Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height$$

Regression Statistics	
Multiple R	0.78801776
R Square	0.62097199
Adjusted R Squar	0.62049342
Standard Error	9.86271666
Observations	1587

ANOVA

	df	SS	MS	F	Significance F
Regression	2	252434.6717	126217.3	1297.555	0
Residual	1584	154080.7171	97.27318		
Total	1586	406515.3888			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	-101.96468	4.466768794	-22.8274	6E-100	-110.726083	-93.203282
Male	5.52761923	0.591069119	9.3519	2.83E-20	4.368259169	6.6869793
Height(cm)	0.96697779	0.026071479	37.08949	2.8E-217	0.915839553	1.018116

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Linear Regression for Business Statistics

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- **Effect of '*Height*' on '*Weight*' is same across Gender.**

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Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height$$

- **Effect of '*Height*' on '*Weight*' is same across Gender.**
- **Dummy variable '*Male*' allows for some overall differences.**

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Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height$$

- Effect of '*Height*' on '*Weight*' is same across Gender.
- Dummy variable '*Male*' allows for some overall differences.
- No Gender difference allowed based on '*Height*'.

Regression Statistics	
Multiple R	0.78801776
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Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height$$

- Effect of ‘*Height*’ on ‘*Weight*’ is same across Gender.
- Dummy variable ‘*Male*’ allows for some overall differences.
- No Gender difference allowed based on ‘*Height*’.

How do we allow impact of ‘Height’ based on ‘Gender’ ?

Regression Statistics	
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Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height$$

Linear Regression for Business Statistics

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Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height + \beta_3 \underbrace{Male * Height}_{\text{Interaction Variable}}$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height + \beta_3 Male * Height$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height + \beta_3 Male * Height$$

Value of Y variable when all X variables are zero.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height + \beta_3 Male * Height$$

Value of Y variable when all X variables are zero.

No ‘managerial’ interpretation.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 Height + \beta_3 Male * Height$$

Value of Y variable when all X variables are zero.

No ‘managerial’ interpretation.

We could mean-center the variable ‘*Height*’.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

Mean Centered

Mean Centered

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

'*Weight*' of an Olympian when all X variables are zero.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

'*Weight*' of an Olympian when all X variables are zero.

$Male = 0,$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

'*Weight*' of an Olympian when all X variables are zero.

$Male = 0,\quad \overline{Height} = 0,$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

'*Weight*' of an Olympian when all X variables are zero.

$$Male = 0, \quad \overline{Height} = 0, \quad Male * \overline{Height} = 0$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

'*Weight*' of an Olympian when all X variables are zero.

$$Male = 0, \quad \overline{Height} = 0, \quad Male * \overline{Height} = 0$$

- **Olympian is a *Female*.**

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

'*Weight*' of an Olympian when all X variables are zero.

$$Male = 0, \quad \overline{Height} = 0, \quad Male * \overline{Height} = 0$$

- Olympian is a *Female*.
- *Actual height* is the average height observed in data.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

'*Weight*' of an Olympian when all X variables are zero.

$$Male = 0, \quad \overline{Height} = 0, \quad Male * \overline{Height} = 0$$

- **Olympian is a *Female*.**
- ***Actual height* is the average height observed in data.**

β_0 is the weight of a Female Olympian whose height is at a level equal to the average height observed in the data.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

The *additional weight* of a Male Olympian as compared to a Female Olympian when the height is equal to the average height observed in the data.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

The *additional weight* of a Male Olympian as compared to a Female Olympian when the height is equal to the average height observed in the data.

$\beta_0 + \beta_1$ = *Total weight* of a Male Olympian whose height is equal to the average height observed in the data.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

The impact of one centimeter increase in height on the weight, all other variables held constant.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

The impact of one centimeter increase in height on the weight, all other variables held constant.

To keep the interaction variable *'at the same level'*, in our interpretation, we need to consider a Female Olympian.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

The impact of one centimeter increase in height on the weight, all other variables held constant.

To keep the interaction variable ***'at the same level'***, in our interpretation, we need to consider a Female Olympian.

β_2 is the impact of one centimeter increase in height on the weight of Female Olympians.

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \underset{\uparrow}{\beta_2}\overline{Height} + \beta_3 Male * \overline{Height}$$

The impact of one centimeter increase in height on the weight, all other variables held constant.

To keep the interaction variable ***'at the same level'***, in our interpretation, we need to consider a Female Olympian.

β_2 is the impact of one centimeter increase in height on the weight of Female Olympians.

What about the impact of height on the weight of Male Olympians?

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

The ***additional impact*** **of Height on weight, for Male Olympians.**

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

The ***additional impact*** **of Height on weight, for Male Olympians.**

The ***total impact*** **of Height on weight for Male Olympians is** $\beta_2 + \beta_3$

Linear Regression for Business Statistics

Interaction Effects in a Regression Model.

$$Weight = \beta_0 + \beta_1 Male + \beta_2 \overline{Height} + \beta_3 Male * \overline{Height}$$

- β_0 is the weight of a Female Olympian whose height is at a level equal to the average height observed in the data.
- β_1 is the *additional weight* of a Male Olympian vis-à-vis a Female Olympian when the height is equal to the average height observed in the data.
- $\beta_0 + \beta_1$ = *Total weight* of a Male Olympian whose height is equal to the average height observed in the data.
- β_2 is the impact of one centimeter increase in height on the weight of Female Olympians.
- β_3 is the *additional impact* of Height on weight, for Male Olympians.
- $\beta_2 + \beta_3$ is the *total impact* of Height on weight for Male Olympians.