Chapter 32 AC Circuits

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AC Sources and Phasors

The instantaneous emf an AC oscillator can be written as $\varepsilon = \varepsilon_0 \cos \omega t$ where ε_0 is the maximum emf and $\omega = 2\pi f$ is the angular frequency.

The value of the emf is decreasing when $0 < \omega t < \pi$ and increasing when $\pi < \omega t < 2\pi$.

The resistor voltage is an AC circuit is $v_R = V_R \cos \omega t$ where V_R is the maximum voltage.

$$v_R = i_R R$$

Capacitor Circuits

$$v_C = V_C \cos \omega t$$

where V_C is the maximum voltage across the capacitor.

$$q = Cv_C = CV_C \cos \omega t$$

The current is the rate of charge, $i_C = dq/dt$.

$$i_C = \frac{dq}{dt} = \frac{d}{dt}(CV_C \cos \omega t) = -\omega CV_C \sin \omega t = \omega CV_C \cos(\omega t + \frac{\pi}{2})$$

The AC current of a capacitor leads the capacitor voltage by $\pi/2$ rad, or $90°$.

Capacitive reactance, $X_C \equiv \frac{1}{\omega C}$, is represented with ohms and shows the relationship between peak voltage and peak current.

$$I_C = \frac{V_C}{X_C} \quad \text{or} \quad V_C = I_C X_C$$

RC Filter Circuits

V_R will increase steadily from 0 to ε_0 as ω is increased from 0 to very high frequencies.

$$\omega_c = \frac{1}{RC}$$

Inductor Circuits

$$i_L = I_L \cos(\omega t - \frac{\pi}{2})$$

where $I_L = V_L/\omega L$ and $V_L = I_L X_L$ is maximum inductor current and voltage.

The AC current through an inductor lags the inductor voltage by $\pi/2$ rad, or $90°$.

The Series RLC Circuit

A circuit where a resistor, inductor, and capacitor are in series.

1. The instantaneous current of all three elements is the same: $i = i_R = i_L = i_C$.
2. The sum of the instantaneous voltages matches the emf: $\varepsilon = v_R + v_L + v_C$.

The peak current in the RLC circuit is

$$I = \frac{\varepsilon_0}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

The three peak voltags are then found from $V_R = IR$, $V_L = IX_L$, and $V_C = IX_C$.

The denominator in the peak current equation is the impedance, Z, of the circuit (measured in ohms):

$$\sqrt{R^2 + (X_L - X_C)^2}$$

The current is not in phase with the emf and the phase angle ϕ between the emf and the current is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The resonance frequency is the frequency for the maximum current, $I_{\max} = \varepsilon_0/R$, in the series RLC circuit,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- At resonance, the capacitive reactance (X_C) and inductive reactance (X_L) cancel each other out, meaning $V_C = V_L$.
- If $V_C > V_L$, the circuit operates below resonance frequency.
- If $V_L > V_C$, the circuit operates above resonance frequency.

Power in AC Circuits

The emf supplies energy to a circuit at the rate $p = i\varepsilon$ where i and ε are the instantaneous current from and potential difference across the emf.

Resistors

For a resistor,

$$p_R = i_R v_R = i_R^2 R = I_R^2 R \cos^2 \omega t$$

The average power, P, is the total energy dissipated per second.

$$P_R = I_R^2 R \cos^2 \omega t = I_R^2 R \Big[\frac{1}{2}(1 + \cos 2\omega t)\Big] = \frac{1}{2} I_R^2 R + \frac{1}{2} I_R^2 R \cos 2\omega t$$

The average of $\cos 2\omega t$ is zero so

$$P_R = \frac{1}{2} I_R^2 R$$

$$P_R = (I_{\mathrm{rms}})^2 R = \frac{(V_{\mathrm{rms}})^2}{R} = I_{\mathrm{rms}} V_{\mathrm{rms}}$$

where $I_{\mathrm{rms}} = \frac{I_R}{\sqrt{2}}$ is the root-mean-square current.

Capacitors and Inductors

The capacitor's and inductor's average power are zero.

The Power Factor

$$P_{\mathrm{source}} = \frac{1}{2} I \varepsilon_0 \cos \phi = I_{\mathrm{rms}} \varepsilon_{\mathrm{rms}} \cos \phi$$

where $\cos \phi$ is the called the power factor and ϕ is the phase between the current and emf.

$$P_{\mathrm{source}} = P_{\mathrm{max}} \cos^2 \phi$$

where $P_{\mathrm{max}} = \frac{1}{2} I_{\mathrm{max}} \varepsilon_0$.