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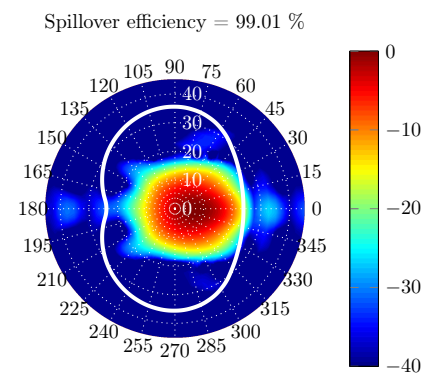
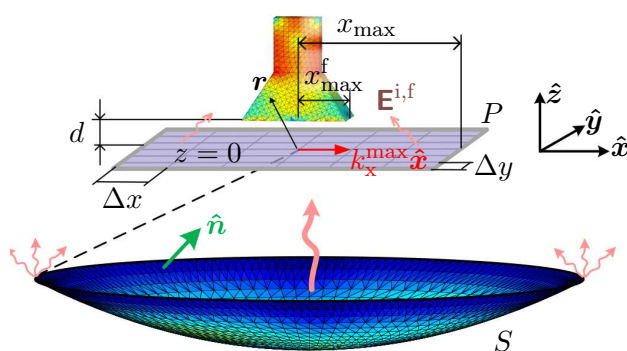
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# Antenna toolbox

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REFERENCE MANUAL

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# Radiation field processing

## 1.1 Field structure

To store a radiation field from an antenna, we use a MATLAB structure with specially named fields. We will refer to such specially crafted structure as *Field structure*.

An example of a typical Field structure is shown below. The field is stored in the spherical grid (`Field.GridType='spherical'`) with 181  $\theta$ -points and 361  $\phi$ -points. This is a far-field (`Field.NearFar='far'`) of a  $Y$ -oriented linear-polarized source (`Field.Ludwig3RefPhi=pi/2`). The field vectors are decomposed into the Ludwig 3rd polarization components (`Field.Polarization='ludwig3'`).

```
Field =
    struct with fields:
        Freq: 6.9000e+09
        E: [181x361x2 double]
        THETA: [181x361 double]
        PHI: [181x361 double]
        Polarization: 'ludwig3'
        NearFar: 'far'
        GridType: 'spherical'
        GridSymmetry: 'unsymmetrical'
        Ludwig3RefPhi: 1.5708
```

The detailed description of the structure fields is given in Table 1.1 below.

