

[3]

OPERATIONAL ESTIMATES OF LAKE EVAPORATION

F.I. MORTON

*National Hydrology Research Institute, Environment Canada, Ottawa, Ont. K1A 0E7
(Canada)*

(Received June 22, 1982; revised and accepted August 11, 1982)

ABSTRACT

Morton, F.I., 1983. Operational estimates of lake evaporation. *J. Hydrol.*, 66: 77–100.

The complementary relationship between areal and potential evapotranspiration takes into account the changes in the temperature and humidity of the air as it passes from a land environment to a lake environment. Minor changes convert the latest version of the complementary relationship areal evapotranspiration (CRAE) models to a complementary relationship lake evaporation (CRLE) model. The ability of the CRLE model to produce reliable estimates of annual lake evaporation from monthly values of temperature, humidity and sunshine duration (or global radiation) observed in the land environment with no locally optimized coefficients is tested against comparable water-budget estimates for 11 lakes in North America and Africa. Maps of annual lake evaporation and annual net reservoir evaporation (i.e. the difference between lake evaporation and areal evapotranspiration) for the part of Canada to the east of the Pacific Divide and for the southern U.S.A. are presented. An approximate routing technique, which takes into account the effects of depth and salinity on the seasonal pattern of monthly lake evaporation, is formulated and tested against comparable water-budget estimates for 10 lakes in North America and Africa. The results indicate that the CRLE model, with its associated routing technique, is much superior to the other techniques in current use that rely on climatological or pan observations in the land environment.

1. INTRODUCTION

Operational estimates of lake evaporation must rely on readily available information and in practice this means climatological data or pan evaporation data that have been observed in the land environment. There are several problems associated with the use of such observations. The first is that seasonal changes in subsurface heat storage are not reflected directly in pan evaporation or climatological data and that such changes are significant in determining seasonal variations in the evaporation from deep lakes. This problem is not too important because annual estimates are adequate for most water planning and management or environmental impact studies. Furthermore, the effects of subsurface heat storage changes can be taken

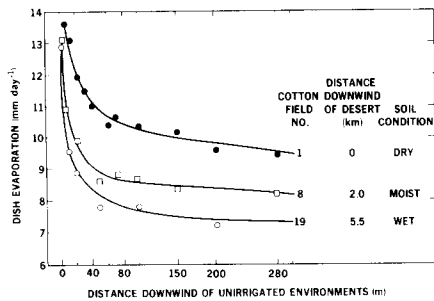


Fig. 1. Comparison of evaporation rates across irrigated cotton fields on December 27, 1963 (Davenport and Hudson, 1967).

into account in an approximate way with routing techniques. A much more serious problem is that pan and climatological observations are influenced significantly by changes in the availability of water for evapotranspiration from the adjacent land and are, therefore, not representative of the environment over the lake.

Reliable information on the transition that takes place when the air passes from the land over a lake is rare if not non-existent. However, Davenport and Hudson (1967) have presented data on the transition from desert to irrigated cotton that can be used to show what happens at the upwind edge of a lake. They measured the variation in evaporation across a series of irrigated and fallow fields in the Sudan Gezira, using fiberglass dishes with black-painted wells 113 mm in diameter and 36 mm in depth. The dish evaporation provided a somewhat distorted reflection of the potential evapotranspiration. The passage of air from the desert (or from the unirrigated fallow fields) over the irrigated cotton caused the dish evaporation above the cotton to decrease rapidly in the downwind direction and to approach a low constant value within 300 m, the width of the fields. Furthermore, as the air passed from irrigated cotton across unirrigated fallow, the dish evaporation above the fallow increased rapidly in the downwind direction and approached but did not reach the value observed at the upwind edge of the irrigation area. Fig. 1 shows the variation of dish evaporation across three irrigated fields on December 27, 1963. The ratio of daily dish evaporation at the downwind edges of the irrigated cotton to that at the upwind edge of the irrigated area was 0.69 for the field with "dry" soil, 0.60 for the field with "moist" soil and 0.53 for the field with "wet" soil.

The decreases in dish evaporation across the cotton fields were associated with decreases in temperature and increases in humidity. The vapour pressures appeared to attain equilibrium values within the 300-m width of the fields, but the temperatures were still decreasing, possibly because the observations were made above the level of the crop and the dishes.

Fig. 1 shows how the dish evaporation and potential evaporation increase when the water available for evapotranspiration from the area upwind decreases and how they decrease when the water available for evapotranspiration

from the area upwind increases. Moreover, the dish evaporation for the “wet” field provides an indication of what happens over a lake in an arid climate. Thus the upwind dish evaporation reflects the potential evaporation in the desert and the low, relatively constant dish evaporation near the downwind edge reflects the potential evaporation over most of the lake. Furthermore, the dish evaporation from the “moist” and “dry” fields provides an analogy for what happens over lakes in progressively more humid climates where the contrasts between lake and land environments are less extreme. Because the transition zone is so narrow, the lake evaporation would approximate the low constant downwind value of potential evaporation.

The effects of changes in the availability of water to upwind areas can be taken into account by the complementary relationship between potential and areal evapotranspiration. A conceptual rationalization and a review of available theoretical knowledge and reliable empirical evidence in a companion paper (Morton, 1983) have indicated that the complementary relationship is a plausible working hypothesis. It is expressed in the following equation:

$$E_T + E_{TP} = 2E_{TW} \quad (1)$$

in which E_T is the areal evapotranspiration, the evapotranspiration from an area so large that the effects of upwind boundary transitions, such as those shown in Fig. 1, are negligible; E_{TP} is the potential evapotranspiration, as estimated from a solution of the vapour transfer and energy-balance equations, representing the evapotranspiration that would occur from a hypothetical moist surface with radiation absorption and vapour transfer characteristics similar to those of the area and so small that the effects of the evapotranspiration on the overpassing air would be negligible; and E_{TW} is the wet-environment areal evapotranspiration, the evapotranspiration that would occur if the soil-plant surfaces of the area were saturated and there were no limitations on the availability of water.

Fig. 2 provides a schematic representation of eq. 1 under conditions of constant radiant-energy supply. The ordinate represents evapotranspiration and the abscissa represents water supply to the soil-plant surfaces of the area, a quantity that is usually unknown. When there is no water available for areal evapotranspiration (extreme left of Fig. 2) it follows that $E_T = 0$, that the air is very hot and dry and that E_{TP} is at its maximum rate of $2E_{TW}$ (the dry environment potential evapotranspiration). As the water supply to the soil-plant surfaces of the area increases (moving to the right in Fig. 2) the resultant equivalent increase in E_T causes the overpassing air to become cooler and more humid which in turn produces an equivalent decrease in E_{TP} . Finally, when the supply of water to the soil-plant surfaces of the area has increased sufficiently, the values of E_T and E_{TP} converge to that of E_{TW} .

The conventional definition for potential evapotranspiration is the same as the definition for the wet-environment areal evapotranspiration. However, the potential evapotranspiration that is estimated from a solution of the

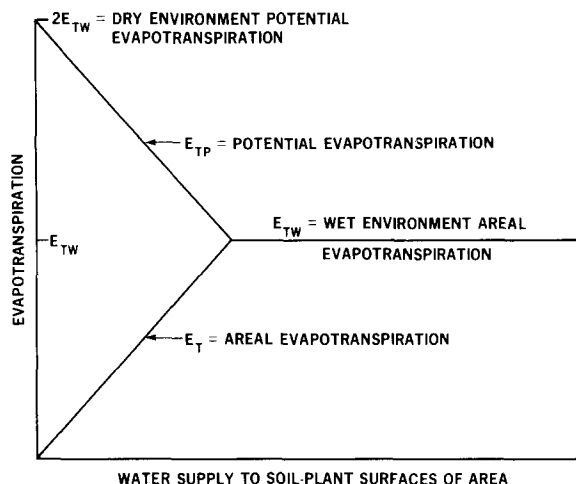


Fig. 2. Schematic representation of complementary relationship between areal and potential evapotranspiration with constant radiant-energy supply.

vapour transfer and energy-balance equations by analytical (Penman, 1948), graphical (Ferguson, 1952) or iterative (Morton, 1983) techniques has reactions to changes in the water supply to the soil-plant surfaces similar to those shown for E_{TP} in Fig. 2, so that what is being estimated can exceed what is being defined by as much as 100%. By taking into account such reactions, the complementary relationship is analogous to the Bernoulli equation for open-channel flow in which the potential energy responds in a complementary way to changes in kinetic energy.

