Double Integrals in Polar Coordinates

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The double integral of the function $f(r, \theta)$ over the poalr rectangular region R in the $r\theta$-plane is defined as

$$\iint\limits_{R} f(r, \theta)\, dA = \lim_{m,n \to \inf} \sum_{i=1}^{m} \sum_{j=1}^{n} f(r_{ij}^*, \theta_{ij}^*)\, \Delta A = \lim_{m,n \to \inf} \sum_{i=1}^{m} \sum_{j=1}^{n} f(r_{ij}^*, \theta_{ij}^*) r_{ij}^*\, \Delta r\, \Delta \theta$$

If $f(r, \theta)$ is continuous on a general polar region D, then

$$\iint\limits_{D} f(r, \theta) r\, dr\, d\theta = \int_{\theta=\alpha}^{\theta=\beta} \int_{r=h_1(\theta)}^{r=h_2(\theta)} f(r, \theta) r\, dr\, d\theta$$