Table 1.1: Fields of the Field structure

<p>GridType</p> <p>Default: 'spherical'</p>	<p>Type of the coordinate system in which the field points are defined. It can be one of the following strings:</p> <ul style="list-style-type: none"> <li>• 'spherical' – spherical coordinate system (<math>R, \theta, \phi</math>). Point coordinates are stored in the corresponding structure fields <code>R</code> (for near-field only), <code>THETA</code> and <code>PHI</code></li> <li>• 'rectangular' – rectangular coordinate system (<math>X, Y, Z</math>). Point coordinates are stored in the corresponding structure fields <code>X</code>, <code>Y</code> and <code>Z</code>. Often field is defined in a plane, and in this case the coordinate <code>Z</code> is absent.</li> <li>• 'AzEl' – azimuth-elevation coordinate system (<math>Az, El, R</math>). Point coordinates are stored in the corresponding structure fields <code>AZ</code>, <code>EL</code> and <code>R</code> (for near-field only).</li> </ul> <p>If field <code>GridType</code> is absent, the 'spherical' grid is considered.</p>
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<p>GridSymmetry</p> <p><i>Default:</i> 'unsymmetrical'</p>	<p>Used only in conjunction with <code>GridType = 'spherical'</code>.  <code>GridSymmetry</code> shows how points are distributed in the polar <math>\theta</math>-cuts (i.e. constant <math>\phi</math>, varying <math>\theta</math>).</p> <p>Can take one of the following values:</p> <ul style="list-style-type: none"> <li>• 'symmetrical' – <math>\theta</math>-cuts are symmetrical about the pole, i.e. <math>-\theta_{\max} \leq \theta \leq +\theta_{\max}</math> for each <math>\phi = \phi_0</math>. To cover whole sphere <math>\theta</math> and <math>\phi</math> should span angles <math>-180^\circ \leq \theta \leq 180^\circ</math> and <math>0^\circ \leq \phi &lt; 180^\circ</math>.</li> <li>• 'unsymmetrical' – <math>\theta</math>-cuts are asymmetrical about the pole, i.e. <math>0^\circ \leq \theta \leq +\theta_{\max}</math> for each <math>\phi = \phi_0</math>. To cover whole sphere <math>\theta</math> and <math>\phi</math> should span angles <math>0^\circ \leq \theta \leq 180^\circ</math> and <math>0^\circ \leq \phi &lt; 360^\circ</math>.</li> <li>• 'custom' – all other cases.</li> </ul> <p>If field <code>GridSymmetry</code> is absent, the 'unsymmetrical' grid is assumed by some functions. However, it is not recommended to omit this field.</p>
<p>R , [m] (optional)          THETA , [rad]          PHI , [rad]</p> <p>OR</p> <p>X , [m]          Y , [m]          Z , [m] (optional)</p> <p>OR</p> <p>AZ , [rad]          EL , [rad]          R , [m] (optional)</p>	<p>Coordinates of the field points in the coordinate system specified in <code>GridType</code>.</p> <p><b><u>Coordinates are defined as a grid</u></b></p> <p>The coordinate grid can be regular or irregular (both are generated by MATLAB <code>meshgrid</code> function).</p> <p>A regular grid (the most common case) should be generated as:</p> <ul style="list-style-type: none"> <li>• for <code>GridType = 'spherical'</code>:  <code>[PHI THETA R] = meshgrid(PhiFirst:PhiStep:PhiLast, ThetaFirst:ThetaStep:ThetaLast, RFirst:RStep:RLast);</code>              Note that R is absent for a far-field.</li> <li>• for <code>GridType = 'rectangular'</code>:  <code>[Y X Z] = meshgrid(YFirst:YStep:YLast, XFirst:XStep:XLast, ZFirst:ZStep:ZLast);</code>              Note that Z is absent for a field in a plane grid.</li> <li>• for <code>GridType = 'AzEl'</code>:  <code>[EL AZ R] = meshgrid(ElFirst:ElStep:ElLast, AzFirst:AzStep:AzLast, RFirst:RStep:RLast);</code>              Note that R is absent for a far-field.</li> </ul> <p><b><u>Coordinates are defined as arbitrary points</u></b></p> <p>Alternatively, arbitrary points can be provided. In this case these structure fields contain just vectors, and <code>GridSymmetry</code> must be 'arbitrary'. Nevertheless this is a very general format, it is not supported by most of the toolbox functions (unless updated :)). Therefore it is strongly recommended to use grids as described above.</p>

E

Values of the electric field components<sup>1</sup>. The format of `E` depends on how the observation points/directions are defined. There are two formats:

### 1) When points are stored in a **grid**

In general `E` is 6-D array:

`Field.E(iCoor1, iCoor2, iComp, iExcit, iFreq, iCoor3),`

where

- `iCoor1` first coordinate of the grid (see [Remark 1](#))
- `iCoor2` second coordinate of the grid (see [Remark 1](#))
- `iCoor3` third coordinate of the grid (see [Remark 1](#) and [Remark 2](#))
- `iComp` component of the field (see [Polarization](#) field of the structure)
- `iExcit` index of the excitation. This allows to store several E-fields in a single structure. For example, these fields can be radiation patterns of an antenna array for different excitation scenarios; or they can be embedded element patterns of an array or other multiport antenna, etc.
- `iFreq` index of the frequency from [Freq](#) structure field.

#### Remark 1

Meaning of `iCoor1`, `iCoor2` and `iCoor3` depends on the `GridType`, i.e.

<code>GridType</code>	<code>(iCoor1, iCoor2, iCoor3)</code>
<code>'spherical'</code>	<code>(iTheta, iPhi, iR)</code>
<code>'rectangular'</code>	<code>(iX, iY, iZ)</code>
<code>'AzEl'</code>	<code>(iAZ, iEL, iR)</code>

#### Remark 2

Please note that index of the 3rd grid coordinate is in the last dimension of `E`, since it is absent in most practical cases, such as far-fields or a field on a planar grid.

