Midterm Exam 1

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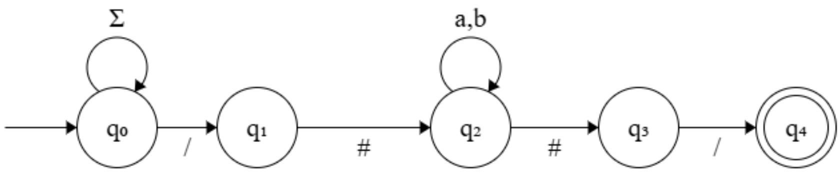
Problem 1

$$(a \cup d)^* c(b(dd)^* \cup c \cup d)^* b$$

Problem 2

Let M be a DFA that recognizes L. We can construct an NFA M' to recognize $h(L)$ by replacing each transition $\delta(q, a)$ in M with a sequence of transitions in M' that process $h(a)$ instead of a. Since NFAs and DFAs both define the class of regular languages, the resulting language $h(L)$ is regular. Therefore, regular languages are closed under homomorphism.

Problem 3



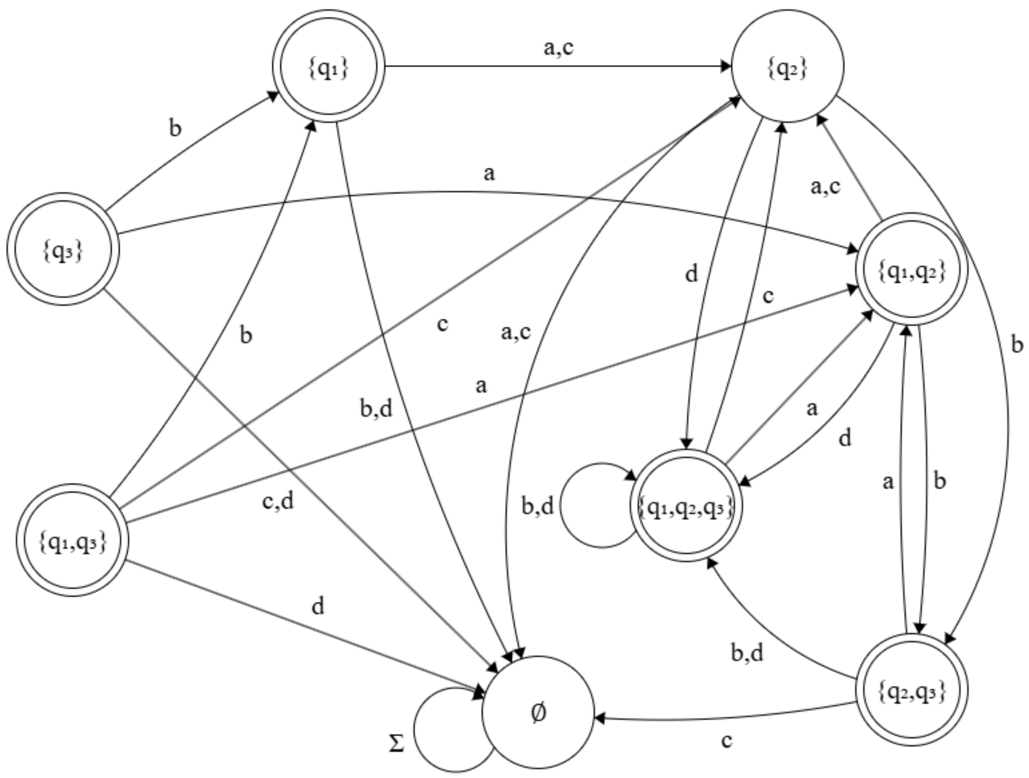
Problem 4

If $L = \{ccc\#cc\#c \text{ with } c \in \{a, b\}^*\}$ is a regular language, then it must satisfy the Pumping Lemma, where there exists a pumping length p such that any string s in L with $|s| \geq p$ can be split into $s = xyz$ where:

