Multi-variable limit check: Check when $x = 0$, $y = 0$, and $x = y$ or check if limit depends on θ when converting to polar coordinates

Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (xy-plane: ellipse, xz-plane: ellipse, yz-plane: ellipse)

Hyperboloid of One Sheet: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ (xy-plane: ellipse, xz-plane: hyperbola, yz-plane: hyperbola)

Hyperboloid of Two Sheets: $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ (xy-plane: hyperbola, xz-plane: hyperbola, yz-plane: ellipse)

Elliptic Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ (xy-plane: ellipse, xz-plane: two intersecting lines, yz-plane: two intersecting lines)

Elliptic Paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$ (xy-plane: ellipse, xz-plane: parabola, yz-plane: parabola)

Hyperbolic Paraboloid (Saddle): $\frac{x^2}{a^2} - \frac{y^2}{b^2} = z$ (xy-plane: hyperbola, xz-plane: parabola, yz-plane: negative parabola)

Clairaut's Theorem: Suppose that $f(x, y)$ is defined on an open desk D that contains the point (a, b). If the functions f_{xy} and f_{yx} are continuous on D, then $f_{xy} = f_{yx}$.

$$\nabla f(x, y) = f_x(x, y)\,\mathbf{i} + f_y(x, y)\,\mathbf{j}$$

Directional derivative of f in the direction of u is

$$D_u f(a, b) = \lim_{h \to 0} \frac{f(a + h\cos\theta, b + h\sin\theta) - f(a, b)}{h}$$

$$D_u f(x, y) = \nabla f(x, y) \cdot \mathbf{u} = f_x(x, y)\cos\theta + f_y(x, y)\sin\theta$$

$$T = L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$D_u f(x_0, y_0)$ is maximized when u points in the same direction as $\nabla f(x_0, y_0)$. The maximum value of $D_u f(x_0, y_0)$ is $\|\nabla f(x_0, y_0)\|$.

$D_u f(x_0, y_0)$ is minimized when u points in the opposite direction from $\nabla f(x_0, y_0)$. The minimum value of $D_u f(x_0, y_0)$ is $-\|\nabla f(x_0, y_0)\|$.

A point is a critical point of a function of two variables if $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$ or either $f_x(x_0, y_0)$ or $f_y(x_0, y_0)$ does not exist.

Second Derivative Test: If the first and second-order partial derivatives are continuous,

$$D = f_{xx}(x_0, y_0)f_{yy}(x_0, y_0) - (f_{xy}(x_0, y_0))^2$$

1. If $D > 0$ and $f_{xx}(x_0, y_0) > 0$, then f has a local minimum at (x_0, y_0).
2. If $D > 0$ and $f_{xx}(x_0, y_0) < 0$, then f has a local maximum at (x_0, y_0).
3. If $D < 0$, then f has a saddle point at (x_0, y_0).
4. If $D = 0$, then the test is inconclusive.

Finding local extema in a bounded region:

1. Test extreme points on boundary
2. Test all boundary lines: Consider a line segment connecting (a, c) and (b, c). It can be parameterized by the equations $x(t) = t$, $y(t) = c$ for $a \leq t \leq b$. Define $g(t) = f(x(t), y(t))$. The critical points are where $g'(t) = 0$.
3. Test whole bounded region