

Camera Model and Calibration

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Applications

Object Transfer

Transfer an object from one image or video to another. For example, inserting a person and their shadow from one video to another.

Pose Estimation

Given a 3D model of an object and its image (2D projection), determine the location and orientation (translation and rotation) of the object, such that when projected on the image plane, it will match the image.

Transformations

3D Translation

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$
$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = T \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

where $T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is the translation matrix

Scaling

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \times S_x \\ Y_1 \times S_y \\ Z_1 \times S_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = S \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

where $S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is the scaling matrix

Rotation

Rotation matrices are orthonormal matrices, so the inverse and transpose of a rotation matrix are equal.

$$r_i \cdot r_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

where r_i and r_j are rows in the rotation matrix.

Rotation around Z axis:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = R \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

where $R = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the rotation matrix

Rotation around an arbitrary axis:

$$R = R_Z^\alpha R_Y^\beta R_Z^\gamma = \begin{bmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma \\ -\sin \beta & \cos \beta \sin \gamma & \cos \beta \cos \gamma \end{bmatrix}$$

The approximation if angles are small ($\cos \theta \approx 1$) is:

$$\begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix}$$