

Indian Institute of Technology, Madras
ACM Winter School on Quantum Computing - 2022

Problem Set 3

05 January 2022

(1) Identify the quantum gates and write down the matrix form of the gates.

(a) $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\} \rightarrow \{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$

(b) $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\} \rightarrow \{|00\rangle, |01\rangle, |11\rangle, |10\rangle\}$

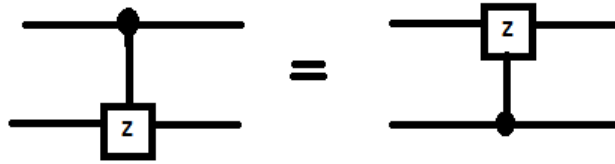
(2) Verify the following circuit identity-

(a) $CX_1C = X_1X_2$, (b) $CY_1C = Y_1X_2$, (c) $CZ_1C = Z_1$,

(d) $CX_2C = X_2$, (e) $CY_2C = Z_1Y_2$, (f) $CZ_2C = Z_1Z_2$

Here C is CNOT gate with qubit 1 being the control qubit and qubit 2 being the target qubit.

(3) Show that-



(4) Verify the following relations-

(a) $HXH = Z$, (b) $HZH = X$, (c) $HYH = -Y$

(5) Construct a CNOT gate from one controlled Z gate and two Hadamard gates. Specify the control and the target qubits.

(6) Verify that the CNOT gate is simple permutation whose action on a density matrix ρ is to rearrange the elements in the matrix.

(7) Construct a fredkin gate (controlled-swap gate) using three Toffoli gates.