考试科目名称 离散数学 （B 卷）

考试方式：开卷 考试日期_____年 ___月____日 教师_____赵建华，姚远_____

系（专业）__软件学院（软件工程）___ 年级_____ 班级_____

学号_____ 姓名_____ 成绩_____

注意：所有作答请写在答题纸上。

1. (10 points) Symbolize the following propositions, and provide the logic reasoning steps:

   "Natural numbers are all integers, and integers are all rational numbers; some rational numbers are not integers. So, natural numbers are all rational numbers, and there are rational numbers that are neither natural numbers nor integers."

2. (8 points) A player rolls three dices. Calculate the probability that two of them have same point and the other one has a strictly larger point.

3. (8 points) Let S be the set $\{a, b, c, d, e, f\}$, and A, B be two relations defined on S as follows.

   A={ (a,c), (a,d), (a,e), (b,a), (b,d), (b,e), (b,f), (c,a), (c,b), (c,e), (c,f), (d,c), (d,d), (e,f), (f,f), (f,a)}

   B ={(a,c), (a,f), (b,b), (b,e), (b,f), (c,b), (c,c), (d,e), (e,c), (e,f)}

   Please give the relations $A°B$ and A^2.

4. (8 points) Given a set A={1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Give the Hasse diagram of A in terms of the divisibility relation on A, and list all the minimal value(s) and maximal value(s) of A on the divisibility.

5. (8 points) Suppose set A has four elements. Give the total number of equivalence relations on A.

6. (8 points) Assume that n is a positive even number.

   (1) How many functions $f:\{0,1\}^n \to \{0,1\}^n$ are there satisfying $\forall x: f(x) \neq x$.

   (2) For $x \in \{0,1\}^n$, let x^r be the reverse string of x. How many strings are there satisfying $x^r = x$.

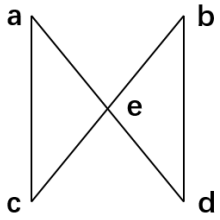
7. (8 points) Let $Z_m=\{0,1,\ldots,m-1\}$, $+_m$ be the addition modulo m, and $*_m$ be the multiplication modular m.

   (1) Prove that $(Z_m, +_m)$ is a group.

   (2) Give and prove the sufficient and necessary condition of $(Z_m-\{0\}, *_m)$ being a group.

8. (8 points) Given a simple graph G with a limited number of vertices, and assume that we can delete vertices in G step by step as follows: we can delete only vertices with degrees less than 2. Prove that: all the vertices in G can be deleted if and only if there is no circuit in G.

9. (10 points) Consider a simple Euler graph G (|G|>2). A vertex *v* in G is called *extendible* if any simple path from *v* can be extended to an Euler circuit. For example, as shown in the figure below, vertex *e* is extensible; vertex *a* is not extendible, as the simple path *aec* cannot be extended to an Euler circuit. Prove that: vertex *v* is extensible if and only if G-*v* is a forest.



10. (8 points) Represent the expression (a-b) * (c + d) + (e / f) * g as a rooted binary tree. Then, give the sequences travelling this tree in preorder and postorder, respectively.

11. (8 points) Let G be an Abelian group. Prove that the function f: G→G defined as $f(a) = a^3$ is a homomorphism.

12. (8 points) Let (L, ≤) be a lattice. For any element x, y, z in L, prove the following formulas:

   (1) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$

   (2) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$

中文参考

1. (10 分) 符号化以下命题，并给出推理的证明过程。
   自然数都是整数，整数都是有理数，有些有理数不是整数，所以自然数都是有理数，并且存在既不是自然数也不是整数的有理数。

2. (8 分) 计算掷出三个骰子后，有两个骰子点数相同，且另外一个的点数大于这个点数的概率。

3. (8 分) 已知集合 S={a,b,c,d,e,f}和 S 上的关系 A，B 如下：
   A={ (a,c), (a,d), (a,e), (b,a), (b,d), (b,e), (b,f), (c,a), (c,b), (c,e), (c,f), (d,c), (d,d), (e,f), (f,f), (f,a)}
   B ={(a,c), (a,f), (b,b), (b,e), (b,f), (c,b), (c,c), (d,e), (e,c), (e,f)}
   请给出关系 A°B，和 A^2。

4. (8 分) 已知整数集合 A={1，2，3，4，5，6，7，8，9，10}，画出 A 相对于整除关系的 Hasse 图。并指出 A 中相对于整除关系的所有极小值和极大值。

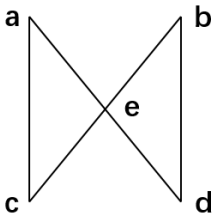
5. (8 分) 若集合 A 有四个元素，给出 A 上的所有等价关系的个数。

6. (8 分) 假设 n 是一个正偶数，试分别回答以下问题：
   (1) 存在多少函数$f:\{0,1\}^n \to \{0,1\}^n$，满足$\forall x: f(x) \neq x$.
   (2) 对于$x \in \{0,1\}^n$，令x^r表示x的倒序串，存在多少这样的x满足$x^r = x$.

7. (8分) 设Z_m={0,1,⋯,m-1}，$+_m$是模m加法运算，$*_m$是模m乘法运算。
   (1) 证明（Z_m，$+_m$）是群。
   (2) 给出（Z_m-{0}, $*_m$）是群的充要条件，并证明之。

8. (8分) 给定一个顶点个数有限的简单图G，假定我们只可以通过如下方式逐步删除G中的顶点：每一步可以删除度数小于2的顶点。试证明：如果G中的所有顶点能被删除当且仅当G中没有回路。

9. (10分) 简单图G是满足|G|>2的欧拉图，定义G中的节点v是可延展的(extendible)指：从节点v出发的任意简单通路都可以继续延展成欧拉回路。例如下图所示，只有节点e是可延展的；对于节点a，aec这条简单通路无法继续延展成欧拉回路。试证明：节点v是可延展的当且仅当G-v是一个森林。



10. (8分) 给出表达式(a-b) * (c + d) + (e / f) * g的二叉树表示，然后分别给出按照preorder和postorder遍历这棵树得到的序列。

11. (8分) 假设G是一个阿贝尔群，函数f:G->G定义为$f(a) = a^3$，请证明f是一个同态映射。

12. (8分) 设(L, ≤)为格，对于L的任意元素x, y, z，证明下式成立：

(1) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$

(2) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$