The T41-EP Quadrature Sampling Detector

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I have been enjoying the process of building and understanding the T41-EP software-defined transceiver as discussed in the book "Digital Signal Processing and Software Defined Radio Transceiver - Theory and Construction of the T41-EP Amateur Radio Software Defined Transceiver", Third Edition by Al Peter and Jack Purdum. At the heart of this transceiver implementation is the quadrature-sampling detector (QSD) which which digitizes the incoming RF signals and converts them to signals close to 48kHz which are then processed using powerful digital-signal-processing techniques

In the book, the QSD is described as a 4-position switch which samples a sinusoidal waveform 4 times a cycle and which therefore produces samples which are separated in phase by 90° and which then can produce signals separated by 90° in phase; referred to as the in-phase (I) and quadrature (Q) signals. As an example, the book discusses the processing of a 7 MHz to produce two signals which are 90-degrees out of phase at a frequency of 48 kHz.

Although the explanation of the QSD in the book is perfectly correct, I got to thinking about the processing of more complex modulated signals such as a 7 MHz SSB signal modulated by a 2.5 kHz audio signal. Realizing that I couldn't think of a simply description of the result when the QSD processes such a signal and being an engineer who tends to think of things analytically ... and likes to explain them that way ... I decided to examine the analytic basis behind the QSD process.

Consider two sinusoidal signals which I will refer to as an information signal

$$s_{\mathrm{i}}(t) = A_{\mathrm{i}} \sin(\omega_{\mathrm{i}} t) \tag{1}$$

and a carrier signal

$$s_{\mathrm{c}}(t) = \sin(\omega_{\mathrm{c}} t) \tag{2}$$

Here

$$\omega_{\mathrm{i}} = 2\pi f_{\mathrm{i}} \tag{3}$$

is the angular frequency of the information signal in radians/second, f_{i} is its frequency in Hz, and A_{i} is its amplitude.

Similarly

$$\omega_{\mathrm{c}} = 2\pi f_{\mathrm{c}} \tag{4}$$

is the angular frequency of the carrier signal in radians/second, f_{c} is its frequency in Hz and, without loss of generality, I have assumed it to have unity amplitude.

If these two signals are multiplied together (a process frequently referred to as "mixing"), the result is a new signal[1]

$$\begin{aligned} m_1(t) &= A_{\mathrm{i}} \sin(\omega_{\mathrm{i}} t) \sin(\omega_{\mathrm{c}} t) \\ &= \frac{A_{\mathrm{i}}}{2}\Big(\cos\big((\omega_{\mathrm{c}} - \omega_{\mathrm{i}})t\big) - \cos\big((\omega_{\mathrm{c}} + \omega_{\mathrm{i}})t\big)\Big) \end{aligned} \tag{5}$$

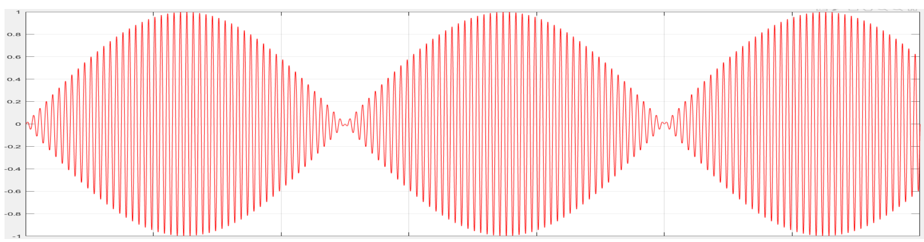

Figure 1: Plot of $m_1(t)$ of Eqn. 5

Notice that $m_1(t)$ can be thought either as the product of two signals of frequencies ω_{c} and ω_{i} with amplitude A_{i} or as the sum of two signals of angular frequencies $(\omega_{\mathrm{c}} - \omega_{\mathrm{i}})$ and $(\omega_{\mathrm{c}} + \omega_{\mathrm{i}})$ ... $(f_c - f_i)$ and $(f_c + f_i)$ in Hz ... and of amplitudes $A_{\mathrm{i}}/2$ and $-A_{\mathrm{i}}/2$ respectively. I have plotted $m_1(t)$ in Fig. 1 for the case that the carrier frequency is 100 times that of the information signal.

