

## **Interaction of particles with matter**

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## 1. Particles and interactions

**Four types of interactions: gravitational, weak, electro-magnetic, strong.** The typical relative magnitudes of these forces:  $10^{-39} : 10^{-7} : 10^{-2} : 1$ . The gravitational force plays no role in the high energy physics.

**Particles.** Consider only relatively long-lived particles so that they could travel some distance and had a chance to interact with matter before decaying. Note that due to  $\gamma$ -factor, the distance particles can travel before decaying will depend on their energy. We will use 100 GeV as a benchmark energy (particles of higher energy hardly ever produced even at Tevatron, the most powerful collider as of today).

### Leptons:

- neutrinos (stable)
- electron (stable), muon ( $\tau=2 \mu\text{s}$ ,  $c\tau=500 \text{ m}$ ,  $\gamma c\tau=500 \text{ km}$ ),  $\tau$ -lepton ( $\tau=0.3 \text{ ps}$ ,  $c\tau=90 \mu\text{m}$ ,  $\gamma c\tau=5 \text{ mm}$ )

**Hadrons:** consider only ground states of quarks (excited states live  $\sim 10^{-21}$ - $10^{-23} \text{ s}$ )

Mesons—quark-antiquark pairs

- lightest mesons made of u,d-quarks,  $M\sim 140 \text{ MeV}$ 
  - $\pi^\pm$  ( $\tau=30 \text{ ns}$ ,  $c\tau=10 \text{ m}$ ,  $\gamma c\tau=6 \text{ km}$ )
  - $\pi^0$  ( $\tau=10^{-16} \text{ s}$ ,  $c\tau=30 \text{ nm}$ ,  $\gamma c\tau=20 \mu\text{m}$ )
- lightest mesons made of s- and u,d-quarks,  $M\sim 500 \text{ MeV}$ 
  - $K^\pm$  ( $\tau=10 \text{ ns}$ ,  $c\tau=4 \text{ m}$ ,  $\gamma c\tau=800 \text{ m}$ )
  - $K_L^0$  ( $\tau=50 \text{ ns}$ ,  $c\tau=20 \text{ m}$ ,  $\gamma c\tau=4 \text{ km}$ ),  $K_S^0$  ( $\tau=0.1 \text{ ns}$ ,  $c\tau=3 \text{ cm}$ ,  $\gamma c\tau=6 \text{ m}$ )
  - $\eta$  ( $\tau=10^{-18} \text{ s}$ , ...)
- lightest mesons made of c- and u,d,s-quarks,  $M\sim 2 \text{ GeV}$ 
  - $D^\pm$  ( $\tau=1 \text{ ps}$ ,  $c\tau=300 \mu\text{m}$ ,  $\gamma c\tau=15 \text{ mm}$ )
  - $D^0$  ( $\tau=0.4 \text{ ps}$ ,  $c\tau=120 \mu\text{m}$ ,  $\gamma c\tau=6 \text{ mm}$ )
  - $D_s^\pm$  ( $\tau=0.5 \text{ ns}$ ,  $c\tau=150 \mu\text{m}$ ,  $\gamma c\tau=8 \text{ mm}$ )
  - $J/\psi$  ( $\tau=10^{-20} \text{ s}$ , ...)
- lightest mesons made of b- and u,d,s,c-quarks,  $M\sim 5 \text{ GeV}$ 
  - $B^\pm$ ,  $B^0$ ,  $B_s^0$  ( $\tau=2 \text{ ps}$ ,  $c\tau=500 \mu\text{m}$ ,  $\gamma c\tau=10 \text{ mm}$ )
  - $B_c^\pm$  ( $\tau=0.5 \text{ ps}$ ,  $c\tau=150 \mu\text{m}$ ,  $\gamma c\tau=3 \text{ mm}$ )
  - $\Upsilon$  (...)
- no mesons are ever formed with t-quarks (they live much less than  $10^{-23} \text{ s}$ , hadron formation time)

Baryons—quark-quark-quark particles

- lightest baryons made of u,d-quarks,  $M\sim 1 \text{ GeV}$ 
  - $p^\pm$  (stable)
  - $n$  ( $\tau=15 \text{ min}$ ,  $c\tau=3\times 10^8 \text{ km}$ ,  $\gamma c\tau=3\times 10^{10} \text{ km}$ )
- lightest baryons made of s- and u,d-quarks,  $M\sim 1 \text{ GeV}$ 
  - $\Lambda$  ( $\tau=0.3 \text{ ns}$ ,  $c\tau=8 \text{ cm}$ ,  $\gamma c\tau=8 \text{ m}$ )
  - ...

