PART # 01

1. Which of the following option is not correct ?

(A) $\sqrt{3\sqrt{6\sqrt{2}}} < \sqrt{6\sqrt{2\sqrt{3}}}$

(B) $\frac{1}{1+2+3+4+...+2019} > \frac{1}{2+3+4+...+2020}$

(C) $\sqrt{3\sqrt[3]{5}} < \sqrt[3]{6\sqrt{2}}$

(D) None of these

2. Which of the following option is correct ?

(A) $\sqrt{3\sqrt{5}} > \sqrt{6\sqrt{2}}$

(B) $\frac{1}{x} > \frac{1}{y} \Rightarrow x < y$

(C) $\frac{987654321}{987654322} > \frac{98765432}{98765433}$

(D) $x > y \Rightarrow x^2 > y^2$

3. Number of real solution of $x^2 + \frac{4}{x^2} + x^4 = 1$ is

(A) 0 (B) 1 (C) 2 (D) 6

4. If (x, y) satisfy the equation $2x^2 + y^2 - 2xy - 4x + 4 = 0$, which of the following option(s) is(are) true

(A) $x + y = 4$ (B) $\frac{x}{y} = 1$ (C) $\frac{3x+y}{x+y} = 2$ (D) $x|y$

5. If x, y and z are real and different and $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$, then u is always

(A) non-negative (B) zero (C) non-positive (D) none of these

6. Find $3x + 2y + 6z$ if x, y, and z satisfy the system of equations

$$\begin{cases} (x+y):(y+z):(z+x) = 3:4:5 \\ 7x + 3y - 5z = 4 \end{cases}$$

(A) 36 (B) 42 (C) 52 (D) 63

7. Set of all the solutions of x when $|x^2 - 1| + |x| = x^2 + x - 1$ is

(A) $[-1, 1]$ (B) $(-\infty, -1] \cup [1, \infty)$ (C) $[1, \infty)$ (D) $[0, 1]$

8. Considers the equation $||x| + a| = 4$, then which of the following option(s) is(are) true

(A) if $a = 4$, equation has exactly one solution in x

(B) if $a = 0$, equation has two solutions in x

(C) if $a = 2019$, equation has no solutions in x

(D) if $a = -2020$, equation has four solutions in x

9. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval $(a, b, c \in \mathbb{R})$

(A) $\left[\frac{1}{2}, 2\right]$ (B) $[-1, 2]$ (C) $\left[-\frac{1}{2}, 1\right]$ (D) $\left[-1, \frac{1}{2}\right]$

10. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

11. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

(A) $-4 < x \le 0$ (B) $0 < x < 1$ (C) $-100 < x < 100$ (D) $-\infty < x < \infty$

12. If α and β are the roots of $10 \times 25^x - 29 \times 10^x + 10 \times 4^x = 0$, then

(A) $\alpha^2 + \beta^2 = 1$ (B) $\alpha^2 + \beta^2 = 2$ (C) $\alpha^2 + \beta^2 = 3$ (D) $\alpha^2 + \beta^2 = 4$