### 2) When points are stored as **arbitrary points**

In this case for any `GridType` the dimensions are:

`Field.E(iPt, iComp, iExcit, iFreq),`

where `iPt` is index of the observation point.

Polarization	<p>Specifies what field components are stored in <code>E</code>. It can have one of the following values:</p> <table border="1" data-bbox="507 324 1453 1391"> <thead> <tr> <th>Polarization</th><th>iComp meaning</th></tr> </thead> <tbody> <tr> <td>'rectangular'</td><td> <code>iComp = 1</code> – <math>x</math>-component of the field;  <code>iComp = 2</code> – <math>y</math>-component of the field;  <code>iComp = 3</code> – <math>z</math>-component of the field. </td></tr> <tr> <td>'spherical'</td><td> <code>iComp = 1</code> – <math>r</math>-component of the field;  <code>iComp = 2</code> – <math>\theta</math>-component of the field;  <code>iComp = 3</code> – <math>\phi</math>-component of the field.   Note that for a far-field <math>r</math>-component of the field is zero, but these useless values are still stored in the structure (because the <math>r</math>-component corresponds to <code>iComp = 1</code>). I.e. <code>size(Field.E,3)=3</code> always. This is done for compatibility reasons. </td></tr> <tr> <td>'ludwig3'</td><td> <code>iComp = 1</code> – <math>Co</math>-component of the field;  <code>iComp = 2</code> – <math>Xp</math>-component of the field;  <code>iComp = 3</code> – <math>r</math>-component of the field.   Note that for a far-field <math>r</math>-component of the field is zero, and there is no sense to keep it in the structure. Therefore <code>size(Field.E,3)=2</code> for a far-field.   Additionally, <b>Polarization = 'ludwig3'</b> requires to specify <code>Ludwig3RefPhi</code> field of the structure.   The <b>second definition</b> from Appendix A is used to describe the Ludwig3 field components. </td></tr> <tr> <td>'circular'</td><td> <code>iComp = 1</code> – <math>RH</math>-component of the field;  <code>iComp = 2</code> – <math>LH</math>-component of the field;  <code>iComp = 3</code> – <math>r</math>-component of the field.   Note that for a far-field <math>r</math>-component of the field is zero, and there is no sense to keep it in the structure. Therefore <code>size(Field.E,3)=2</code> for a far-field. </td></tr> </tbody> </table>	Polarization	iComp meaning	'rectangular'	<code>iComp = 1</code> – $x$ -component of the field; <code>iComp = 2</code> – $y$ -component of the field; <code>iComp = 3</code> – $z$ -component of the field.	'spherical'	<code>iComp = 1</code> – $r$ -component of the field; <code>iComp = 2</code> – $\theta$ -component of the field; <code>iComp = 3</code> – $\phi$ -component of the field.  Note that for a far-field $r$ -component of the field is zero, but these useless values are still stored in the structure (because the $r$ -component corresponds to <code>iComp = 1</code> ). I.e. <code>size(Field.E,3)=3</code> always. This is done for compatibility reasons.	'ludwig3'	<code>iComp = 1</code> – $Co$ -component of the field; <code>iComp = 2</code> – $Xp$ -component of the field; <code>iComp = 3</code> – $r$ -component of the field.  Note that for a far-field $r$ -component of the field is zero, and there is no sense to keep it in the structure. Therefore <code>size(Field.E,3)=2</code> for a far-field.  Additionally, <b>Polarization = 'ludwig3'</b> requires to specify <code>Ludwig3RefPhi</code> field of the structure.  The <b>second definition</b> from Appendix A is used to describe the Ludwig3 field components.	'circular'	<code>iComp = 1</code> – $RH$ -component of the field; <code>iComp = 2</code> – $LH$ -component of the field; <code>iComp = 3</code> – $r$ -component of the field.  Note that for a far-field $r$ -component of the field is zero, and there is no sense to keep it in the structure. Therefore <code>size(Field.E,3)=2</code> for a far-field.
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Ludwig3RefPhi [rad]	<p>This angle describes polarization of the field source, and equal to <math>\xi</math> in Appendix A (see, in particular, Fig. A.2). It is used only in conjunction with <b>Polarization = 'ludwig3'</b>.</p> <p>For example, if the far-field source is <math>X</math>-oriented dipole, the co-polar unit vector coincides with the <math>X</math>-axis of the field coordinate system <code>CoordSys</code> for the broadside direction (<math>\theta = 0</math>), and it coincides with the <math>x'</math>-axis for all directions (see Fig. A.2). In this case <math>\xi = \text{Ludwig3RefPhi} = 0</math>. Then, the co-polar component is stored in <code>E(:, :, 1, :, :, :)</code> and the cross-polar component is in <code>E(:, :, 2, :, :, :)</code>. If we now change the <math>(\widehat{co}, \widehat{xp})</math>-basis by executing <code>Field = ConvertFieldPolarization(Field, 'ludwig3', 90)</code>, <math>\xi = \text{Ludwig3RefPhi}</math> will become equal to <math>\pi/2</math>, which means that vectors <math>\widehat{co}</math> and <math>\widehat{xp}</math> are <math>90^\circ</math> rotated in the primed coordinate system. Since our real co-polarized vector (radiated by the <math>X</math>-oriented dipole) coincides with <math>x'</math>-axis, its complex magnitude will be stored in <code>-E(:, :, 2, :, :, :)</code> (note the “<math>-</math>” sign), and the complex magnitude of the radiated cross-polarized vector – in <code>E(:, :, 1, :, :, :)</code>.</p>										

