**Indian Institute of Technology, Madras**

**ACM Winter School on Quantum Computing - 2022**

Problem Set 4 06 January 2022

---

1. Check whether the following states are entangled or not using the PPT criteria:

a)$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ ;

b)$|\chi\rangle = |++\rangle$ where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

c)$\rho = \frac{1}{8}I_{4X4} + \frac{1}{2}|\phi^+\rangle\langle\phi^+|$

d)$\rho = \frac{3}{16}I_{4X4} + \frac{1}{4}|\phi^+\rangle\langle\phi^+|$ Here $I_{4X4}$ is the 4X4 Identity matrix.

2. Quantify the amount of entanglement present in the states given above using the Negativity and log Negativity of the system.

3. Calculate the entanglement of the following states using von-Neumann entropy:

a)$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ ;

b)$|\chi\rangle = |++\rangle$ where $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

c)$\rho = \frac{1}{8}I_{4X4} + \frac{1}{2}|\phi^+\rangle\langle\phi^+|$

d)$\rho = \frac{3}{16}I_{4X4} + \frac{1}{4}|\phi^+\rangle\langle\phi^+|$ Here $I_{4X4}$ is the 4X4 Identity matrix.

4.Let us consider the tripartite quantum states

a)$|\psi\rangle = \frac{1}{\sqrt{2}}|000\rangle + |111\rangle$

b) $|\psi\rangle = \frac{1}{\sqrt{2}}|001\rangle + |010\rangle + |100\rangle$

and calculate the entanglement of the system using the von-Neumann entropy.

5. Use the concurrence measure to calculate the entanglement of the states given below

a) $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - i|10\rangle)$

b) $\rho = \frac{1}{4}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

6. Consider a quantum state of the following form: $\rho = (1-\mu)\frac{I_{4X4}}{4} + \mu|\psi_s\rangle\langle\psi_s|$ where $|\psi_s\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
Find the value of $\mu_s$ below which the state is not tangled. Here $I_{4X4}$ is the 4X4 Identity matrix.

7. Let us consider a quantum state with a density matrix of the form: $\rho = \begin{bmatrix} a & 0 & 0 & g \\ 0 & b & e & 0 \\ 0 & f & c & 0 \\ h & 0 & 0 & d \end{bmatrix}$

This state is known as the 'X' state. Find an expression for the concurrence of the system.