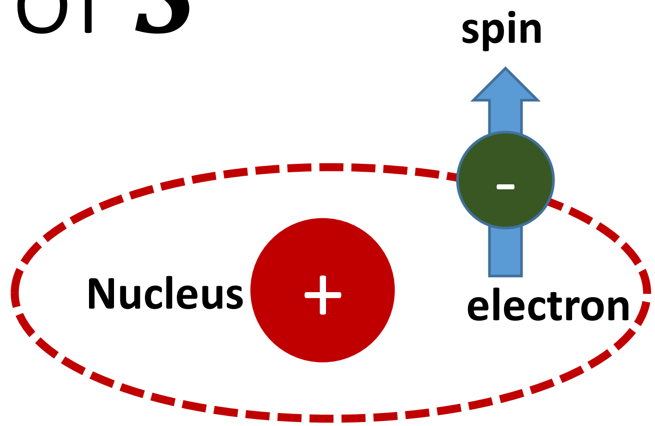
$$\boldsymbol{q} = \boldsymbol{E} \times \boldsymbol{p}$$

$$[\boldsymbol{\sigma}, \boldsymbol{E} \cdot (\boldsymbol{\sigma} \times \boldsymbol{p})] = 2i\, \boldsymbol{\sigma} \times \boldsymbol{q} = 2i\, \boldsymbol{\sigma} \times (\boldsymbol{E} \times \boldsymbol{p})$$

$$\frac{g\mu_B \boldsymbol{B_{Rashba}}}{\hbar} \times \langle \boldsymbol{S}(t) \rangle = \frac{1}{i\hbar} \frac{a_R}{\hbar} 2i\, (\boldsymbol{E} \times \boldsymbol{p}) \times \langle \boldsymbol{S}(t) \rangle$$

$$\boldsymbol{B_{Rashba}} = \frac{2a_R}{g\mu_B} (\boldsymbol{E} \times \boldsymbol{k}) \qquad \boldsymbol{B_{Rashba}} = \frac{2a_R E_z}{g\mu_B} \left(-k_y \hat{\boldsymbol{x}} + k_x \hat{\boldsymbol{y}} \right)$$

Time evolution of $\boldsymbol{S}$



$$\boldsymbol{B_{Rashba}} = \frac{\boldsymbol{E}\times\boldsymbol{v}}{2c^2\sqrt{1-v^2/c^2}} \simeq \frac{\boldsymbol{E}\times\boldsymbol{v}}{2c^2}$$

$$H_R = -\frac{a_R}{\hbar}E_z\hat{\boldsymbol{z}}\cdot[\boldsymbol{\sigma}\times\boldsymbol{p}] \qquad \boldsymbol{E} = E_z\hat{\boldsymbol{z}}$$

$$\boxed{\frac{d\boldsymbol{S}}{dt} = \frac{g\mu_B\boldsymbol{B_{Rashba}}}{\hbar}\times\boldsymbol{S}}$$

$$\boldsymbol{B_{Rashba}} = \frac{2a_RE_z}{g\mu_B}\left(-k_y\hat{\boldsymbol{x}} + k_x\hat{\boldsymbol{y}}\right) = B_x\hat{\boldsymbol{x}} + B_y\hat{\boldsymbol{y}}$$

$$\frac{dS_x}{dt} = \frac{g\mu_B}{\hbar}B_yS_z = \frac{2a_RE_z}{\hbar}k_xS_z$$

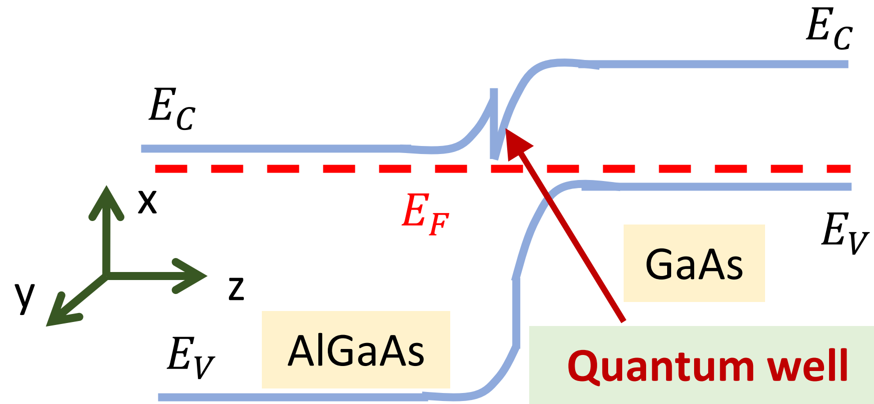
$$\frac{dS_y}{dt} = -\frac{g\mu_B}{\hbar}B_xS_z = \frac{2a_RE_z}{\hbar}k_yS_z$$

$$\frac{dS_z}{dt} = \frac{g\mu_B}{\hbar}\left(B_xS_y - B_yS_x\right) = -\frac{2a_RE_z}{\hbar}\left(k_yS_y + k_xS_x\right)$$

