

FIGURE 1. Diagram of the function rlm\_dpl1\_extreme\_3a (blue graphs on the top and the left) and its polyhedral complex  $\Delta \mathcal{P}$  (gray solid lines). The set  $E(\pi)$  is the union of the faces shaded in green. The heavy diagonal green line x + y = 1 + f corresponds to the symmetry condition (the line x + y = f appears as an edge of  $F_1$ ). Vertices of  $\Delta \mathcal{P}$  do not necessarily project (dotted gray lines) to breakpoints. At the borders, the projections  $p_i(F)$  of two-dimensional additive faces are shown as gray shadows:  $p_1(F)$  at the top border,  $p_2(F)$  at the left border,  $p_3(F)$  at the bottom and the right borders.

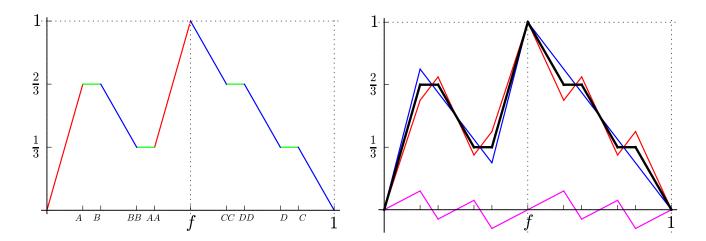


FIGURE 2. The function chen\_3\_slope\_not\_extreme is minimal, but not extreme, as proved by extremality\_test(h, show\_plots=True). The procedure first shows that for any distinct minimal  $\pi^1 = \pi + \bar{\pi}$  (blue),  $\pi^2 = \pi - \bar{\pi}$  (red) such that  $\pi = \frac{1}{2}\pi^1 + \frac{1}{2}\pi^2$ , the functions  $\pi^1$  and  $\pi^2$  are continuous piecewise linear with the same breakpoints as  $\pi$ . A finite-dimensional extremality test then finds a perturbation  $\bar{\pi}$  (magenta), as shown.

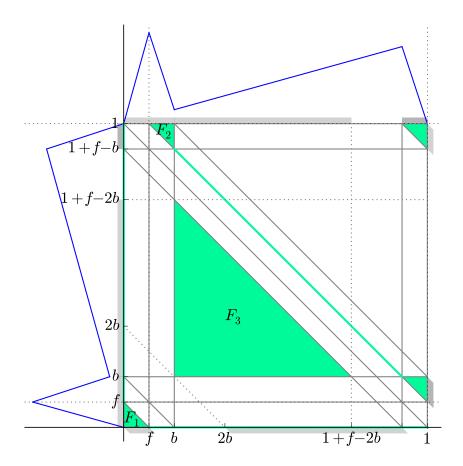


FIGURE 3. The  $drlm_backward_3_slope$  function

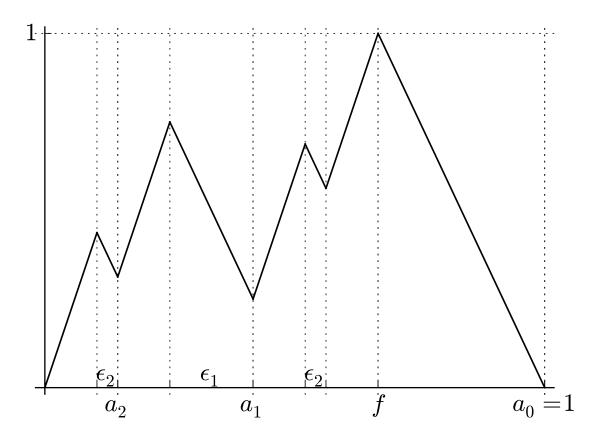


FIGURE 4. The  $kf_n_step_mir$  function