Spintronics and Nanomagnetics

ECS 521/641

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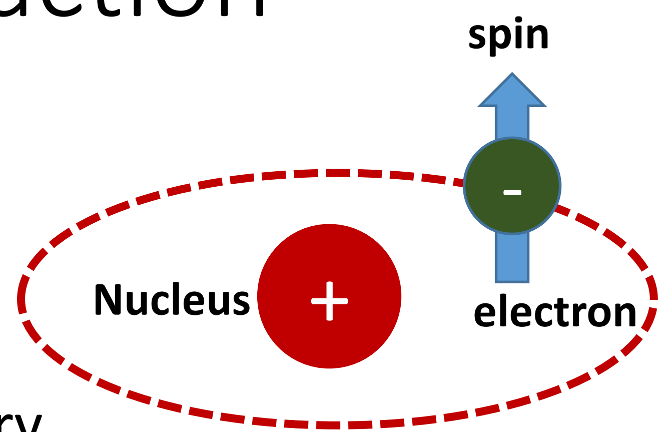
Spin-Orbit interaction

Spin-Orbit Interaction

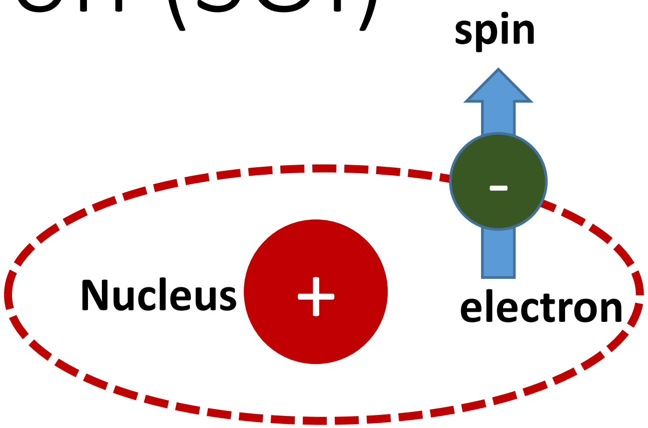


- The negatively charged electron in an atom feels an electric field due to positively charged nucleus
- A magnetic field (did not exist in laboratory frame) will appear in the rest frame of the electron through Lorentz transformation
- According to Einstein's relativity theory, the flux density associated with the magnetic field

$$\boldsymbol{B} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}}$$

- Thomas's correction: The direction of velocity is always changing, even if the magnitude is not

Spin-Orbit Interaction (SOI)



$$\boldsymbol{B} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}}$$

$$E_{rel} = -\boldsymbol{\mu_e} \cdot \boldsymbol{B}$$

spin Orbit

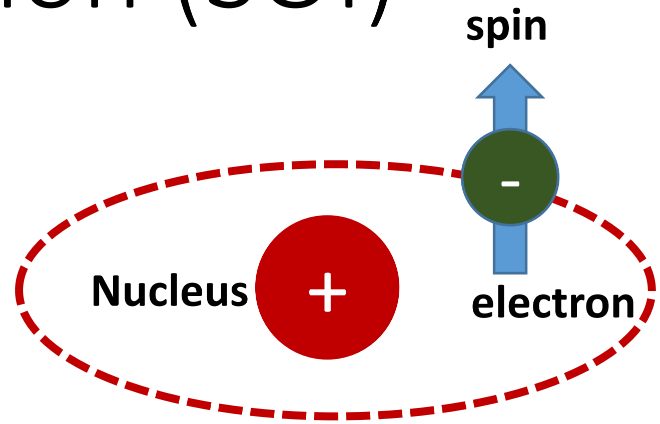
Landé g-factor

$$\frac{Magnetic\ moment\ \mu_e\ (in\ units\ of\ the\ Bohr\ magneton\ \mu_B)}{Angular\ momentum\ \boldsymbol{s}\ (in\ units\ of\ \hbar)} = -g$$

$$\boldsymbol{\mu_e} = -g\mu_B\boldsymbol{s}$$

$$E_{rel} = g\mu_B\boldsymbol{s} \cdot \boldsymbol{B}$$

Spin-Orbit Interaction (SOI)



