

Matrices

A matrix is in **echelon form** if:

1. All nonzero rows are above any rows of all zeros
2. Each leading entry of a row is in a column to the right of the leading entry of the row above it
3. All entries in a column below a leading entry are zeros

A matrix is in **reduced echelon form** if:

1. It is in echelon form
2. The leading entry in each nonzero row is 1

A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot column. That is, if and only if an echelon form of the augmented matrix has no row of the form $[0 \ \cdots \ 0 \ b]$ with b being nonzero.

Let A be an $m \times n$ matrix. Then the following statements are logically equivalent. That is, for a particular A , either they are all true statements or they are all false.

1. For each \mathbf{b} in \mathbb{R}^m , the equation $A\mathbf{x} = \mathbf{b}$ has a solution
2. Each \mathbf{b} in \mathbb{R}^m is a linear combination of the columns of A
3. The columns of A span \mathbb{R}^m
4. A has a pivot position in every row

Homogeneous Equation

A linear equation in the form $Ax = 0$ where:

- A is an $m \times n$ matrix
- \mathbf{x} is a vector in \mathbb{R}^n
- $\mathbf{0}$ is the zero vector in \mathbb{R}^m

Properties

1. The homogeneous equation always has at least one solution (the trivial solution), where $\mathbf{x} = \mathbf{0}$
2. If the matrix \mathbf{A} has more columns than rows ($n > m$), the system often has infinitely many solutions
3. If \mathbf{A} has n pivot columns, the columns of \mathbf{A} are linearly independent, since every variable is a basic variable

If $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a linear transformation, then there exists a unique matrix A such that the following equation is true.

$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \text{ in } \mathbb{R}^n$$

In fact, A is the $m \times n$ matrix whose j th column is the vector $T(\mathbf{e}_j)$ where \mathbf{e}_j is the j th column of the identity matrix in \mathbb{R}^n , as shown in the equation, $A = [T(e_1) \quad \cdots \quad T(e_n)]$

The mapping $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be one-to-one if each \mathbf{b} in \mathbb{R}^m is the image of at most one \mathbf{x} in \mathbb{R}^n .

The mapping $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is said to be onto \mathbb{R}^m if each \mathbf{b} in \mathbb{R}^m is the image of at least one \mathbf{x} in \mathbb{R}^n .