

Chapter 31 Electromagnetic Fields and Waves

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The Transformation of Electric and Magnetic Fields

There is a single electromagnetic field that presents different faces, in terms of \vec{E} and \vec{B} , to different viewers.

$$\begin{aligned}\vec{E}_B &= \vec{E}_A + \vec{v}_{BA} \times \vec{B}_A \\ \vec{B}_B &= \vec{B}_A - \frac{1}{c^2} \vec{v}_{BA} \times \vec{E}_A\end{aligned}$$

where \vec{v}_{BA} is the velocity of reference frame B relative to frame A and where the fields are measured at the same point in space. (Only valid if $\vec{v}_{BA} \ll c$)

The Displacement Current

A changing magnetic field causes an induced electric field and vice versa.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \left(I_{\text{through}} + I_{\text{disp}} \right) = \mu_0 \left(I_{\text{through}} + \epsilon_0 \frac{d\Phi_e}{dt} \right)$$

$$I_{\text{disp}} = \epsilon_0 \frac{d\Phi_e}{dt}$$

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0} \quad \text{Gauss's Law}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{Gauss's Law for Magnetism}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \quad \text{Faraday's Law}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}} + \epsilon_0 \mu_0 \frac{d\Phi_e}{dt} \quad \text{Ampere-Maxwell Law}$$

General Force Equation

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \text{Lorentz Force Law}$$

1. **Gauss's Law:** Charged particles create an electric field.
2. **Gauss's Law for Magnetism:** There are no isolated magnetic poles.
3. **Faraday's Law:** An electric field can also be created by a changing magnetic field.
4. **Ampere-Maxwell Law:** Currents and a changing electric field can each create a magnetic field.
5. **Lorentz Force Law:** An electric force is exerted on a charged particle in an electric field and a magnetic force is exerted on a charge moving in a magnetic field.

Electromagnetic Waves

$$\frac{\partial^2 E_y}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 E_y}{\partial x^2} \quad (\text{the wave equation for electromagnetic waves})$$

$$v_{\text{em}} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s} = c$$

1. \vec{E} and \vec{B} are perpendicular to each other and each to the direction of travel.
2. $E = cB$ at any point on the wave.

Right-hand rule

:

1. Point index finger in the direction of electric field
2. Point middle finger in the direction of magnetic field
3. Point thumb in the direction of motion

Wave Key Points

$$E(x, t) = E_0 \cos(kx - \omega t + \phi)$$

1. $k = \frac{2\pi}{\lambda}$ where k is wave number and λ is wavelength
2. $T = \frac{2\pi}{\omega}$ where T is period and ω is angular frequency
3. $f = \frac{1}{T}$ where f is frequency
4. $v = f\lambda$ where v is the propagation speed
5. $v = \frac{E_0}{B_0}$ where E_0 and B_0 are the electric and magnetic field components

Properties of Electromagnetic Waves

The intensity, average energy transfer, of an electromagnetic wave is

$$I = \frac{P}{A} = S_{\text{avg}} = \frac{1}{2c\mu_0} E_0^2 = \frac{c\epsilon_0}{2} E_0^2$$

$$\frac{E_1}{E_2} = \frac{d_2}{d_1}$$

Polarization

$$I_{\text{transmitted}} = I_0 \cos^2 \theta$$