Chapter 3 Determinants

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Introduction to Determinants

A $n \times n$ matrix is invertible if and only if its determinant is nonzero.
For $n \geq 2$, the **determinant** of an $n \times n$ matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is the sum of n terms of the form $\pm a_{1j} \det A_{1j}$, with plus and minus signs alternating, where the entries $a_{11}, a_{12}, \ldots, a_{1n}$ are from the first row of A.

$$\det A = \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det A_{1j}$$

The determinant of an $n \times n$ matrix A can be computed by a cofactor expansion across any row or down any column. The expansion across the ith row using the cofactor, $C_{ij} = (-1)^{i+j} \det A_{ij}$, is

$$\det A = a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in}$$

The cofactor expansion down the jth column is

$$\det A = a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj}$$

If A is a triangular matrix, then $\det A$ is the product of the entries on the main diagonal of A.