Spintronics and Nanomagnetics

ECS 521/641

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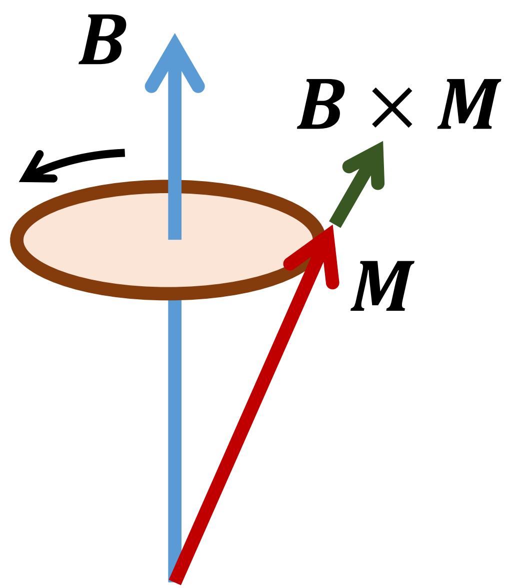
Evolution of spinor on Bloch sphere

Larmor precession

$$\frac{d\boldsymbol{M}}{dt} = |\gamma|(\boldsymbol{B} \times \boldsymbol{M}) \qquad |\gamma| = \frac{g\mu_B}{\hbar}$$



$$\frac{d\langle \boldsymbol{S} \rangle}{dt} = |\gamma|(\boldsymbol{B} \times \langle \boldsymbol{S} \rangle)$$

$$\boldsymbol{S} = \frac{\hbar}{2}\boldsymbol{\sigma}$$

$$\frac{d\langle \boldsymbol{\sigma} \rangle}{dt} = |\gamma|(\boldsymbol{B} \times \langle \boldsymbol{\sigma} \rangle)$$

$$\langle \sigma_n \rangle = \langle \xi_n^+ | \sigma_n | \xi_n^+ \rangle$$

Spinor in Bloch sphere

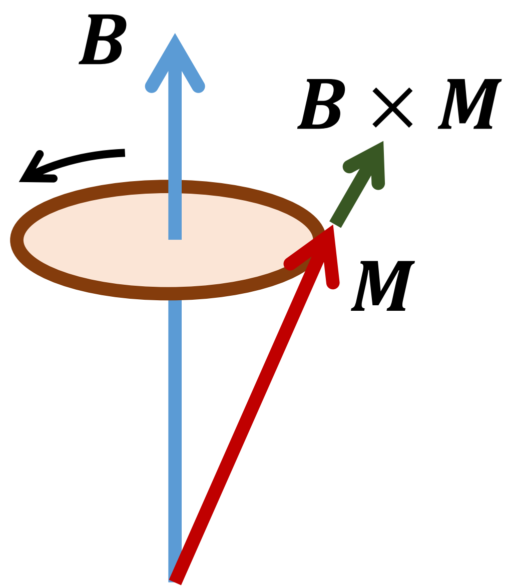
$$\frac{d}{dt}\begin{bmatrix} \langle \sigma_x \rangle \\ \langle \sigma_y \rangle \\ \langle \sigma_z \rangle \end{bmatrix} = |\gamma| \begin{bmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{bmatrix} \begin{bmatrix} \langle \sigma_x \rangle \\ \langle \sigma_y \rangle \\ \langle \sigma_z \rangle \end{bmatrix}$$

Larmor precession

$$\frac{d\langle\sigma_x\rangle}{dt} = |\gamma|\big(B_y\langle\sigma_z\rangle - B_z\langle\sigma_y\rangle\big)$$

$$\frac{d\langle\sigma_y\rangle}{dt} = |\gamma|\big(B_z\langle\sigma_x\rangle - B_x\langle\sigma_z\rangle\big)$$

$$\frac{d\langle\sigma_z\rangle}{dt} = |\gamma|\big(B_x\langle\sigma_y\rangle - B_y\langle\sigma_x\rangle\big)$$



$$\langle\sigma_n\rangle = \langle\xi_n^+|\sigma_n|\xi_n^+\rangle$$

Spinor in Bloch sphere

$$\frac{d}{dt}\begin{bmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{bmatrix} = |\gamma|\begin{bmatrix}0 & -B_z & B_y\\ B_z & 0 & -B_x\\ -B_y & B_x & 0\end{bmatrix}\begin{bmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{bmatrix}$$

Ehrenfest theorem

- Time evolution of the expectation value of a time-dependent observable for a quantum-mechanical system

$$\frac{d}{dt}\langle\psi(t)|A(t)|\psi(t)\rangle$$
$$= \left[\frac{d}{dt}\langle\psi(t)|\right]A(t)|\psi(t)\rangle + \langle\psi(t)|A(t)\left[\frac{d}{dt}|\psi(t)\rangle\right] + \langle\psi(t)|\frac{dA(t)}{dt}|\psi(t)\rangle$$

$$\frac{d}{dt}\langle\psi(t)| = -\frac{1}{i\hbar}H(t)\langle\psi(t)| \qquad \frac{d}{dt}|\psi(t)\rangle = +\frac{1}{i\hbar}H(t)|\psi(t)\rangle$$

$$\frac{d}{dt}\langle\psi(t)|A(t)|\psi(t)\rangle$$
$$= \frac{1}{i\hbar}\langle\psi(t)|A(t)H(t) - H(t)A(t)|\psi(t)\rangle + \langle\psi(t)|\frac{dA(t)}{dt}|\psi(t)\rangle$$

$$\frac{d\langle A(t)\rangle}{dt} = \frac{1}{i\hbar}\langle[A(t)H(t)]\rangle + \left\langle\frac{dA(t)}{dt}\right\rangle$$

Deriving Larmor precession from Ehrenfest theorem

$$\frac{d\langle A(t)\rangle}{dt} = \frac{1}{i\hbar}\langle[A(t)H(t)]\rangle + \left\langle\frac{dA(t)}{dt}\right\rangle$$

$$H_B = -\frac{g\mu_B}{2}\boldsymbol{\sigma}\cdot\boldsymbol{B}$$

$$\frac{d\langle\sigma_x\rangle}{dt} = -\frac{1}{i\hbar}\frac{g\mu_B}{2}\left(B_y\langle[\sigma_x,\sigma_y]\rangle + B_z\langle[\sigma_x,\sigma_z]\rangle\right)$$

$$\frac{d\langle\sigma_y\rangle}{dt} = -\frac{1}{i\hbar}\frac{g\mu_B}{2}\left(B_x\langle[\sigma_y,\sigma_x]\rangle + B_z\langle[\sigma_y,\sigma_z]\rangle\right)$$

$$\frac{d\langle\sigma_z\rangle}{dt} = -\frac{1}{i\hbar}\frac{g\mu_B}{2}\left(B_x\langle[\sigma_z,\sigma_x]\rangle + B_y\langle[\sigma_z,\sigma_y]\rangle\right)$$