### Carriers of the forces:

- photons (stable)
- gluons (do not occur as free particles)
- W/Z (live less than  $10^{-25} \text{ s}$ )

	Weak	Electromagnetic	Strong
relative strength	$10^{-7}$	$10^{-2}$	1
<b>neutrinos</b>	Yes	-	-
<b>e, <math>\mu</math>, <math>\tau</math></b>	Yes	Yes	-
<b>hadrons: charged</b>	Yes	Yes	Yes
<b>neutral</b>	Yes	-	Yes
<b>photons</b>	-	Yes	-

## **2. Weak interactions (neutrinos)**

The only particles for which this interaction is of any significance are neutrinos, as they do not participate in the other interactions. The force is extremely weak: the mean free path of 1 MeV anti-neutrino in water is about  $10^{20}$  cm or 50 light years! So detection of neutrino is actually detection of missing energy-momentum in the balance of the outgoing particles.

We never observe multiple neutrino interactions along its passage through matter. Whenever an extremely rare single interaction happens, other particles are produced (for example,  $\bar{\nu}_e + p \rightarrow e^+ + n$ ) or knocked off from the media (for example,  $\nu_e + e^- (\text{atomic}) \rightarrow \nu_e + e^- (\text{scattered})$ )—it is these particles that we will detect by making use of their interactions with matter via electromagnetic or strong force.

### 3. Electromagnetic interactions (charged particles)

#### 3.1. Ionization energy losses for a charged particle passing through media

##### Bethe-Bloch formula for energy losses $dE/dx$ (1930s)

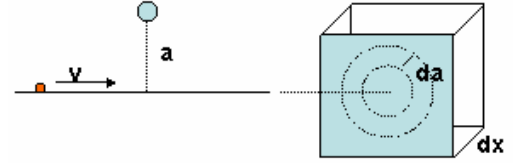
$$-\frac{dE}{dx} = 4\pi \frac{z^2 \alpha^2}{\beta^2} \frac{Z\rho}{A m_N m_e} \left[ \frac{1}{2} \ln \frac{2m_e \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2} \right],$$

where:

- $m_e, m_N, \alpha$ —universal constants: electron and nucleon masses; fine structure constant;
- $z, \beta, \gamma$ —incoming particle parameters (charge in units of  $e$ , velocity  $\beta=v/c$ , gamma factor);
- $Z, A, \rho, I$ —media properties: charge and atomic number of media atoms, density, average ionization potential for the media ( $I \sim 16 \cdot Z^{0.9}$  eV for  $Z > 1$ )
- $\delta$ —small correction due to media polarization (for gasses, it is negligibly small).

To understand the origin of the Bethe-Bloch formula, we will consider the following simplified model.

Momentum transfer from an incoming particle of charge  $ze$  to an atomic electron is:



$$dq_z = F_z dt = F_z \frac{dx}{v} = \frac{ze^2}{4\pi(x^2 + a^2)} \frac{dx}{v} \frac{a}{\sqrt{x^2 + a^2}},$$

where  $a$ —the shortest distance between particle's trajectory and an atomic electron;  $v$ —particle velocity,  $ze$ —particle charge,  $x$ —charged particle's coordinate along its trajectory. After integration from  $-\infty$  to  $+\infty$  in  $x$  we will have the total momentum transferred to an electron

$$q = 2 \frac{z\alpha}{av}.$$

Although the incoming particle can be relativistic, the recoiling electrons are typically not. Therefore, we can use non-relativistic kinematics to describe them. The energy transferred to electron (and, consequently, lost by the incoming particle) is then:

$$T = \frac{q^2}{2m_e} = 2 \frac{z^2 \alpha^2}{m_e v^2} \frac{1}{a^2}.$$

Summing up over all electrons in the media, the average energy losses experienced by the incoming particle after passing distance  $dx$  of the media will be:

$$dE = \int_{a_{\min}}^{a_{\max}} T 2\pi a n_e da dx = 4\pi \frac{z^2 \alpha^2}{m_e v^2} n_e dx \ln \left( \frac{a_{\max}}{a_{\min}} \right),$$