The chief advantage of the complementary relationship is that it permits the areal evapotranspiration, a product of complex processes and interactions in the soil-plant-atmosphere system, to be estimated by its effects on the routine climatological observations that are used to compute potential evapotranspiration. Because the model avoids the complexities of the soil-plant system it requires no local optimization of coefficients and is, therefore, falsifiable. This means that it can be tested rigorously so that errors in the associated assumptions and relationships can be detected and corrected by progressive testing over an ever-widening range of environments. Such a methodology uses the entire world as a laboratory and requires that a correction made to obtain agreement between model and river-basin water-budget estimates in one environment must be applicable without modification in all other environments. The estimates resulting from the most recent application of this methodology (Morton, 1983) agree closely with comparable long-term water-budget estimates for 143 river basins in North America, Africa, Ireland, Australia and New Zealand.

The evaporation from a shallow lake, E_W , differs from the wet-environment areal evapotranspiration, E_{TW} , only because the radiation absorption and vapour transfer characteristics of water differ from those of vegetated land surfaces. The potential evaporation, E_P , differs from the potential evapo-

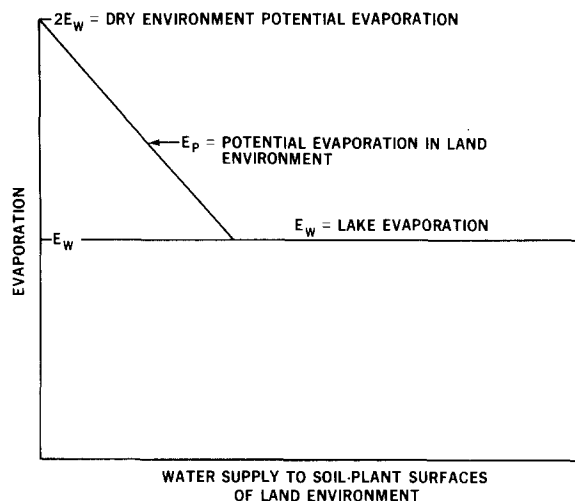


Fig. 3. Schematic representation of relationship between shallow-lake evaporation and potential evaporation in the land environment with constant radiant-energy supply.

transpiration, E_{TP} , for the same reasons. Although the lake evaporation is equal to the potential evaporation in the lake environment it can differ significantly from the potential evaporation in the land environment.

Fig. 3 provides a schematic representation of the relationship between shallow-lake evaporation and potential evaporation in the land environment under conditions of constant radiant-energy supply. The ordinate represents evaporation and the abscissa represents the water supply to the soil-plant surfaces of the land environment. Since a lake is defined to be so wide that the effects of the kind of upwind transition shown in Fig. 1 are negligible, the lake evaporation is independent of variations in the water supply to the soil-plant surfaces of the land environment. However, the complementary relationship predicts that the potential evaporation in a completely dry land environment would be twice the lake evaporation and that it would decrease in response to increases in the water supply to the soil-plant surfaces until it reached a minimum equal to the lake evaporation as shown in Fig. 3.

The conventional techniques for estimating lake evaporation are based on the assumption that the evaporation estimated from pan or climatological observations in the land environment can be transposed to a nearby lake by applying a simple coefficient. The apparent lack of alternatives has led to the practice of ignoring the increasing body of evidence that these techniques are unrealistic. Such evidence is implicit in a tabulation published by Hounam (1973) which shows that in the U.S.A. the annual class-A pan coefficient is 0.81 for Lake Okeechobee in Florida, where the average annual precipitation is ~ 1400 mm; 0.70 for Lake Hefner in Oklahoma, where the average annual precipitation is ~ 800 mm; and 0.52 for the Salton Sea in California, where the average annual precipitation is ~ 60 mm. These kind of variations undermine the foundations of the conventional techniques

because they indicate that lakes create their own environments which differ more and more from the land environments as the land environments become more arid. However, they are compatible with the complementary relationship and the kind of interactions shown in Figs. 1 and 3, which predict that the lake evaporation would be equal to the potential evaporation in a wet land environment and would be equal to half the potential evaporation in a dry land environment.

The transposability of coefficients from one lake to another will always be open to doubt because of the infinite variety of land environments and the lack of knowledge on how they influence the pan evaporation and the climatological observations. In this context, the complementary relationship seems an attractive alternative. Its use as the basis for the latest version of what are now referred to as the CRAE (i.e. complementary relationship areal evapotranspiration) models; the application of this latest version to provide operational estimates of areal evapotranspiration from routine observations of temperature, humidity and sunshine duration (or global radiation); the test of the results against comparable water-budget estimates for 143 river basins in North America, Africa, Ireland, Australia and New Zealand; and examples of how reliable operational estimates of areal evapotranspiration can be a major factor in the transformation of hydrology from a descriptive to a predictive science are described in a companion paper (Morton, 1983). The same reference presents as spinoff the minor changes required to transform the CRAE model to a CRLE (i.e. complementary relationship lake evaporation) model. Described herein are the use of the resultant CRLE model to provide operational estimates of shallow-lake evaporation from routine observations of temperature, humidity and sunshine duration (or global radiation) in the land environment; the development of a routing procedure to take into account the effects of seasonal subsurface heat storage changes in converting estimates of shallow-lake evaporation to estimates of deep-lake evaporation; a test of the results against comparable water-budget estimates for eleven lakes in North America and Africa; the development of a procedure for converting lake evaporation to pond evaporation; and ways by which operational estimates of lake evaporation, in combination with operational estimates of areal evapotranspiration, can be applied to problems of water planning and management.

The Appendix presents descriptions and data sources for the ten lakes with water-budget estimates of evaporation that are used in the development and testing of the CRLE model.

2. SHALLOW-LAKE EVAPORATION

The sequential operations for the latest version of the CRAE (complementary relationship areal evapotranspiration) models are presented in detail in a companion paper (Morton, 1983). The required station characteristics are

the latitude in degrees, the altitude in metres and a rough estimate of average annual precipitation in millimetres per year. The required climatological inputs are monthly values of dew-point temperature in degrees Celsius, air temperature in degrees Celsius and the ratio of observed to maximum possible sunshine duration. The outputs are monthly values of the net radiation that would occur if the surface were at air temperature, the potential evapotranspiration and the areal evapotranspiration, all in millimetres of evaporation or, in the case of net radiation, of evaporation equivalent. The minor changes that are needed for humidity, temperature and insolation input options and for shorter time-period options are presented in detail thereafter. The minor modifications needed to produce estimates of the net radiation for a water surface at air temperature, the potential evaporation and the shallow-lake evaporation, using the same land environment inputs, are also presented as an option (Morton, 1983).

The changes required to convert the CRAE model to a CRLE model are minor because potential evaporation differs from potential evapotranspiration and shallow-lake evaporation differs from wet-environment areal evapotranspiration only by reason of the differing effects of water surfaces and vegetated land surfaces on radiation absorption and vapour transfer characteristics. Thus the technique developed to compute potential evapotranspiration, using a rapidly converging iterative solution of the vapour transfer and energy-balance equations, is identical to the technique used to compute potential evaporation; and the once-only calibration of equations for estimating the land-surface vapour transfer coefficient and the wet-environment areal evapotranspiration, using monthly climatological data from stations in arid environments, is almost identical to the once-only calibration of equations for estimating the water-surface vapour transfer coefficient and the shallow-lake evaporation (Morton, 1983).

The net radiation for a water surface tends to be higher than the net radiation for a vegetated land surface because the effects of a lower albedo normally outweigh the effects of a higher emissivity. However, the effects of the radiation differences in making potential evaporation higher than potential evapotranspiration are offset to a large extent by the effects of a calibrated value for the land-surface vapour transfer coefficient that is 12% higher than the calibrated value for the water-surface vapour transfer coefficient. Furthermore, the effects of the radiation differences in making the shallow-lake evaporation exceed the wet-environment areal evapotranspiration are offset to a large extent by the effects of calibrated coefficients for computing wet-environment areal evapotranspiration that are 7–8% higher than the calibrated coefficients for computing shallow-lake evaporation.

The latest versions of the CRAE and CRLE models have been documented in FORTRAN and in RPN notation for the Hewlett-Packard® HP-67 hand-held calculator (Morton et al., 1980). They have also been documented in RPN notation for the Hewlett-Packard® HP-41C hand-held calculator and

this can be made available on request. The greater storage capacity of HP-41C eliminates the need for inserting extra program cards during individual computations and permits the full range of options.