Deriving Larmor precession from Ehrenfest theorem

$$|\gamma| = \frac{g\mu_B}{\hbar}$$

$$\frac{d\langle\sigma_x\rangle}{dt} = -\frac{1}{i\hbar}\frac{g\mu_B}{2}\left(B_y\langle[\sigma_x,\sigma_y]\rangle + B_z\langle[\sigma_x,\sigma_z]\rangle\right)$$
$$= |\gamma|\left(B_y\langle\sigma_z\rangle - B_z\langle\sigma_y\rangle\right)$$

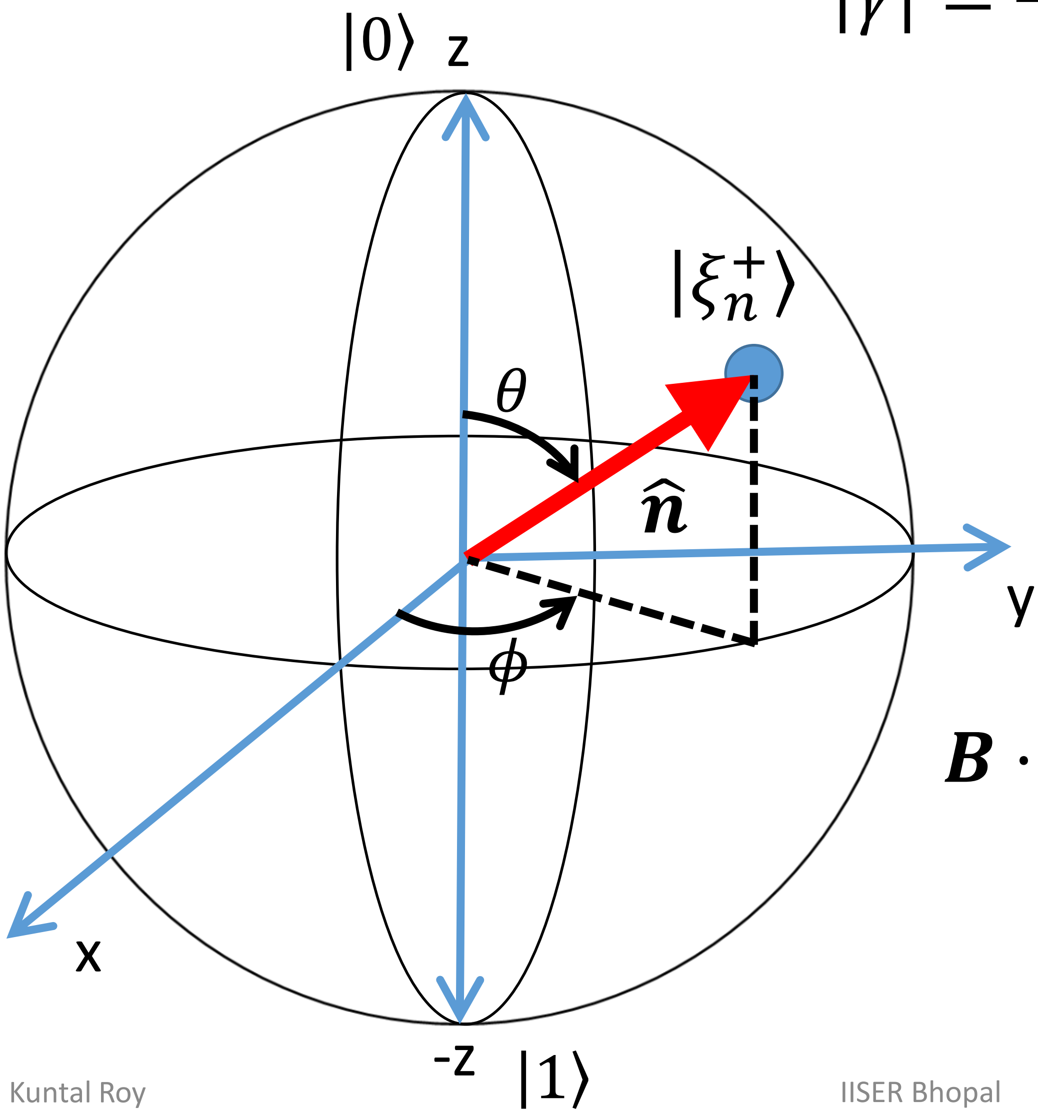
$$\frac{d\langle\sigma_y\rangle}{dt} = -\frac{1}{i\hbar}\frac{g\mu_B}{2}\left(B_x\langle[\sigma_y,\sigma_x]\rangle + B_z\langle[\sigma_y,\sigma_z]\rangle\right)$$
$$= |\gamma|\left(B_z\langle\sigma_x\rangle - B_x\langle\sigma_z\rangle\right)$$

$$\frac{d\langle\sigma_z\rangle}{dt} = -\frac{1}{i\hbar}\frac{g\mu_B}{2}\left(B_x\langle[\sigma_z,\sigma_x]\rangle + B_y\langle[\sigma_z,\sigma_y]\rangle\right)$$
$$= |\gamma|\left(B_x\langle\sigma_y\rangle - B_y\langle\sigma_x\rangle\right)$$

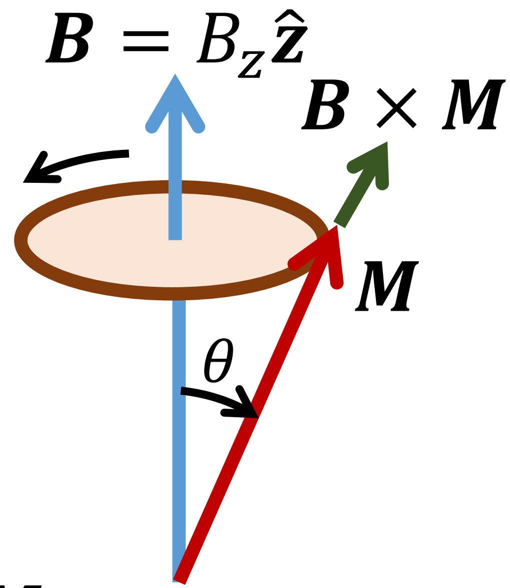
$$\frac{d}{dt}\begin{bmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{bmatrix} = |\gamma|\begin{bmatrix}0 & -B_z & B_y\\ B_z & 0 & -B_x\\ -B_y & B_x & 0\end{bmatrix}\begin{bmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{bmatrix}$$

$$\frac{d\langle\boldsymbol{\sigma}\rangle}{dt} = |\gamma|(\boldsymbol{B}\times\langle\boldsymbol{\sigma}\rangle)$$