Note the two cutoffs in the integration  $a_{\min}$  and  $a_{\max}$ . At large distances, for which the calculated  $T$  becomes smaller than the ionization/excitation potential  $I$ , no energy transfer becomes possible—this imposes a cutoff  $a_{\max}$ :

$$I = 2 \frac{z^2 \alpha^2}{m_e v^2} \frac{1}{a_{\max}^2}.$$

The maximum energy an incoming particle can transfer to an electron is limited by the energy-momentum conservation laws. For an incoming particle of mass  $M$ , velocity  $v$  (and gamma-factor  $\gamma$ ):

$$T_{\max} = \frac{2m\beta^2\gamma^2}{1 + 2\gamma\left(\frac{m}{M}\right) + \left(\frac{m}{M}\right)^2}$$

This can be accounted for introducing a cutoff  $a_{\min}$ :  $T_{\max} = 2 \frac{z^2 \alpha^2}{m_e v^2} \frac{1}{a_{\min}^2}.$

Therefore, the formula for energy losses that we are deriving can be re-written as follows (now the negative sign is also included to reflect that the energy is being lost):

$$-\frac{dE}{dx} = 4\pi \frac{z^2 \alpha^2}{\beta^2} \frac{1}{m_e} n_e \left[ \frac{1}{2} \ln \left( \frac{T_{\max}}{I} \right) \right]$$

The number of electrons in the media of density  $\rho$  and made of elements (Z,A) is  $n_e = Z \frac{\rho}{Am_N}$ , from where:

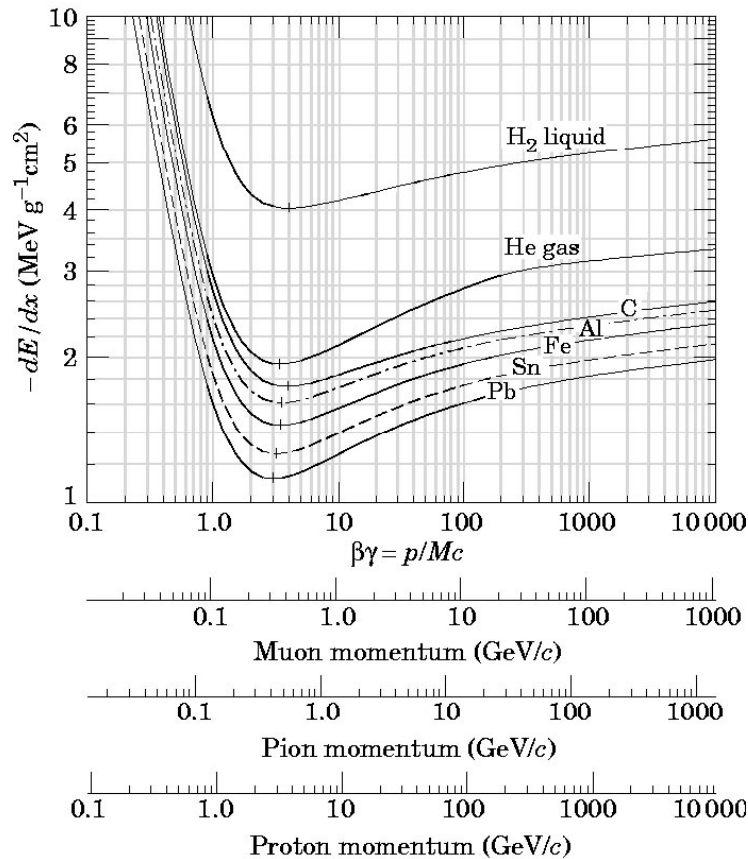
$$-\frac{dE}{dx} = 4\pi \frac{z^2 \alpha^2}{\beta^2} \frac{Z\rho}{Am_N m_e} \left[ \frac{1}{2} \ln \left( \frac{T_{\max}}{I} \right) \right]$$

- Dependence on the charge  $ze$  of the incoming particle is as  $z^2$ .
- The energy losses decrease as  $1/v^2$ , where  $v$  is the velocity of the incoming particle. Slow moving particles would lose more energy and as their momenta increases (and the velocity saturates at the speed of light), one expects flattening out of  $dE/dx$ .
- If  $dE/dx$  is normalized on  $\rho$ , the dependence on media becomes very weak as  $Z/A \sim 1/2$  for most elements and  $=1$  for hydrogen.

