Indian Institute of Technology, Madras

ACM Winter School on Quantum Computing - 2022

Problem Set 5 10 January 2022

---

1. Verify the concept, working and use of the following gates: (a) NOT Gate; (b) Identity Gate; (c) T and $T^\dagger$ gates; (d) S and $S^\dagger$ gates; (e) Pauli Z gate; (f) phase gate for $\pi/2$, $\pi/4$ and π; (g) R_z gate; (h) Hadamard gate; (i) CNOT gate; (j) Toffoli gate; (k) SWAP gate;

2. Using IBM Q composer create the following four bell states and display your output result.

$$\begin{aligned}
(a)\quad |\phi^+\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\
(b)\quad |\phi^-\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \\
(c)\quad |\psi^+\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \\
(d)\quad |\psi^-\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)
\end{aligned}$$

3. A GHZ state is a multipartite state of the form $\frac{1}{\sqrt{2}}(|0...0\rangle + |1...1\rangle)$. It is a maximally entangled state with a property that loss of even a single qubit removes all the entanglement in the system. Give the circuit diagram for a GHZ state with 3 qubits.

4. When we look into three qubit system another special kind of entangled state is the W state. The W-state is $\frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$. Draw the quantum circuit for this state. Demostrate teleportation using GHZ and W state and give the respective circuit diagrams and outputs.

5. Demonstrate Quantum teleportation using the Bell state $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

6. Using IBM Composer demonstrate the QKD protocol (a) without Interception and (b) with interception. .