Polar Coordinates

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Converting Points between Coordinate Systems

Given a point P in the plane with Cartesian coordinates (x, y) and polar coordinates (r, θ), the following conversion formulas hold true.

$$x = r\cos\theta \quad y = r\sin\theta \quad r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$$

Common Polar Equations

- Line passing through the pole with slope $\tan K$.

$$\theta = K$$

- Circle

$$r = a\cos\theta + b\sin\theta$$

- Spiral

$$r = a + b\theta$$

- Cardioid

$$r = a(1+\cos\theta) \quad \text{or} \quad r = a(1+\sin\theta) \quad \text{or} \quad r = a(1-\cos\theta) \quad \text{or} \quad r = a(1-\sin\theta)$$

- Limaçon

$$r = a\cos\theta + b \quad \text{or} \quad r = a\sin\theta + b$$

- Rose

$$r = a\cos(b\theta) \quad \text{or} \quad r = a\sin(b\theta)$$

Symmetry in Polar Curves and Equations

Consider a curve generated by the function $r = f(\theta)$ in polar coordinates.

1. The curve is symmetric about the polar axis if for every point (r, θ) on the graph, the point $(r, -\theta)$ is also ont eh graph. Similarly, the equation $r = f(\theta)$ is unchanged by replacing θ with $-\theta$.
2. The curve is symmetric about the pole if for every point (r, θ) on the graph, the point $(r, \pi+\theta)$ is also on the graph. Similarly, the equation $r = f(\theta)$ is unchanged when replacing r with $-r$, or θ with $\pi + \theta$.
3. The curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$ if for every point (r, θ) on the graph, the point $(r, \pi - \theta)$ is also on the graph. Similarly, the equation $r = f(\theta)$ is unchanged when θ is replaced by $\pi - \theta$.