Precession angle and rate



$$|\gamma| = \frac{g\mu_B}{\hbar}$$



$$\frac{d\boldsymbol{M}}{dt} = |\gamma|(\boldsymbol{B} \times \boldsymbol{M})$$

$$\boldsymbol{B} \cdot \frac{d\boldsymbol{M}}{dt} = 0 \quad \Rightarrow \frac{d(\boldsymbol{B} \cdot \boldsymbol{M})}{dt} = 0$$

$$\frac{d\phi}{dt} = \frac{g\mu_B B_z}{\hbar}$$

Rotation on Bloch sphere

$$\hat{\boldsymbol{n}} = \left(n_x, n_y, n_z\right) = (sin\theta cos\phi, sin\theta sin\phi, cos\theta)$$

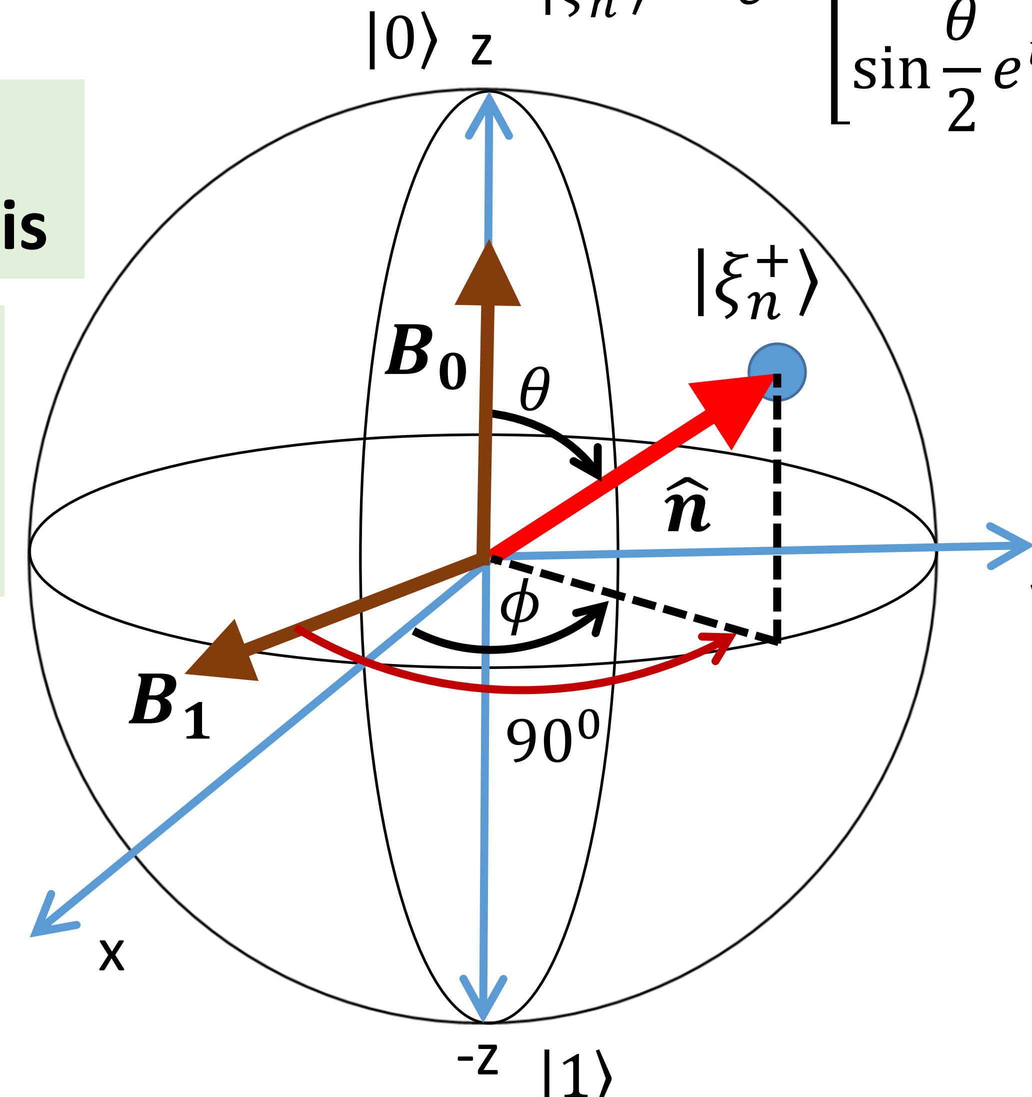
$$|\xi_n^+\rangle = e^{i\gamma}\begin{bmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}e^{i\phi} \end{bmatrix}$$

$\boldsymbol{B_0}$: Time-independent magnetic field along z-axis

$\boldsymbol{B_1}$: A rotating magnetic field in the (x, y) plane chasing the spinor

Nuclear Magnetic Resonance (NMR)

$$|0\rangle = |\xi_n^+(\theta = 0, \phi, \gamma)\rangle$$
$$|1\rangle = |\xi_n^+(\theta = \pi, \phi, \gamma)\rangle$$

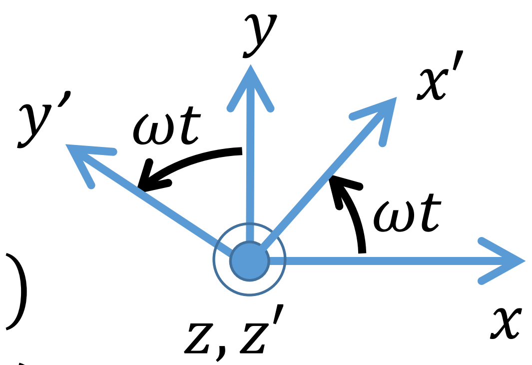


Probability of spin flip: I. I. Rabi (1940s)

$$\widehat{\boldsymbol{x}'} = \cos\omega t\,\hat{\boldsymbol{x}} + \sin\omega t\,\hat{\boldsymbol{y}}$$

$$\widehat{\boldsymbol{y}'} = -\sin\omega t\,\hat{\boldsymbol{x}} + \cos\omega t\,\hat{\boldsymbol{y}}$$

$$\widehat{\boldsymbol{z}'} = \hat{\boldsymbol{z}}$$



$$\hat{\boldsymbol{n}} = (n_x, n_y, n_z)$$

$$\widehat{\boldsymbol{n}'} = (n'_x, n'_y, n'_z)$$

$$\widehat{\boldsymbol{n}'} = \begin{bmatrix} n'_x \\ n'_y \\ n'_z \end{bmatrix} = [A]\begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = [A]\hat{\boldsymbol{n}}$$

$$[A] = \begin{bmatrix} \cos\omega t & \sin\omega t & 0 \\ -\sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\boldsymbol{n}} = [A]^{-1}\,\widehat{\boldsymbol{n}'}$$

$$[A]^{-1} = \begin{bmatrix} \cos\omega t & -\sin\omega t & 0 \\ \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{d\hat{\boldsymbol{n}}}{dt} = [A]^{-1}\frac{d\,\widehat{\boldsymbol{n}'}}{dt} + \frac{d[A]^{-1}}{dt}\,\widehat{\boldsymbol{n}'}$$