A lake has been defined as a body of water so wide that the effects of the kind of upwind transitions shown in Fig. 1 are negligible. A shallow lake is one for which seasonal subsurface heat storage changes are insignificant. Estimates of subsurface heat storage changes can be incorporated into the CRLE models by adding them to the net radiation estimates in equation (C-27) of the companion paper (Morton, 1983). In practical terms, this is not possible because of the huge amounts of time, money and effort that are required for the measurement of temperature profiles with adequate spatial and temporal resolution. Seasonal subsurface heat storage changes can be taken into account in an approximate way with a routing technique that is described in Section 3. Furthermore, they can be ignored if the period of interest is one or more integral years. In other words, any lake is a shallow lake if one is interested only in the annual or mean annual evaporation.

The most recent version of the CRLE model should provide reliable estimates of shallow-lake evaporation anywhere in the world from records of temperature, humidity and sunshine duration (or global radiation) observed in the land environment with no need for locally optimized coefficients. This capability has been tested with water-budget estimates of evaporation from East Africa (Lake Victoria), from the U.S.A. (Salton Sea and Silver Lake in California, Lake Hefner in Oklahoma, Pyramid and Winnemucca Lakes in Nevada and Utah Lake in Utah), from the North American Great Lakes System (Lake Ontario) and from Canada (Last Mountain Lake in Saskatchewan and Dauphin Lake in Manitoba). Descriptions and data sources for these lakes are presented in the Appendix. The CRLE model was used to provide monthly estimates of shallow-lake evaporation for these ten lakes. To avoid the problems involved with seasonal changes in subsurface heat storage, the monthly model estimates were accumulated to provide annual or mean annual values and plotted against the comparable water-budget estimates in Fig. 4. The maximum and average absolute deviations of the model estimates from the line of equality are 100 and 49 mm yr.⁻¹, respectively, while the maximum and average absolute percentage deviations are 5.6 and 3.9%, respectively.

Pouyaud (1979) has published water-budget estimates and the required climatological inputs for Lac de Bam, a long narrow lake located at 13°20'N and 1°30'W on an intermittent tributary of the White Volta River in Upper Volta. The water-budget estimates of evaporation are questionable and incomplete because of difficulties in estimating seepage outflows and rainy-season inflows. In fact there was only one year, the year ending on September 30, 1976, with continuous water-budget estimates and the required climatological data. Based on monthly temperature, vapour pressure and sunshine duration data observed a few kilometres from the lake, the CRLE model estimates of shallow-lake evaporation for that year total 2153 mm, ~ 8% less

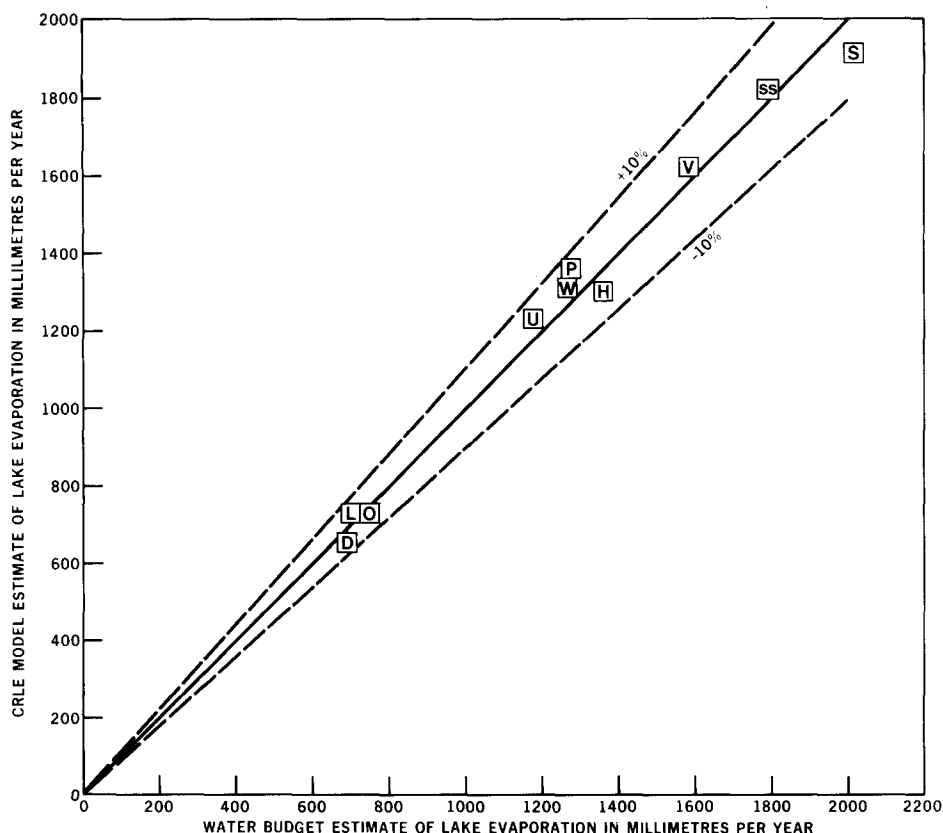


Fig. 4. Comparison of model estimates with water-budget estimates of evaporation from Lake Victoria (V), Salton Sea (ss), Silver Lake (S), Lake Hefner (H), Pyramid Lake (P), Winnemucca Lake (W), Lake Ontario (O), Last Mountain Lake (L) and Dauphin Lake (D).

than the comparable water-budget estimate of 2346 mm. Furthermore, global radiation observations for eight of the months and global radiation estimates for the remaining four months (obtained by adjusting observations from the same months of other years in accordance with differences in sunshine duration) have been used to replace the sunshine duration observations as input. The resultant CRLE model estimates of shallow-lake evaporation total 2304 mm, $\sim 2\%$ less than the comparable water-budget estimate. The estimates for Lac de Bam are not included in Fig. 4 because the difference between the estimates based on sunshine duration inputs and those based on global radiation inputs is too large to be ignored.

The comparisons between model and water-budget estimates for eleven lakes, as presented in Fig. 4 and the preceding paragraph, demonstrate that the most recent version of the CRLE models can use routine climatological observations to provide reliable estimates of annual lake evaporation over a wide range of environments with no locally derived coefficients. However, the model results do not agree nearly so well with published energy-budget

estimates for Lake Mead on the Colorado River and Lake Nasser on the Nile River. Thus the energy-budget estimate for Lake Mead during the year ending September 30, 1953 (U.S.G.S., 1958) was 2163 mm whereas the comparable model estimate, based on monthly dew-point and air temperature observations at Boulder City and monthly global radiation observations at Boulder Island, was only 1680 mm. The two quantities are not strictly comparable because the energy-budget estimates include the effects of net water-borne heat inputs, an energy term that is significant only for deep reservoirs on large rivers in hot climates, and releases of subsurface heat storage, an energy term that is usually insignificant on an annual basis. However, when these two terms are used in the CRLE model, by adding them to the net radiation estimates in equation (C-27) of the companion paper (Morton, 1983), the estimate is increased to 1855 mm yr.^{-1} , which is still almost 15% less than energy-budget estimate. This type of discrepancy is even more evident in a comparison for Lake Nasser where Omar and El-Bakry (1981) have used monthly average values of different meteorological elements over the lake to produce an energy-budget estimate of 2689 mm yr.^{-1} . This is much higher than the value of 2087 mm yr.^{-1} that is derived by using monthly averages of dew-point temperature, air temperature and global radiation recorded near the south end of the lake at Wadi Halfa (Griffiths, 1972), as input to the CRLE model. By incorporating the monthly values of net water-borne heat inputs and releases of subsurface heat storage that have been estimated by Omar and El-Bakry (1981), the model estimate is increased to 2160 mm yr.^{-1} , which is still almost 20% less than the energy-budget value.

In evaluating the foregoing results it should be noted that although the energy-budget technique is based on the law of conservation of energy, it has a number of drawbacks. These are: (1) it relies on the questionable assumption that the Bowen-ratio provides an adequate estimate of the ratio of sensible to latent heat fluxes under all the peculiar conditions of atmospheric stability that can occur over a lake; (2) the extrapolation of the inputs required for the energy-budget technique from a few measurement points to an entire lake can lead to significant error; and (3) the energy-budget technique had never been rigorously tested by applying an *identical* version to a number of lakes in different environments and comparing the results with the applicable water-budget estimates. The nearest approach to such a test has been performed in Australia (Hoy and Stephens, 1979) but this was not satisfactory because the lakes were unsuitable for water-budget studies and the energy-budget estimates were assumed to be superior. The word "identical" has been stressed because it is easy to obtain significantly different results by seemingly arbitrary selection of techniques. For example, Omar and El-Bakry (1981) computed net long-wave radiation from an equation that unrealistically ignores the effects of atmospheric water vapour, thereby producing estimates of Lake Nasser evaporation significantly higher than those that would have resulted from the use of any one of the more realistic published equations.

