Camera Model and Calibration

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Applications

Object Transfer

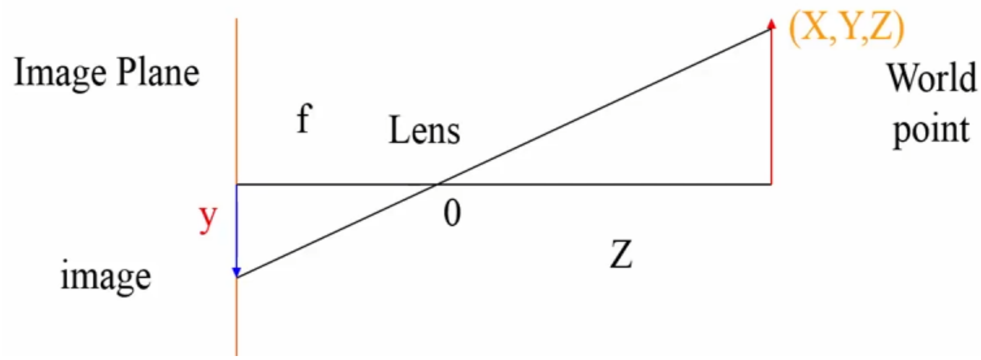
Transfer an object from one image or video to another. For example, inserting a person and their shadow from one video to another.

Pose Estimation

Given a 3D model of an object and its image (2D projection), determine the location and orientation (translation and rotation) of the object, such that when projected on the image plane, it will match the image.

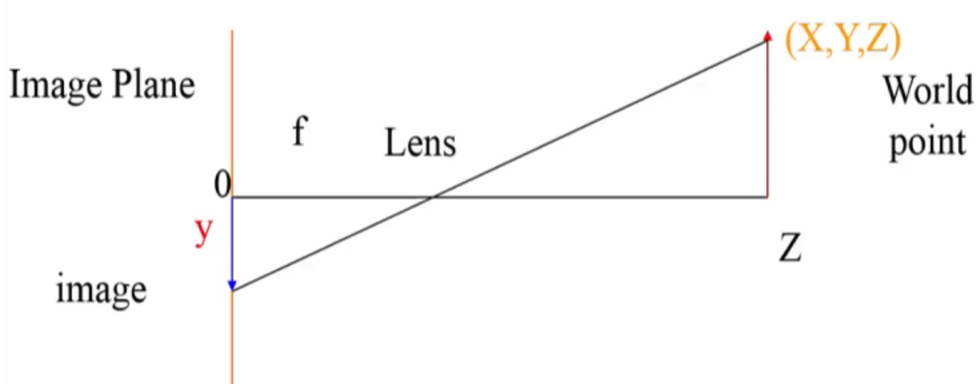
Perspective Projection

Origin at the lens center



$$\frac{-y}{Y} = \frac{f}{Z} \rightarrow y = -\frac{fY}{Z} \qquad \frac{-x}{X} = \frac{f}{Z} \rightarrow x = -\frac{fX}{Z}$$

Origin at the image center



$$\frac{-y}{Y} = \frac{f}{Z-f} \rightarrow y = -\frac{fY}{Z-f} \qquad \frac{-x}{X} = \frac{f}{Z-f} \rightarrow x = -\frac{fX}{Z-f}$$

Transformations

3D Translation

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = T \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

where $T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is the translation matrix

Scaling

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \times S_x \\ Y_1 \times S_y \\ Z_1 \times S_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = S \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

where $S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is the scaling matrix

Rotation

Rotation matrices are orthonormal matrices, so the inverse and transpose of a rotation matrix are equal.

$$r_i \cdot r_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

where r_i and r_j are rows in the rotation matrix.