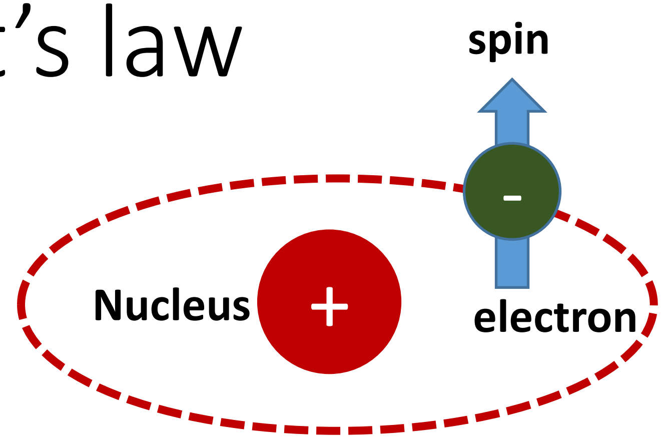
$$\boldsymbol{B} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}}$$

$$E_{rel} = g\mu_B \boldsymbol{s} \cdot \boldsymbol{B}$$

$$\mu_B = \frac{e\hbar}{2m} \qquad \boldsymbol{s} = \frac{1}{2}\sigma$$

$$E_{rel} = \frac{g}{2}\frac{e\hbar}{2m}\frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - \frac{v^2}{c^2}}} \cdot \boldsymbol{\sigma}$$

Non-relativistic approximation Using Biot-Savart's law



$$\boldsymbol{B} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - v^2/c^2}}$$

$$E_{rel} = \frac{g}{2}\frac{e\hbar}{2m}\frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2\sqrt{1 - \frac{v^2}{c^2}}} \cdot \boldsymbol{\sigma}$$

Non-relativistic approximation

$$\frac{v^2}{c^2} \ll 1$$

Thomas's correction: 2 factor

An observer sitting on the electron

Nucleus is revolving around it with a velocity $-\boldsymbol{v}$

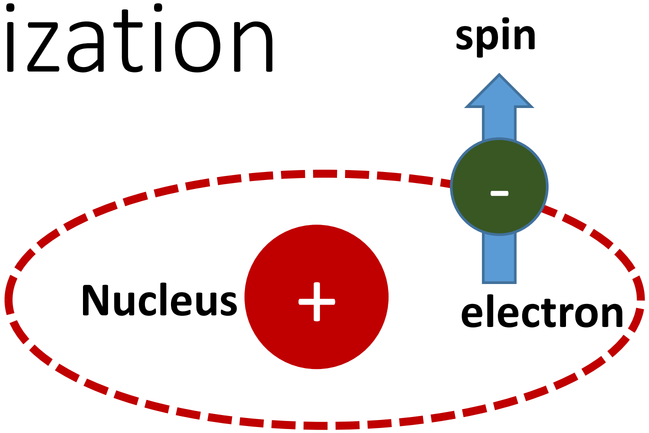
Radius of the orbit $-\boldsymbol{r}$

Biot Savart's law $\boldsymbol{B} = Ze\frac{\boldsymbol{r} \times \boldsymbol{v}}{4\pi\epsilon_0 c^2 r^3}$

$$\boldsymbol{B} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{2c^2}$$

$$\boldsymbol{E} = \frac{Ze\boldsymbol{r}}{4\pi\epsilon_0 r^3}$$

Spin-orbit interaction from angular momentum quantization



Biot Savart's law $$\boldsymbol{B} = Ze\frac{\boldsymbol{r}\times\boldsymbol{v}}{4\pi\epsilon_0 c^2 r^3}$$

$$\boldsymbol{B} = Ze\frac{\boldsymbol{K}}{4\pi\epsilon_0 m c^2 r^3}$$

Non-relativistic approximation

$$\frac{v^2}{c^2} \ll 1$$

Orbital angular momentum

$$\boldsymbol{K} = \boldsymbol{r}\times m\boldsymbol{v}$$

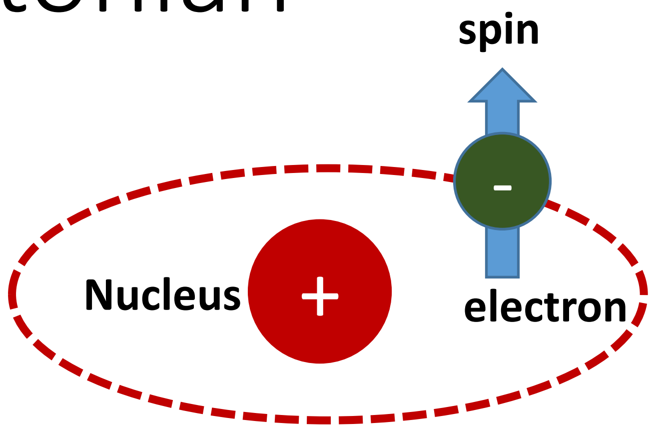
Quantization: $\boldsymbol{K} = \hbar\boldsymbol{l}$