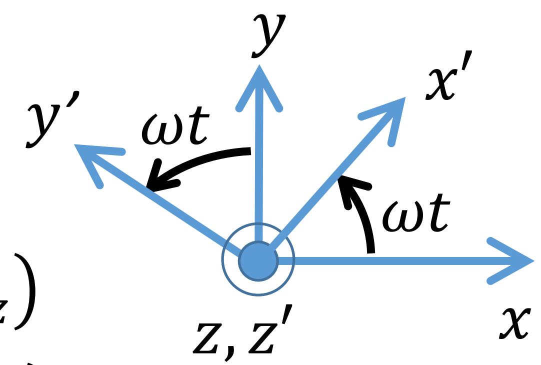
$$= [X]\hat{\boldsymbol{n}} = [X][A]^{-1}\widehat{\boldsymbol{n}'}$$

Rabi formula

$$\widehat{x'} = \cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}$$

$$\widehat{y'} = -\sin\omega t\,\hat{x} + \cos\omega t\,\hat{y}$$

$$\widehat{z'} = \hat{z}$$



$$\hat{n} = (n_x, n_y, n_z)$$

$$\widehat{n'} = (n'_x, n'_y, n'_z)$$

$$[X'] = [A][X][A]^{-1} - [A]\frac{d[A]^{-1}}{dt}$$

$$\frac{d\,\widehat{n'}}{dt} = [X']\,\widehat{n'}$$

$$\widehat{n'} = [A]\hat{n}$$

$$\hat{n} = [A]^{-1}\,\widehat{n'}$$

$$[A]^{-1}\frac{d\,\widehat{n'}}{dt} + \frac{d[A]^{-1}}{dt}\,\widehat{n'} = [X][A]^{-1}\widehat{n'}$$

$$\frac{d\hat{n}}{dt} = [A]^{-1}\frac{d\,\widehat{n'}}{dt} + \frac{d[A]^{-1}}{dt}\,\widehat{n'}$$

$$= [X]\hat{n} = [X][A]^{-1}\widehat{n'}$$

$$\frac{d\,\widehat{n'}}{dt} = \left\{[A][X][A]^{-1} - [A]\frac{d[A]^{-1}}{dt}\right\}\widehat{n'}$$

Rabi formula

$$\frac{d\,\widehat{\boldsymbol{n}'}}{dt} = [X']\,\widehat{\boldsymbol{n}'}$$

$$[X'] = [A][X][A]^{-1} - [A]\frac{d[A]^{-1}}{dt}$$

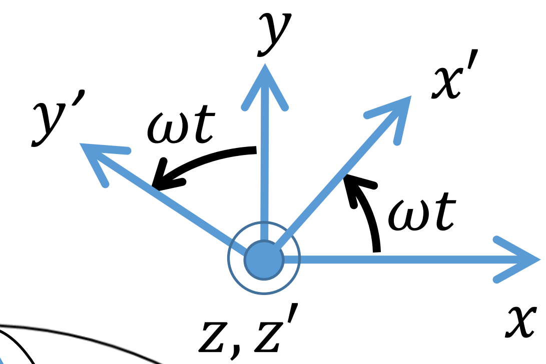
$$\frac{d}{dt}\begin{bmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{bmatrix} = \frac{g\mu_B}{\hbar}\begin{bmatrix}0 & -B_z & B_y\\ B_z & 0 & -B_x\\ -B_y & B_x & 0\end{bmatrix}\begin{bmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{bmatrix}$$

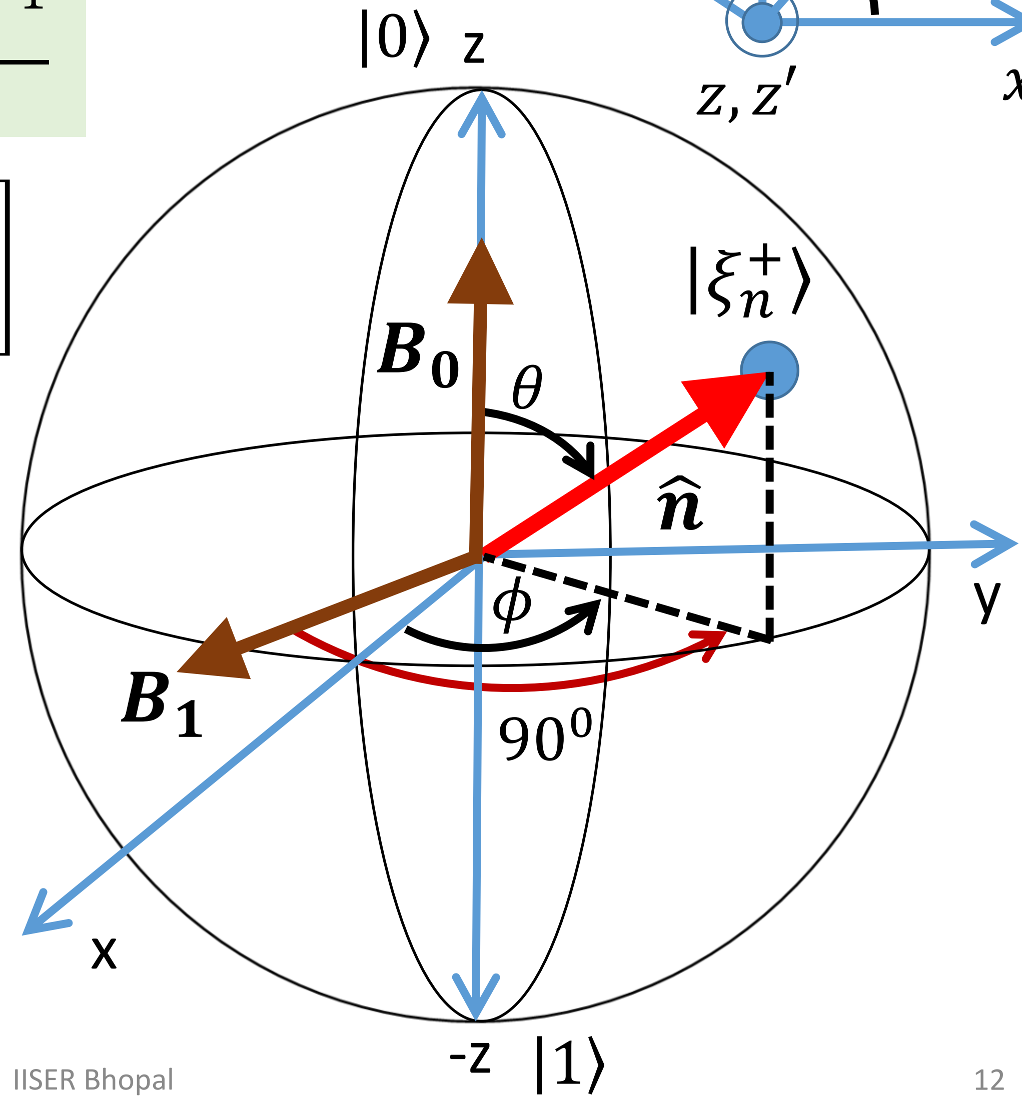
$$\frac{d\widehat{\boldsymbol{n}}}{dt} = [X]\,\widehat{\boldsymbol{n}}$$

$$\boldsymbol{B} = (B_x, B_y, B_z)$$
$$= (B_1\cos\omega t\,, B_1\sin\omega t\,, B_0)$$

$$[A] = \begin{bmatrix}\cos\omega t & \sin\omega t & 0\\ -\sin\omega t & \cos\omega t & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$[A]^{-1} = \begin{bmatrix}\cos\omega t & -\sin\omega t & 0\\ \sin\omega t & \cos\omega t & 0\\ 0 & 0 & 1\end{bmatrix}$$





