Mini notes

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1. Matrices
2. 定理型環境
3. 文字の色を変える

Norm on matrices

The standard norm of vector is the sum of quadratics of componets. What would be the matrix version? Since a matrix is also a sequence of numbers, the first guess is the same: the sum of quadratics of elements:

$$||A||^2 = \sum_{i,j} (A_{ij})^2. \tag{1}$$

We can write this formula using trace:

$$\sum_{i,j} (A_{ij})^2 = \sum_{i,j} A^T_{ji} A_{ij} \tag{2}$$

$$= \mathrm{tr} A^T A. \tag{3}$$

It is easier to see the norm is invariant under orthogonal transformation:

$$\begin{aligned} \mathrm{tr} A^T A &\to \mathrm{tr} U A U^{-1}{}^T U A U^{-1} && (4) \\ &= \mathrm{tr} U^{-1}{}^T A^T U^T U A U^{-1} && (5) \\ &= \mathrm{tr} A^T A. && (6) \end{aligned}$$

Inner product

By analogy, we can define inner product via vector norm.

$$(A, B) := \frac{1}{4}\left(||A+B||^2 - ||A-B||^2\right) \tag{7}$$

Explicitely,

$$\begin{aligned} ||A+B|| &= \mathrm{tr}(A+B)^T(A+B) && (8)\\ &= \mathrm{tr}\left(A^TA + A^TB + B^TA + B^TB\right) && (9)\\ &= ||A|| + ||B|| + 2\mathrm{tr}A^TB, && (10) \end{aligned}$$

where we used the symetry property of trace:

$$\mathrm{tr}A^TB = \mathrm{tr}B^TA. \tag{11}$$

Thus the inner product of matrices becomes

$$(A, B) = \mathrm{tr}A^TB. \tag{12}$$

Dual matrix

Inner product $(\cdot, \cdot)$ on vector space defines linear functional $\tilde{A} = (\cdot, A)$

$$(A, B) = A^* B, A^* = \mathrm{tr} \circ A^T. \tag{13}$$

Derivative with respect to matrix

We have seen that matrices can be treated as vector, and they are naturally endowed with inner product. By utilizing this structure, we can consider derivative of matrix-arg function quite naturally.
Let $f : \mathrm{Mat} \to \mathbb{R}$ and Δ be an infinitesimal matrix. Then the derivative of f is a matrix L that satisfies the following:

$$f(A+\Delta) = f(A) + (\Delta, L) + O(\Delta^2). \tag{14}$$

Theorem 2.1

定理型環境が使える。使い方は普通の LaTeX と同じ

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Proof.

証明も書ける。 □

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Example 2.2

example

文字の色を変えてみよう！

赤

文字の色を変えてみよう！

赤青

文字の色を変えてみよう！

赤青緑