13. If $x^4 - 3x^3 + 4x^2 - 3x + 1 = 0$ and $x \in \mathbb{R}$, then number of x is

(A) 1 (B) 2 (C) 3 (D) 4

14. Sita's age on her birthday in 1993 is equal to the sum of the digits of her birth year. How old she was in 1993?

15. Solve $2(2\sqrt{6}+5)^x - (\sqrt{3}+\sqrt{2})^x - 1 = 0$ for real value x.

16. Solve the cubic equation $x^3 - 12x^2 + 29x - 18 = 0$.

17. Solve the equation $4\sqrt{x+1} = x + 4$.

18. Compute the largest solution to $x^2 - 5x + 2\sqrt{x^2 - 5x + 3} = 12$.

19. Find the sum of the real roots of the equation $x^2 + 12x + 16 = 2\sqrt{x^2 + 12x + 19}$.

20. Compute the largest solution to $3x^2 + 15x + 2\sqrt{x^2 + 5x + 1} = 2$.

21. Compute the smallest solution to $x^2 + 3 - \sqrt{2x^2 - 3x + 2} = \frac{3}{2}(x + 1)$.

22. Solve the equation $\sqrt{1 + \frac{9}{x}} + 4\sqrt{\frac{x}{x+9}} - 5 = 0$.

23. Find the product of all real solutions of x that satisfy $x^2 + 3x - \frac{3}{x^2 + 3x - 7} = 9$.

24. Find all real values of x that satisfy $(9x^2 - 4)^2 + (9x^2 - 4)(4x^2 - 9) + (4x^2 - 9)^2 = (13x^2 - 13)^2$.

25. Compute the largest solution to $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

26. Solve $15x^5 + 34x^4 + 15x^3 - 15x^2 - 34x - 15 = 0$.

27. Solve the real value x : $(2x^2 - 3x + 1)(2x^2 + 5x + 1) = 9x^2$.

28. Find the smallest solution of the equation $\frac{8(x^2+2x)}{x^2-1} + \frac{3(x^2-1)}{x^2+2x} = 11$.

29. Solve the real x : $(2x^2 - x - 6)^4 + (2x^2 - x - 8)^4 = 16$

30. Find all real values of x that satisfy $(16x^2 - 9)^3 + (9x^2 - 16)^3 = (25x^2 - 25)^3$.

31. Find the real solutions to the equation $(x + y)^2 = (x + 1)(y - 1)$.

32. If $x - 1 = \frac{y+1}{2} = \frac{z-2}{3}$, determine real values of x, y, and z such that $x^2 + y^2 + z^2$ has the smallest value.

33. Find $9x + 3y + 9z$ if x, y, and z satisfy the system of equations

$$\begin{cases} \frac{x}{2} = \frac{y}{3} = \frac{z}{4} \\ x + y + z = \sqrt{x + y + z + 1} + 5 \end{cases}$$

34. If $\frac{a-b}{c} = \frac{b+c}{a} = \frac{a-c}{b}$, compute all possible values of $\frac{a}{a+b+c}$.

35. Find $p = \frac{3(x-1)(y-1)(z-1)}{(x-1)^3 + (y-1)^3 + (z-1)^3}$ if $x + y + z = 3$.

36. If $\frac{(a-b)(c-d)}{(b-c)(d-a)} = -\frac{5}{3}$, find $\frac{(a-c)(b-d)}{(a-b)(c-d)}$.

37. If $x = \frac{\sqrt{2a+3b} + \sqrt{2a-3b}}{\sqrt{2a+3b} - \sqrt{2a-3b}}$, then prove that $3bx^2 - 4ax + 3b = 0$

38. If $x = \frac{8ab}{a+b}$, then value of $\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}$

39. If $x = \frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$ then the value of $\frac{x+2\sqrt{5}}{x-2\sqrt{5}} + \frac{x+2\sqrt{3}}{x-2\sqrt{3}}$ equals to

40. Solve for real x

(i) $|x^2 - 4x| = x + 1$

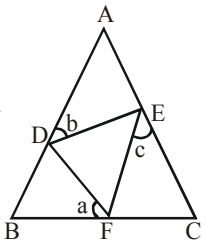
(ii) $|x^2 + 1| = |x - 2|$

(iii) $|x^2 - 16| - 8|x - 2| = x(8 - x)$

(iv) $a^2|x + a| + |a^2x + 1| = 1 - a^3$, where a is real constant.

PART # 02

1. In this diagram AB and AC are the equal sides of an isosceles triangle ABC, in which equilateral triangle DEF is inscribed . Designate angle BFD by a, angle ADE by b, and angle FEC by c. Then:



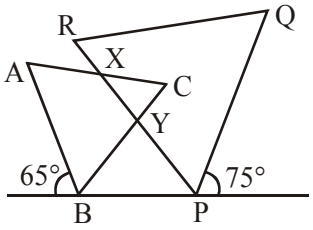
(A) $b = \frac{a+c}{2}$ (B) $b = \frac{a-c}{2}$

(C) $a = \frac{b-c}{2}$ (D) $a = \frac{b+c}{2}$

2. Three parallel lines ℓ_1, ℓ_2 and ℓ_3 (ℓ_2 in between ℓ_1 & ℓ_3) are drawn through the vertices A, B and C of a square ABCD. If the distance between ℓ_1 and ℓ_2 is 7 and between ℓ_2 and ℓ_3 is 12, then the area of the square ABCD is