From Eqn. 5 we see that with both the information and carrier signals sinusoidal functions of time the result is two cosinusoidal signals at frequencies $(f_{\mathrm{c}} - f_{\mathrm{i}})$ and $(f_{\mathrm{c}} + f_{\mathrm{i}})$; i.e. the difference between the carrier and information frequencies and the sum of the carrier and information frequencies. However, if the carrier signal is shifted by 90° in phase to become cosinusoidal, e.g.

$$s_{\mathrm{c}}(t) = \cos(\omega_{\mathrm{i}} t) \tag{6}$$

multiplication of the information signal with the carrier signal results in[2]

$$\begin{aligned} m_2(t) &= A_{\mathrm{i}} \sin(\omega_{\mathrm{i}} t) \cos(\omega_{\mathrm{c}} t) \\ &= \frac{A_{\mathrm{i}}}{2}\Big(\sin\big((\omega_{\mathrm{c}} + \omega_{\mathrm{i}})t\big) - \sin\big((\omega_{\mathrm{c}} - \omega_{\mathrm{i}})t\big)\Big) \end{aligned} \tag{7}$$

[1]Here I make use of the trigonometric identity: $\sin a \sin b = 1/2(\cos(a-b) - \cos(a+b))$
[2]Here I make use of the trigonometric identity: $\sin a \cos b = 1/2(\sin(a-b) - \sin(a+b))$

In this case we see that the result is two sinusoidal signals at frequencies $(f_\mathrm{c} - f_\mathrm{i})$ and $(f_\mathrm{c} + f_\mathrm{i})$. Comparing Eqs. 5 and 7, we see that the 90° phase shift of the carrier signal carries through to a 90° phase shift in the each of the two signals in the summation result.

Thus, we see that when the same signal is multiplied by two carrier signals at the same frequency but 90° apart in phase, e.g. $\sin(\omega_\mathrm{c})$ vs. $\cos(\omega_\mathrm{c})$, the result is two sets of signals which are 90° apart in phase. **In other words, the resultant signals are in quadrature! This result is key to understanding the QSD and the signal-processing capability which it enables and we will refer to it explicitly later in this memo.**

At this point, it is most useful to switch the discussion to focus on the frequency content of signals and to talk about the frequency spectrum of the various signals. First of all, let us consider the spectrum of a single sinusoidal signal $s_1(t)$ at frequency f_0 (radian frequency $\omega_0 = 2\pi f_0$) such as

$$s_1(t) = A_\mathrm{s} \sin(\omega_0 t) \tag{8}$$

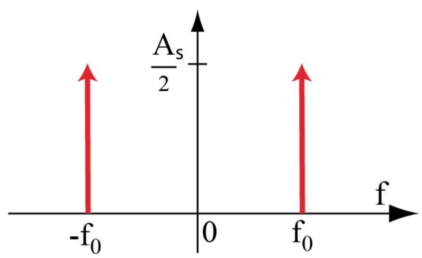

Figure 2: Amplitude spectrum of a sinusoidal signal at frequency f_0.

The amplitude spectrum of this signal is shown in Fig. 2. Notice that the plot is symmetric and shows equal components at both positive and negative frequencies. The concept of a negative frequencies may seem a bit strange at this point but it is actually a mathematical requirement of the formal spectral representation of sinusoidal signals. For example, consider a signal $s_2(t)$ of amplitude A_s but of negative frequency $-\omega_0$

$$s_2(t) = A_\mathrm{s} \sin(-\omega_0 t) = -A_\mathrm{s} \sin(\omega_0 t) \tag{9}$$

where we have used a trigonometric identity to show that $s_2(t)$ is of the same amplitude as $s_1(t)$ but shifted by 180° in phase.