Time evolution of $\boldsymbol{S}$

2D electron gas in $\boldsymbol{x-y}$ plane

$\boldsymbol{E}$ in z-direction → Rashba SOI



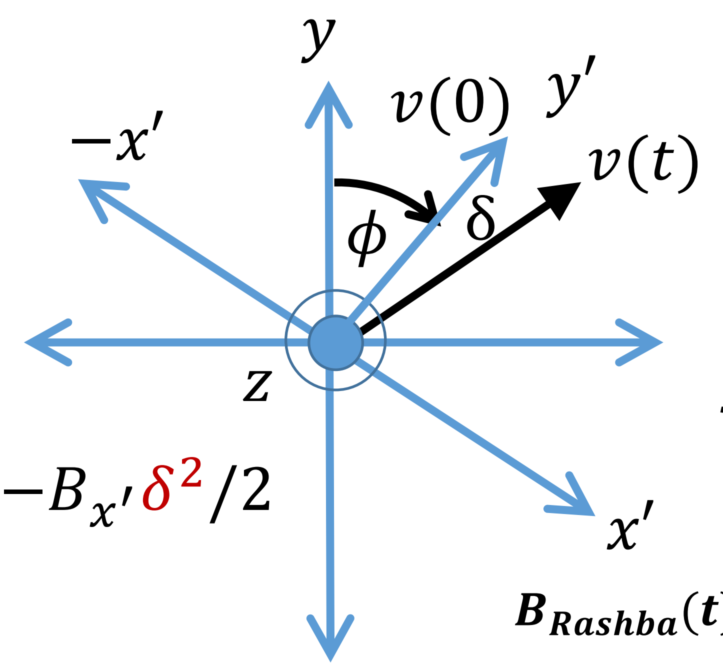
$$\boldsymbol{B_{Rashba}} = \frac{2a_R E_z}{g\mu_B}\left(-k_y\widehat{\boldsymbol{x}} + k_x\widehat{\boldsymbol{y}}\right)$$

$$= B_x\widehat{\boldsymbol{x}} + B_y\widehat{\boldsymbol{y}}$$

E_x in x-direction

$t = 0$: $\quad B_{x'}(0) \neq 0 \quad B_{y'}(0) = 0$

$B_{Rashba}(0)$ is along the x'-direction



$$S_{x'}(0) = \pm\frac{\hbar}{2}$$

$$S_{y'}(0) = 0$$

$$S_z(0) = 0$$

$t = \tau$:

$$dB_{x'} = B_{x'}(\cos\delta - 1) \simeq -B_{x'}\delta^2/2$$

$$dB_{y'} = B_{x'}\sin\delta \simeq B_{x'}\delta$$

$$\frac{dB_{x'}}{dt} \simeq 0 \qquad \frac{dB_{y'}}{dt} \neq 0$$

$$S_{x'}(t) \simeq \pm\frac{\hbar}{2}$$

Time evolution of $\boldsymbol{S}$

$t = \tau$:

$$dB_{x'} = B_{x'}(cos\delta - 1) \simeq -B_{x'}\delta^2/2$$

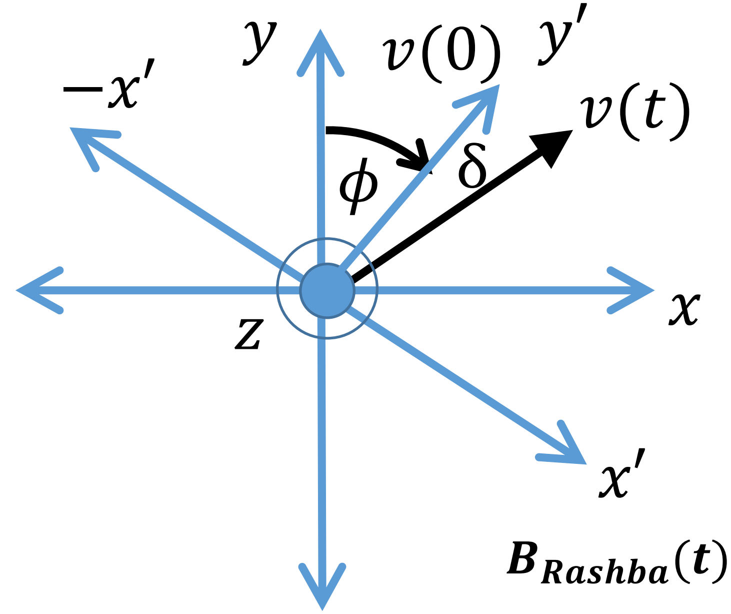
$$dB_{y'} = B_{x'}sin\delta \simeq B_{x'}\delta$$

$$\frac{dB_{x'}}{dt} \simeq 0 \qquad S_{x'}(t) \simeq \pm\frac{\hbar}{2}$$

$$\frac{dB_{y'}}{dt} \neq 0 \Rightarrow \frac{dS_{y'}}{dt} \neq 0$$

$$\frac{d^2S_{y'}}{dt^2} = 0$$

Assumption: Change is linear in time



$$\boldsymbol{B_{Rashba}} = B_x\hat{\boldsymbol{x}} + B_y\hat{\boldsymbol{y}}$$

$$\frac{dS_{x'}}{dt} = \frac{g\mu_B}{\hbar}B_{y'}S_z$$

$$\frac{dS_{y'}}{dt} = -\frac{g\mu_B}{\hbar}B_{x'}S_z$$

$$\frac{dB_{x'}}{dt}S_z + \frac{dS_z}{dt}B_{x'} = 0 \Leftarrow$$

$$B_{x'}(\tau)S_{y'}(\tau) - B_{y'}(\tau)S_{x'}(\tau) = 0$$

$$\frac{dS_z}{dt} = \frac{g\mu_B}{\hbar}\left(B_{x'}S_{y'} - B_{y'}S_{x'}\right)$$

Time evolution of $\boldsymbol{S}$

$t = \tau$:

$$dB_{x'} = B_{x'}(cos\delta - 1) \simeq -B_{x'}\delta^2/2$$

$$dB_{y'} = B_{x'}sin\delta \simeq B_{x'}\delta$$

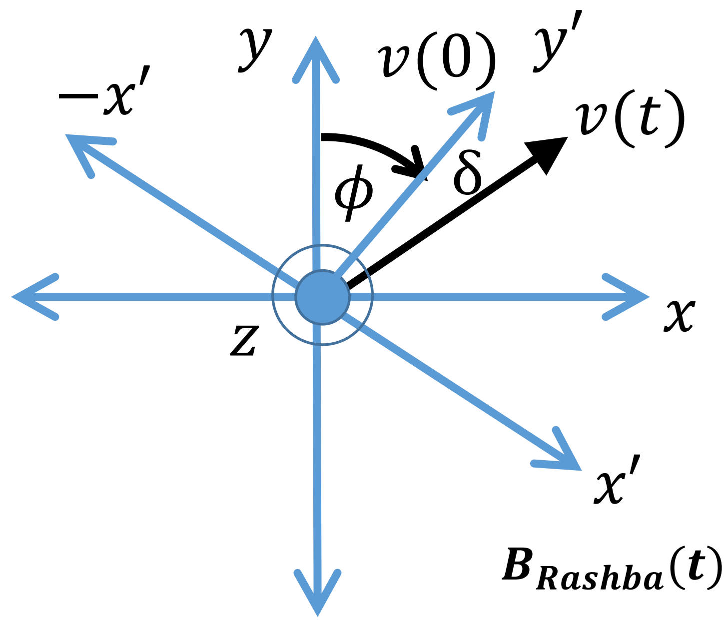
$$\frac{dB_{x'}}{dt} \simeq 0 \qquad \boxed{S_{x'}(t) \simeq \pm\frac{\hbar}{2}}$$

$$B_{x'}(\tau)S_{y'}(\tau) - B_{y'}(\tau)S_{x'}(\tau) = 0$$

$$S_{y'}(\tau) = \pm\frac{B_{y'}(\tau)}{B_{x'}(\tau)}\frac{\hbar}{2}$$

$$S_z(t) = \mp\frac{\hbar}{g\mu_B}\frac{1}{B_{x'}(t)}\frac{d}{dt}\left(\frac{B_{y'}(t)}{B_{x'}(t)}\right)\frac{\hbar}{2}$$