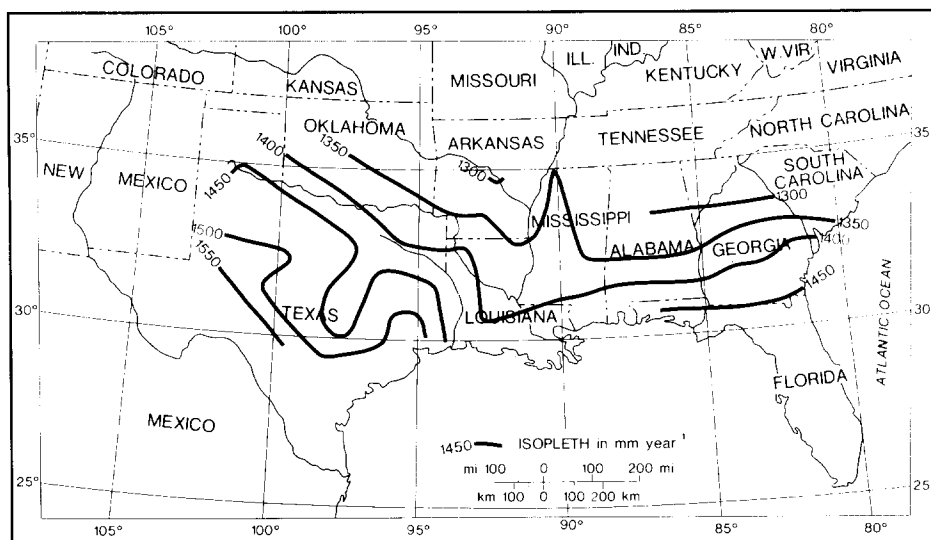


Fig. 5. Lake evaporation for the southern U.S.A. during the 5 years ending September 30, 1965.

It is easy to recognize that CRLE model estimates of annual lake evaporation would be orders of magnitude easier, faster and cheaper than energy-budget estimates. What is not so easy to accept is that such estimates may also be more realistic. However, the considerations outlined above indicate that such a conclusion is not only possible but also probable.

The test of the CRAE model (Morton, 1983) required the preparation of evapotranspiration maps for the part of Canada to the east of the Pacific Divide and for the southeastern U.S.A. This involved punching computer cards with 5 years of monthly air temperatures, dew-point temperatures, and sunshine duration ratios for 37 climatological stations in the U.S.A. and 153 climatological stations in Canada. The availability of the cards provided the incentive to prepare similar maps for lake evaporation and for the difference between lake evaporation and areal evapotranspiration.

Fig. 5 is a map of the southeastern U.S.A. showing the average annual lake evaporation for the 5 years ending September 30, 1965, and Fig. 6 is a map of the part of Canada to the east of the Pacific Divide showing the average annual lake evaporation for the 5 years ending December 31, 1969. They are based on the accumulation of monthly CRLE model estimates for all climatological stations in the two areas that report both air and dew-point temperatures. The isopleths were plotted by linear interpolation between the stations. It should be noted that the estimates shown in the two maps do not apply to bodies of water so small that the type of edge effect shown in Fig. 1 would be significant or to lakes where there are large net water-borne heat inputs.

A comparison of Fig. 5 with the map on Plate 18 of the *Water Atlas of the United States* (D.W. Miller et al., 1963) and a comparison of Fig. 6 with the

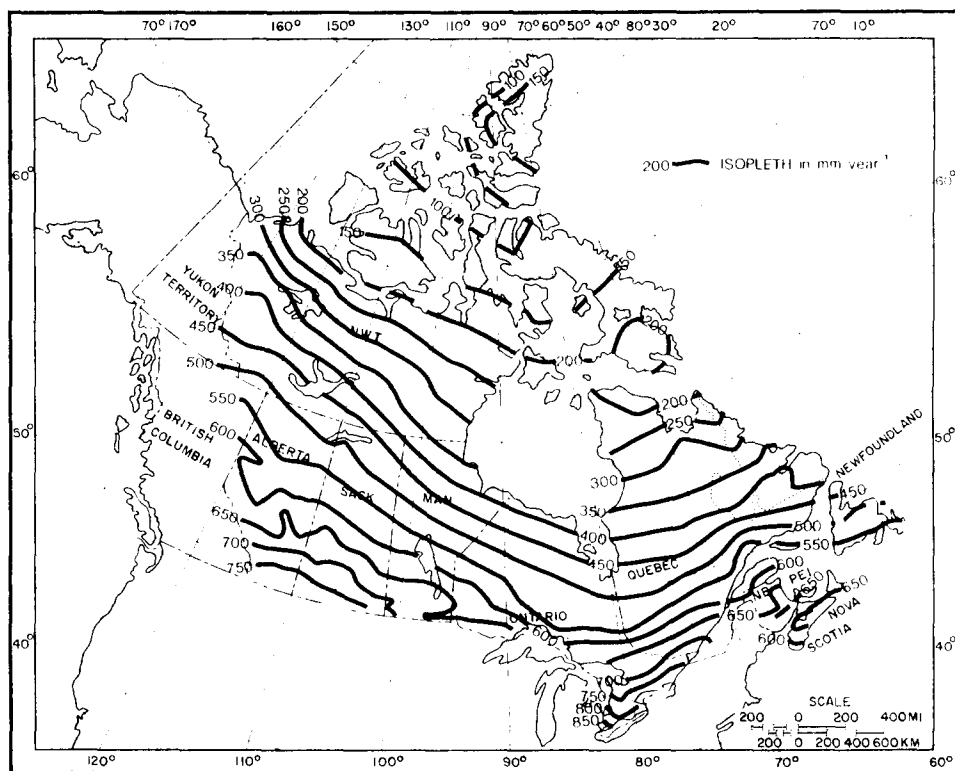


Fig. 6. Lake evaporation for the part of Canada to the east of the Pacific Divide during the 5 years ending December 31, 1969.

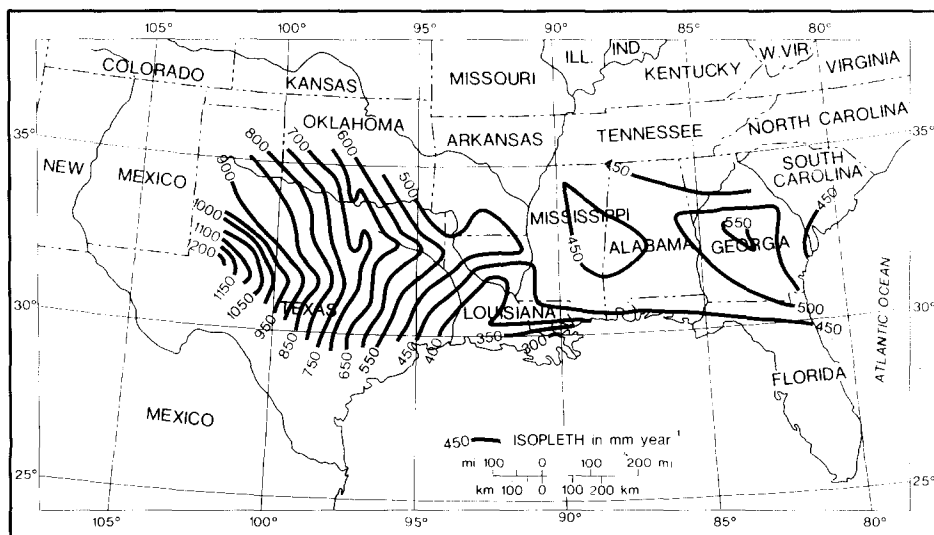


Fig. 7. Net reservoir evaporation for the southern U.S.A. during the 5 years ending September 30, 1965.