Ignoring phase information greatly simplifies our discussion without the loss of any generality so we will stick with spectral amplitude representations in which all signals of a given frequency have the representation (equal amplitude positive and negative frequency components) independent of phase. Furthermore, given that we need only be concerned with the magnitude of the spectrum at any given frequency, from this point on we will only draw the spectrum for positive frequencies, much as it is displayed on the display of the T41-EP

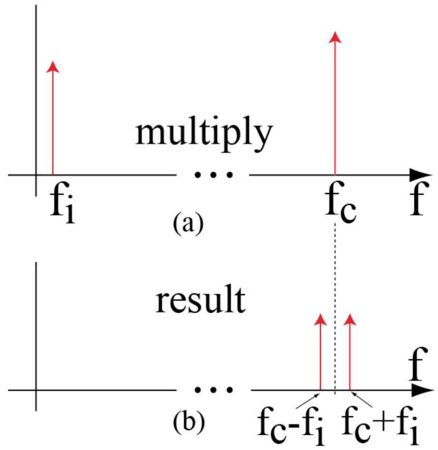

Figure 3: Spectrum of (a) the two signals $s_{\mathrm{i}}(t)$ and $s_{\mathrm{c}}(t)$ and (b) their product $m(t)$.

and to further simplify the discussion, we will also not explicitly label the amplitudes of the various frequency components.

Figure 3 (a) shows the amplitude spectrum of the signals $s_{\mathrm{i}}(t)$ and $s_{\mathrm{c}}(t)$. In Fig. 3 (b) we see the amplitude spectrum of the product signal $m(t)$. Notice that the frequency components are equal in magnitude and symmetrically located around the carrier frequency f_{c}. A signal of this type is referred to as a carrier-suppressed double-sideband signal and the two components are the lower-sideband and upper-sideband at frequencies $(f_{\mathrm{c}} - f_{\mathrm{i}})$ and $(f_{\mathrm{c}}+f_{\mathrm{i}})$ respectively. As a practical matter, the information of interest consists of information spread over a range of frequencies, for example the band of frequencies in the range produced by a human voice. Analogous to the mixing of signal $s_{\mathrm{i}}(t)$ and a carrier signal $s_{\mathrm{c}}(t)$ as shown in Fig. 3, Fig 4 shows the result of mixing a band of signals with a carrier signal at frequency f_{c}.

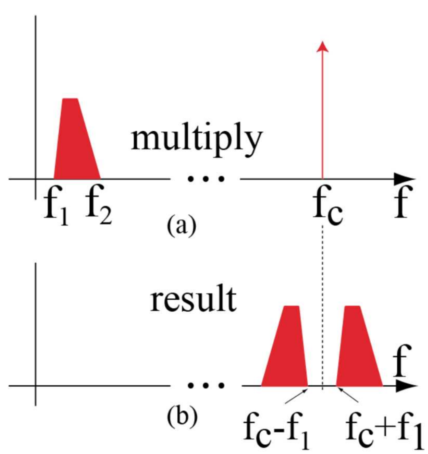

Figure 4: Analogous to Fig. 3, the spectrum of (a) a band of information signals and a carrier signal $s_{\mathrm{c}}(t)$ and (b) their product.

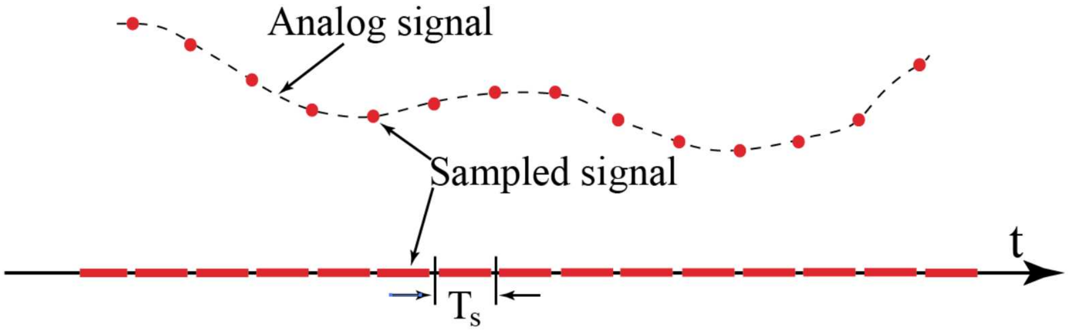

Figure 5: Sampling of an analog signal.

A key aspect of digital signal processing is that analog signals of interest are sampled prior to processing. Sampling means that (essentially) instantaneous samples of the analog signal are taken periodically in time and retained sequentially for processing by various algorithms. Let us begin by considering a sampling process where an information signal is sampled once every T_s seconds. In this case the resultant sampled signal corresponds to the value of the information signal at times equal to integral multiples of T_s and zero elsewhere as shown in Fig. 5.