Thomas's correction: 2 factor

$$E_{rel} = g\mu_B \boldsymbol{s}\cdot\boldsymbol{B}$$

$$E_{rel} = g\mu_B\hbar\frac{Ze}{8\pi\epsilon_0 m c^2 r^3}\boldsymbol{s}\cdot\boldsymbol{l}$$

Spin-Orbit Hamiltonian



$$E_{rel} = \frac{g}{2}\frac{e\hbar}{2m}\frac{\boldsymbol{E}\times\boldsymbol{v}}{2c^2\sqrt{1-\frac{v^2}{c^2}}}\cdot\boldsymbol{\sigma}$$

$g = 2$ for free electrons

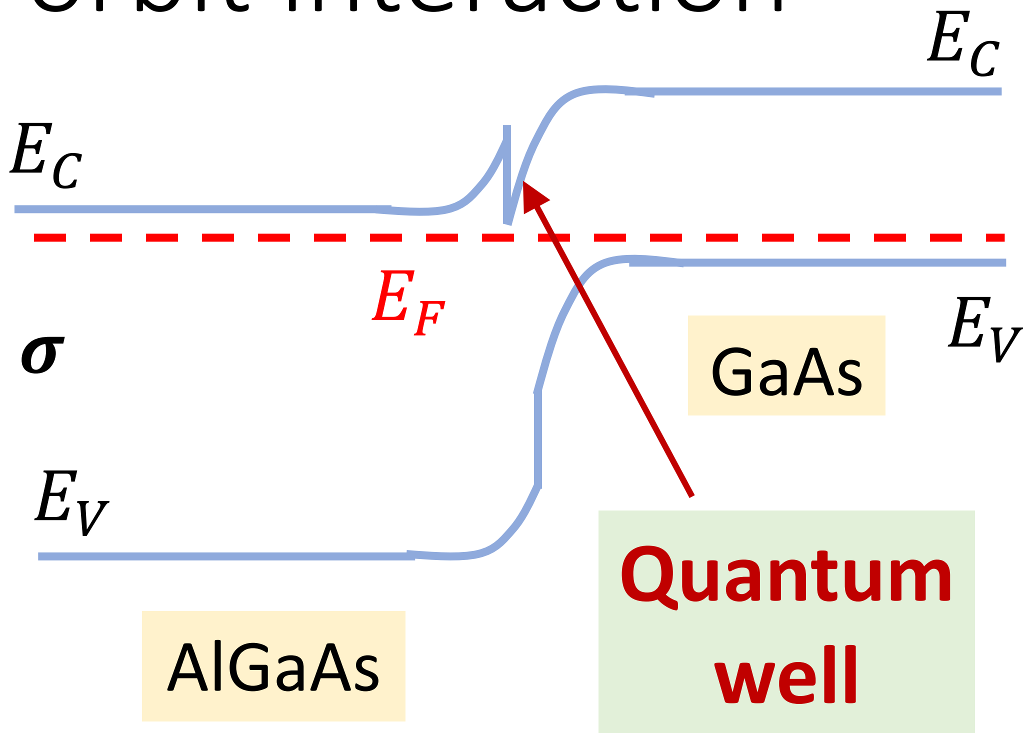
$$H_{SO} = -\frac{e\hbar}{4m^2c^2\sqrt{1-\frac{v^2}{c^2}}}(\boldsymbol{\nabla}V\times\boldsymbol{p})\cdot\boldsymbol{\sigma}$$

$$H_{SO} \cong -\frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla}V\times\boldsymbol{p})\cdot\boldsymbol{\sigma}$$

Non-relativistic approximation

Rashba spin-orbit interaction

$$H_{SO} \cong \frac{ge\hbar}{8m^{*2}c^2}(\boldsymbol{E}\times\boldsymbol{p})\cdot\boldsymbol{\sigma}$$



$$H_{Rashba} \cong -\frac{ge\hbar}{8m^{*2}c^2}\boldsymbol{E}(\boldsymbol{r})\cdot(\boldsymbol{\sigma}\times\boldsymbol{p})$$

