Vectors

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Vectors in the Plane

A vector is a quantity that has both magnitude and direction.

Component Form of a Vector

A vector with initial points (x_i, y_i) and terminal point (x_t, y_t), can be represented in component form as $\langle x_t - x_i, y_t - y_i \rangle$

Properties of Vector Operations

Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors in a plane. Let r and s be scalars.

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	Commutative property
$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$	Associative property
$\mathbf{u} + \mathbf{0} = \mathbf{u}$	Additive identity property
$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$	Additive inverse property
$r(s\mathbf{u}) = (rs)\mathbf{u}$	Associativity of scalar multiplication
$(r + s)\mathbf{u} = r\mathbf{u} + s\mathbf{u}$	Distributive property
$r(\mathbf{u} + \mathbf{v}) = r\mathbf{u} + r\mathbf{v}$	Distributive property
$1\mathbf{u} = \mathbf{u}$, $0\mathbf{u} = \mathbf{0}$	Identity and zero properties

Vectors in Three Dimensions

Distance Between Two Points in Space

$$d_{p_1 \to p_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a Sphere

Let the sphere have a center (a, b, c) and radius r.

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$