Freq , [Hz]	<p><b>Freq</b> contains the frequency corresponding to the field stored in this structure. It can be a vector if there are several fields stored in <b>E</b>.</p> <p>The condition <code>length(Field.Freq)==size(Field.E,5)</code> must be always satisfied.</p>						
NearFar (optional)  Default: 'far'	<p>Shows if a near-field of far-field is stored in the structure. Can take one of the values:</p> <table> <tr> <th>NearFar</th><th>Description</th></tr> <tr> <td>'near'</td><td>A near-field stored in E</td></tr> <tr> <td>'far'</td><td>A far-field stored in E.  In this case 3rd component (<i>r</i>-component) is absent in 'ludwig3' and 'circular' polarization notations. The 'spherical' notation always contains <i>r</i>-component (= 0), but it is ignored in functions.</td></tr> </table>	NearFar	Description	'near'	A near-field stored in E	'far'	A far-field stored in E.  In this case 3rd component ( <i>r</i> -component) is absent in 'ludwig3' and 'circular' polarization notations. The 'spherical' notation always contains <i>r</i> -component (= 0), but it is ignored in functions.
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CoordSys							

It worth to note that nevertheless the structure format supports storage of both far- and near-fields, most of the functions in this toolbox are designed to work only with far-fields. Therefore, some fields of the structure are omitted in most cases.

## Two definitions of Ludwig3 Field components

If we assume that the radiating antenna is X-polarized, then according to the Ludwig's 3rd definition the primed coordinate system can be defined for every direction as

$$\begin{aligned}\hat{x}' &= \hat{\theta} \cos \phi - \hat{\phi} \sin \phi \\ \hat{y}' &= \hat{\theta} \sin \phi + \hat{\phi} \cos \phi \\ \hat{z}' &= \hat{r}\end{aligned}\tag{A.1}$$

This coordinate system is depicted in Fig. A.1.