$$\frac{B_{x'}(t)\, dB_{y'}(t)/dt - B_{y'}(t)\, dB_{x'}(t)/dt}{B_{x'}^2(t)}$$



$$\boldsymbol{B_{Rashba}} = B_x\hat{\boldsymbol{x}} + B_y\hat{\boldsymbol{y}}$$

$$\frac{dS_{x'}}{dt} = \frac{g\mu_B}{\hbar}B_{y'}S_z$$

$$\frac{dS_{y'}}{dt} = -\frac{g\mu_B}{\hbar}B_{x'}S_z$$

$$\frac{dS_z}{dt} = \frac{g\mu_B}{\hbar}(B_{x'}S_{y'} - B_{y'}S_{x'})$$

Time evolution of $\boldsymbol{S}$

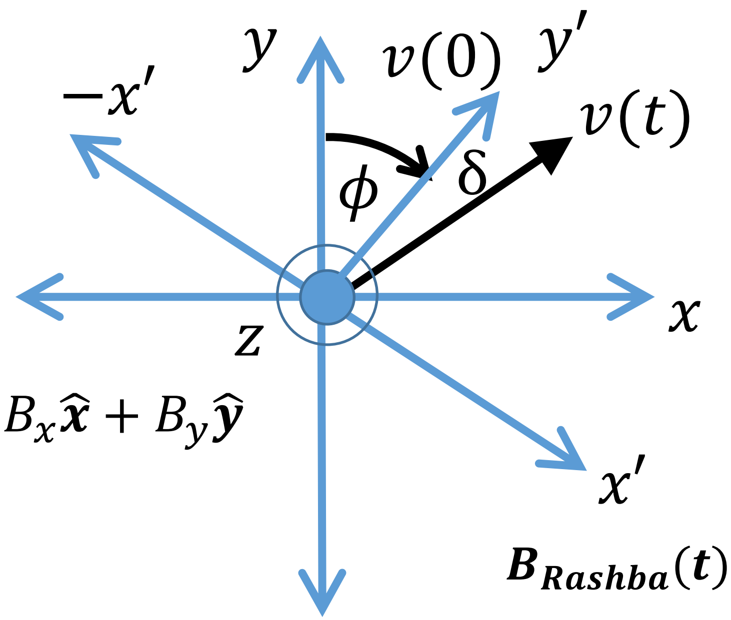
$t = \tau$:

$$dB_{x'} = B_{x'}(\cos\delta - 1) \simeq -B_{x'}\delta^2/2$$

$$dB_{y'} = B_{x'}\sin\delta \simeq B_{x'}\delta$$

$$\frac{dB_{x'}}{dt} \simeq 0 \qquad S_{x'}(t) \simeq \pm\frac{\hbar}{2}$$

$$\boldsymbol{B}_{\boldsymbol{Rashba}} = B_x\hat{\boldsymbol{x}} + B_y\hat{\boldsymbol{y}}$$



$$S_z(t) = \mp\frac{\hbar}{g\mu_B}\frac{1}{B_{x'}(t)}\frac{d}{dt}\left(\frac{B_{y'}(t)}{B_{x'}(t)}\right)\frac{\hbar}{2}$$