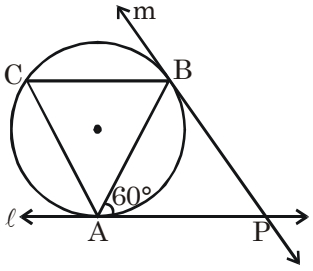
(A) 193 (B) 169 (C) 196 (D) 225

3. In the diagram if ΔABC and ΔPQR are equilateral. The $\angle CXY$ equals



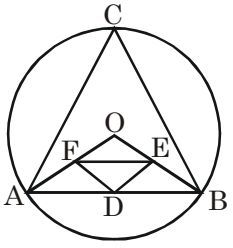
(A) 35° (B) 40° (C) 45° (D) 50°

4. In the diagram below, if ℓ and m are two tangents and AB is a chord making an angle of 60° with the tangent ℓ, then the angle between ℓ and m is



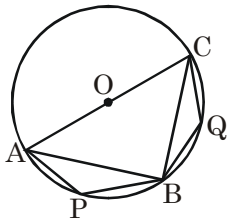
(A) 45° (B) 30° (C) 60° (D) 90°

5. In the diagram, O is the centre of the circle and D, E and F are mid points of AB, BO and OA respectively. If $\angle DEF = 30°$, then $\angle ACB$ is



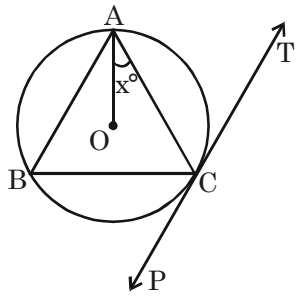
(A) 30° (B) 60° (C) 90° (D) 120°

6. In the below diagram, O is the centre of the circle, AC is the diameter and if $\angle APB = 120°$, then $\angle BQC$ is



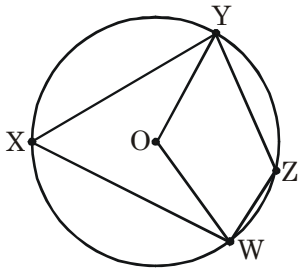
(A) 30° (B) 150° (C) 90° (D) 120°

7. In the adjoining figure, PT is a tangent at point C of the circle. O is the circumcentre of ΔABC. If $\angle ACP = 118°$, then the measure of $\angle x$ is



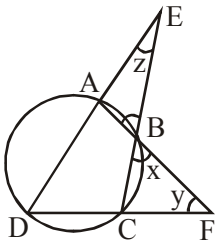
(A) 28° (B) 32° (C) 42° (D) 38°

8. In the cyclic quadrilateral WXYZ on the circle centered at O, $\angle ZYW = 10°$ and $\angle YOW = 100°$. What is the measure of $\angle YWZ$?



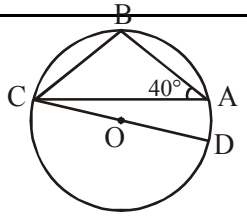
(A) 30° (B) 40° (C) 50° (D) 60°

9. In the adjacent figure, if $\angle y + \angle z = 100°$ then the measure of $\angle x$ is :



(A) 50° (B) 40° (C) 45° (D) Cannot be determined

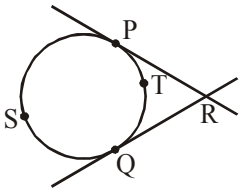
10. In the diagram, O is the centre of the circle. If $\angle BAC = 40°$, then $\angle BCD$ equals



(A) 40° (B) 60°

(C) 10° (D) 50°

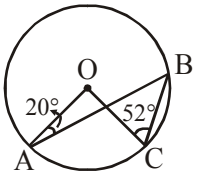
11. PR and QR are tangents to the given circle. If arc PSQ = 4 arc PTQ, then $\angle PRQ$, in degrees is



(A) 90 (B) 72

(C) 80 (D) 108

12. In the diagram, O is the centre of the circle $\angle OAB = 20°$ and $\angle OCB = 52°$. The measure of $\angle ABC$, in the degrees, is



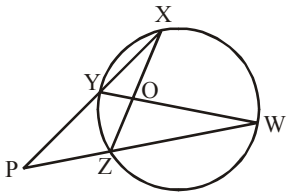
(A) 20 (B) 32

(C) 40 (D) 52

13. Two circles of unit radius touch each other and each of them touches internally a circle of radius 2 units. The radius of the circle which touches all the three circles