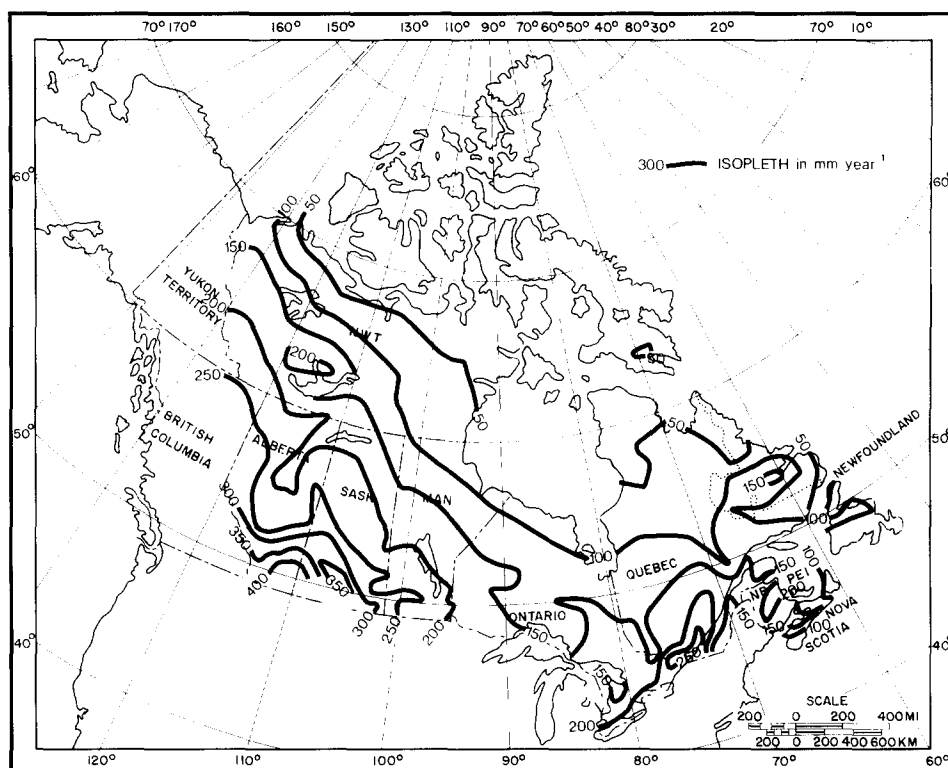


Fig. 8. Net reservoir evaporation for the part of Canada to the east of the Pacific Divide during the 5 years ending December 31, 1969.

map on plate 17 of the *Hydrological Atlas of Canada* (C.N.C.—I.H.D., 1978) indicate that the model estimates are generally higher in areas with high relative humidities and lower in areas with low relative humidities. The reasons for these discrepancies, which can be highly significant, are that all four maps are based on data from the land environment and that the CRLE model takes into account the differences between the land and the lake environments.

Fig. 7 is a map of the southeastern U.S.A., showing the difference between the average annual lake evaporation and the average annual areal evapotranspiration for the 5 years ending September 30, 1965, and Fig. 8 is a map of the part of Canada to the east of the Pacific Divide showing the difference between the average annual lake evaporation and the average annual areal evapotranspiration for the 5 years ending December 31, 1969. The lake evaporation estimates are those used to prepare Figs. 5 and 6, and the areal evapotranspiration estimates are those used in testing the areal evapotranspiration model (Morton, 1983). The isopleths were plotted by interpolation between the 190 climatological stations in the two areas that report both air and dew-point temperatures.

The purpose of Figs. 7 and 8 is to provide an estimate of net reservoir

evaporation, or the effect on the long-term water balance of constructing a reservoir when edge effects and net water-borne heat inputs are insignificant. Such estimates are often needed for water resource or environmental impact investigations. The maps probably give values that are somewhat too high because the vegetation flooded by the reservoir would tend to transpire at a higher rate than the vegetation around the airports, where climatological stations are normally located.

3. DEEP-LAKE EVAPORATION

The CRLE models do not take into account the effects of seasonal changes in subsurface heat storage so that the monthly estimates of the evaporation are realistic only for shallow lakes or when accumulated to provide annual totals. It is expected that the data required to provide physically based short-term estimates will seldom be available on a routine basis so it is fortunate that annual estimates are adequate for most engineering and hydrologic applications. However, it is possible to take subsurface heat storage into account in an approximate way by applying storage routing techniques similar to those used in routing water through natural reservoirs in hydrology. The storage (V) is related to the deep-lake evaporation (E_L) in:

$$V = kE_L [1 + 7 \exp(-E_L/12)] \quad (2)$$

in which k is the storage constant in units of months; and the constant 12 in the argument of the exponential term is in units of mm month^{-1} . The non-linearity represented by the exponential term is of significance only during high-latitude winters.

The routing process also requires a time delay (t) which is taken into account by using input data for the preceding months in the following way:

$$E_W^t = E_W^{[t]} + (t - [t])(E_W^{[t+1]} - E_W^{[t]}) \quad (3)$$

in which the E_W are the CRLE model estimates of shallow-lake evaporation and the superscripts refer to the delay time or the number of months backward it is necessary to go to obtain a value of E_W for the computations of the current month. The equation is used to estimate the value of E_W with a delay time that has both integral and fractional components, i.e. $[t]$ and $(t - [t])$, respectively, from the values of E_W for two of the preceding integral months.

The solution of the storage- and water-balance equation must be iterative because of the exponential term in eq. 2. A rapidly converging solution may be obtained using the following relationships:

$$\delta E_{LE} =$$

$$\frac{E_W^t - \frac{1}{2}(E_{LB} + E_{LE}^*) + kE_{LB}[1 + 7 \exp(-E_{LB}/12)] - kE_{LE}^*[(1 + 7 \exp(-E_{LE}^*/12))]}{\frac{1}{2} + k + 7k(1 - E_{LE}^*/12) \exp(-E_{LE}^*/12)} \quad (4)$$

$$E_{LE} = \delta E_{LE} + E_{LE}^* \quad (5)$$

in which E_{LB} is the deep-lake evaporation at the end of the preceding month; E_{LE} is the deep-lake evaporation at the end of the current month; E_{LE}^* is a trial value of E_{LE} ; and δE_{LE} is the estimated correction. Eqs. 4 and 5 are repeated until $|\delta E_{LE}| \leq 0.01 \text{ mm month}^{-1}$, a goal that is reached within four iterations. The initial value of E_{LE}^* is E_{LB} and subsequent values are those estimated from eq. 5 during the preceding iteration. In eq. 4 the numerator is the error in the water balance and the denominator is the derivative of the numerator with respect to E_{LE}^* with the sign changed.

The monthly value of deep-lake evaporation is estimated from:

$$E_L = \frac{1}{2} (E_{LB} + E_{LE}) \quad (6)$$

The routing constant k and the delay time t (both in months) are estimated from:

$$k = d [0.04 + 0.11/\{1 + (\frac{1}{16} d)^2\}] \quad (7)$$

$$t = 0.50k \quad (8)$$

in which d is the effective depth of the lake in metres. It is estimated from the average depth (d_A) in metres and the concentration of total dissolved solids (s) in ppm, using:

$$d = d_A / (1 + 0.00003s) \quad (9)$$

The derivation of eqs. 7–9 is described subsequently.

The results of any routing procedure depend on a reasonable estimate of the starting point. Although errors resulting from an arbitrary starting point wear off quite quickly it is usually worthwhile to repeat the computations for the first year a number of times and use the end-of-year results for the last trial as the final starting point. When using monthly values for a single year or average monthly values for an average year, it is worthwhile to repeat the computations until E_{LE} for the last month approaches quite closely the value for the preceding trial. This makes the annual deep-lake evaporation equal to the annual shallow-lake evaporation. The nature of the routing equation produces fast convergence so that this criterion, which has been used to produce the results presented herein, can be met with few repetitions.

Eqs. 7–9 were derived by using the monthly or average monthly water-budget estimates of evaporation and model estimates of shallow-lake

evaporation for Lake Ontario ($d_A = 86$ m and $s = 100$ ppm), Pyramid Lake ($d_A = 61$ m and $s = 3500$ ppm), Lake Hefner ($d_A = 8.2$ m and $s = 800$ ppm), Salton Sea ($d_A = 8.0$ m and $s = 37,000$ ppm) and Last Mountain Lake ($d_A = 7.6$ mm and $s = 1700$ ppm) in accordance with the procedure set out hereunder.

(1) The iterative procedure of eqs. 3–6 was used to estimate monthly or average monthly values of deep-lake evaporation for Lake Ontario and Pyramid Lake with many different combinations of storage constant (k) and delay time (t). For the two combinations that gave the best fit to the comparable water-budget estimates of each of these deep lakes, the delay time was very close to half the storage constant as shown in eq. 8.