$$\frac{B_{x'}(t)\; dB_{y'}(t)/dt - B_{y'}(t)\; dB_{x'}(t)/dt}{B_{x'}^2(t)}$$

$$B_{y'}(\tau) - B_{y'}(0) = B_{y'}(\tau) \simeq B_{x'}(\tau)\delta(\tau)$$

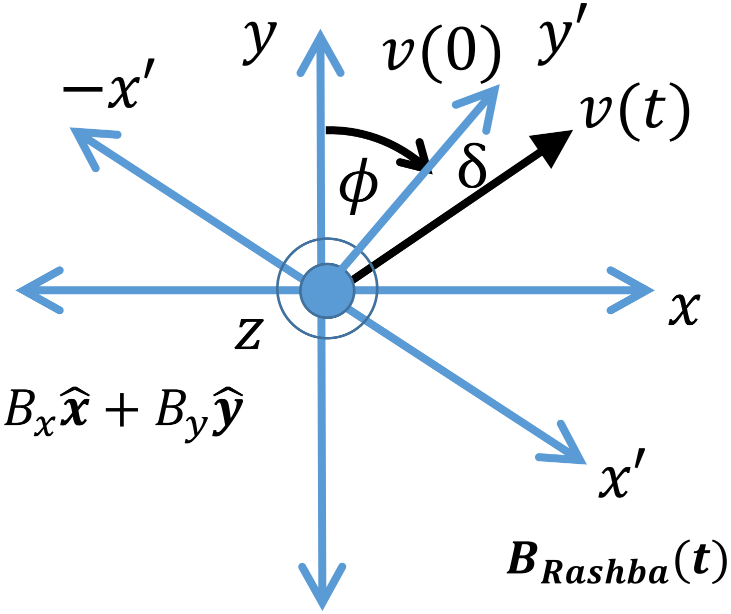
$$S_z(t) = \mp\frac{\hbar}{g\mu_B}\frac{1}{B_{x'}^2(t)}\frac{dB_{y'}(t)}{dt}\frac{\hbar}{2} \qquad S_z(t) = \mp\frac{\hbar^2}{2g\mu_B}\frac{1}{B_{x'}(t)}\frac{d\delta(t)}{dt}$$

$$\sin(\phi + \delta(t)) = \frac{v_x(t)}{v(t)} = \frac{k_x(t)}{k(t)}$$

Time evolution of $\boldsymbol{S}$

$$S_z(t) = \mp \frac{\hbar^2}{2g\mu_B}\frac{1}{B_{x'}(t)}\frac{d\delta(t)}{dt}$$

$$sin(\phi + \delta(t)) = \frac{v_x(t)}{v(t)} = \frac{k_x(t)}{k(t)}$$



$$\boldsymbol{B_{Rashba}} = B_x\hat{\boldsymbol{x}} + B_y\hat{\boldsymbol{y}}$$

$$\frac{d\delta(t)}{dt} = \frac{d(\phi + \delta(t))}{dt} = \frac{d}{dt}\left[\sin^{-1}\frac{k_x(t)}{k(t)}\right]$$

$$= \frac{1}{\sqrt{1 - k_x^2(t)/k^2(t)}}\frac{k(t)\ dk_x(t)/dt - k_x(t)\ \ dk(t)/dt}{k^2(t)}$$

$$k_y(t) = k_y(0)$$

E_x in x-direction

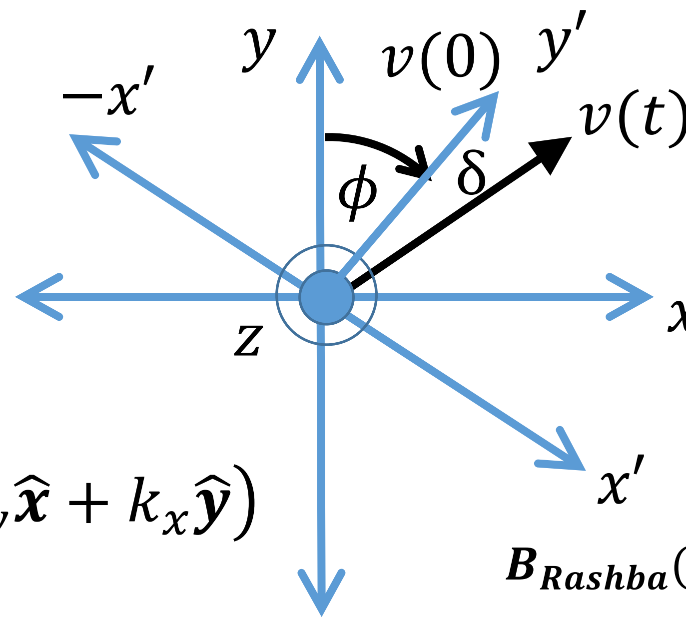
$$= \frac{1}{k_y(0)/k(t)}\frac{k(t)dk_x(t)/dt - \ (k_x^2(t)/k(t))\ dk_x(t)/dt}{k^2(t)}$$

$$= \frac{k_y(0)}{k^2(t)}\frac{dk_x(t)}{dt}$$

$$S_z(t) = \mp \frac{\hbar^2}{2g\mu_B}\frac{1}{B_{x'}(t)}\frac{k_y(0)}{k^2(t)}\frac{dk_x(t)}{dt}$$