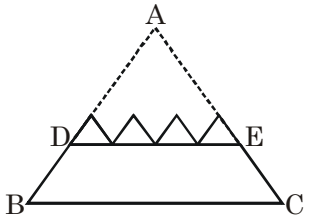
(A) 5 (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) None of these

14. Given that $\overset{\frown}{XY} = 75°$, $\overset{\frown}{XYZ} = 117°$ and $\overset{\frown}{YZW} = 173°$ in the diagram given below, then



(A) $\angle YPZ = 45°$ (B) $\angle YOZ = 77°$ (C) $\angle XYW = 56°$ (D) $\angle OWZ = 21°$

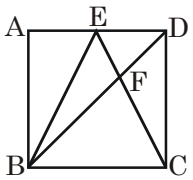
15. All the triangles in the diagram below are similar to isosceles triangle ABC in which AB = AC. Each of the smallest triangle has area 1 unit^2 and ΔABC has area 40 unit^2, then area of trapezium DBCE



(A) 16 (B) 8 (C) 24 (D) 32

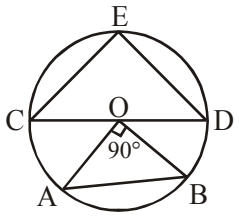
16. ABCD is a square and E is mid point of AD. If area of square is given by A_1.

(A) $\frac{\text{area of } \Delta EFD}{\text{area of } \Delta BFC} = \frac{1}{4}$ (B) $\frac{\text{area of } \Delta BEF}{\text{area of } \Delta DFC} = 1$



(C) area of $\Delta EFD = \frac{A_1}{12}$ (D) area of $\Delta BEF = \frac{A_1}{6}$

17. In the diagram 'O' is the centre of the circle. The point E lies on the circumference of the circle such that the area of the ΔECD is maximum. If $\angle AOB$ is a right angle then the ratio of the area of ΔECD to the area of the ΔAOB is :



(A) 2 : 1 (B) 3 : 2 (C) 4 : 3 (D) 4 : 1

18. If the orthocentre and circumcentre of a ΔABC do not lie inside or on the side of the triangle then the nature of the triangle can be :

(A) acute angled (B) obtuse angled

(C) right angled (D) nothing definite can be said

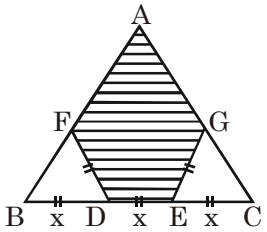
19. From a point P in the base BC of an isosceles triangle ABC, a straight line is drawn at right angles to the base cutting AB in Q & CA produced in R. Then the triangle AQR is

(A) isosceles (B) right angled (C) right isosceles (D) equilateral

20. The bases of an isoceles trapezium are 3 cm & 5 cm and one of its non-parallel sides is 7 cm length of the diagonal is :

(A) 7 (B) 8 (C) 9 (D) 10

21. ΔABC is an equilateral triangle. If BD = EG = DF = DE = EC, then the ratio of area of the shaded portion to area of ΔABC is-

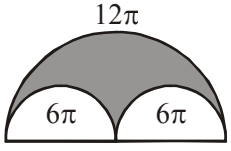


(A) $\frac{4}{11}$ (B) $\frac{7}{9}$ (C) $\frac{5}{12}$ (D) $\frac{6}{7}$

22. If the circumference of a circle is equal to perimeter of a square, then the ratio of area of circle to area of square is

(A) $\pi : 2$ (B) $2 : \pi$ (C) $4 : \pi$ (D) $\pi : 4$

23. The arc lengths of three semicircles are as shown.

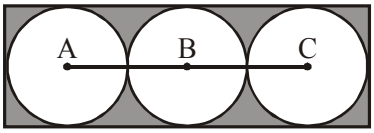


The area of the shaded region

(A) 18π (B) 54π

(C) 36π (D) 72π

24. Three equal circles, with centres A, B and C are inscribed in a rectangle, as shown. If AC = 4x, then the area of the shaded region is



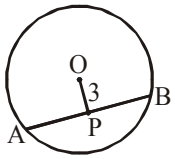
(A) $9x^2 - 3\pi x^2$ (B) $12x^2 - 3\pi x^2$