One can see that in comparison to the formula we have derived, the Bethe-Bloch formula has an extra factor of  $\frac{2m_e \beta^2 \gamma^2}{I}$  under the log, which actually leads to a slow rise of ionization losses with the particle momentum. This

is due to accounting for the relativistic flattening of the electric field of the incoming particle. As the result, the particle can ionize atoms at farther and farther distance as its field becomes more and more squashed at larger momenta. This rise eventually flattens out due to polarization effects in the media. The plots below show the actual density-normalized  $dE/dx$  curves for a few materials.

- The particles with velocities corresponding to  $\beta\gamma > 3$  are usually called **minimum-ionizing particles (mip)**.
- The typical value of energy losses at the minimum is about 2 MeV/(g/cm<sup>2</sup>).



### 3.2. Scintillation

A charged particle traversing matter leaves behind it a wake of excited molecules. Certain types of molecules, will release a small fraction of this energy in the form of optical or close UV photons, for which the media may be quite transparent with attenuation lengths reaching as much as a few meters. Amount of energy carried away by scintillation light is typically 1% or less of  $dE/dx$ , or about  $10^3$ - $10^4$  photons per MeV of ionization losses. This light may be used for detecting the fact of a particle traversing the media.

Scintillating materials are classified in inorganic and organic.

#### Inorganic (e.g., NaI, $PbWO_4$ )

- relatively slow (10-1000 ns)
- high-Z, very high density (good for  $\gamma$  and  $e$  detection and energy measurement)

#### Organic (plastics, liquid)

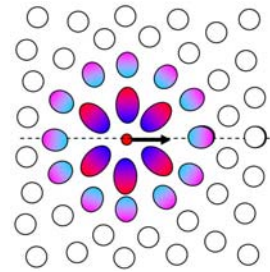
- fast (2 ns)
- low cost and ease of fabricating various shapes
- Hydrogen reach (good neutron detection)

Below is an example of scintillation mechanism in noble gases (note that the original excitations have much larger energy than the energy of the scintillation light).

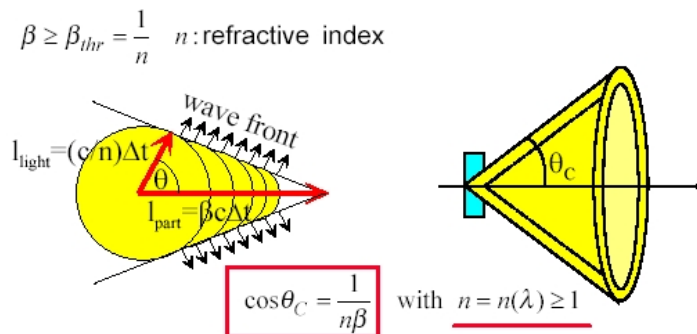
### 3.3 Cherenkov radiation

As a charged particle traverse matter, it produces a wake of polarized molecules along its path. As the molecules get polarized and then de-polarized, they emit radiation in all directions.

If the particle moves with a speed  $v$  exceeding speed of light in the media  $c/n$  ( $n$  is the refractive index for the media), in a certain direction the radiation becomes *coherent* and results in significant amount of light produced. The direction in which waves would add up coherently can be easily constructed using Huygens technique of circular fronts. It is an electromagnetic radiation shock wave.



This radiation is called Cherenkov (often spelled Čerenkov) after Cherenkov who has studied it in mid-1930s and showed that it was not due to luminiscense or any other known radiation mechanism. The first observation is actually attributed to Mallet in 1926. The theory behind the Cherenkov radiation was put together by Frank and Tamm in 1937.



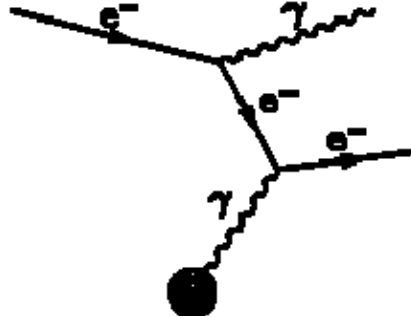
One can see that there is a threshold minimal velocity  $v=1/n$ , below which there is no radiation. As the particle surpasses this threshold velocity, it starts emitting small amount of light in the forward direction ( $\theta \sim 0$ ). As its velocity approaches speed of light, the intensity reaches its maximum and opens up to the maximum angle  $\theta_{\max} = \arccos(1/n)$ . Amount of energy emitted is about  $10^{-4}$  of  $dE/dx$  losses.

