Electric Field Lines

Electric field lines are continuous curves which have the same direction as the electric field

- Electric field lines are continuous curves tangent to the electric field vectors.
- Closely spaced field lines indicate a greater field strength.
- Electric field lines start on positive charges and end on negative charges.
- Electric field lines never cross.

Electric Field Produced by a Continuous Distribution of Charge

$$\vec{E}(\vec{r}) = \int \frac{kdq}{r^2}\hat{r}$$

where dq is a small element of charge at each point in a electric field produced by a continuous distribution of charge and r is the distance from that small point in the electric field to the point.

Linear Charge Density

- The linear charge density of an object of length L and charge Q is defined as $\lambda = \frac{Q}{L}$
- Linear charge density, which has units of C/m, is the amount of charge per meter of length

If the chraged line is infinitely long:

$$E = \frac{2k\lambda}{r}$$

Surface Charge Density

- The surface charge density of a two-dimensional distribution of charge across a surface of area A is defined as $\eta = \frac{Q}{A}$.
- Surface charge density, which has units of C/m^2, is the amount of charge per square meter.

Capacitors

$$E = \frac{V}{d}$$

- The electric field inside an ideal parallel plate capacitor is uniform and directed from the postiively charged plate to the negatively charged plate.
- If you know the surface charge density $\sigma = \frac{Q}{A}$ on the plates, the electric field can also be calculated using: $E = \frac{\sigma}{\epsilon_0}$.
- Inside the capacitor, the net field points toward the negative plate.

$$\vec{E}_{capacitor} = \begin{cases} \left(\frac{Q}{\epsilon_0 A}, \text{from positive to negative}\right) & \text{inside} \\ \vec{0} & \text{outside} \end{cases}$$

Electric Dipoles

Dipole moment $p = q \times d$ where d is a vector directed from negative charge to the positive one and q is the magnitude of charge.

$$E = \frac{2kp}{r^3}$$

where $\epsilon_0 = 8.854 \times 10^-12\, C^2/N \cdot m^2$ is the vacuum permittivity

When the dipole moment $\vec{p}$ is at an angle θ to the field, it causes the dipole to experience a torque, $\tau = r \times F = p \times E$. Each charge experiences force $F = qE$. The net force on the dipole is zero because the forces are of opposite directions so dipole will not move as a whole in electric field.

$$|\tau| = pE\sin(\theta)$$

where θ is the angle between the dipole moment and the electric field.