

Table 12.19

	Price option 1	Price option 2	Price option 3	
First	25	30	40	Period 1
Business	50	40	45	
Economy	50	53	65	
First	22	45	50	Period 2
Business	45	55	75	
Economy	50	60	80	
First	45	60	75	Period 3
Business	20	40	50	
Economy	55	60	75	

Use the actual demands that resulted from the prices you set in period 1 to rerun the model at the beginning of period 2 to set price levels for period 2 and provisional price levels for period 3.

Repeat this procedure with a rerun at the beginning of period 3. Give the final operational solution.

Contrast this solution to one obtained at the beginning of period 1 by pricing to maximise yield based on expected demands.

12.25 Car rental 1

A small ('cut price') car rental company, renting one type of car, has depots in Glasgow, Manchester, Birmingham and Plymouth. There is an estimated demand for each day of the week except Sunday when the company is closed. These estimates are given in Table 12.20. It is not necessary to meet all demand.

Table 12.20

	Glasgow	Manchester	Birmingham	Plymouth
Monday	100	250	95	160
Tuesday	150	143	195	99
Wednesday	135	80	242	55
Thursday	83	225	111	96
Friday	120	210	70	115
Saturday	230	98	124	80

Cars can be rented for one, two or three days and returned to either the depot from which rented or another depot at the start of the next morning. For example, a 2-day rental on Thursday means that the car has to be returned on Saturday morning; a 3-day rental on Friday means that the car has to be returned on Tuesday morning. A 1-day rental on Saturday means that the car has to be returned on Monday morning and a 2-day rental on Tuesday morning.

Table 12.21

From	To			
	Glasgow	Manchester	Birmingham	Plymouth
Glasgow	60	20	10	10
Manchester	15	55	25	5
Birmingham	15	20	54	11
Plymouth	8	12	27	53

Table 12.22

From	To			
	Glasgow	Manchester	Birmingham	Plymouth
Glasgow	—	20	30	50
Manchester	20	—	15	35
Birmingham	30	15	—	25
Plymouth	50	35	25	—

The rental period is independent of the origin and destination. From past data, the company knows the distribution of rental periods: 55% of cars are hired for one day, 20% for two days and 25% for three days. The current estimates of percentages of cars hired from each depot and returned to a given depot (independent of day) are given in Table 12.21.

The marginal cost, to the company, of renting out a car ('wear and tear', administration etc.) is estimated as follows:

1-Day hire	£20
2-Day hire	£25
3-Day hire	£30

The 'opportunity cost' (interest on capital, storage, servicing, etc.) of owning a car is £15 per week.

It is possible to transfer undamaged cars from one depot to another depot, irrespective of distance. Cars cannot be rented out during the day in which they are transferred. The costs (£), per car, of transfer are given in Table 12.22.

Ten percent of cars returned by customers are damaged. When this happens, the customer is charged an excess of £100 (irrespective of the amount of damage that the company completely covers by its insurance). In addition, the car has to be transferred to a repair depot, where it will be repaired the following day. The cost of transferring a damaged car is the same as transferring an undamaged one (except when the repair depot is the current depot, when it is zero). Again

the transfer of a damaged car takes a day, unless it is already at a repair depot. Having arrived at a repair depot, all types of repair (or replacement) take a day.

Only two of the depots have repair capacity. These are (cars/day) as follows:

Manchester	12
Birmingham	20

Having been repaired, the car is available for rental at the depot the next day or may be transferred to another depot (taking a day). Thus, a car that is returned damaged on a Wednesday morning is transferred to a repair depot (if not the current depot) during Wednesday, repaired on Thursday and is available for hire at the repair depot on Friday morning.

The rental price depends on the number of days for which the car is hired and whether it is returned to the same depot or not. The prices are given in Table 12.23 (in £).

Table 12.23

	Return to Same Depot	Return to Another Depot
1-Day hire	50	70
2-Day hire	70	100
3-Day hire	120	150

There is a discount of £20 for hiring on a Saturday so long as the car is returned on Monday morning. This is regarded as a 1-day hire.

For simplicity, we assume the following at the beginning of each day:

1. Customers return cars that are due that day
2. Damaged cars are sent to the repair depot
3. Cars that were transferred from other depots arrive
4. Transfers are sent out
5. Cars are rented out
6. If it is a repair depot, then the repaired cars are available for rental.

In order to maximise weekly profit, the company wants a ‘steady state’ solution in which the same expected number will be located at the same depot on the same day of subsequent weeks.

How many cars should the company own and where should they be located at the start of each day?

This is a case where the integrality of the cars is not worth modelling. Rounded fractional solutions are acceptable

12.26 Car rental 2

In the light of the solution to the problem stated in Section 12.25, the company wants to consider where it might be most worthwhile to expand repair capacity. The weekly fixed costs, given below, include interest payments on the necessary loans for expansion.

The options are as follows:

1. Expand repair capacity at Birmingham by 5 cars per day at a fixed cost per week of £18 000.
2. Further expand repair capacity at Birmingham by 5 cars per day at a fixed cost per week of £8000.
3. Expand repair capacity at Manchester by 5 cars per day at a fixed cost per week of £20 000.
4. Further expand repair capacity at Manchester by 5 cars per day at a fixed cost per week of £5000.
5. Create repair capacity at Plymouth of 5 cars per day at a fixed cost per week of £19 000.

If any of these options is chosen, it must be carried out in its entirety, that is, there can be no partial expansion. Also, a further expansion at a depot can be carried out only if the first expansion is also carried out, so for example option (2) at Birmingham cannot be chosen unless option (1) is also chosen. If option (2) is chosen, thereby also choosing option (1), these count as two options. Similar stipulations apply regarding the expansions at Manchester. At most three of the options can be carried out.

12.27 Lost baggage distribution

A small company with six vans has a contract with a number of airlines to pick up lost or delayed baggage, belonging to customers in the London area, from Heathrow airport at 6 p.m. each evening. The contract stipulates that each customer must have their baggage delivered by 8 p.m. The company requires a model, which they can solve quickly each evening, to advise them what is the minimum number of vans they need to use and to which customers each van should deliver and in what order. There is no practical capacity limitation on each van. All baggage that needs to be delivered in a two-hour period can be accommodated in a van. Having ascertained the minimum number of vans needed, a solution is then sought, which minimises the maximum time taken by any van.

On a particular evening, the places where deliveries need to be made and the times to travel between them (in minutes) are given in Table 12.24. No allowance