Rabi formula

$$\frac{d\,\widehat{\boldsymbol{n}'}}{dt} = [X']\,\widehat{\boldsymbol{n}'}$$

$$[X'] = [A][X][A]^{-1} - [A]\frac{d[A]^{-1}}{dt}$$

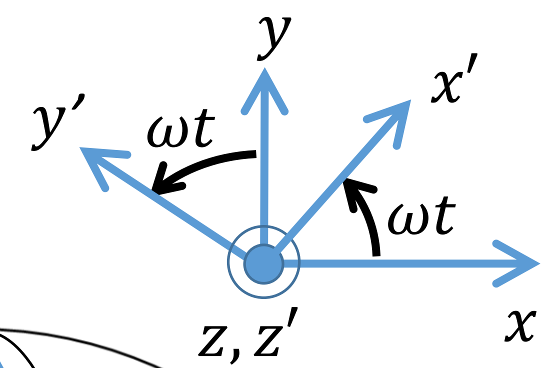
$$[X'] = \begin{bmatrix} 0 & \omega - \omega_0 & 0 \\ -(\omega - \omega_0) & 0 & -\omega_1 \\ 0 & \omega_1 & 0 \end{bmatrix}$$

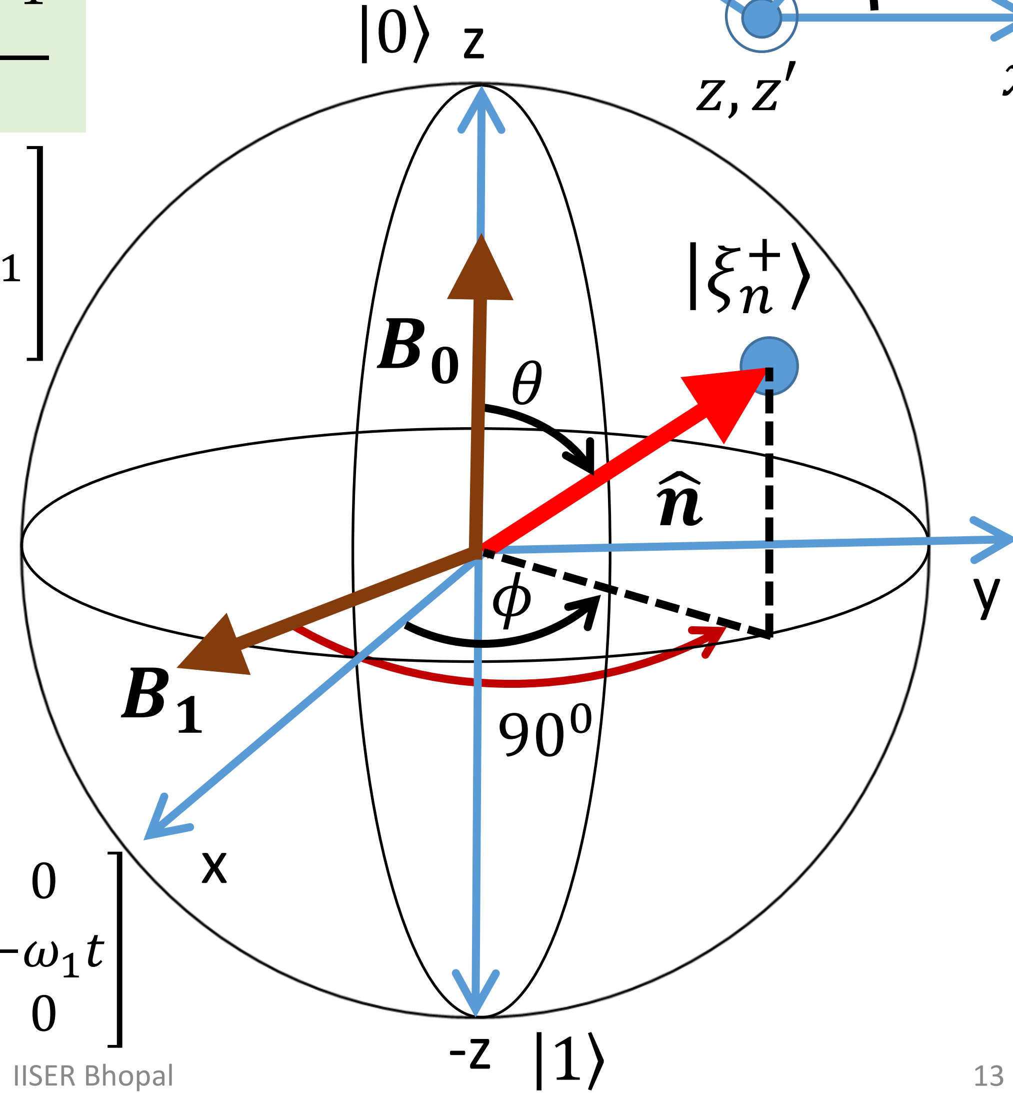
$$\omega_0 = \frac{g\mu_B B_0}{\hbar} \qquad \omega_1 = \frac{g\mu_B B_1}{\hbar}$$

$$\widehat{\boldsymbol{n}'}(t) = e^{[Q](t)}\,\widehat{\boldsymbol{n}'}(0)$$

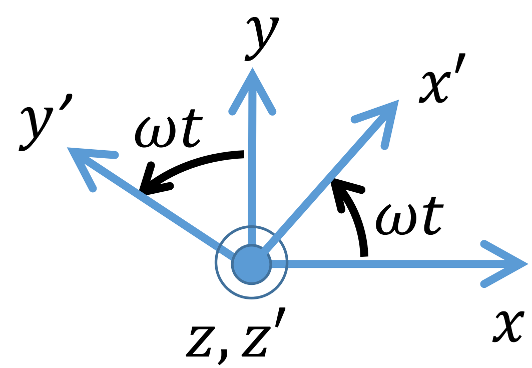
$$[Q](t) = \int_0^t [X'](t')dt'$$

$$[Q](t) = \begin{bmatrix} 0 & (\omega - \omega_0)t & 0 \\ -(\omega - \omega_0)t & 0 & -\omega_1 t \\ 0 & \omega_1 t & 0 \end{bmatrix}$$