(C) $12x^2 - 6\pi x^2$ (D) $6x^2 - 6\pi x^2$

25. In ΔABC, AB = 12, BC = 10 and AD = 8, where D is mid point of side BC, then area of ΔABC is

(A) $\frac{15\sqrt{15}}{4}$ (B) $\frac{15\sqrt{15}}{2}$ (C) $\frac{15\sqrt{5}}{2}$ (D) $\frac{15\sqrt{5}}{4}$

26. In the given diagram, radius of circle is 10 and point P is mid point of chord AB, then length of AB is -

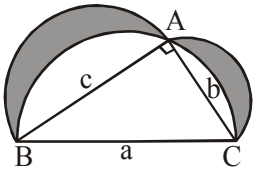


(Circle with centre O and OP = 3)

(A) $2\sqrt{82}$ (B) 4

(C) $2\sqrt{91}$ (D) 8

27. ABC is a right-angled triangle with vertex A on the semicircle drawn, with 'a' as diameter. Semicircle are also constructed as shown, with diameters 'b' and 'c'. The area of the shaded region is



(A) $\frac{\pi b^2}{2}+\frac{\pi c^2}{2}-\frac{\pi a^2}{2}$ (B) $\frac{\pi b^2}{4}+\frac{\pi c^2}{4}-\frac{\pi a^2}{4}$

(C) $\frac{\pi b^2}{8}+\frac{\pi c^2}{8}-\frac{\pi a^2}{8}+\frac{bc}{2}$ (D) $\frac{\pi a^2}{4}-\frac{bc}{2}$

28. Let a point P lies inside an equilateral ΔABC such that its perpendicular distances from sides are P_1, P_2, P_3. If side length of ΔABC is 2 unit then

(A) $(P_1 + P_2 + P_3)$ is equal to $\sqrt{3}$ (B) $(P_1 + P_2 + P_3)$ is equal to $4\sqrt{3}$

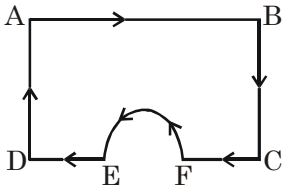
(C) Area of $\Delta ABC = \sqrt{3}$ Sq. unit (D) Area of ΔABC = 3 Sq. unit

29. In a ΔABC, if $\angle C = 90°$ and area is $\frac{1}{2}$ square units. Let minimum possible value of hypotenuse is λ then

(A) λ is irrational number (B) λ is a rational number

(C) $\lambda^4 + \lambda^2 = 6$ (D) $\lambda^4 + \lambda^2 = 3$

30. If an insect moves on the path as shown in the figure, where AB = BC = AD = 3; FC = DE = 1 and given circular segment is semicircle, then the distance covered by the insect in one complete round is

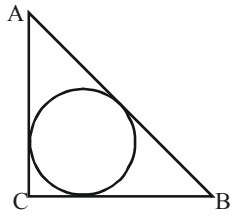


(A) $\left(11+\frac{\pi}{2}\right)$ (B) $13 + \pi$ (C) $12-\frac{\pi}{2}$ (D) greater than 12

31. Let S_1, S_2, ... be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1 sq. cm ?

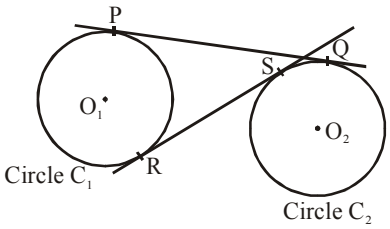
(A) 7 (B) 8 (C) 9 (D) 10

32. In triangle $ABC, \angle C = 90^\circ. AC = 6, BC = 8.$ Find the area of the regions outside the circle but inside the triangle.



(A) $12\sqrt{2}$ (B) $24 - 4\pi$ (C) 24 (D) $24 - 2\pi$

33. Let C_1 and C_2 be two circles of radius 5 and 4 respectively. Distance between centre of these circles is 11, then which of the following is/are correct ?



(A) PQ < RS (B) PQ > 11 (C) PQ < 11 (D) RS = $\sqrt{40}$

Comprehension Type :

Paragraph for Question 34 to 35

A circumcircle is a circle which passes through all vertices of a triangle and an incircle is a circle which is inscribed in a triangle touching all sides of a triangle. Let ABC be a right angled triangle whose radius of circumcircle is 5 and its one side AB = 6. The radius of incircle of triangle ABC is r.