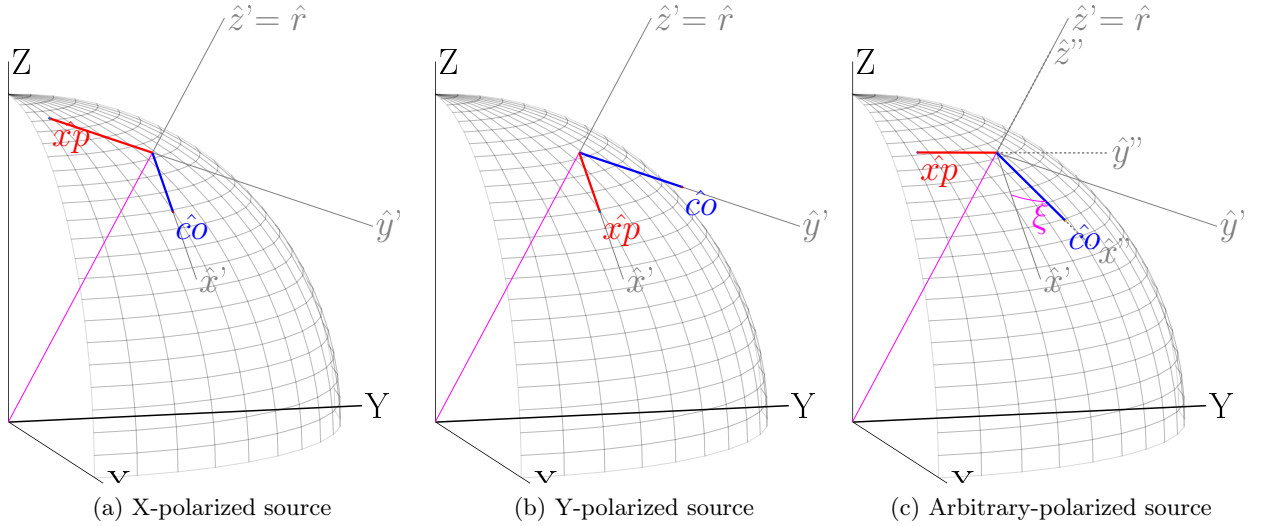


Figure A.1: Co- and Xp- basis vectors definition 1 (as in Per-Simon's book)

If we consider an antenna pattern measurement setup, the axis  $\hat{z}'$  points to the probe, and axis  $\hat{x}'$  is parallel to the probe polarization axis. In such setup the co-polarization unit vector  $\hat{c}\hat{o}$  will be aligned with the  $\hat{x}'$  axis (Fig. A.1a)

Now the cross-polarization unit vector  $\hat{x}\hat{p}$  should be defined. Since it must be orthogonal to both  $\hat{c}\hat{o}$  and  $\hat{r}$ , only two possible possibilities exist:

1. **Definition 1** (see Fig. A.1(a,b)):  $\hat{x}\hat{p} \times \hat{c}\hat{o} = \hat{r}$ . This definition is used in the Kildal's book [1]. According to it, for x-polarized sources  $\hat{c}\hat{o} = \hat{x}'$  and  $\hat{x}\hat{p} = -\hat{y}'$ ; for y-polarized sources  $\hat{c}\hat{o} = \hat{y}'$  and  $\hat{x}\hat{p} = \hat{x}'$  (see also [1, p.24]).
2. **Definition 2** (see Fig. A.2(a,b)):  $\hat{c}\hat{o} \times \hat{x}\hat{p} = \hat{r}$ . This definition is used in GRASP and in many other publications, e.g. [2], so it is more commonly used. According to it, for x-polarized sources  $\hat{c}\hat{o} = \hat{x}'$  and  $\hat{x}\hat{p} = \hat{y}'$ ; for y-polarized sources  $\hat{c}\hat{o} = \hat{y}'$  and  $\hat{x}\hat{p} = -\hat{x}'$ .

In order to define the arbitrary linear polarization of the radiated field, it is convenient to introduce the double-prime coordinate system which is rotated at angle  $\xi$  about z-axis of the primed coordinate system, see Fig. A.1c and Fig. A.2c. In this case the angle  $\xi$  is chosen such that the field polarization