Time evolution of $\boldsymbol{S}$

$$S_z(\tau) = \mp \frac{\hbar^2}{2g\mu_B} \frac{1}{B_{x'}(\tau)} \frac{k_y(0)}{k^2(\tau)} \frac{dk_x}{dt}$$



$$\frac{d(\hbar k_x)}{dt} = -eE_x$$

$$\boldsymbol{B}_{\boldsymbol{x}'} = \boldsymbol{B}_{\boldsymbol{Rashba}} = \frac{2a_R E_z}{g\mu_B}\left(-k_y \hat{\boldsymbol{x}} + k_x \hat{\boldsymbol{y}}\right)$$

$$= B_x \hat{\boldsymbol{x}} + B_y \hat{\boldsymbol{y}}$$

$$k_y(t) = k_y(0)$$

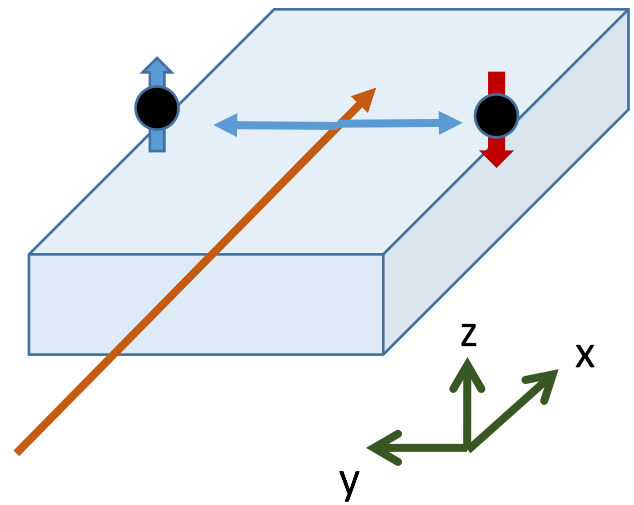
E_x in x-direction

$$B_{x'}^2 = B_x^2 + B_y^2 = \left(\frac{2a_R E_z}{g\mu_B}\right)^2 k^2$$

$$\boxed{S_z(\tau) = \pm \frac{e\hbar}{4a_R E_z} \frac{k_y(0)}{k^3(\tau)} E_x}$$

Spin Hall current

$$S_z(\tau) = \pm \frac{e\hbar}{4a_R E_z} \frac{k_y(0)}{k^3(\tau)} E_x$$



$$\boldsymbol{B_{Rashba}} = \frac{2a_R E_z}{g\mu_B}\left(-k_y \boldsymbol{\hat{x}} + k_x \boldsymbol{\hat{y}}\right)$$

$$= B_x \boldsymbol{\hat{x}} + B_y \boldsymbol{\hat{y}}$$

$k_y(t) = k_y(0)$

$$S_z = \frac{\hbar}{2}\sigma_z$$

E_x in x-direction

$$j_{s_z}^{y} = \frac{1}{2}\{S_z, v_y\} = \frac{p_y}{m^*} S_z$$

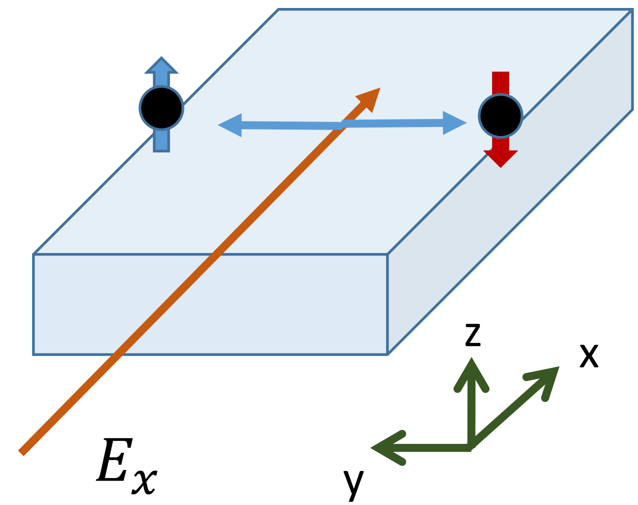
$$v_y = \frac{p_y}{m^*}[I] - \frac{a_R}{\hbar} E_z \sigma_x$$

$$j_{sy}(\tau) = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{k_F} d\phi k(\tau) dk(\tau) \frac{\hbar k_y(0)}{m^*} S_z(\tau)$$