(2) The iterative procedure was then used again to estimate monthly or average monthly values of deep-lake evaporation for Lake Ontario, Pyramid Lake, Lake Hefner, Salton Sea and Last Mountain Lake, using many different trial estimates of the storage constant and the delay time estimated from eq. 8. The five trial estimates of the storage constant that gave the best fit to the monthly water-budget estimates of each lake were used in the formulation of eqs. 7 and 9. Note that eq. 9 depends almost entirely on data for the Salton Sea where the concentration of total dissolved solids is more than ten times higher than that for Pyramid Lake, the lake with the next highest concentration.

Fig. 9 has been prepared to provide comparisons between monthly or average monthly values of shallow-lake evaporation as estimated with the latest version of the CRLE models; of deep-lake evaporation as estimated from E_w and the previously described routing procedure; and of the lake evaporation as estimated from the water budget. The comparisons are for the ten lakes described in the Appendix and used in the preparation of Fig. 4. These include the five lakes used to derive eqs. 7–9. In evaluating the results, the uncertainties in the measurement of end-of-month lake levels and their contribution to error in the monthly water-budget estimates should be kept in mind. Thus the Lake Hefner water-budget estimates for the months of February and March provide grounds for suspicion that the estimated water level for the end of February was 4–5 cm too high. From this point of view Fig. 9 shows that the application of the previously described routing procedure to the CRLE model estimates of shallow-lake evaporation can provide reasonably reliable estimates of monthly deep-lake evaporation. In this regard it may be of interest to note that the average absolute deviation from the water-budget estimates is $16.7 \text{ mm month}^{-1}$ for the deep-lake estimates as compared to $32.3 \text{ mm month}^{-1}$ for the shallow-lake estimates.

4. POND EVAPORATION

The CRLE model does not take into account the effects of increased evaporation at the upwind transition. These effects can be ignored for lakes

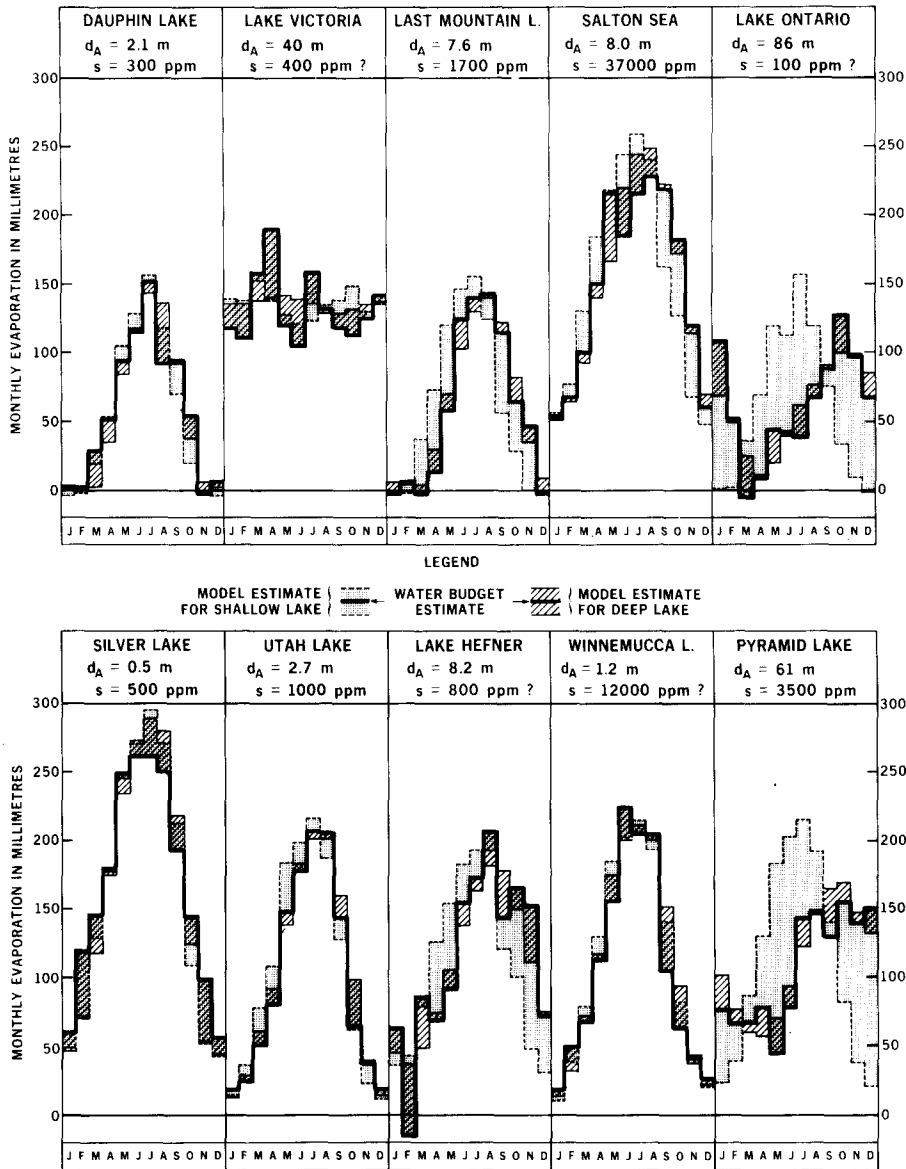


Fig. 9. Comparison of monthly model estimates of shallow-lake and deep-lake evaporation with comparable monthly water-budget estimates.

but they could be significant for ponds or other small bodies of water. The nature of the transition is shown in Fig. 1 where the dish evaporation at the upwind edge of the cotton fields is analogous to the potential evaporation in the land environment and the dish evaporation at the downwind edge of the cotton fields is approaching a low constant value that is analogous to the lake evaporation. The transition can be approximated by:

$$E_{PX} = E_L + (E_P - E_L)/(1 + X/C) \quad (10)$$

in which E_{PX} is the potential evaporation at distance X downwind of the upwind shoreline; E_P is the potential evaporation in the land environment; E_L is the deep-lake evaporation (i.e. the potential evaporation downwind of the transition); and C is a constant. In Fig. 1, the value of C is 8 m for the "wet" field, 10 m for the "moist" field and 30 m for the "dry" field. Superficially, it would seem that the value for the "wet" field is more appropriate for a lake. However, the transition may be wider when the environmental contrasts are less extreme and when there is an increase in wind speed over water rather than a decrease over cotton. Therefore the value of C is conservatively estimated to be 13 m, the geometric mean of the value for the three fields.

The average evaporation for a lake that is X m wide in the crosswind direction (E_{LX}) can be estimated from the following integration of eq. 10:

$$E_{LX} = E_L + (E_P - E_L) \frac{\ln(1 + X/C)}{X/C} \quad (11)$$

and the percent error involved in using E_L as the lake evaporation is:

$$100 (E_{LX}/E_L - 1) = 100 (E_P/E_L - 1) \frac{\ln(1 + X/C)}{X/C} \quad (12)$$

Lac de Bam, the lake in Upper Volta discussed in a preceding section, has a length of ~ 30 km roughly perpendicular to the prevailing winds and a maximum width of 800 m. The latter figure provides a reasonable estimate of the average crosswind width X . The ratio E_P/E_L for the twelve months averaged 1.56 and this provides a good estimate of the annual value for E_P/E_L . When used in eq. 13 these figures indicate that the annual CRLE evaporation estimates for Lac de Bam are $\sim 3.8\%$ too low.

5. CONCLUDING DISCUSSION

The CRLE model estimates are most sensitive to errors in the required sunshine duration or radiation inputs. They are relatively insensitive to errors in the dew-point temperature, or other optional humidity inputs, and in the air temperature inputs. Furthermore it does not matter much where in the vicinity of the lake the temperature and humidity inputs are observed because the complementary relationship automatically takes into account the effects of differing surroundings. Thus the difference between estimates derived from observations in the land environment and estimates derived from observations over the lake would be due primarily to the relatively minor effect of the difference in humidity on the estimates of net radiation.

One of the objections to the CRLE models is that they do not take into account the effects of wind speed on lake evaporation. Such an objection ignores the following considerations:

(1) Vapour is transported away from a lake in a vertical direction by wind-induced turbulence and heat-induced turbulence. Heat-induced turbulence preponderates over wind-induced turbulence during periods of high evaporation and becomes more pronounced at low wind speeds. This is one of the reasons that attempts to plot evaporation per unit vapour pressure difference against wind speed always resemble buckshot on a barn door.

(2) The effects on evaporation of an increase in wind speed are partially offset by the effects on evaporation of the resultant decrease in surface temperature.