Rabi formula



$$\widehat{\boldsymbol{n}'}(t) = e^{[Q](t)}\,\widehat{\boldsymbol{n}'}(0)$$

$$\hat{\boldsymbol{n}}(t) = [U](t)\,\hat{\boldsymbol{n}}(0)$$

$$[Q](t) = \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & \beta \\ 0 & -\beta & 0 \end{bmatrix}$$

$$\alpha = (\omega - \omega_0)t$$

$$\beta = -\omega_1 t$$

$$[P](t) = [S]^{-1}(t)[Q](t)[S](t) = diag(\lambda_1(t), \lambda_2(t), \lambda_3(t))$$

$$\lambda_1 = 0$$

$$\lambda_2 = +i\sqrt{(\alpha^2 + \beta^2)}$$

$$\lambda_3 = -i\sqrt{(\alpha^2 + \beta^2)}$$

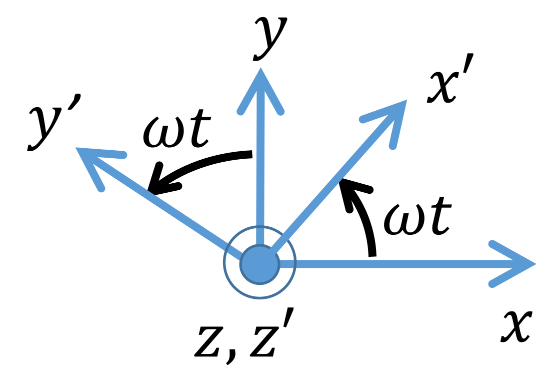
$$\boldsymbol{q_1} = \left[\frac{\beta}{\alpha}, 0, 1\right]$$

$$\boldsymbol{q_2} = \left[-\frac{\alpha}{\beta}, -\frac{i\sqrt{\alpha^2 + \beta^2}}{\beta}, 1\right]$$

$$\boldsymbol{q_3} = \left[-\frac{\alpha}{\beta}, +\frac{i\sqrt{\alpha^2 + \beta^2}}{\beta}, 1\right]$$

Rabi formula

$$\widehat{\boldsymbol{n}'}(t) = e^{[Q](t)}\,\widehat{\boldsymbol{n}'}(0) \qquad \widehat{\boldsymbol{n}}(t) = [U](t)\,\widehat{\boldsymbol{n}}(0)$$

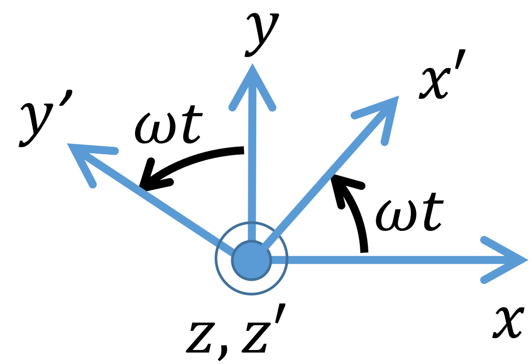


$$[Q](t) = \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & \beta \\ 0 & -\beta & 0 \end{bmatrix} \qquad \begin{aligned} \alpha &= (\omega - \omega_0)t \\ \beta &= -\omega_1 t \end{aligned}$$

$$[P](t) = [S]^{-1}(t)[Q](t)[S](t) = diag(\lambda_1(t), \lambda_2(t), \lambda_3(t))$$

$$[S](t) = \begin{bmatrix} \dfrac{\beta}{\alpha} & -\dfrac{\alpha}{\beta} & -\dfrac{\alpha}{\beta} \\ 0 & -\dfrac{i\sqrt{\alpha^2+\beta^2}}{\beta} & +\dfrac{i\sqrt{\alpha^2+\beta^2}}{\beta} \\ 1 & 1 & 1 \end{bmatrix}$$

Rabi formula



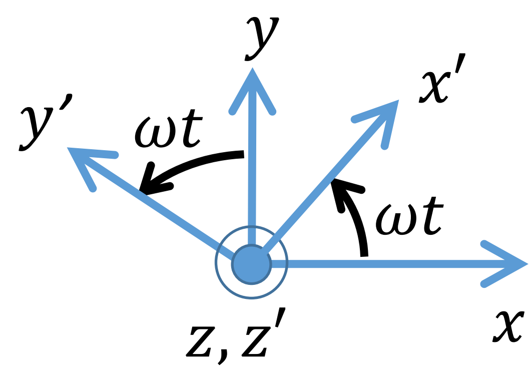
$$\widehat{\boldsymbol{n}'}(t) = e^{[Q](t)}\,\widehat{\boldsymbol{n}'}(0) \qquad \hat{\boldsymbol{n}}(t) = [U](t)\,\hat{\boldsymbol{n}}(0)$$

$$[Q](t) = \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & \beta \\ 0 & -\beta & 0 \end{bmatrix} \qquad \begin{aligned} \alpha &= (\omega - \omega_0)t \\ \beta &= -\omega_1 t \end{aligned}$$

$$[P](t) = [S]^{-1}(t)[Q](t)[S](t) = diag(\lambda_1(t), \lambda_2(t), \lambda_3(t))$$

$$[S]^{-1}(t) = \begin{bmatrix} \dfrac{\alpha\beta}{\alpha^2 + \beta^2} & 0 & \dfrac{\alpha^2}{\alpha^2 + \beta^2} \\ -\dfrac{\alpha\beta}{2(\alpha^2 + \beta^2)} & \dfrac{i\beta}{2\sqrt{\alpha^2 + \beta^2}} & \dfrac{0.5\beta^2}{\alpha^2 + \beta^2} \\ -\dfrac{\alpha\beta}{2(\alpha^2 + \beta^2)} & -\dfrac{i\beta}{2\sqrt{\alpha^2 + \beta^2}} & \dfrac{0.5\beta^2}{\alpha^2 + \beta^2} \end{bmatrix}$$

Rabi formula



$$\widehat{\boldsymbol{n}'}(t) = e^{[Q](t)}\,\widehat{\boldsymbol{n}'}(0)$$

$$\hat{\boldsymbol{n}}(t) = [U](t)\,\hat{\boldsymbol{n}}(0)$$

$$[Q](t) = \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & \beta \\ 0 & -\beta & 0 \end{bmatrix}$$

