

Permittivity of free space:  $\epsilon_0 = 8.854 \times 10^{-12}$   
 Elementary charge:  $e = 1.602 \cdot 10^{-19}$   
 Proton mass:  $1.673 \cdot 10^{-27}$  Electron mass:  $9.11 \cdot 10^{-31}$   
 Speed of light:  $c = 3 \cdot 10^8$  Permeability constant:  $\mu_0 = 1.26 \times 10^{-6}$

### Moving Conductor

$$I = \frac{\Delta V}{R} = \frac{\varepsilon}{R} = \frac{vIB}{R}$$

$$F_{\text{mag}} = F_{\text{pull}} = IIB = \frac{vI^2 B^2}{R}$$

$$P_{\text{input}} = P_{\text{dissipated}} = I^2 R = \frac{v^2 I^2 B^2}{R}$$

$$\Phi_m = \vec{A} \cdot \vec{B} = |A||B| \cos \theta \quad (\text{uniform magnetic field})$$

- Increasing flux: The induced magnetic field points opposite the applied magnetic field.
- Decreasing flux: The induced magnetic field points in the same direction as the applied magnetic field.
- Steady flux: There is no induced magnetic field.

$$\varepsilon_{\text{induced}} = -N \frac{d\Phi_m}{dt} \quad I_{\text{induced}} = \frac{\varepsilon_{\text{induced}}}{R}$$

$$E_{\text{inside}} = \frac{r}{2} \left| \frac{dB}{dt} \right| \quad \text{Solenoid}$$

### Transformers

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \quad P_1 = P_2 \quad V_1 I_1 = V_2 I_2 \quad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

### Inductors

$$L = \frac{\Phi_m}{I} \quad \text{henry (H)}$$

$$\Delta V_L = -L \frac{dI}{dt} \quad U_L = L \int_0^I IdI = \frac{1}{2} LI^2$$

### LC Circuits

$$I = -\frac{dQ}{dt} \quad Q(t) = Q_0 \cos \omega t \quad \omega = \frac{1}{\sqrt{LC}}$$

### LR Circuits

$$I = I_0 e^{-t/(L/R)}$$

$$\tau = \frac{L}{R} \quad \text{where current has decreased to } e^{-1}$$

### Right-hand rule (wire)

1. Point thumb in the direction of current
2. Point fingers in the direction of magnetic field
3. Point palm in the face of force on wire

$$\vec{E}_B = \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A \quad \vec{B}_B = \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A$$

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Maxwell changed Ampere's Law because it only applied to steady current.

$$I_{\text{disp}} = \epsilon_0 \frac{d\Phi_e}{dt} = \epsilon_0 \frac{dE \cdot A}{dt}$$

Displacement current is from changing electric field rather than flow of charges.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \quad \text{Gauss's Law}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss's Law for Magnetism}$$

$$\oint \vec{E} \cdot d\vec{s} = -N \frac{d\Phi_m}{dt} \quad \text{Faraday's Law}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} \quad \text{Ampere-Maxwell Law}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{Lorentz Force Law}$$

$$E(x, t) = E_0 \cos(kx - \omega t + \phi)$$

$$c = \frac{\omega}{k} = \frac{\omega \lambda}{2\pi} = f\lambda = \frac{E}{B}$$

$$I = \frac{P}{4\pi r^2} = \frac{c\epsilon_0 E_0^2}{2}$$

$$\langle S \rangle = \frac{1}{\mu_0} EB \sin \theta \quad (\text{Poynting Vector}) \quad \text{energy flux of an electromagnetic wave}$$

### Right-hand rule (electromagnetic waves)

Index: electric field, Middle: magnetic field, Thumb: motion

$$I_{\text{transmitted}} = \frac{1}{2} I_0 \quad (\text{unpolarized}) \quad I_{\text{transmitted}} = I_0 \cos^2 \theta \quad (\text{polarized})$$

$$\varepsilon = \varepsilon_0 \cos \omega t \quad \omega = 2\pi f$$

$v_R = i_R R = V_R \sin \omega t$  (AC circuit) and  $V_R$  is the maximum voltage

### Capacitor circuit

$$v_C = V_C \cos \omega t \quad q = C v_C$$

$$i_C = -\omega C V_C \sin \omega t = \omega C V_C \cos(\omega t + \frac{\pi}{2})$$

$$X_C = \frac{1}{\omega C} \quad I_C = \frac{V_C}{X_C} \quad (X_C \text{ is Capacitive reactance})$$

$$\omega_C = \frac{1}{RC} \quad (\text{RC Circuit})$$

### Inductor circuit

An inductor is a coil of wire that generates a magnetic field when current flows through it and resists changes in current by inducing an emf opposite to the change.

$$i_L = I_L \cos(\omega t - \frac{\pi}{2})$$

$$X_L = \omega L \quad I_L = \frac{V_L}{X_L} \quad (X_L \text{ is Inductive reactance})$$

### Series RLC Circuit

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad (\text{impedance}) \quad I_{\text{peak}} = \frac{\varepsilon_0}{Z}$$

$$\phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) \quad (\text{angle between emf and current})$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (\text{resonance frequency}) \quad \text{when } X_L = X_C \text{ and } Z = R$$

- If  $V_C > V_L$ , the circuit operates below resonance frequency
  - If  $V_L > V_C$ , the circuit operates above resonance frequency
- $p = i\varepsilon$  (AC circuit)  $i$  and  $\varepsilon$  are current and potential difference

$$P_R = \frac{1}{2} I_R^2 R = I_{\text{rms}} V_{\text{rms}}$$

$$x_{\text{rms}} = \frac{x}{\sqrt{2}}$$

$$P_{\text{source}} = \frac{1}{2} I \varepsilon_0 \cos \phi = I_{\text{rms}} \varepsilon_{\text{rms}} \cos \phi = P_{\text{max}} \cos^2 \phi$$

$\cos \phi$  is the power factor,  $\phi$  is the phase between current and emf,  $P_{\text{mas}} = \frac{1}{2} I_{\text{max}} \varepsilon_0$ .

- AC circuit with capacitor: current leads voltage by  $\frac{\pi}{2}$  (current reaches maximum  $\frac{T}{4}$  before voltage)
- AC circuit with inductor: current lags voltage by  $\frac{\pi}{2}$  (current reaches maximum  $\frac{T}{4}$  after voltage)
- AC circuit with resistor: current is in phase with voltage