This sort of sampling is equivalent to multiplying the information signal by a carrier signal which consists of impulses of magnitude 1 at times equal to integral multiples of the sampling time T_s and zero elsewhere. Such a carrier signal is illustrated by the impulse train in Fig. 6. Notice that the impulse train shown in Fig. 6 extends for all times. This is necessary for the mathematics presented here to work. Of course, practical sampling occurs over a finite period of time ... starting at the earliest when you power on your T41-EP!. An exact analysis taking this into account would greatly increase the mathematical complexity. However, as a practical matter, as long as there are a sufficient number of samples, which is easily achievable in practice, the actual performance of the sampled system does not vary significantly from the ideal.

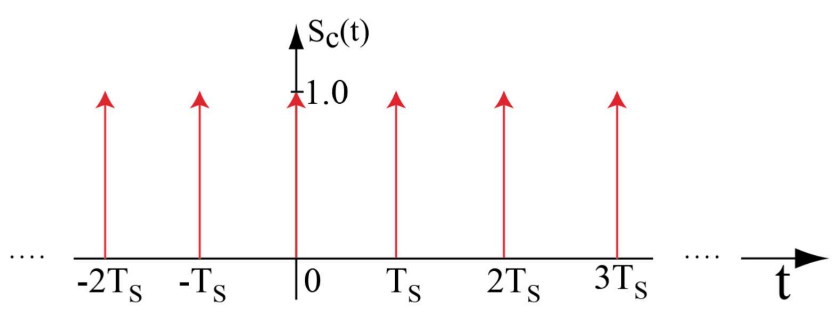

Figure 6: A unity-amplitude impulse train corresponding to a signal which produces samples periodically at every T_s seconds.

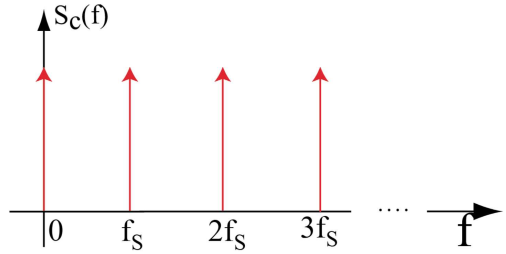

Figure 7: An impulse train in frequency corresponding to a signal which samples periodically at every T_s seconds.

NOTE: As discussed in Al Peter's and Jack Purdum's book, it is important to insure that the sampling frequency is at least twice the maximum frequency of the signal(s) being sampled (the Nyquist criterion). If this is not the case, spurious signals are generated which degrade the capability of the sampled system to reproduce the signals of interest. In the case of the T41-EP, this constraint is insured by the use of band-pass filters for each band which insure that higher-frequency signals which violate the Nyquist criterion are eliminated.

It can be shown that the frequency spectrum of a infinite train of impulses separated in time by T_s seconds is an impulse train in frequency with the impulses separated in frequency by $f_s = 1/T_s$ and which extends over all frequencies, positive, negative and zero. Figure 7 shows the spectrum of such an impulse train where, as we have discussed, we have not included the impulses at negative frequency.

In Fig. 7, we see that the sampling spectrum is simply a picket fence of sampling frequencies similar to the single sampling frequency discussed previously. Thus we see that analogous to the derivation which produced the frequency spectrum of Fig. 4, the spectrum of sampling an information signal is shown in Fig. 8, with the spectrum of the information signal found around each of the components of the sampling-signal spectrum.