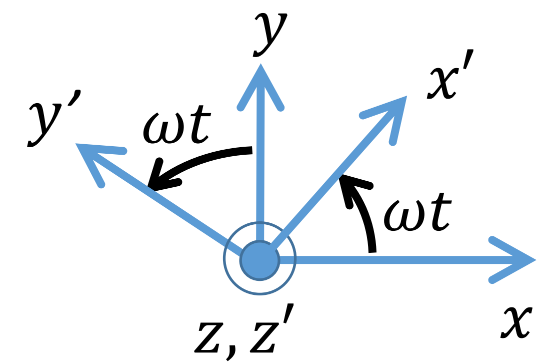
$$\alpha = (\omega - \omega_0)t$$

$$\beta = -\omega_1 t$$

$$[P](t) = [S]^{-1}(t)[Q](t)[S](t) = diag(\lambda_1(t), \lambda_2(t), \lambda_3(t))$$

$$\begin{aligned} e^{[Q](t)} &= e^{[S](t)[P](t)[S]^{-1}(t)} \\ &= [I] + [S][P][S]^{-1} + \frac{1}{2!}([S][P][S]^{-1})^2 + \cdots \\ &= [I] + [S]\left([P] + \frac{[P]^2}{2!} + \frac{[P]^3}{3!} + \cdots\right)[S]^{-1} \\ &= [S](t)e^{[P](t)}[S]^{-1}(t) \end{aligned}$$

Rabi formula



$$\widehat{\boldsymbol{n}'}(t) = e^{[Q](t)}\,\widehat{\boldsymbol{n}'}(0)$$

$$\boxed{\hat{\boldsymbol{n}}(t) = [U](t)\,\hat{\boldsymbol{n}}(0)}$$

$$[Q](t) = \begin{bmatrix} 0 & \alpha & 0 \\ -\alpha & 0 & \beta \\ 0 & -\beta & 0 \end{bmatrix}$$

$$\alpha = (\omega - \omega_0)t$$

$$\beta = -\omega_1 t$$

$$[P](t) = [S]^{-1}(t)[Q](t)[S](t) = diag(\lambda_1(t), \lambda_2(t), \lambda_3(t))$$

$$e^{[Q](t)} = [S](t)e^{[P](t)}[S]^{-1}(t)$$

$$\widehat{\boldsymbol{n}'}(t) = [S](t)e^{[P](t)}[S]^{-1}(t)\,\widehat{\boldsymbol{n}'}(0)$$

$$\widehat{\boldsymbol{n}'} = [A]\hat{\boldsymbol{n}}$$

$$\widehat{\boldsymbol{n}'}(t) = [A](t)\hat{\boldsymbol{n}}(t)$$

$$\widehat{\boldsymbol{n}'}(0) = [A](0)\hat{\boldsymbol{n}}(0)$$

$$\boxed{\hat{\boldsymbol{n}}(t) = \underbrace{\left\{[A]^{-1}(t)\left[[S](t)e^{[P](t)}[S]^{-1}(t)\right]A(0)\right\}}_{[U](t)}\,\hat{\boldsymbol{n}}(0)}$$

Rabi formula

$$\hat{\boldsymbol{n}}(t) = U(t)\,\hat{\boldsymbol{n}}(0)$$

$$[U](t) = [A]^{-1}(t)\left[[S](t)e^{[P](t)}[S]^{-1}(t)\right]A(0)$$

$$[U](t) = \begin{bmatrix} g(\delta,\chi)\sin\omega t + h(\delta,\chi)\cos\omega t & g(\delta,\chi)\cos\omega t - cos\delta\sin\omega t & [f(\delta)\cos\omega t\, cos\chi - sin\delta\sin\omega t]\sin\chi \\ -g(\delta,\chi)\cos\omega t + h(\delta,\chi)\, sin\,\omega t & g(\delta,\chi)\sin\omega t + cos\delta\cos\omega t & [f(\delta)\sin\omega t\, cos\chi - sin\delta\cos\omega t]\sin\chi \\ f(\delta)\cos\chi\sin\chi & \sin\delta\sin\chi & \cos^2\chi + \sin^2\chi cos\delta \end{bmatrix}$$

$$\delta = \sqrt{\alpha^2+\beta^2} = \sqrt{(\omega-\omega_0)^2+{\omega_1}^2}\;\; t$$

$$f(\delta) = 1 - cos\delta$$

$$h(\delta,\chi) = cos\delta\cos^2\chi + \sin^2\chi$$

$$g(\delta,\chi) = \sin\delta\cos\chi$$

$$\chi\;= \tan^{-1}\frac{\omega_1}{\omega_0-\omega}$$

$$\sin^2\chi = \frac{\omega_1^2}{\omega_1^2+(\omega_0-\omega)^2}$$

Rabi formula

$$|\langle 1|\xi_n^+\rangle|^2 = \sin^2\frac{\theta(t)}{2} = \frac{1-cos\theta(t)}{2}$$

$$|\xi_n^+\rangle = e^{i\gamma}\begin{bmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\phi}\end{bmatrix}$$