Heterostructure

$$H_{Rashba} \cong \boldsymbol{\eta}(\boldsymbol{r})\cdot(\boldsymbol{\sigma}\times\boldsymbol{p})$$

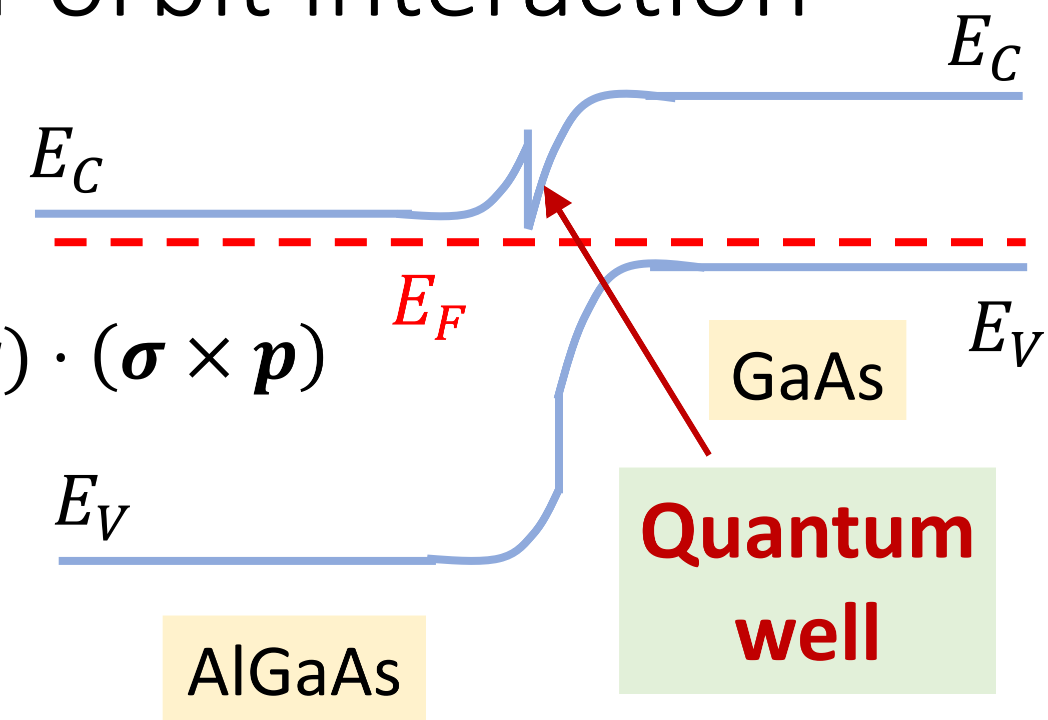
HIGH E-field

$$\boldsymbol{\eta}(\boldsymbol{r}) = -\frac{ge\hbar}{8m^{*2}c^2}\boldsymbol{E}(\boldsymbol{r})$$

Rashba SOI

Rashba spin-orbit interaction

Band-structure effects



$$H_{Rashba} \cong -\frac{ge\hbar}{8{m^*}^2c^2}\boldsymbol{E}(\boldsymbol{r})\cdot(\boldsymbol{\sigma}\times\boldsymbol{p})$$

$$H_{Rashba} \cong \boldsymbol{\eta}(\boldsymbol{r})\cdot(\boldsymbol{\sigma}\times\boldsymbol{p})$$

$$\boldsymbol{\eta}(\boldsymbol{r}) = -\frac{ge\hbar}{8{m^*}^2c^2}\boldsymbol{E}(\boldsymbol{r})$$

$$\boldsymbol{\eta}(\boldsymbol{r}) = -\frac{e\hbar}{m^*(\boldsymbol{r})}\frac{\pi\Delta_s(2E_g+\Delta_s)}{E_g(E_g+\Delta_s)(3E_g+2\Delta_s)}\boldsymbol{E}(\boldsymbol{r})$$

a_R is a material constant

Bandgap: E_g

SO Splitting: Δ_s

$$H_{Rashba} \cong -\frac{a_R}{\hbar}\boldsymbol{E}(\boldsymbol{r})\cdot[\boldsymbol{\sigma}\times(\boldsymbol{p}+e\boldsymbol{A})]$$

