Chapter 27 Current and Resistance

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The Electron Current

Current is the flow of charge through a conductor.

Charge Carriers

The drift speed is the net motion of the individual electrons through a conductor, which is typically $v_d = 10^{-4} m/s$. The number of electrons N_e of electrons that pass through the cross section during the time interval Δt is

$$N_e = i_e \Delta t$$

where i_e is the electron current which is the number of electrons per second that passes through a cross section of a conductor.

The electrons travel distance $\Delta x = v_d \Delta t$, forming a cylinder of charge with volume $V = A\Delta x$. If the electron density is n_e electrons per cubic meter, then the total number of electrons in the cylinder is

$$N_e = n_e V = n_e A \Delta x = n_e A v_d \Delta t$$

resulting in

$$i_e = n_e A v_d$$

Electron density in metals

Metal	Electron density (m^{-3})
Aluminum	18×10^{28}
Iron	17×10^{28}
Copper	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

Creating a Current

An electron current is a net motion of charges sustained by an internal electric field.

At the moment a positive charged wire and a negative charged wire are connected, the surface charge density varies from positive at the positive capacitor plate through zero at the midpoint to negative at the negative plate. The varying surface charge distribution creates an internal electric field inside the wire, causing a current.

$$i_e = \frac{n_e e \tau A}{m} E$$

where τ is the mean time between collisions.

Current and Current Density

If Q is the total amount of charge that has moved past a point in the wire, we define the current I in the wire to be the rate of charge flow

$$I \equiv \frac{dQ}{dt} A = \text{ampere} = C/s$$

For a steady current, the current I during the time interval Δt is

$$I = \frac{Q}{\Delta t} = \frac{eN_e}{\Delta t} = ei_e$$

The current density J in a wire is the current per square meter of cross section

$$J = \text{current density} \equiv \frac{I}{A} = n_e e v_d$$

Key Points

- The current I is opposite of the direction of motion of the electrons in a metal.
- The current into a junction between wires must equal the current out of it.

Conductivity and Resistivity

$J = n_e e v_d$ and $v_d = e\tau E/m$, resulting in the current density being

$$J = \frac{n_e e^2 \tau}{m} E$$

Since $n_e e^2 \tau/m$ depends only on the conducting material, the conductivity σ of a material is

$$\sigma = \frac{n_e e^2 \tau}{m}$$

Using conductivity,

$$J = \sigma E$$

Metal	Resistivity (Ωm)	Conductivity ($\Omega^{-1} m^{-1}$)
Aluminum	2.8×10^{-8}	3.5×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}

where the resistivity $\rho = 1/\sigma$ is the inverse of the conductivity.

Metals become better conductors at lower temperatures since they have higher conductivity and lower resistivity. The complete loss of resistance at low temperatures is called superconductivity.

Resistance and Ohm's Law

Since $E = \Delta V/L$,

$$I = \frac{A}{\rho L} \Delta V$$

showing that the current is directly proportional to the potential difference between the ends of a conductor.

The resistance is $r = \rho L/A$, which results in current the current through a conductor being

$$I = \frac{\Delta V}{R} \quad \text{(Ohm's Law)}$$

1. The potential difference ΔV_{wire} from the battery causes an electric field $E = \Delta V_{\text{wire}} L$ in the wire.
2. The electric field establishes a current $I = JA = \sigma A E$ in the wire.
3. The magnitude of the current is determined jointly by the battery and the wire's resistance to be $I = \Delta V_{\text{wire}}/R$.

Temperature Coefficient of Resistance

$$R = R_0 \times (1 + \alpha \times (T - T_0))$$

where R is the resistance at temperature T, R_0 is the resistance at temperature T_0, and α is the temperature coefficient of resistance