**1958 Cherenkov, Frank, Tamm** awarded Nobel Prize for discovery and explanation of the effect.

### 3.4 Bremsstrahlung (radiation losses or “breaking radiation”)

Bethe and Heitler, 1934

Relativistic charged particles, as they propagate through matter and wiggle due to multiple scattering on nuclei, experience accelerations and, therefore, must be radiating electromagnetic waves—emission of such photons is called “breaking” radiation, or bremsstrahlung.



The energy losses turn out to be proportional to the incoming particle energy, while the proportionality coefficient  $1/X_0$  depends on the composition of the material as well as the incoming particle mass and charge.

$$-\frac{dE}{dx} = \frac{E}{X_0},$$

Typical values of  $X_0$ :

- 0.6 cm (lead), 1.8 cm (iron)
- 36 cm (water)
- 30 m (air).

The energy at which bremsstrahlung radiation becomes equal to ionization losses is called critical energy  $E_c$ . Below  $E_c$ , the ionization losses dominate; above  $E_c$ , the main source of energy losses is bremsstrahlung. For media atoms with  $Z \geq 13$  the critical energy values for incident electrons can be estimated as follows:

$$E_c = \frac{550 \text{ MeV}}{Z}.$$

The critical energy for electrons in iron ( $Z=26$ ) is  $\sim 20$  MeV, 7 MeV in lead ( $Z=82$ ).

The amount of radiation is proportional to  $\sim 1/m^2$ . For example, breaking radiation for muons, the lightest charged particle after an electron, will be 40,000 times smaller than for electrons of the same momentum. The critical energy for muons is therefore  $\sim 1$  TeV.

## 4. Electromagnetic interactions (photons)

There are three very distinct processes of photon interacting with media:

- $\gamma + atom \rightarrow atom^+ + e^-$  (photoelectric effect; dominant for  $\epsilon_\gamma < 0.1$  MeV)
- $\gamma + e^- \rightarrow \gamma + e^-$  (Compton effect; dominant for  $0.1 \text{ MeV} < \epsilon_\gamma < 10 \text{ MeV}$ )
- $\gamma + nuclear \rightarrow e^+ + e^- + nuclear$  (Pair production; dominant for  $\epsilon_\gamma > 10 \text{ MeV}$ )

Photon beam intensity in a media falls as  $I = I_0 e^{-x/\lambda}$ , where  $\lambda$  is a free mean path:  $\lambda = \frac{1}{n_A \sigma_{total}} = \frac{1}{n_A \sum_i \sigma_i}$ .

### 4.1 Photoelectric process, or photo-effect

This is the process of photo absorption leading to ionization of an atom. If photon energy is sufficient, an electron from atomic shells. Approximately, the photo-effect cross section away from the characteristic energy peaks depends on the element properties and photon energy as

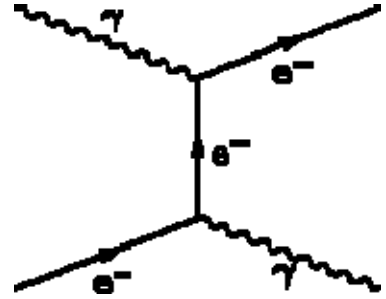
$$\sigma_{ph} \propto \frac{Z^5 \alpha^4}{\epsilon^3}.$$

### 4.2 Compton scattering

This is the process of scattering of photons on atomic electrons. Approximately, the Compton cross section has the following dependence on media properties and photon energy as:

$$\sigma_c \propto Z \alpha^2 \frac{\ln \epsilon}{\epsilon}$$

The t-channel diagram is shown on the right. There is also a similar s-channel diagram. One can see from these diagrams that the matrix element will be proportional to  $e^2$ . Therefore, the atomic cross section will be proportional to  $Z \cdot (e^2)^2$ , where the extra factor  $Z$  accounts for the number of electrons in an atom.



The energy spectrum of scattered photons and knocked off electron is approximately flat from  $\epsilon=0$  to  $\epsilon=\epsilon_\gamma$ .

### 4.3 Pair Production

This is the process  $e^+e^-$ -pair production by photons in the nuclear field.

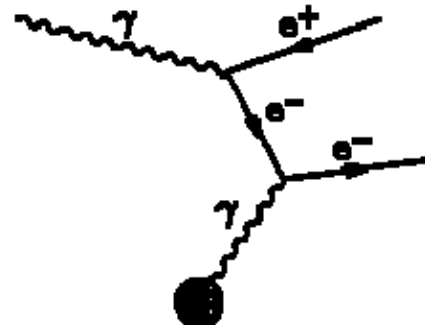
This process has an energy threshold and only possible for  $\epsilon_\gamma > 2m_e = 1 \text{ MeV}$ .