Rashba SOI: Band splitting

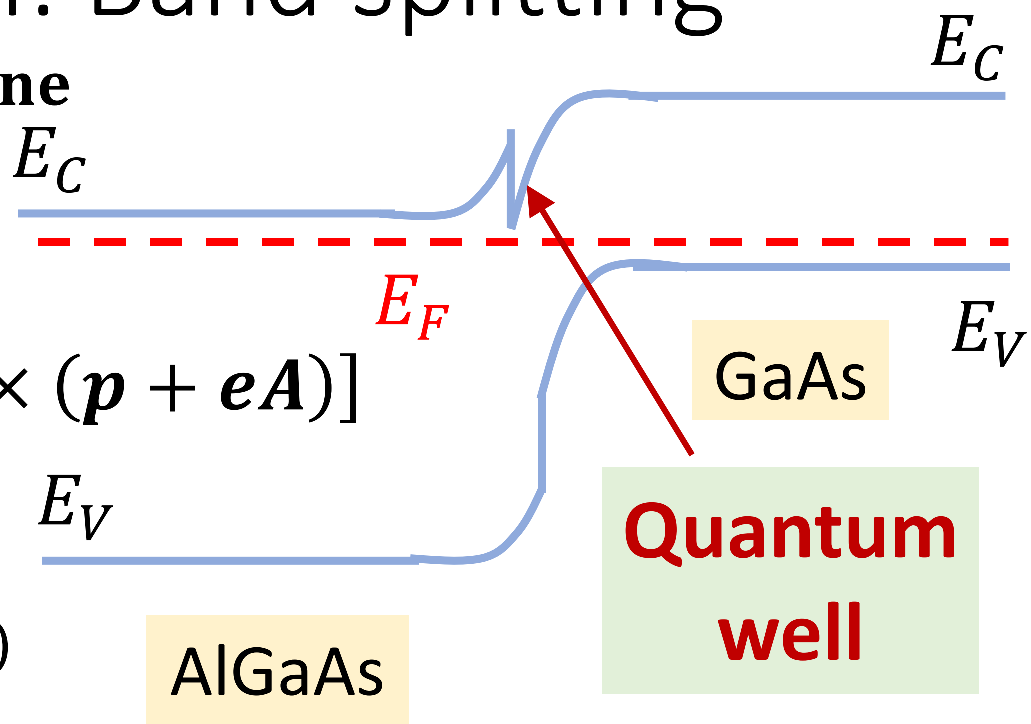
2D electron gas in $x - z$ plane

$\boldsymbol{E}$ **in y-direction**

$$H_{Rashba} \cong -\frac{a_R}{\hbar}\boldsymbol{E}(\boldsymbol{r}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})]$$

$\boldsymbol{B}$ **in y-direction**

Landau gauge $\boldsymbol{A} = (\boldsymbol{Bz}, \boldsymbol{0}, \boldsymbol{0})$



Exercise: *Determine H_{Rashba}*

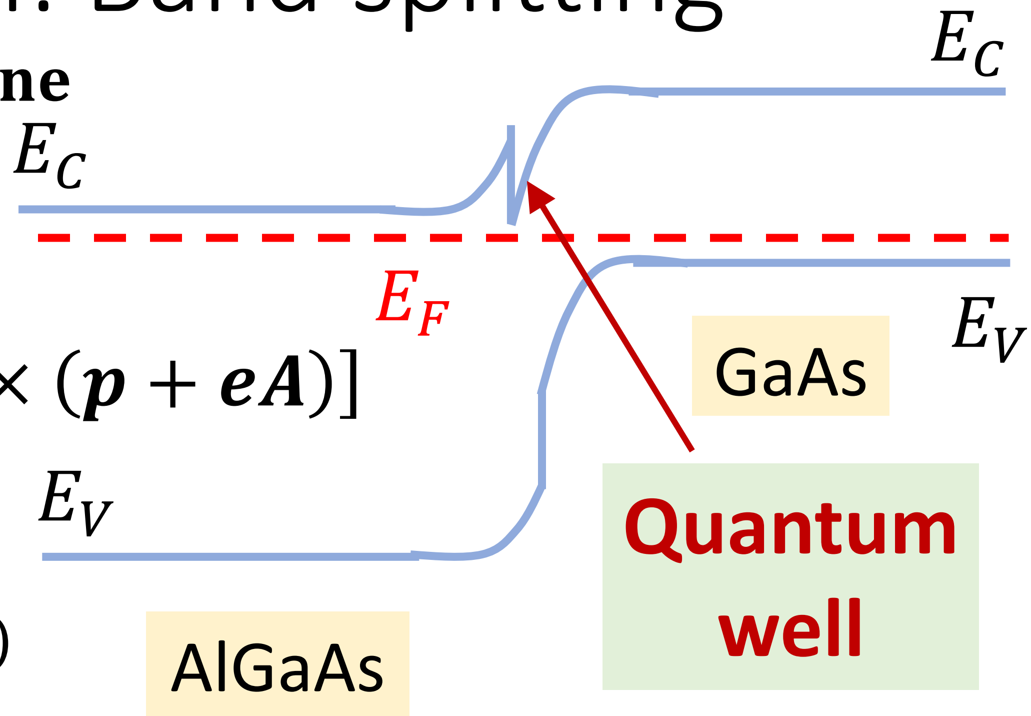
$$H_{Rashba} \cong -\frac{a_R}{\hbar}E_y[(p_x + eBz)\sigma_z - p_z\sigma_x]$$