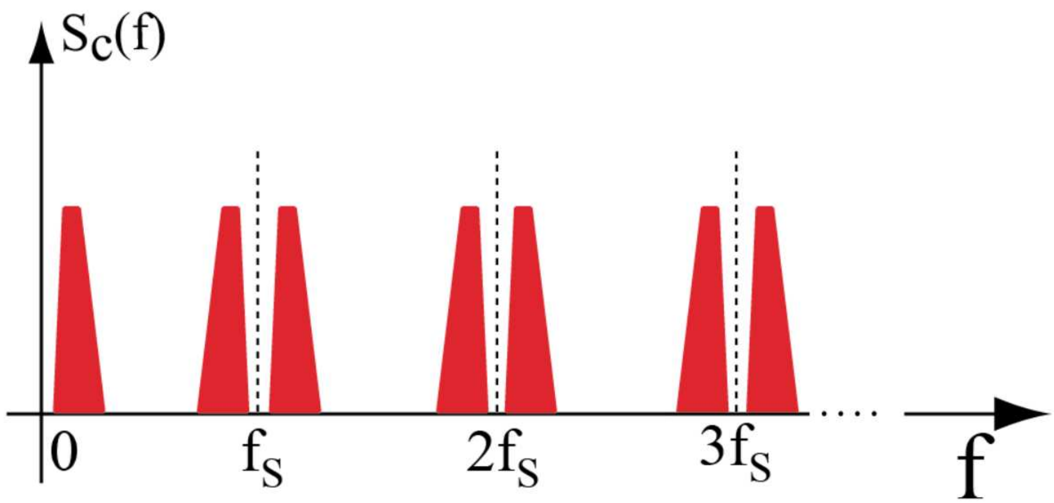

Figure 8: The spectrum of an information signal sampled at a frequency of f_s

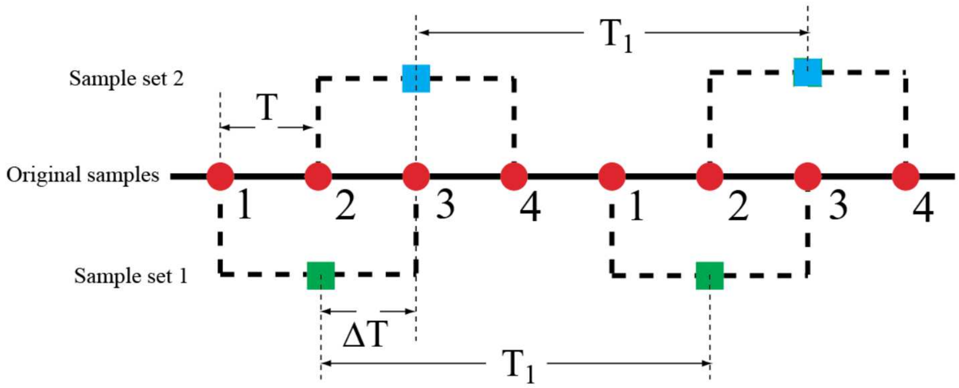

Figure 9: Timing of the sampling process in the T41-EP QSD.

At this point, we are prepared to specifically discuss the QSD of the T41-EP. Following the discussion of SDRT, let us consider an 7.0 MHz signal. The T41-EP QSD is driven by a carrier which produces samples at frequency of 28.192 MHz. The first and third samples are then combined into a single sample as are the second and fourth samples. Note that because the sampling frequency is not exactly 4 times the signal frequency, the samples will not be exactly 90° and the samples which are combined are not exactly 180° apart. As we will see, this is not a problem since the T41-EP quadrature-sampling process does not depend on the the phase spacing of the signal samples.

The in-phase (I) and quadrature (Q) signals processed in the T41-EP are produced by a clever combination of samples of the analog RF signal. Referring to Fig. 9, this will be illustrated assuming that the T41-EP is tuned to 7.0 MHz where there is a 7.0 MHz rf signal. Tuning to this frequency sets the sampling frequency of the T41-EP to 28.192 MHz, corresponding to a sampling time of $T = 1/28.192$ MHz $= 35.471$ nsec. With reference to Fig. 9, the basic process discussed in terms of processing consecutive sets of 4 samples.

Note the following:

- From each set of 4 samples, samples 1 and 3 are combined (averaged) to produce what is referred to in the figure as Sample Set 1 and samples 2 and 4 are combined (averaged) to produces Sample Set 2.
- From Fig. 9 we see that both Sample Set 1 and Sample Set 2 are themselves sets of samples of the analog RF signal separated in time by $T_1 = 4 \times T = 141.884$ nsec, correspond to a sampling frequency of 7.048 MHz.
- Finally note that the samples of Sample Sets 1 and 2 are separated by in time by one quarter of their sampling time ($\Delta T = T_1/4$), corresponding to a phase shift of 90°.