Rotation around Z axis:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = R \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

where $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the rotation matrix

Euler Angles — Rotation around an arbitrary axis:

$$R = R_Z^{\alpha} R_Y^{\beta} R_Z^{\gamma} = \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

The approximation if angles are small ($\cos\theta \approx 1$) is:

$$\begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix}$$

Perspective

Homogenous transformation

$$(X, Y, Z) \to (kX, kY, kZ, k)$$

Inverse Homogenous transformation

$$(C_{h1}, C_{h2}, C_{h3}, C_{h4}) \to \left(\frac{C_{h1}}{C_{h4}}, \frac{C_{h2}}{C_{h4}}, \frac{C_{h3}}{C_{h4}}\right)$$

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = P \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix} = \begin{bmatrix} kX \\ kY \\ kZ \\ k - \frac{kZ}{f} \end{bmatrix}$$

where $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}$ is the perspective matrix

$$x = \frac{C_{h1}}{C_{h4}} = \frac{kX}{k - \frac{kZ}{f}} = \frac{fX}{f - Z}$$

$$y = \frac{C_{h2}}{C_{h4}} = \frac{kY}{k - \frac{kZ}{f}} = \frac{fY}{f - Z}$$

Camera Model

1. Camera is at the origin of the world coordinates
2. Translate by G
3. Rotate around Z axis in counter clockwise direction
4. Rotate again around X in counter clockwise direction
5. Translate by C

Since we are moving the camera instead of object we need to use inverse transformations.

$$C_h = PT_C R^X_{-\phi} R^Z_{-\theta} T_G W_h$$

Using the previous values for P, C, and R:

$$x = f \frac{(X - X_0)\cos\theta + (Y - Y_0)\sin\theta - r_1}{-(X - X_0)\sin\theta\sin\phi + (Y - Y_0)\cos\theta\sin\phi - (Z - Z_0)\cos\phi + r_3 + f}$$

$$y = f \frac{(X - X_0)\sin\theta\cos\phi + (Y - Y_0)\cos\theta\cos\phi + (Z - Z_0)\sin\phi - r_2}{-(X - X_0)\sin\theta\sin\phi + (Y - Y_0)\cos\theta\sin\phi - (Z - Z_0)\cos\phi + r_3 + f}$$

Camera Calibration

Finding the A matrix using 3D points and corresponding 2D points.

$$C_h = AW_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$
$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$
$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

$$x = \frac{C_{h1}}{C_{h4}} \quad y = \frac{C_{h2}}{C_{h4}}$$

Solve for matrix using least squares fit.

1. Using the value for Ch_4 in terms of the unknown variables, the following homogenous equations can be formed.
$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$
$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$
2. Since these are homogenous equations, there are infinite solutions.
3. By setting one of the unknown values, a_{44} to 1, the leftover x and y terms can be moved to the right side of the equations, resulting in a single solution.
$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx = x$$
$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy = y$$
4. Since the two equations are for one point, you can solve for the unknown values with enough points.

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2Y_2 & -x_2Z_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1Y_1 & -y_1Z_1 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2Y_2 & -y_2Z_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nY_n & -y_nZ_n \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$DQ = R \longrightarrow D^T DQ = D^T R \longrightarrow Q = (D^T D)^{-1} D^T R$$

Camera Parameters

- Extrinsic Parameters — Parameters that define the location and orientation of the camera reference frame with respect to a known world reference frame
- Intrinsic Parameters — Parameters necessary to link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame

Image and Camera Coordinates

- $(x_{\mathrm{im}}, y_{\mathrm{im}})$ — image coordinates
- (x, y) — camera coordinates
- (o_x, o_y) — image center (in pixels)
- (s_x, s_y) — effectives size of pixels (in millimeters)

$$x_{\mathrm{im}} = -\frac{x}{s_x} + o_x \quad y_{\mathrm{im}} = -\frac{y}{s_y} + o_y$$

$$\begin{bmatrix} x_{\mathrm{im}} \\ y_{\mathrm{im}} \\ 1 \end{bmatrix} = C \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where $\begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix}$ is the camera matrix