$$k_y(0) \simeq k(\tau)\, cos\phi$$

Spin Hall current and conductivity

$$S_z(\tau) = \pm \frac{e\hbar}{4a_R E_z} \frac{k_y(0)}{k^3(\tau)} E_x$$



$$j_{sy}(\tau) = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{k_F} d\phi k(\tau) dk(\tau) \frac{\hbar k_y(0)}{m^*} S_z(\tau)$$

$$k_y(0) \simeq k(\tau)\, cos\phi$$

$$j_{sy}(\tau) = \pm \frac{e\hbar^2}{16\pi^2 m^* a_R E_z} E_x \int_0^{2\pi} \int_0^{k_F} d\phi\, dk(\tau) \cos^2\phi$$

$$k_y(t) = k_y(0)$$

$$j_{sy1}(\tau) = \frac{e\hbar^2}{16\pi\, m^* a_R E_z} E_x k_{F_1} \qquad j_{sy2}(\tau) = -\frac{e\hbar^2}{16\pi\, m^* a_R E_z} E_x k_{F_2}$$

$$k_{F_2} - k_{F_1} = 2m^* a_R E_z / \hbar^2$$

$$j_{sy}(\tau) = -\frac{e\hbar^2}{16\pi\, m^* a_R E_z} E_x \left(k_{F_2} - k_{F_1}\right)$$

Spin Hall conductivity

$$j_{sy}(\tau) = -\frac{e}{8\pi} E_x$$

Intrinsic SHE

$$\sigma_{SH} = \frac{j_{sy}(\tau)}{-E_x} = \frac{e}{8\pi}$$

Spin Hall effect: Intrinsic versus extrinsic

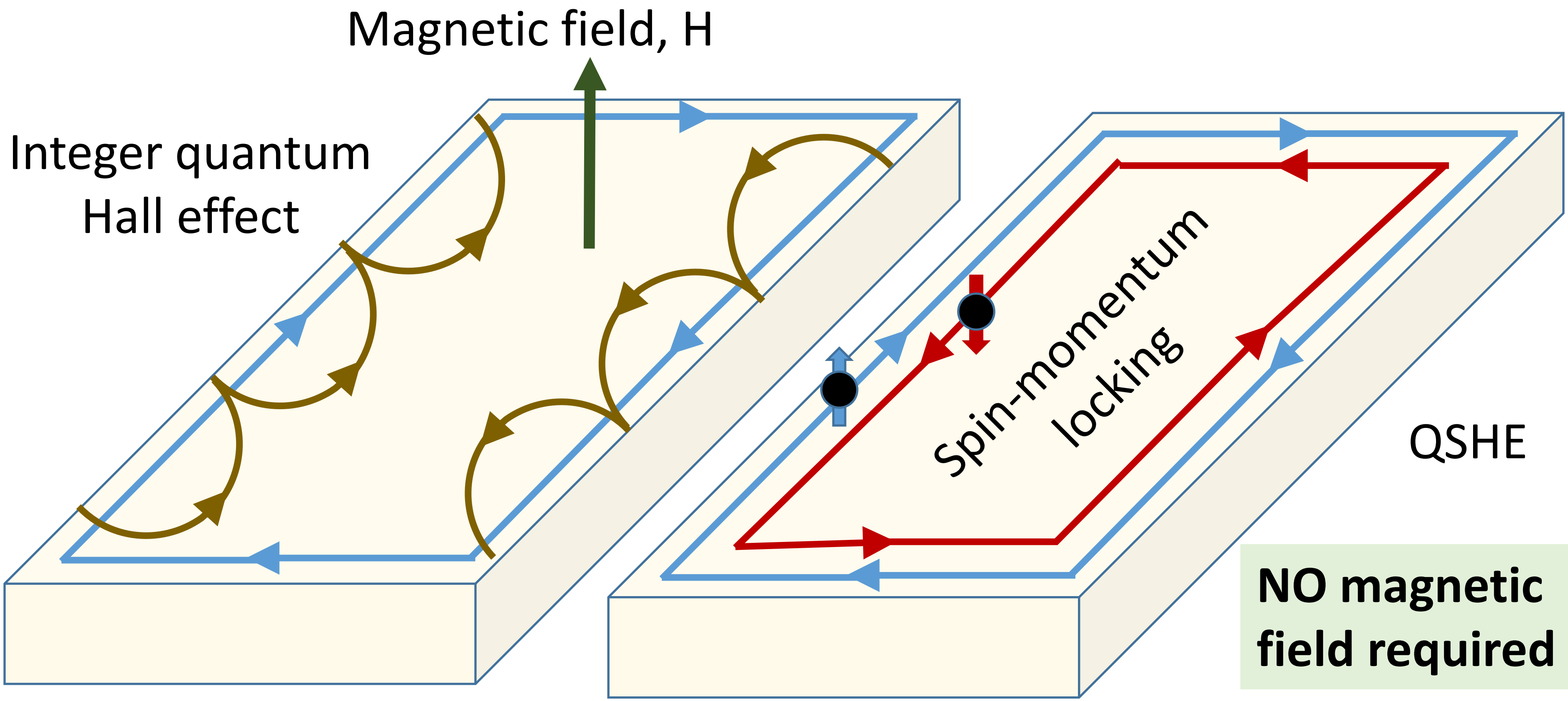
- Intrinsic spin Hall effect
  - Rashba spin orbit coupling
  - No electric field in the y-direction (Hall direction)