From this we see that the net effect is that the QSD has effectively generated two sets of samples of the 7.0 MHz signal sampled at a frequency of 7.048 MHz. The resultant QSD

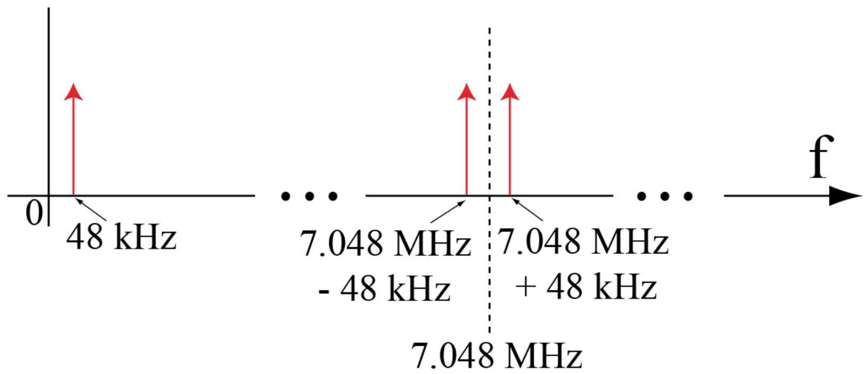

Figure 10: Spectrum of the two QSD output signals,

output spectrum is a set of upper and lower sidebands separated by 48 kHz from frequencies which are integral multiples of the derived sampling frequency of 7.048 MHz and including 0 Hz. This first two components of this spectrum are shown in Fig. 10. It is the signals around 48 kHz which will them be further processed in software to ultimately produce the desired audio output of the SDT.

As we have seen, the amplitude spectra for each of the two sets of sampled data are the same since they are both samples at the same frequency of the same signal. However, as we have seen, there is one important difference. Specifically, as we have seen, the two sample sets are separated in time corresponding to a 90° degrees phase shift. As was emphasized previously, the net result is that the QSD has generated two sets of samples which are themselves in quadrature!

Fig. 11 shows the QSD output signal spectrum for an input signal which contains a wide range of frequencies. As with the 7.0 MHz signal discussed above the spectra of the two sampled signals are in quadrature. Note that it is the spectra themselves which are in quadrature ... not the individual frequency components of the spectra.

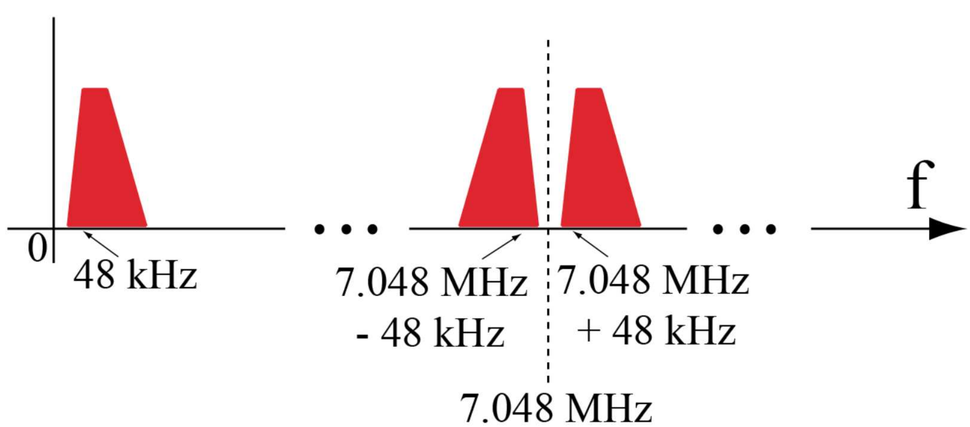

Figure 11: Spectrum of the two QSD output signals with a more complex input signal.

