Chapter 23 The Electric Field

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Electric Field Models

1. A point charge (small charged objects):

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

2. An infinitely long line of charge (wires):

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2|\lambda|}{r}$$

3. An infinitely wide plane of charge (electrodes):

$$\vec{E} = \frac{\eta}{2\epsilon_0}\begin{cases}\text{away from plane if charge } + \\ \text{toward plane if charge } -\end{cases}$$

4. A sphere of charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{R} \text{ for } r > R$$

The Electric Field of Point Charges

The Electric Field of a Dipole

Two equal but opposite charges separated by a small distance form an electric dipole. The dipole moment $\vec{p} = qs$, where q is the positive charge and s is the distance between the charges, determines the orientation of the dipole and electric field strength.

The electric field on a point on the axis between the two charges:

$$\vec{E}_{\text{dipole}} = -\frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \text{ (on the axis)}$$

where r is the distance measured from the center of the dipole.

$$\vec{E}_{\text{dipole}} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3} \text{ (in the bisecting plane)}$$

Electric Field Lines

- Electric field lines are continuous curves tangent to the electric field vectors.
- Closely spaced field lines indicate a greater field strength.
- Electric field lines start on positive charges and end on negative charges.
- Electric field lines never cross.

The Electric Field of a Continuous Charge Distribution

Charge Density

- The linear charge density of an object of length L and charge Q is defined as $\lambda = \frac{Q}{L}$.

$$Q = \int_0^L \lambda dx$$

- Linear charge density, which has units of C/m, is the amount of charge per meter of length.
- The surface charge density of a surface with area A and charge Q is defined as $\eta = \frac{Q}{A}$.
- Surface charge density, which has units of C/m^2, is the amount of charge per square meter.
- The volume charge density of an object with volume V and charge Q is defined as $\rho = \frac{Q}{V}$.
- Volume charge density, which has units of C/m^3, is the amount of charge per cubic meter.

The Parallel-Plate Capacitor

A parallel-plate capacitors is the arrangement of two electrodes (metal plates) face-to-face and charged equally but oppositely.

Properties

$$\vec{E}_{\text{capacitor}} = \begin{cases} \left(\frac{Q}{\epsilon_0 A}, \text{ from positive to negative}\right) & \text{inside} \\ \vec{0} & \text{outside} \end{cases}$$

- Capacitors are charged by transferring electrons from one plate to the other.
- Capacitors have a uniform electric field between the electrodes, meaning that the electric field from the electrodes is the same in strength and direction.

Motion of a Dipole in an Electric Field

When the dipole moment $\vec{p}$ is at an angle θ to the field, it causes the dipole to experience a torque, $\tau = r \times F = p \times E$. Each charge experiences force $F = qE$. The net force on the dipole is zero because the forces are of opposite directions so dipole will not move as a whole in electric field.

$$|\tau| = pE \sin\theta$$

where θ is the angle between the dipole moment and the electric field.