Indian Institute of Technology, Madras

ACM Winter School on Quantum Computing - 2022

Problem Set 2 | 04 January 2022

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1. Using the tests of purity verify if the following density matrices are pure

a) $\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ b) $\frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ c) $\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ d) $\frac{1}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. Calculate the Von Neumann entropy of the following two qubit states.

a) $|\phi\rangle = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$ b) $|\phi\rangle = \frac{|01\rangle+|10\rangle}{\sqrt{2}}$ c) $|\phi\rangle = \frac{|00\rangle+i|11\rangle}{\sqrt{2}}$ d) $|\phi\rangle = \frac{|01\rangle+i|10\rangle}{\sqrt{2}}$

3. Similar to two level system (qubit), we can have a three level system known as qutrit. The basis states of a qutrit are

$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$; $|1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$; $|2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

For these states write the matrix form of

a) $|\psi_0\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$

b) $|\psi_1\rangle = \frac{1}{\sqrt{6}}(-2|00\rangle + |11\rangle + |22\rangle)$

And calculate the reduced density matrices of ρ_a and ρ_b. (Hint: $\rho_{AB} = |\psi_0\rangle\langle\psi_0|$ and $\rho_a = \mathrm{Tr}_b(\rho_{ab})$)

4. Calculate the Von Neumann entropy of the following states and their reduced states.

a) $|\psi\rangle = \frac{|01\rangle-|10\rangle}{\sqrt{2}}$;

b) $|\chi\rangle = |++\rangle$ where $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$

Calculate $S(\rho_a)$ and $S(\rho_b)$, the entropy of the reduced states ρ_a and ρ_b.

5. Show that for a pure bibartite separable state $|\psi_{ab}\rangle = |\psi_a\rangle \otimes |\psi_b\rangle$ implies $\rho_{ab} = \rho_a \otimes \rho_b$

6. Show that if U and V are unitary, then $(U \otimes V)$ is also unitary.

7. Which of the following state vectors are valid representation of a qubit?

a) $0.70|0\rangle + 0.3|1\rangle$

b)$\cos^2 x|0\rangle - \sin^2 x|1\rangle$

c)$0.8|0\rangle + 0.6|1\rangle$

8. Measurement is made on each of the following qubits. What are the probabiliteies that qubits are in state $|0\rangle$ and $|1\rangle$?

a) $\frac{i|0\rangle+|1\rangle}{\sqrt{2}}$

b) $\frac{(1+i)|0\rangle+i|1\rangle}{\sqrt{3}}$

c) $\frac{|0\rangle+2|1\rangle}{\sqrt{5}}$

9. Assume that the second qubit of the state is $|\psi_0\rangle = \frac{1}{\sqrt{30}}(|00\rangle + 2i|01\rangle - 3|10\rangle - 4i|11\rangle)$ is measured and observed to be in state $|1\rangle$. What is the probability that a subsequent measurement of the first qubit will yield $|1\rangle$?