Entanglement quantification with Von-Neumann entropy-

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

=> Density matrix. $\rho = |\psi\rangle = \frac{1}{2}(|01\rangle - |10\rangle)_{AB}(\langle 01| - \langle 10|)_{AB}$

$$= \frac{1}{2}\left[|01\rangle\langle 01|_{AB} - |01\rangle\langle 10|_{AB} - |10\rangle\langle 01|_{AB} + |10\rangle\langle 10|_{AB}\right]$$

• Finding reduced density matrix of A using partial Trace..

$$\rho_A = \frac{1}{2}\left[|0\rangle\langle 0| \, Tr_B(|1\rangle\langle 1|) - |0\rangle\langle 1| \, Tr_B(|1\rangle\langle 0|)\right.$$

$$\left. - |1\rangle\langle 0| \, Tr_B(|0\rangle\langle 1|) + |1\rangle\langle 1| \, Tr_B(|0\rangle\langle 0|)\right]$$

Here $Tr(|1\rangle\langle 0|) = Tr.\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = 0$

& millarly $Tr(|0\rangle\langle 1|) = 0$

$Tr(|1\rangle\langle 1|) = Tr(|0\rangle\langle 0|) = 1.$

$$\rho_A = \frac{1}{2}(|0\rangle\langle 0| + |1\rangle\langle 1|) = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

* Eigen values of ρ_A. $\rightarrow$ $\begin{vmatrix} \frac{1}{2}-\lambda & 0 \\ 0 & \frac{1}{2}-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1/2, 1/2.$

* Von-neumann Entropy of $\rho_A \Rightarrow S_A = -\sum \lambda \log_2 \lambda = 1.$

$S_A = 1. \Rightarrow$ Maximally entangled. state

$= 0 \Rightarrow$ Product stat

$0 < S_A < 1 \Rightarrow$ Entangled state.

Q.5a).

- For $|\phi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$.

$$C(|\phi\rangle) = 2|c_0c_3 - c_1c_2|$$

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left[|01\rangle - i|10\rangle\right] \qquad c_1 = \frac{1}{\sqrt{2}} \qquad c_2 = \frac{-i}{\sqrt{2}}$$

$$C|\phi\rangle = 2.|c_0c_3 - c_1c_2| = 2.\left|\frac{i}{2}\right| = 1.$$

$\Rightarrow$ Entanglement $= h.\left(\frac{1+\sqrt{1-c}}{2}\right) = h(\frac{1}{2}) = 1 =$ max.

Entang.

Entanglement by concurrence → $\rho = \frac{1}{4}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

$$\check{\rho}_s = (\sigma_y \otimes \sigma_y)\, \rho^* \,(\sigma_y \otimes \sigma_y)$$

Eigen values of $\check{\rho}_s \to \alpha_1\ \alpha_2\ \alpha_3\ \alpha_4$

Square root of eigen values $= \underbrace{\sqrt{\alpha_1}}\ \underbrace{\sqrt{\alpha_2}}\ \underbrace{\sqrt{\alpha_3}}\ \underbrace{\sqrt{\alpha_4}}$
$= \lambda_1\ \lambda_2\ \lambda_3\ \lambda_4$

Concurrence –

$$C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

Entanglement $E = h\left(\frac{1+\sqrt{1-C}}{2}\right)$

$$h(x) = -x \log_2 x - (1-x)\log_2(1-x)$$