(3) Wind speed observations in the land environment are extremely sensitive to instrument height, upwind surface roughness and nearby obstructions, so they do not necessarily provide a good estimate of wind speed over a lake. Therefore the use of routinely observed wind speeds can lead to significant error.

The foregoing observations indicate that the use of routinely observed wind speeds in estimating lake evaporation does not significantly reduce error and may quite possibly increase it.

The CRLE models do not implicitly take into account seasonal change in subsurface heat storage. In this, they are in no way inferior to the more conventional potential evaporation, pan evaporation and bulk aerodynamic techniques that also rely on data observed in the land environment. In fact, they are superior because they can readily accommodate explicit estimates of subsurface heat storage changes that have been determined from temperature soundings and when, as is usual, the results of such soundings are not available, they can be coupled with the routing technique presented herein to produce estimates of deep-lake evaporation that reflect the storage changes. Comparisons between monthly routed values of deep-lake evaporation and the corresponding water-budget values for ten lakes indicates that the routing technique provides reasonably realistic estimates.

The CRLE models and the routing technique do not take into account the kind of upwind shoreline transition shown in Fig. 1. Therefore they are applicable only to lakes. However, the results can be applied to ponds or other small bodies of water when modified using eq. 11.

The limitations and disadvantages of the CRLE models seem minor in comparison with the advantages that are summarized below:

(1) They require as input only land environment observations of temperature, humidity and sunshine duration and the results are relatively insensitive to errors in temperature and humidity.

(2) They require no local optimization or fudging of coefficients. This means that the results are falsifiable and can be tested rigorously against comparable water-budget estimates anywhere in the world. The results presented herein show good agreement with the corresponding water-budget estimates for nine lakes in North America and two lakes (including Lac de Bam) in Africa.

(3) They have a sound physical basis and are, therefore, easily adaptable

to unusual applications. Thus it is easy to make the adaptations needed to estimate the effects of heat rejection from thermal-power plants and to estimate the effects of net water-borne heat inputs to deep reservoirs on large rivers in hot, arid climates.

(4) The same input data and an almost identical model can be used to provide an estimate of the evapotranspiration that has taken place in the area where a reservoir is planned or the evapotranspiration that would have taken place if a reservoir did not exist. The difference between the estimated lake evaporation and the estimated evapotranspiration, the net reservoir evaporation, is an important quantity because it represents the effect of an existing or a planned reservoir on the water balance of a basin.

The foregoing advantages make the CRLE models much superior to the conventional potential evaporation, pan evaporation or bulk aerodynamic techniques that also rely on data observed in the land environment. With regard to the second advantage it should be noted that no other technique (including the energy-budget technique) has been tested so rigorously and therefore no other technique can be used with such confidence to provide estimates of lake evaporation anywhere in the world without the need for locally calibrated coefficients.

APPENDIX — DOCUMENTATION OF LAKES

The lakes used to develop and test the CRLE model in this and a companion paper (Morton, 1983) are documented herein in order of increasing latitude.

Lake Victoria is located in the headwaters of the White Nile River in East Africa with its centre near 1°S and 33°E . It has an altitude of ~ 1130 m above sea level, an average depth of 40 m, an area of $68,000\text{ km}^2$ and a concentration of total dissolved solids that is believed to be ~ 400 ppm. The average annual rainfall is ~ 1670 mm. Monthly water-budget estimates of evaporation and the required climatological data for the 5 years ending December 31, 1974, have been supplied by the Chief Technical Advisor of the W.M.O. Hydrometeorological Survey of Lakes Victoria, Kyoga and Mobuto Sese Seko. Kite (1981) has tabulated the average monthly values of water-budget evaporation. The CRLE model estimates are based on the averages of the monthly temperatures and dew-point temperatures recorded at Entebbe and Mwanza and the monthly values of sunshine duration averaged over the lake, using shoreline and island stations. A second set of CRLE model estimates that use the lake average values of global radiation as an input option provided results only 2% less on an average annual basis and are therefore not used separately in the test of the model.

The *Salton Sea* is located at $33^{\circ}15'\text{N}$ and $115^{\circ}50'\text{W}$ in the State of California, U.S.A. During 1961 and 1962 it was ~ 71 m below sea level with an average depth of 8.0 m and an area of 920 km^2 . The load of total dissolved solids has been observed at $\sim 37,000$ ppm. The climate is arid with an average annual precipitation between 20 and 60 mm. Monthly water-budget estimates of evaporation and the required climatological inputs for the years 1961 and 1962 have been presented by Hughes (1967). Two different sets of CRLE model estimates have been made, both based on dew-point and air temperatures observed at Sandy Beach. They differ in that one uses the sunshine duration observed at Yuma as input whereas the other uses the global and hemispheric radiation observed at Sandy Beach

as input. Since the former set of model estimates exceeds the latter set by less than 2% on an average annual basis it has been selected for use in the preparation of Figs. 4 and 9.

Silver Lake is located at $35^{\circ}25'N$ and $116^{\circ}05'W$ in the State of California, U.S.A., at an altitude of ~ 280 m above sea level. In March, 1938, the lake had an area exceeding 50 km^2 and a maximum depth of ~ 2 m but by September, 1939, most of the water in the lake had evaporated. The average depth and total dissolved solids for this period are assumed to have been 0.5 m and 500 ppm, respectively. The climate is arid with an annual rainfall of ~ 100 mm. Water-budget estimates of evaporation and observations of temperature and relative humidity for the twelve months ending April 30, 1939, have been presented by Blaney (1957). There are no sunshine duration or global radiation records available for use as input. However, the lake is almost equidistant from Las Vegas, where later global radiation observations were high but within the expected range, and Inyokern (China Lake), where later global radiation observations were abnormally high. These later records indicated that there is little year-to-year variation in global radiation in the vicinity of the lake. Therefore, the CRLE model estimates of the evaporation from Silver Lake are based on the temperature and relative humidities observed at the lake and on the monthly values of global radiation observed at Las Vegas averaged over the 5 years ending December 31, 1964.

Lake Hefner is located in the State of Oklahoma, U.S.A., at $35^{\circ}35'N$ and $97^{\circ}40'W$. It is ~ 365 m above sea level, has an average depth of 8.2 m, an average area of 10.5 km^2 and the concentration of total dissolved solids is estimated to be ~ 800 ppm. The climate is subhumid with an average annual precipitation near 790 mm. Water-budget estimates of evaporation from the lake for the 16 months ending August 31, 1951, were an important part of the well-known Lake Hefner studies (U.S.G.S., 1954). The CRLE model estimates are based on dew-point temperature, air temperature and sunshine duration records for Oklahoma City. The values of water-budget and model estimates of lake evaporation for the months of May–August are averages for two years while those for the remaining eight months are for one year only.

Pyramid Lake and *Winnemucca Lake* are located between $119^{\circ}W$ and $120^{\circ}W$ at a latitude of $40^{\circ}N$ and an altitude of ~ 1160 m above sea level in the State of Nevada, U.S.A. The climate is arid with an average annual precipitation of ~ 165 mm. The depth and area of Winnemucca Lake in early 1935 were ~ 5 m and $\sim 60 \text{ km}^2$, respectively, but by late 1938 the lake was practically dry. The average depth and the concentration of total dissolved solids are unknown but can be assumed to be 1.2 m and 13,000 ppm, respectively, without contributing to significant error. The average depth and average area of Pyramid Lake during 1935 and 1936 were estimated to be ~ 61 m and 460 km^2 , respectively, taking into account an apparent error of 9 ft. (~ 2.75 m) in the gauge readings prior to 1971 (U.S.G.S., 1974). The concentration of total dissolved solids for Pyramid Lake has been measured and the reported value was ~ 3500 ppm. Harding (1962) has presented water-budget estimates of evaporation from Pyramid Lake for the two complete years ending on December 31, 1936, and from Winnemucca Lake for 37 of the 41 months ending on September 30, 1938. The comparable CRLE model estimates are based on dew-point temperatures, air temperatures, and sunshine duration recorded at Reno, Nevada. The monthly average values of water-budget and model estimates used in Figs. 4 and 9 are for the calendar years 1935 and 1936 in the case of Pyramid Lake and for from one to four years, depending on the availability of water-budget data, in the case of Winnemucca Lake.