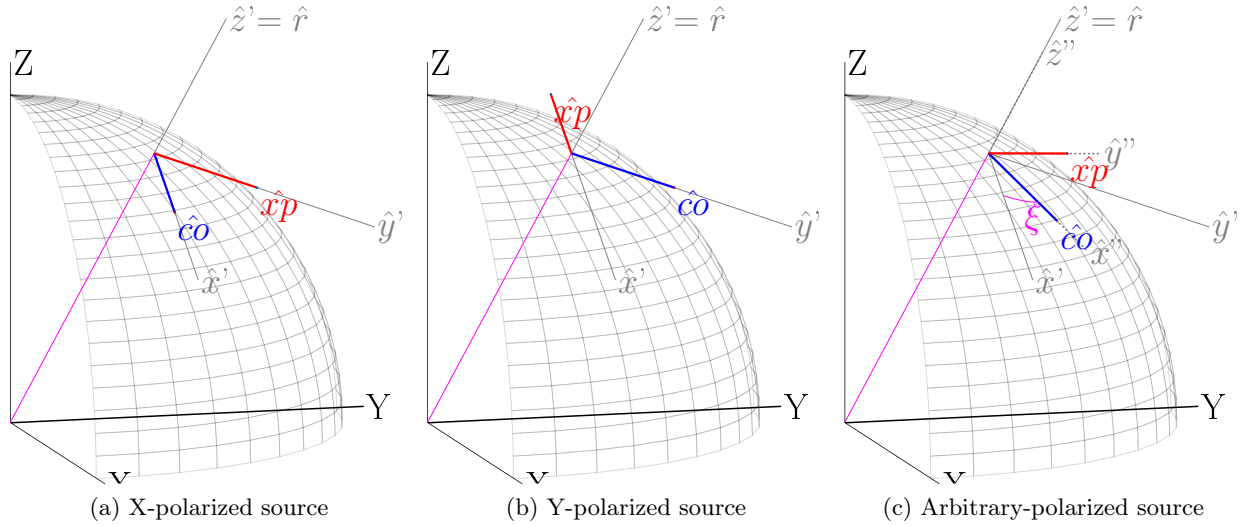


Figure A.2: Co- and Xp- basis vectors definition 2 (as in GRASP and many articles)

vector is parallel to the axis  $\hat{x}''$ . In this case for the ideal linear polarization the cross-polar component is equal to zero.

In practice the angle  $\xi$  is determined only for the boreside direction to correct the probe polarization misalignment in a measurement setup, and then this angle is used for all other directions where the radiation pattern have been measured.

Below we show the co- and cross-polar components of the field are related to the spherical components for the arbitrary polarized source.

### Definition 1

It can be seen from the Fig. A.1c that

$$\begin{aligned}\widehat{co} &= \hat{x}' \cos \xi + \hat{y}' \sin \xi \\ \widehat{xp} &= \hat{x}' \sin \xi - \hat{y}' \cos \xi\end{aligned}\tag{A.2}$$

If we substitute (A.1) into (A.2), we get

$$\begin{aligned}\widehat{co} &= \hat{\theta} \cos(\phi - \xi) - \hat{\phi} \sin(\phi - \xi) \\ \widehat{xp} &= -\hat{\theta} \sin(\phi - \xi) - \hat{\phi} \cos(\phi - \xi)\end{aligned}\tag{A.3}$$

### Definition 2

In the same way, with the help of Fig. A.2c we can write

$$\begin{aligned}\widehat{co} &= \hat{x}' \cos \xi + \hat{y}' \sin \xi \\ \widehat{xp} &= -\hat{x}' \sin \xi + \hat{y}' \cos \xi\end{aligned}\tag{A.4}$$

and if we substitute (A.1) into (A.4), we get

$$\begin{aligned}\widehat{co} &= \hat{\theta} \cos(\phi - \xi) - \hat{\phi} \sin(\phi - \xi) \\ \widehat{xp} &= \hat{\theta} \sin(\phi - \xi) + \hat{\phi} \cos(\phi - \xi)\end{aligned}\tag{A.5}$$





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## Bibliography

- [1] P.-S. Kildal. *Foundations of Antenna Engineering – A Unified Approach for Line-Of-Sight and Multipath*. Kildal Antenn AB, 2015.
- [2] Allen C. Newel and Greg Hindman. Antenna spherical coordinate systems and their application in combining results from different antenna orientations. Technical report, Nearfield Systems Incorporated, 1999. [https://www.nearfield.com/aboutus/documents/1999ESTEC\\_AN\\_GH\\_Antenna\\_Spherical\\_Coordinate\\_Systems\\_and\\_Application\\_in\\_Combining\\_Result\\_000.pdf](https://www.nearfield.com/aboutus/documents/1999ESTEC_AN_GH_Antenna_Spherical_Coordinate_Systems_and_Application_in_Combining_Result_000.pdf).