Indian Institute of Technology, Madras

ACM Winter School on Quantum Computing - 2022

Problem Set 1 03 January 2022

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(1) Are the following set of vectors linearly independent or dependent? (In the three dimensional vector space.)

(a) $\overrightarrow{A} = \begin{pmatrix} 3 & 0 & 0 \end{pmatrix}$, $\overrightarrow{B} = \begin{pmatrix} 0 & -2 & 0 \end{pmatrix}$, $\overrightarrow{C} = \begin{pmatrix} 0 & 0 & -1 \end{pmatrix}$

(b) $\overrightarrow{A} = \begin{pmatrix} 6 & -9 & 0 \end{pmatrix}$, $\overrightarrow{B} = \begin{pmatrix} -2 & 3 & 0 \end{pmatrix}$

(c) $\overrightarrow{A} = \begin{pmatrix} 2 & 3 & -1 \end{pmatrix}$, $\overrightarrow{B} = \begin{pmatrix} 0 & 1 & 2 \end{pmatrix}$, $\overrightarrow{C} = \begin{pmatrix} 0 & 0 & -5 \end{pmatrix}$

(d) $\overrightarrow{A} = \begin{pmatrix} 1 & -2 & 3 \end{pmatrix}$, $\overrightarrow{B} = \begin{pmatrix} -4 & 1 & 7 \end{pmatrix}$, $\overrightarrow{C} = \begin{pmatrix} 0 & 10 & 11 \end{pmatrix}$, $\overrightarrow{D} = \begin{pmatrix} 14 & 3 & -4 \end{pmatrix}$

(2) Consider the following two kets-

$$|\psi\rangle = \begin{pmatrix} -3i \\ 2+i \\ 4 \end{pmatrix}, \quad |\phi\rangle = \begin{pmatrix} 2 \\ -i \\ 2-3i \end{pmatrix}$$

(a) Find the $\langle\phi|$.

(b) Evaluate the scalar product $\langle\phi|\psi\rangle$.

(3) Consider the states $|\psi\rangle = 3i\,|\phi_1\rangle - 7i\,|\phi_2\rangle$ and $|\chi\rangle = -\,|\phi_1\rangle + 2i\,|\phi_2\rangle$, where $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal.

(a) Calculate $|\psi + \chi\rangle$ and $\langle\psi + \chi|$?

(b) Calculate the scalar products $\langle\chi|\psi\rangle$ and $\langle\psi|\chi\rangle$. Are they equal?

(4) Consider two states $|\psi_1\rangle = 2i\,|\phi_1\rangle + |\phi_2\rangle - a\,|\phi_3\rangle + 4\,|\phi_4\rangle$ and $|\psi_2\rangle = 3\,|\phi_1\rangle - i\,|\phi_2\rangle + 5\,|\phi_3\rangle - |\phi_4\rangle$, where $|\phi_1\rangle$, $|\phi_2\rangle$, $|\phi_3\rangle$ and $|\phi_4\rangle$ are orthonormal kets and a is a constant. Find a such that $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal.

(5) Consider a state $|\psi\rangle = \frac{1}{\sqrt{2}}\,|\phi_1\rangle + \frac{1}{\sqrt{5}}\,|\phi_2\rangle + \frac{1}{\sqrt{10}}\,|\phi_3\rangle$, which is given in terms of three orthonormal eigen states $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$ of an operator $\hat{B}$ such that $\hat{B}\,|\phi_n\rangle = n^2\,|\phi_n\rangle$. Find the expectation value of $\hat{B}$ for the state $|\psi\rangle$.

(6) Evaluate the following commutators-

(a) $[x, p_x]$, (b) $[x^2, p_x]$, (c) $[x, p_x^2]$, (d) $[x, p_z^2]$

(7) For the Pauli Spin matrices

$$\sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Find the commutators-

$[\sigma_X, \sigma_Z], \quad [\sigma_Z, \sigma_Y], \quad [\sigma_Y, \sigma_X]$

(b) Find the anticommutators-

$\{\sigma_X, \sigma_Z\}, \quad \{\sigma_Z, \sigma_Y\}, \quad \{\sigma_Y, \sigma_X\}$

(c) Compute the quantity $\sum_i \sigma_i^2$ where $i = \{X, Y, Z\}$.