For photon energies  $\epsilon_\gamma > (70 \text{ MeV}) / Z^{1/3}$ , the cross section reaches the plateau level and become practically energy independent (the dependence on  $\alpha$  and  $Z$  is obvious):

$$\sigma_{pair} = \frac{7}{9} \left( \frac{4\alpha^3 Z^2}{m_e^2} \ln \frac{183}{Z^{1/3}} \right).$$

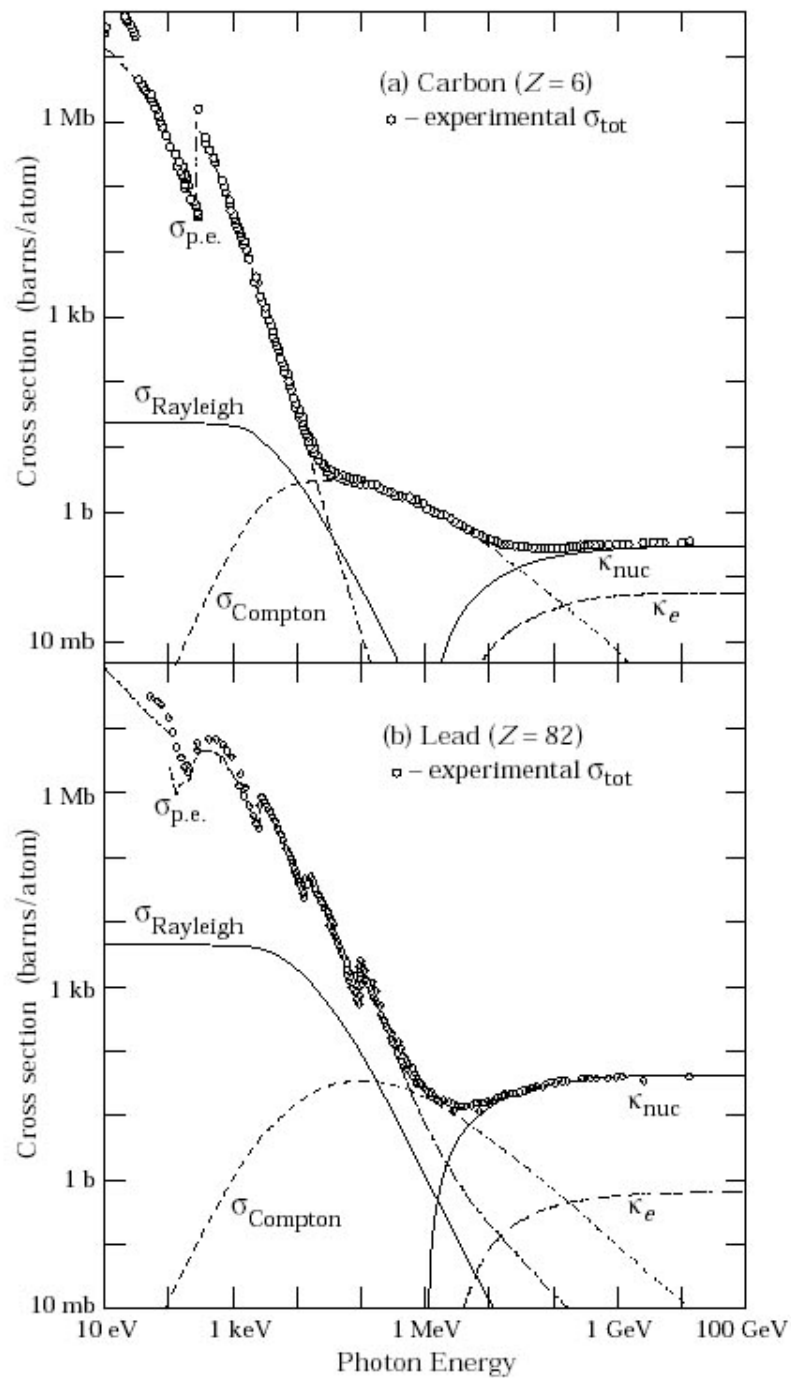
Note the interesting relationship that follows from this equation:

$$\lambda_{pair} \sim (9/7) \cdot X_0$$



From the obvious electron-positron symmetry in this process, the average energies of electrons and positrons must be equal to  $\epsilon_\gamma/2$ .

