Functions of Several Variables

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A **function of two variables** $z = f(x, y)$ maps each ordered pair (x, y) in a subset D of the real plane $\mathbb{R}^2$ to a unique real number z. The set D is called the **domain** of the function. The **range** of f is the set of all real numbers z that has at least one ordered pair $(x, y) \in D$ such that $f(x, y) = z$.

Level Curves

Given a function $f(x, y)$ and a number c in the range of f, a **level curve of a function of two variables** for the value c is defined to be the set of points satisfying the equation $f(x, y) = c$.

Consider a function $z = f(x, y)$ with domain $D \subseteq \mathbb{R}^2$. A **vertical trace** of the function can be either the set of points that solves the equation $f(a, y) = z$ for a given constant $x = a$ or $f(x, b) = z$ for a given constant $y = b$.

Functions of More Than Two Variables

Given a function $f(x, y, z)$ and a number c in the range of f, a **level surface of a function of three variables** is defined to be the set of points satisfying the equation $f(x, y, z) = c$.