  Sinova et al, Universal Intrinsic Spin Hall Effect, Phys. Rev. Lett. 92, 126603 (2004)

- Extrinsic spin Hall effect
  - Spin dependent scattering
  - Band structure effects

  Hirsh, Spin Hall Effect, Phys. Rev. Lett. 83, 1834 (1999)

Quantum spin Hall effect (QSHE)



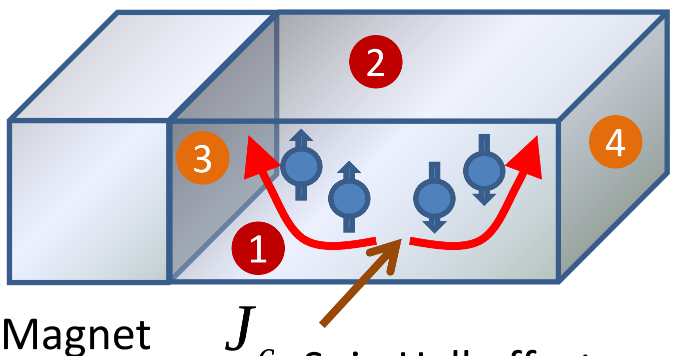
NO magnetic field required

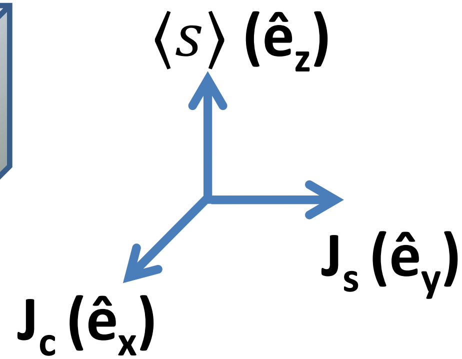
Edge states conduct the current

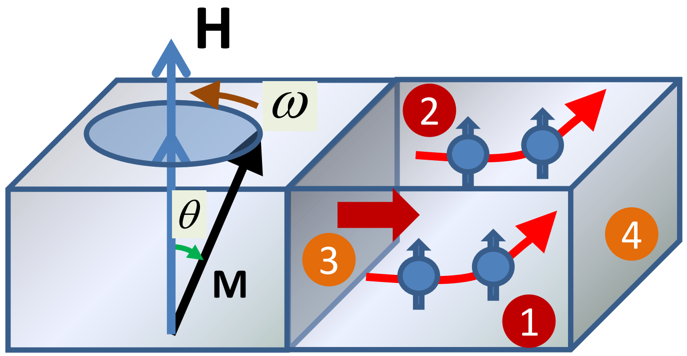
Requirement of a magnetic field is an issue for technology

Topological Insulators
BULK insulating,
SURFACE conducting
(Bi_2Se_3, Bi_2Te_3)

Reciprocity: Spin-transfer-torque (Direct SHE) and Spin pumping (Inverse SHE)







- **Charge current generates spin current via direct SHE and**
- **Spin current exerts spin-transfer-torque on magnet**

- **Precessing magnet injects pure spin current and**
- **Spin current generates charge current via inverse SHE**

$$J_s = \theta_{SH} \langle s \rangle \times J_c$$

Onsager's reciprocity

$$J_c = \theta_{SH} J_s \times \langle s \rangle$$