34. Area of ΔABC is

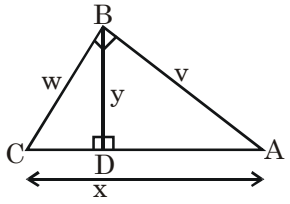
(A) 12 sq. units (B) 24 sq. units (C) 48 sq. units (D) 6 sq. units

35. The value of r is

(A) 1 (B) 2 (C) 3 (D) 4

Paragraph for Question 36 to 38

Consider the figure shown $\angle ABC$ and $\angle BDA$ are both right angles. If v + w = 35 and x + y = 37 then



On the basis of above information, answer the following questions :

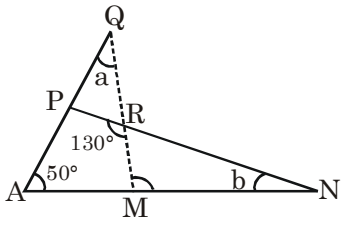
36. The value of y is

(A) 11 (B) 12 (C) 13 (D) 14

37. Area of Δ ABC is

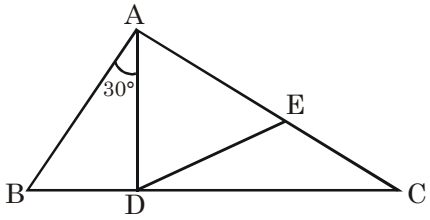
(A) 143 (B) 150 (C) 156 (D) 161

38. In the diagram, the value of (a + b) is

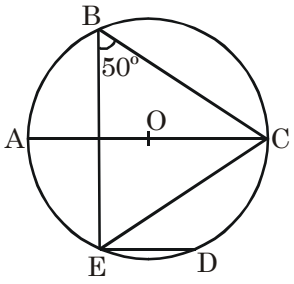


Subjective :

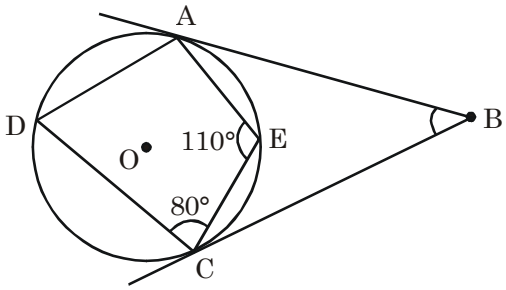
39. In the figure drawn AB = AC, ∠BAD = 30° and AE = AD, then ∠EDC is equal to



40. In the given figure, the chord ED is parallel to the diameter AC. Find ∠CED.

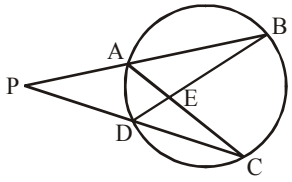


41. In the figure below, BA and BC are tangents to the circle at points A and C, respectively. If ∠AEC = 110° and ∠DCE = 80°, find ∠ABC.



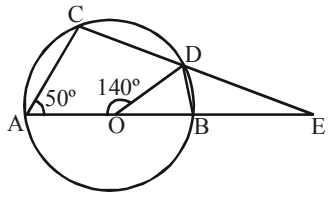
42. Given that $\overset{\frown}{AD} = 40°$, $\overset{\frown}{AB} = 120°$ and $\overset{\frown}{DC} = 100°$, find $\angle BPC$ and $\angle BEC$.

(Where $\overset{\frown}{XY}$ = Angle subtended by arc XY at centre.)



43. An acute isosceles triangle, ABC is inscribed in a circle. Through B and C, tangents to the circle are drawn, meeting at point D. If $\angle ABC = \angle ACB = 2\angle D$, find the measure of $\angle A$.

44. In the given figure, O is the centre of a circle. If $\angle AOD = 140°$ and $\angle CAB = 50°$, $\angle EDB = x°$, then value of $\left(\frac{x}{10}\right)$ is



45. In regular polygon ABCDE ... we have $\angle ACD = 120°$. How many sides does the polygon have?

46. In the given figure if C_1, C_2, C_3 are three concentric circles such that radius of C_1 and C_2 is 1 and 3 unit respectively, then radius of C_3 is