Reduced Density Matrix

Basis

$\{a_i\}\ A \rightarrow \{|0\rangle_A, |1\rangle_A\}$

$\{b_i\}\ B \rightarrow \{|0\rangle_B, |1\rangle_B\}$

$$|\psi\rangle_{AB} = \frac{|00\rangle_{AB} + |11\rangle_{AB}}{\sqrt{2}}$$

$$\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}| = \frac{1}{2}\Big[|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\Big]$$

Idea $\longrightarrow$ find the density matrix of A.

$$\rho_A = Tr_B[\rho_{AB}]$$

$$= \sum_i \langle b_i|\rho_{AB}|b_i\rangle$$

$$= \langle 0_B|\rho_{AB}|0_B\rangle + \langle 1_B|\rho_{AB}|1_B\rangle$$

$$= \frac{1}{2}\Big[|0\rangle\langle 0|_A + |1\rangle\langle 1|_A\Big]$$

Partial transpose

$$\rho_{AB} = \sum_{ijkl} \rho_{ijkl} \, |i_A j_B\rangle\langle k_A l_B|$$

$$= \sum_{ijkl} \rho_{ijkl} \, |i\rangle\langle k|_A \otimes |j\rangle\langle l|_B .$$

Partial trace over B

$$\rho_{AB}^{T_B} = \sum_{ijkl} \rho_{ijkl} \, |i\rangle\langle k|_A \otimes |l\rangle\langle j|_B .$$

Eg $$|\psi_{AB}\rangle = \frac{1}{\sqrt{2}} \left[|10\rangle - |01\rangle \right]$$

$$\rho_{AB} = \frac{1}{2} \Big[|1_A 0_B\rangle\langle 1_A 0_B| - |1_A 0_B\rangle\langle 0_A 1_B|$$

$$- |0_A 1_B\rangle\langle 1_A 0_B| + |0_A 1_B\rangle\langle 0_A 1_B| \Big]$$

Transpose B

$$\rho_{AB}^{T_B} = \frac{1}{2} \Big[|10\rangle\langle 10| - |11\rangle\langle 00|$$

$$- |00\rangle\langle 11| + |01\rangle\langle 01| \Big]$$

$$\rho_{AB}^{T_A} = \frac{1}{2} \Big[|10\rangle\langle 10| - |00\rangle\langle 11|$$

$$- |11\rangle\langle 00| + |01\rangle\langle 01| \Big]$$

$$\rho_{AB} = \frac{1}{2} \begin{array}{c} \\ \langle 00| \\ \langle 01| \\ \langle 10| \\ \langle 11| \end{array} \begin{array}{c} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\ \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ \hline 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\rho_{AB}^{T_B} = \frac{1}{2} \left[\begin{array}{cc|cc} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right]$$

$$\rho_{AB}^{T_A} = \frac{1}{2} \left[\begin{array}{cc|cc} 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right]$$

Problem set 2

(9) $|\psi_0\rangle = \frac{1}{\sqrt{30}} \left(|00\rangle + 2i\,|01\rangle - 3\,|10\rangle - 4i\,|11\rangle \right)$

given:

Second qubit is in state $|1\rangle$ after measurement; so the state post measurement is:

$$|\psi_0\rangle_{\substack{\text{measured}\,|1\rangle \\ \text{on 2nd qubit}}} = \left(2i\,|01\rangle - 4i\,|11\rangle \right)$$

$\downarrow$

normalize!!

$$|\psi_0\rangle = \frac{i}{\sqrt{5}} \left(|01\rangle - 2\,|11\rangle \right)$$

Now, probability that first qubit is in $|1\rangle$ is $\left(\frac{2}{\sqrt{5}} \right)^2 = \frac{4}{5}$ //