$$H_{Rashba} \cong -\frac{a_R}{\hbar}E_y\begin{bmatrix} p_x + eBz & -p_z \\ -p_z & -p_x - eBz \end{bmatrix}$$

Rashba SOI: Band splitting

2D electron gas in $x - z$ plane

$\boldsymbol{E}$ **in y-direction**



$$H_{Rashba} \cong -\frac{a_R}{\hbar}\boldsymbol{E}(\boldsymbol{r}) \cdot [\boldsymbol{\sigma} \times (\boldsymbol{p} + e\boldsymbol{A})]$$

$\boldsymbol{B}$ **in y-direction**

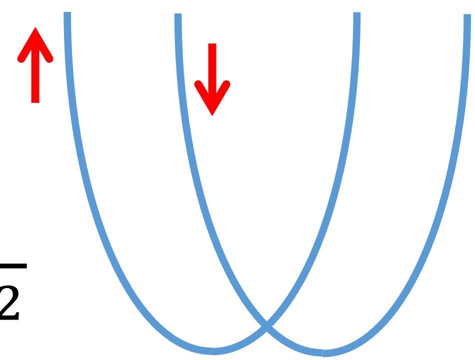
Landau gauge $\boldsymbol{A} = (\boldsymbol{Bz}, \boldsymbol{0}, \boldsymbol{0})$

$$H_{Rashba} \cong -\frac{a_R}{\hbar}E_y \begin{bmatrix} p_x + eBz & -p_z \\ -p_z & -p_x - eBz \end{bmatrix}$$

$$C = p_x + eBz$$

$$D = p_z$$

$$\lambda = \pm\sqrt{C^2 + D^2}$$



Shubnikov–de Haas (SdH) Oscillations

TWO frequencies

Dresselhaus spin-orbit interaction

Dresselhaus SOI arises due to crystallographic inversion asymmetry

$$\left[\frac{|\boldsymbol{p}|^2}{2m} + V_{lattice} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla} V \times \boldsymbol{p}) \cdot \boldsymbol{\sigma}\right][\Psi] = E[\Psi]$$

Bloch wave $[\Psi] = e^{ik.r}[u_{\boldsymbol{k}}(\boldsymbol{r})]$ $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla}$

Exercise: Show

$$\left[\frac{|\boldsymbol{p}|^2}{2m} + V_{lattice} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla} V \times \boldsymbol{p}) \cdot \boldsymbol{\sigma}\right][u_{\boldsymbol{k}}(\boldsymbol{r})]$$

$$+ \hbar\boldsymbol{k} \cdot \left[\frac{|\boldsymbol{p}|}{m} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\sigma} \times \boldsymbol{\nabla} V)\right][u_{\boldsymbol{k}}(\boldsymbol{r})]$$

$$= \left[E_{\boldsymbol{k}} - \frac{\hbar^2k^2}{2m}\right][u_{\boldsymbol{k}}(\boldsymbol{r})]$$

Dresselhaus spin-orbit interaction

Dresselhaus SOI arises due to crystallographic inversion asymmetry

$$\left[\frac{|\boldsymbol{p}|^2}{2m} + V_{lattice} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla} V \times \boldsymbol{p}) \cdot \boldsymbol{\sigma}\right][\Psi] = E[\Psi]$$