Utah Lake is located at $40^{\circ}10'N$ and $111^{\circ}50'W$ at an altitude of ~ 1370 m in the State of Utah, U.S.A. It has an area of 380 km^2 , an average depth of 2.7 m and concentration of total dissolved solids of ~ 1000 ppm. The climate is semi-arid with an average annual precipitation of ~ 340 mm. The water-budget estimates of monthly evaporation, averaged

over the 3 years ending June 30, 1973, have been presented by A.W. Miller and Merritt (1980). The comparable CRLE model estimates have been derived from dew-point temperature, air temperature and sunshine duration data recorded at Salt Lake City.

Lake Ontario, the furthest downstream of the North American Great Lakes, is centred at $43^{\circ}39'N$ and $77^{\circ}47'W$ with an altitude of ~ 75 m above sea level. It has an area of $19,100 \text{ km}^2$, an average depth of 86 m and a concentration of total dissolved solids of ~ 100 ppm. Water-budget estimates of the evaporation for the year ending March 31,

TABLE A-I

Components of the Last Mountain Lake water budget

Year	Month	Precipitation* ¹ (mm)	Gauged inflow* ² (mm)	Ungauged inflow* ³ (mm)	Outflow* ⁴ (mm)	Decrease in level (mm)	Evaporation (mm)
1973	Jan.	2	0	0	6	-1	-5
	Feb.	10	0	0	11	-1	-2
	Mar.	23	6	5	10	-32	-8
	Apr.	62	8	7	-19	-85	11
	May	38	10	8	-44	-93	7
	Jun.	114	17	15	-9	-14	141
	Jul.	38	11	9	-8	93	159
	Aug.	40	3	2	-5	97	147
	Sep.	41	5	4	-8	67	125
	Oct.	12	3	3	27	70	61
	Nov.	32	0	0	23	33	42
	Dec.	37	0	0	13	-11	13
Total		449	63	53	-3	123	691
1977	Jan.	8	0	0	21 ^E	13	0
	Feb.	1	0	0	10 ^E	24	15
	Mar.	10	0	0	9	3	4
	Apr.	7	1	0	-9	-1	16
	May	145	3	3	-28	-69	110
	Jun.	28	2	1	-18	57	106
	Jul.	58	0	0	-11	52	121
	Aug.	26	0	0	1	111	136
	Sep.	48	0	0	0	53	101
	Oct.	5	1	0	-2	59	67
	Nov.	21	0	0	-2 ^E	28	51
	Dec.	30	0	0	5 ^E	-40	-15
Total		387	7	4	-24	290	712

E = estimated from differences in discharges along Qu'Appelle River.

*¹ Average for Dilke, Duval, Last Mountain Wildlife Area, Lumsden, Nokomis and Strasbourg.

*² Total for Lanigan, Arm, Lewis and Saline Creeks (5793 km^2).

*³ Gauged value multiplied by ($5207 \text{ km}^2/5793 \text{ km}^2$).

*⁴ Based on Rowans Ravine averages for 10 days before and after end of month.

1973, were prepared for the I.F.Y.G.L. (International Field Year on the Great Lakes) (Witherspoon, 1978). During this year, the precipitation on the lake as estimated from shoreline stations and radar was 1025 mm. The comparable CRLE model estimates are based on dew-point and air temperatures that are averages for Toronto International, Trenton, and Kingston airports and on sunshine duration ratios that are averages for Kingston, Morven, Smithfield, Toronto, Hamilton and Vineland.

Last Mountain Lake is a long, narrow north—south lake centred at $51^{\circ}06'N$ and $105^{\circ}12'W$ in the province of Saskatchewan, Canada. It is located at an altitude of 490 m above sea level, has an area of 185 km^2 , an average width of 2.5 km, an average depth of 7.6 m and a concentration of total dissolved solids of ~ 1700 ppm. Details of the water-budget estimates of evaporation for the two relatively dry years of 1973 and 1977 are presented in Table A-I. The comparable CRLE model estimates are based on dew-point temperatures, air temperatures and sunshine duration ratios that are averages for Wynyard and the Regina, Moose Jaw and Saskatoon airports.

Dauphin Lake is located at $51^{\circ}15'N$ and $99^{\circ}46'W$ in the Province of Manitoba, Canada, at an altitude of 260 m above sea level. It has an area of 521 km^2 , an average depth of 2.0 m and the concentration of total dissolved solids has been observed at ~ 300 ppm. The average annual precipitation for the years 1967 and 1968 was between 390 and 400 mm. Details of the water-budget estimates of evapotranspiration for 1967 and 1968 have been presented elsewhere (Morton, 1979). The comparable CRLE model estimates are based on the records of dew-point temperature, air temperature and sunshine duration at Dauphin Airport.

REFERENCES

- Blaney, H.F., 1957. Evaporation study at Silver Lake in the Mojave Desert, California. EOS (Trans. Am. Geophys. Union), 38(2): 209—215.
- C.N.C.—I.H.D. (Canadian National Committee for the International Hydrological Decade), 1975. Hydrological Atlas of Canada. Supply and Services Canada, Ottawa, Ont., Plate 17.
- Davenport, D.C. and Hudson, J.P., 1967. Changes in evaporation rates along a 17-km transect in the Sudan Gezira. Agric. Meteorol., 4(5): 339—352.
- Ferguson, J., 1952. The rate of natural evaporation from shallow ponds. Aust. J. Sci. Res., A5: 315—330.
- Griffiths, J.F. (Editor), 1972. Climates of Africa, World Survey of Climatology, 10. Elsevier, Amsterdam, 604 pp. (see especially p. 122).
- Harding, S.T., 1962. Evaporation from Pyramid and Winnemucca Lakes, Nevada. J. Irrig. Drain. Div., Proc. Am. Soc. Civ. Eng., 88(IR1): 1—13.
- Hounam, C.E., 1973. Comparison between pan and lake evaporation. World Meteorol. Org. (W.M.O.), Geneva, Tech. Note No. 126, p. 15.
- Hoy, R.D. and Stephens, S.K., 1979. Field study of lake evaporation — analysis of data from phase 2 storages and summary of phase 1 and phase 2. Aust. Water Resour. Council, Canberra, A.C.T., Tech. Pap., No. 41.
- Hughes, G.H., 1967. Analysis of techniques used to measure evaporation from Salton Sea, California. U.S. Geol. Surv., Prof. Pap. 272-H.
- Kite, G.W., 1981. Recent changes in level of Lake Victoria. Hydrol. Sci. Bull., 26(3): 233—243.
- Miller, A.W. and Merritt, L.B., 1980. Utah Lake evaporation study. Eyring Res. Inst., Inc., Provo, Utah.
- Miller, D.W., Geraghty, J.J. and Collins, R.S., 1963. Water Atlas of the United States. Water Info. Cent., Port Washington, N.Y., Plate 18.

- Morton, F.I., 1979. Climatological estimates of lake evaporation. *Water Resour. Res.*, 15(1): 64–76.
- Morton, F.I., 1983. Operational estimates of areal evapotranspiration and their significance to the science and practice of hydrology. *J. Hydrol.*, 66: 1–76 (this issue; in this paper referred to as the companion paper).
- Morton, F.I., Goard, R. and Piwowar, J., 1980. Programs REVAP and WEVAP for estimating areal evapotranspiration and lake evaporation from climatological observations. *Inland Waters Dir., Environ. Can., Ottawa, Ont., NHRI Pap. No. 12.*
- Omar, M.H. and El-Bakry, M.M., 1981. Estimation of evaporation from the lake of the Aswan High Dam (Lake Nasser) based on measurements over the lake. *Agric. Meteorol.*, 23: 293–308.
- Penman, H.L., 1948. Natural evaporation from open water, bare soil and grass. *Proc. R. Soc. London, Ser. A*, 193: 120–145.
- Pouyaud, B., 1979. Étude de l'évaporation d'un lac en climat soudano-sahélien: le Lac de Bam (Haute-Volta). *Cah. O.R.S.T.O.M. (Off. Rech. Sci. Tech. Outre-Mer), Sér. Hydrol.*, 16(2): 89–143.
- U.S.G.S. (U.S. Geological Survey), 1954. Water loss investigations, Lake Hefner studies. *U.S. Geol. Surv., Prof. Pap. No. 269*, 158 pp.
- U.S.G.S. (U.S. Geological Survey), 1958. Water loss investigations, Lake Mead studies. *U.S. Geol. Surv., Prof. Pap. No. 298*, 100 pp.
- U.S.G.S. (U.S. Geological Survey), 1974. Geological Survey Water Supply-Paper 2127, 1143 pp. (see especially p. 941).
- Witherspoon, D.F., 1978. Hydrology of Lake Ontario. *Verh. Int. Ver. Limnol.*, 20: 276–279.