Context Free Languages

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Context Free Grammar (CFG)

A **context-free grammar** is a serios of subsitution rules or productions, where each rule is a transition from one variable to any combination of variables and terminal symbols. The start symbol can be S or the left-hand side of the topmost rule if it is not explcit.

A context-free grammar is defined by $G = (V, \Sigma, R, S)$ where:

- V is a finite set called the variables
- Σ is a finite set, disjoint from V, called the terminals
- R is a finite set of rules, each rule allowing a variable to be rewritten as a string of variables and terminals
- $S \in V$ is the start variable

The chain to follow to get the string is called a **derivation**. A **leftmost derivation** is a derivation where the leftmost remaining variable is the one replaced. A string w is derived ambiguously in grammar G if it has two or more different leftmost derivations. A grammar G is **ambiguous** if it generates some string ambiguously.

Chomsky Normal Form

A context-free grammar is in **Chomsky Normal Form** if every rule is in one of the following forms:

- $A \to BC$
- $A \to \mathbf{a}$
- $S \to \varepsilon$

where **a** is a terminal and A, B, and C are variables.

Pumping Lemma for Context-Free Languages

If A is a CFL, there is a pumping length p where, if $s \in A$ and $|s| \geq p$, $s = uvxyz$ so that:

1. $uv^ixy^iz \in A$ for all non-negative integers i
2. $|vy| > 0$
3. $|vxy| \leq p$

Pushdown Automata (PDA)

A **pushdown automaton** is an NFA with a stack, where on any transition:

- Push if $a, \varepsilon \to a$
- Pop if $a, a \to \varepsilon$
- Both if $a, a \to b$
- Neither if $a, \varepsilon \to \varepsilon$

A pushdown automaton is defined by $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where:

- Q is the set of states
- Σ is the input alphabet
- Γ is the stack alphabet
- $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to P(Q \times \Gamma_\varepsilon)$ is the transition function
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the accept states

Most PDAs will push \$ to the bottom of the stack to determine if the stack is empty.

Deterministic Context Free Language (DCFL)

A deterministic pushdown automaton (DPDA) is like a PDA, but it only has one legal move in any situation. A **deterministic context free language** is a language that can be recognized by a DPDA.

Key Points

- PDAs and CFGs are equal in power
- Every regular language is also a context-free language since a PDA is just an NFA with a stack.