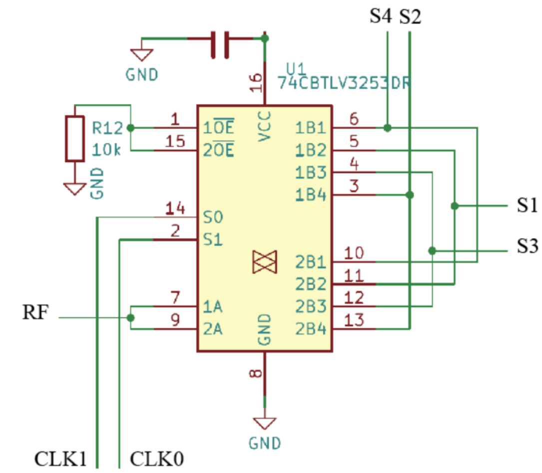

Figure 12: V12 Multiplexer.

Finally, to complete this discussion, I will discuss briefly how the QSD is implemented in the hardware of the V12 version of the T41-EP. In the context of our discussion of a 7.0 MHz signal, Fig. 12 shows the multiplexer (U11) which is used to generate the 28.192 sample set. The input RF signal is supplied simultaneously to the inputs labeled 1A and 2A. The SI5351 clock generator chip (U11) produces 2 sets of 28.192 MHz square waves labeled CLK0 and CLK1, with the CLK1 square wave being delayed by 90° from that of CLK0 as shown in Fig. 13 which also shows period of time during which each of the samples, S1, S2, S3 and S4 are output from the multiplexer[3].

[3]Note that this system varies somewhat from the description above in that the RF signal is sampled by a square wave train rather than an impulse train, somewhat complicating the analysis but not the overall picture of how the QSD works

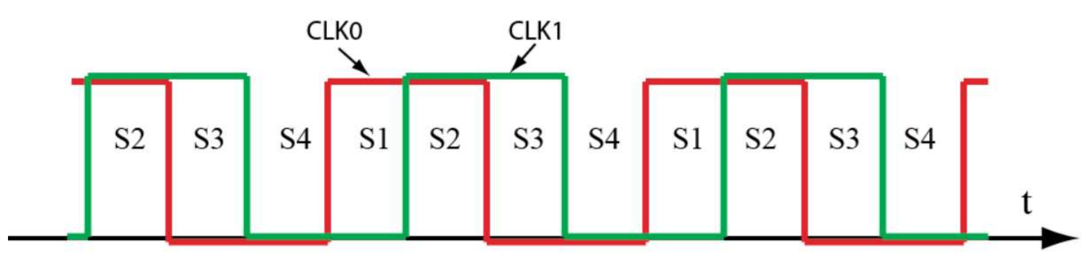

Figure 13: CLK0 and CLK1 output square waves from the SI5351 clock generator.

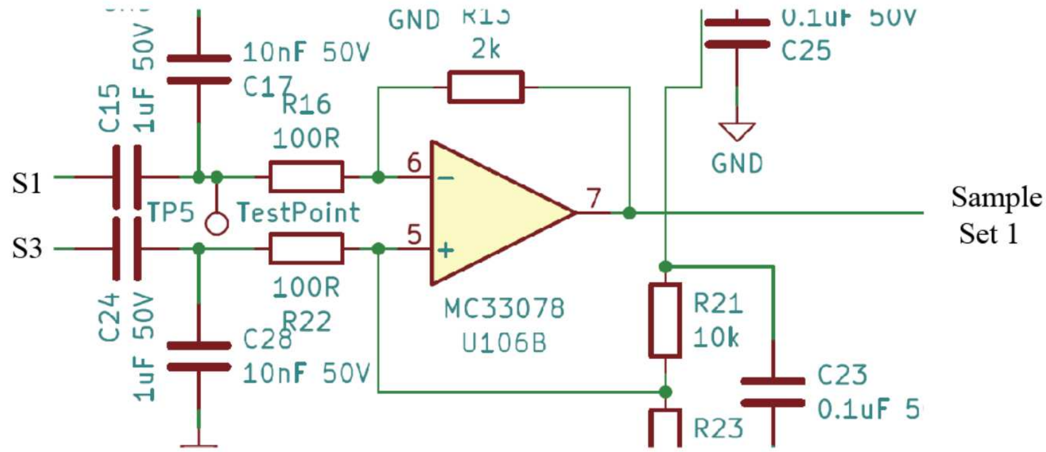

Figure 14: Combining/averaging circuitry.

Finally, Fig 14 shows the circuitry that combines samples S1 and S3 to produce Sample Set 1.