Bloch wave $[\Psi] = e^{i\boldsymbol{k}.\boldsymbol{r}}[u_{\boldsymbol{k}}(\boldsymbol{r})]$ $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla}$

$$\nabla\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) = i\boldsymbol{k}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) + e^{i\boldsymbol{k}.\boldsymbol{r}}\,\nabla\, u_{\boldsymbol{k}}(\boldsymbol{r})$$

$$\nabla^2\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) = i\boldsymbol{k} \cdot \left(i\boldsymbol{k}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) + e^{i\boldsymbol{k}.\boldsymbol{r}}\,\nabla\, u_{\boldsymbol{k}}(\boldsymbol{r})\right) + i\boldsymbol{k} \cdot \left(e^{i\boldsymbol{k}.\boldsymbol{r}}\nabla u_{\boldsymbol{k}}(\boldsymbol{r})\right) + e^{i\boldsymbol{k}.\boldsymbol{r}}\nabla^2\, u_{\boldsymbol{k}}(\boldsymbol{r})$$

$$\nabla^2\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) = -k^2\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) + 2i\boldsymbol{k} \cdot \left(e^{i\boldsymbol{k}.\boldsymbol{r}}\nabla u_{\boldsymbol{k}}(\boldsymbol{r})\right) + e^{i\boldsymbol{k}.\boldsymbol{r}}\nabla^2 u_{\boldsymbol{k}}(\boldsymbol{r})$$

Dresselhaus spin-orbit interaction

Dresselhaus SOI arises due to crystallographic inversion asymmetry

$$\left[\frac{|\boldsymbol{p}|^2}{2m} + V_{lattice} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla} V \times \boldsymbol{p}) \cdot \boldsymbol{\sigma}\right][\Psi] = E[\Psi]$$

Bloch wave $[\Psi] = e^{i\boldsymbol{k}.\boldsymbol{r}}[u_{\boldsymbol{k}}(\boldsymbol{r})]$ $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla}$

$$(\boldsymbol{\nabla} V \times \boldsymbol{p}) \cdot \boldsymbol{\sigma} = (\boldsymbol{\sigma} \times \boldsymbol{\nabla} V) \cdot \boldsymbol{p}$$

$$\left[-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) + V_{lattice}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\sigma} \times \boldsymbol{\nabla} V)(-i\hbar)\boldsymbol{\nabla}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right)\right] = E_{\boldsymbol{k}}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right)$$

Dresselhaus spin-orbit interaction

Dresselhaus SOI arises due to crystallographic inversion asymmetry

$$\left[\frac{|\boldsymbol{p}|^2}{2m} + V_{lattice} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla}V \times \boldsymbol{p}) \cdot \boldsymbol{\sigma}\right][\Psi] = E[\Psi]$$

Bloch wave $[\Psi] = e^{i\boldsymbol{k}.\boldsymbol{r}}[u_{\boldsymbol{k}}(\boldsymbol{r})]$ $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla}$

$$\left[-\frac{\hbar^2}{2m}\left\{-k^2\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) + 2i\boldsymbol{k}\cdot\left(e^{i\boldsymbol{k}.\boldsymbol{r}}\nabla u_{\boldsymbol{k}}(\boldsymbol{r})\right)\right.\right.$$
$$\left. + e^{i\boldsymbol{k}.\boldsymbol{r}}\nabla^2 u_{\boldsymbol{k}}(\boldsymbol{r})\right\} + V_{lattice}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right)$$
$$\left. - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\sigma} \times \boldsymbol{\nabla}V)(-i\hbar)\left\{i\boldsymbol{k}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right) + e^{i\boldsymbol{k}.\boldsymbol{r}}\,\nabla\, u_{\boldsymbol{k}}(\boldsymbol{r})\right\}\right]$$
$$= E_{\boldsymbol{k}}\left(e^{i\boldsymbol{k}.\boldsymbol{r}}u_{\boldsymbol{k}}(\boldsymbol{r})\right)$$

