Chapter 31 Electromagnetic Fields and Waves

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The Transformation of Electric and Magnetic Fields

There is a single electromagnetic field that presents different faces, in terms of $\vec{E}$ and $\vec{B}$, to different viewers.

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A$$
$$\vec{B}_B = \vec{B}_A - \frac{1}{c^2}\vec{v}_{BA} \times \vec{E}_A$$

where $\vec{v}_{BA}$ is the velocity of reference frame B relative to frame A and where the fields are measured at the same point in space. (Only valid if $\vec{v}_{BA} \ll c$)

The Displacement Current

A changing magnetic field causes an induced electric field and vice versa.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0\Big(I_{\text{through}} + I_{\text{disp}}\Big) = \mu_0\Big(I_{\text{through}} + \epsilon_0\frac{d\Phi_e}{dt}\Big)$$

$$I_{\text{disp}} = \epsilon_0\frac{d\Phi_e}{dt}$$

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \quad \text{Gauss's Law}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss's Law for Magnetism}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \quad \text{Faraday's Law}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} \quad \text{Ampere-Maxwell Law}$$

General Force Equation

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{Lorentz Force Law}$$

1. **Gauss's Law**: Charged particles create an electric field.
2. **Gauss's Law for Magnetism**: There are no isolated magnetic poles.
3. **Faraday's Law**: An electric field can also be created by a changing magnetic field.
4. **Ampere-Maxwell Law**: Currents and a changing electric field can each create a magnetic field.
5. **Lorentz Force Law**: An electric force is exerted on a charged particle in an electric field and a magnetic force is exerted on a charge moving in a magnetic field.

Electromagnetic Waves

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 E_y}{\partial x^2} \quad \text{(the wave equation for electromagnetic waves)}$$

$$v_{\text{em}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s} = c$$

1. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and each to the direction of travel.
2. $E = cB$ at any point on the wave.

Right-hand rule

1. Point index finger in the direction of electric field
2. Point middle finger in the direction of magnetic field
3. Point thumb in the direction of motion

Wave Key Points

$$E(x,t) = E_0 \cos(kx - \omega t + \phi)$$

1. $k = \frac{2\pi}{\lambda}$ where k is wave number and λ is wavelength
2. $T = \frac{2\pi}{\omega}$ where T is period and ω is angular frequency
3. $f = \frac{1}{T}$ where f is frequency
4. $v = f\lambda$ where v is the propagation speed
5. $v = \frac{E_0}{B_0}$ where E_0 and B_0 are the electric and magnetic field components

Properties of Electromagnetic Waves

The intensity, average energy transfer, of an electromagnetic wave is

$$I = \frac{P}{A} = S_{\text{avg}} = \frac{1}{2c\mu_0}{E_0}^2 = \frac{c\epsilon_0}{2}{E_0}^2$$

$$\frac{E_1}{E_2} = \frac{d_2}{d_1}$$

Polarization

$$I_{\text{transmitted}} = I_0 \cos^2\theta$$