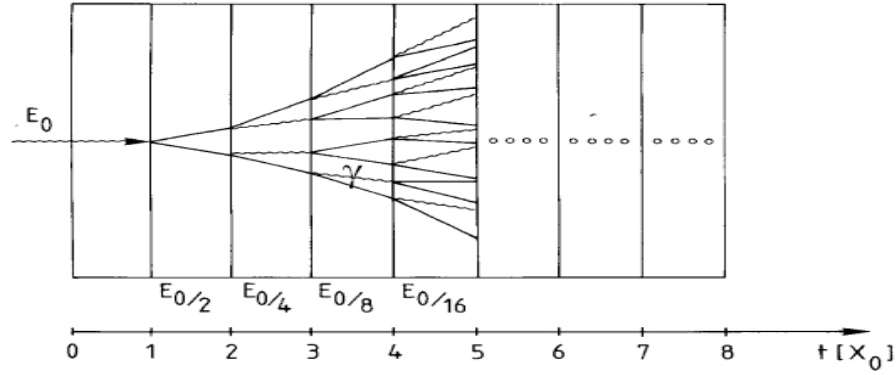




Total photon cross section  $\sigma_{\text{tot}}$  in lead, as a function of energy:

- $\sigma_{\text{p.e.}}$ , atomic photo-effect (electron ejection, photon absorption);
- $\sigma_{\text{Rayleigh}}$ , coherent scattering (Rayleigh scattering—atom neither ionized nor excited);
- $\sigma_{\text{Compton}}$ , incoherent scattering (Compton scattering off an electron);
- $\kappa_{\text{nuc}}$ , pair production, nuclear field;
- $\kappa_e$ , pair production, electron field.

## 5. Electromagnetic showers (electrons and photons)



The picture above illustrates a simplified model of an electromagnetic shower. Only bremsstrahlung and pair production processes are considered. For simplicity, we will assume  $\lambda_{pair} = X_0$ .

One can see that the number of particles grows with the distance as:  $N(t) = 2^t$  and energy per particle falls at each step as  $E(t) = E_0 2^{-t}$ .

Process continues until  $E(t) < E_c$  --after this point electrons will be losing their energy predominantly via ionization losses and photons will be re-scattered a few times (Compton) and finally absorbed (photo-effect). The process of particle multiplication stops when  $\frac{E_0}{2^{t_{max}}} = E_c$ , from where  $t_{max} = \frac{\ln E_0 / E_c}{\ln 2}$ .

*Note the shower size in length grows with energy only logarithmically.*

Total number of particles produced in the shower is *proportional to the incident particle energy*:

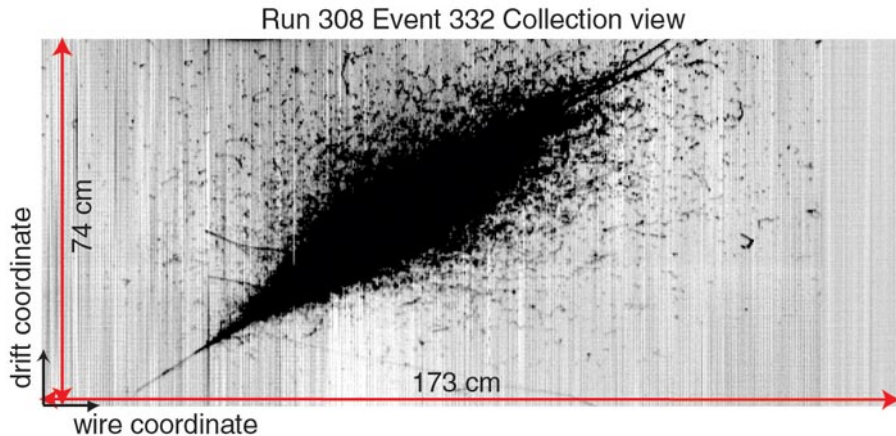
$$N_{total} = \sum_{t=0}^{t_{max}} 2^t = \frac{a_n q - a_0}{q - 1} = \frac{2^{(t_{max}+1)} - 1}{2 - 1} \approx 2 \cdot 2^{t_{max}} = 2 \frac{E_0}{E_c}.$$