Dresselhaus spin-orbit interaction

$$\Big[-\frac{\hbar^2}{2m}\{-k^2\big(e^{ik.r}u_{\boldsymbol{k}}(\boldsymbol{r})\big)+2i\boldsymbol{k}\cdot\big(e^{ik.r}\nabla u_{\boldsymbol{k}}(\boldsymbol{r})\big)$$

$$+e^{ik.r}\nabla^2 u_{\boldsymbol{k}}(\boldsymbol{r})\}+V_{lattice}\big(e^{ik.r}u_{\boldsymbol{k}}(\boldsymbol{r})\big)$$

$$-\frac{e\hbar}{4m^2c^2}(\boldsymbol{\sigma}\times\boldsymbol{\nabla}V)(-i\hbar)\{i\boldsymbol{k}\big(e^{ik.r}u_{\boldsymbol{k}}(\boldsymbol{r})\big)+e^{ik.r}\,\nabla\,u_{\boldsymbol{k}}(\boldsymbol{r})\}\Big]$$

$$=E_{\boldsymbol{k}}\big(e^{ik.r}u_{\boldsymbol{k}}(\boldsymbol{r})\big)$$

$e^{ik.r}\neq 0$

$$\left[\frac{|\boldsymbol{p}|^2}{2m}+V_{lattice}-\frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla}V\times\boldsymbol{p})\cdot\boldsymbol{\sigma}\right][u_{\boldsymbol{k}}(\boldsymbol{r})]$$

Proved

$$+\hbar\boldsymbol{k}\cdot\left[\frac{|\boldsymbol{p}|}{m}-\frac{e\hbar}{4m^2c^2}(\boldsymbol{\sigma}\times\boldsymbol{\nabla}V)\right][u_{\boldsymbol{k}}(\boldsymbol{r})]$$

$$=\left[E_{\boldsymbol{k}}-\frac{\hbar^2k^2}{2m}\right][u_{\boldsymbol{k}}(\boldsymbol{r})]$$

Dresselhaus spin-orbit interaction

Reciprocal lattice vector $\boldsymbol{k} \rightarrow \boldsymbol{k} + \boldsymbol{K}$ $\quad E_{\boldsymbol{k}} = E_{\boldsymbol{k}+\boldsymbol{K}}$

$$\left[\frac{|\boldsymbol{p}|^2}{2m} + V_{lattice} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\nabla}V \times \boldsymbol{p}) \cdot \boldsymbol{\sigma}\right][u_{\boldsymbol{k}+\boldsymbol{K}}(\boldsymbol{r})]$$

$$+ \hbar\boldsymbol{k} \cdot \left[\frac{|\boldsymbol{p}|}{m} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\sigma} \times \boldsymbol{\nabla}V)\right][u_{\boldsymbol{k}+\boldsymbol{K}}(\boldsymbol{r})]$$

$$+\hbar\boldsymbol{K} \cdot \left[\frac{|\boldsymbol{p}|}{m} - \frac{e\hbar}{4m^2c^2}(\boldsymbol{\sigma} \times \boldsymbol{\nabla}V)\right][u_{\boldsymbol{k}+\boldsymbol{K}}(\boldsymbol{r})]$$

$$= \left[E_{\boldsymbol{k}} - \frac{\hbar^2|\boldsymbol{k}+\boldsymbol{K}|^2}{2m}\right][u_{\boldsymbol{k}+\boldsymbol{K}}(\boldsymbol{r})]$$

Perturbation term

Dresselhaus derived the SOI Hamiltonian and spin-splitting energies along crystallographic directions

Dresselhaus spin-orbit interaction

DSOI Hamiltonian

$$H_D = a_D \boldsymbol{\sigma} \cdot \boldsymbol{\kappa}$$

a_D is material constant

$$\kappa_x = \frac{1}{2\hbar^3}\left[p_x\left(p_y^2 - p_z^2\right) + \left(p_y^2 - p_z^2\right)p_x\right]$$

$$\kappa_y = \frac{1}{2\hbar^3}\left[p_y(p_z^2 - p_x^2) + (p_z^2 - p_x^2)p_y\right]$$

$$\kappa_z = \frac{1}{2\hbar^3}\left[p_z\left(p_x^2 - p_y^2\right) + \left(p_x^2 - p_y^2\right)p_z\right]$$

$$p \rightarrow p + eA$$

Papers

- Thomas, L. H., The motion of the spinning electron, Nature 117, 514 (1926).
- Pikus, F. G. and Pikus, G. E., Conduction band spin splitting and negative magnetoresistance in A3B5 heterostructures, Phys. Rev. B 51, 16928 (1995).
- Dresselhaus, G., Spin orbit coupling effects in zinc blende structures, Phys. Rev. 100, 580 (1955).