$$|\langle 1|\xi_n^+\rangle|^2 = \frac{1-n_z(t)}{2}$$

$$\omega_0 = \frac{g\mu_B B_0}{\hbar}$$

$$\hat{\boldsymbol{n}}(t) = U(t)\,\hat{\boldsymbol{n}}(0)$$

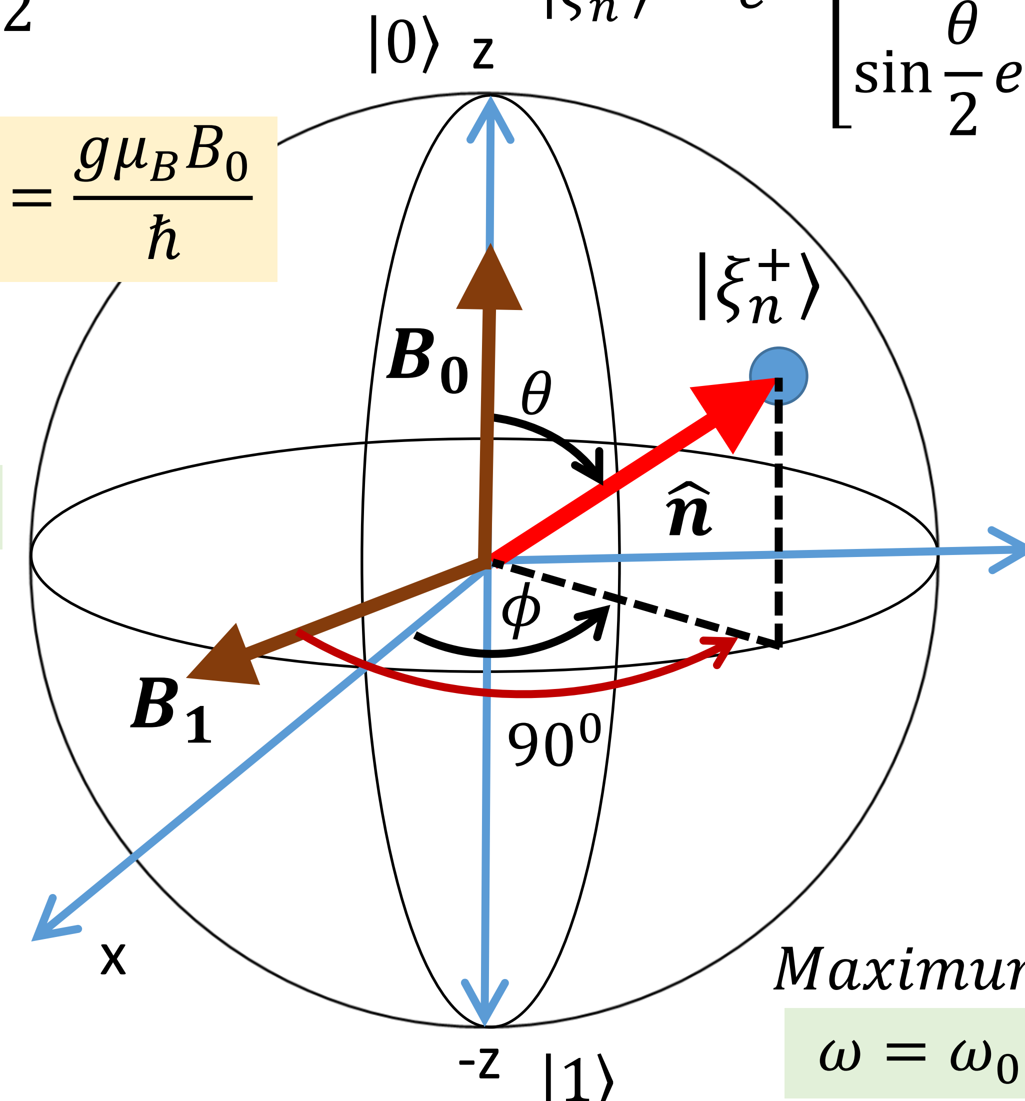
$$\hat{\boldsymbol{n}}(0) = (0,0,1)$$

$$n_z(t) = \cos^2\chi + \sin^2\chi cos\delta$$

$$|\langle 1|\xi_n^+\rangle|^2 = \frac{\sin^2\chi}{2}[1-cos\delta(t)]$$

$$|0\rangle = |\xi_n^+(\theta=0,\phi,\gamma)\rangle$$

$$|1\rangle = |\xi_n^+(\theta=\pi,\phi,\gamma)\rangle$$



Maximum

$$\omega = \omega_0$$

Rabi formula: Spin-flip time

$$|\langle 1|\xi_n^+\rangle|^2 = \sin^2\frac{\theta(t)}{2} = \frac{1-cos\theta(t)}{2}$$

$$|\xi_n^+\rangle = e^{i\gamma}\begin{bmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\phi}\end{bmatrix}$$

$$|\langle 1|\xi_n^+\rangle|^2 = \frac{1-n_z(t)}{2}$$

$$\omega_0 = \frac{g\mu_B B_0}{\hbar}$$

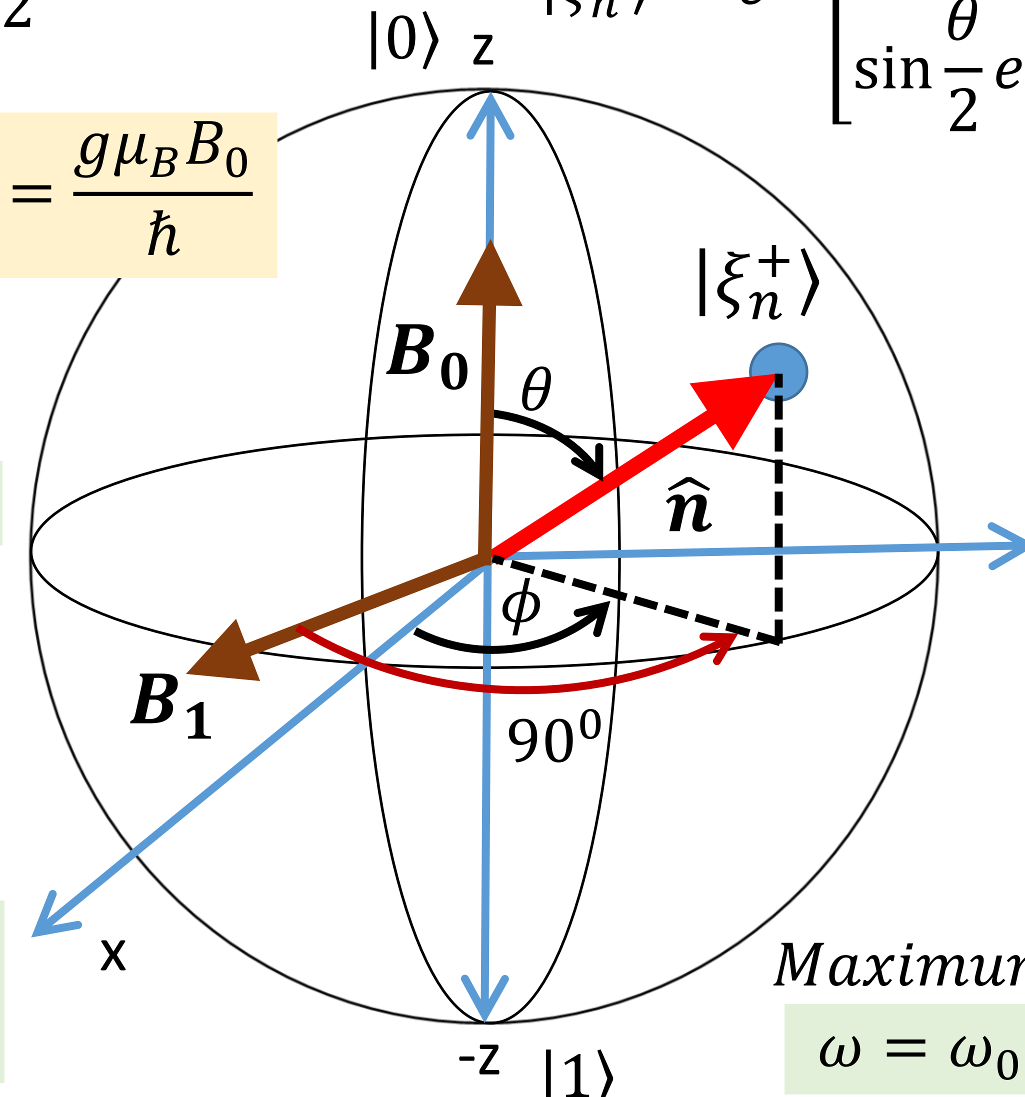
$$\hat{\boldsymbol{n}}(t) = U(t)\,\hat{\boldsymbol{n}}(0)$$

$$\hat{\boldsymbol{n}}(0) = (0,0,1)$$

$$n_z(t) = \cos^2\chi + \sin^2\chi cos\delta$$

$$|\langle 1|\xi_n^+\rangle|^2 = \frac{\sin^2\chi}{2}[1-cos\delta(t)]$$

$$t_s = \frac{T}{2} = \frac{\pi}{\omega_1} = \frac{\pi\hbar}{g\mu_B B_1}$$



Maximum

$$\omega = \omega_0$$