95% of the shower core is contained in a cylinder of radius  $R_{95\%} = 2R_M$  and length  $L_{95\%} \sim t_{max} + 10$ , where

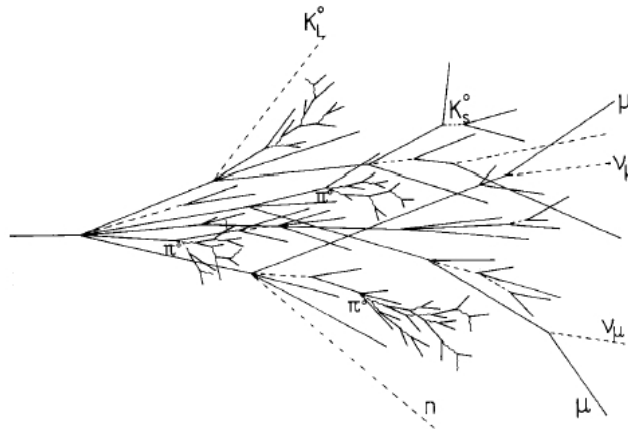
$$R_M = \frac{21 \text{ MeV}}{E_c} X_0 \left[ g / \text{cm}^2 \right] \text{ is Molier radius.}$$

For example, for lead and 100-GeV electrons,  $R_{95\%} \sim 4$  cm and  $L_{95\%} \sim 16$  cm.

Picture below shows an electromagnetic shower recorded by the ICARUS experiment in their Liquid Argon Time Projection Chamber.



## 6. Strong interactions: Hadronic shower (hadrons)



High energy collisions of hadrons typically result in prolific production of pions, the lightest particles made of quarks. Strong interactions of a kind  $\pi + p \rightarrow k \text{ pions} + p$  will, therefore, naturally lead to a hadronic cascade similar in many respects to an electromagnetic shower produced by electrons and photons. The main differences will be in

- mean free path between collisions;
- number of particles produced per collision;
- minimum energy after which the shower development terminates.

From point of view of strong interactions, protons and neutrons approximately can be viewed as non-transparent spheres of radius  $r \sim 1$  fm. Therefore,  $\pi + p$  cross section is  $\sigma_0 \sim \pi r^2 \sim 30$  mb. A nucleus of  $A$  protons and neutrons, in its turn, can be viewed as a bag of tightly packed nucleons and its cross section, therefore, can be estimated as  $\sigma = \sigma_0 \cdot A^{2/3}$ .

Mean free pass for a hadron in media of atoms ( $Z, A$ ) is then  $\lambda = \frac{1}{n\sigma}$ , where  $n$  is atomic density:  $n = \rho / (Am_N)$ .

Thus, one can estimate mean free pass between interactions  $\lambda = \frac{1}{n\sigma} = \frac{Am_N}{\rho\sigma} = \frac{A^{1/3}m_N}{\rho\sigma_0}$ .

For iron, this gives  $\lambda \sim 19$  cm. The actual number is close and somewhat smaller (17 cm)—this is because we ignored interactions that would lead to nucleus excitations.

Number of particles produced per one collision  $k$  typically will be somewhat larger than 2 as in the case electromagnetic shower ( $e \rightarrow e + \gamma$  or  $\gamma \rightarrow e^+ + e^-$ ).

The hadron cascade will stop when the energy of products reaches  $\sim 200$  MeV, after which no more pions can be made (pion mass  $\sim 140$  MeV). The total number of particles in the shower and the point at which it reaches its maximum can be estimated the same way as was done for em-shower (albeit small modification:  $2 \rightarrow k$ ):

$L_{\max} = \lambda \frac{\ln E_0 / E_{\min}}{\ln k}$ , which for 100 GeV hadrons gives  $L_{\max} \sim 6\lambda$ . The transverse shower size  $\sim \lambda$ .

There are two more important differences from the em showers that were not mentioned above:

- The number of particle species that can in principal be produced is large
- The most prolifically produced particles are pions:  $\pi^+$ ,  $\pi^-$ ,  $\pi^0$ . However,  $\pi^0$  is a short lived particle that decays into two photons. Therefore, all  $\pi^0$ s, once produced, decay to photons, which start dense and relatively short electromagnetic showers.

Therefore, although the number of charged tracks produced in a hadronic cascade is proportional to the energy of an incoming particle, the fluctuations are typically much larger than those in the electromagnetic shower initiated by an electron or photon.