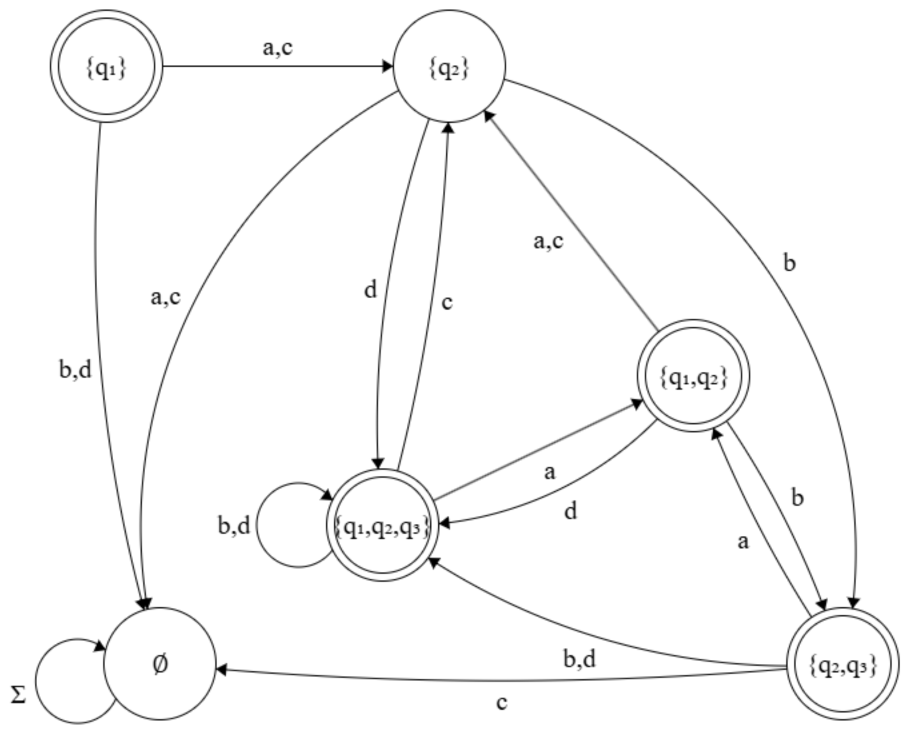
1. $xy^k z \in L$ for all $k \geq 0$
2. $|y| > 0$
3. $|xy| \leq p$

Consider a string $s = a^p b^p a^p b^p a^p b^p \# a^p b^p a^p b^p \# a^p b^p$ in L where $c = a^p b^p$. Pumping y increases the number of a's only in the first c, while the other instances of c are unchanged. Since the resulting string no longer follows the pattern $ccc\#cc\#c$, it contradicts the Pumping Lemma and L is not a regular language.

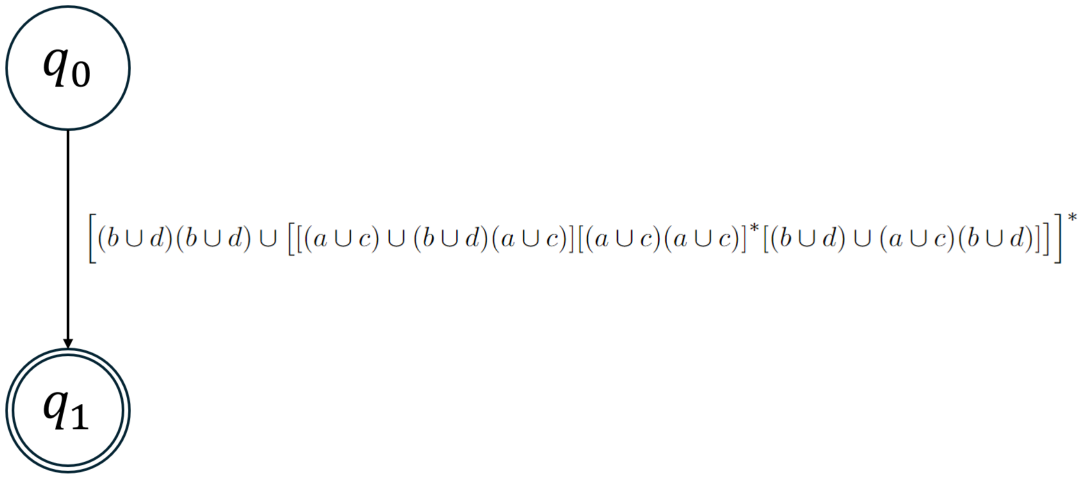
Problem 5



Problem 6



Problem 7


q_0
$\Big[(b \cup d)(b \cup d) \cup \big[[(a \cup c) \cup (b \cup d)(a \cup c)][(a \cup c)(a \cup c)]^*[(b \cup d) \cup (a \cup c)(b \cup d)]\big]\Big]^*$
q_1