

Linear Equations in Linear Algebra

A matrix is in **echelon form** if:

1. All nonzero rows are above any rows of all zeros
2. Each leading entry of a row is in a column to the right of the leading entry of the row above it
3. All entries in a column below a leading entry are zeros

A matrix is in **reduced echelon form** if:

1. It is in echelon form
2. The leading entry in each nonzero row is 1

Properties

- Two matrices are row equivalent if there exists a sequence of elementary row operations that transforms one matrix into the other
- Each matrix is row equivalent to only one reduced echelon matrix
- The echelon form of a matrix is not unique, but the reduced echelon form is unique

Existence and Uniqueness Theorem

A linear system is consistent if the rightmost column of echelon form of the augmented matrix is not a pivot column.

Row Reduction Algorithm

The row reduction algorithm leads directly to an explicit description of the solution set of a linear system when the algorithm is applied to the augmented matrix of the system, leading to a general solution of a system.

1. Forward Phase (reducing a matrix to echelon form)
 - (a) Begin with the leftmost nonzero column. This is a pivot column. The pivot position is at the top
 - (b) Select a nonzero entry in the pivot column as a pivot. If necessary, interchange rows to move this entry into the pivot position
 - (c) Use row replacement operations to create zeros in all positions below the pivot
 - (d) Ignore the row containing the pivot position and all rows above it
 - (e) Repeat until there are no more nonzero rows to modify

2. Backward Phase (reducing a matrix to reduced echelon form)

- (a) Beginning with the rightmost pivot and working upward and to the left, create zeros above each pivot. If a pivot is not 1, make it 1 by a scaling operation

Span (Linear Combination)

- The span of two vectors, $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$, represents all vectors that can be reached by scaling and adding the two vectors
- If the system consisting of vectors \mathbf{v}_1 , \mathbf{v}_2 , and \mathbf{b} is consistent, then \mathbf{b} is in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$

Other

- Any list of five real numbers is a vector in \mathbb{R}^n
- $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ consists of the vectors, \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3