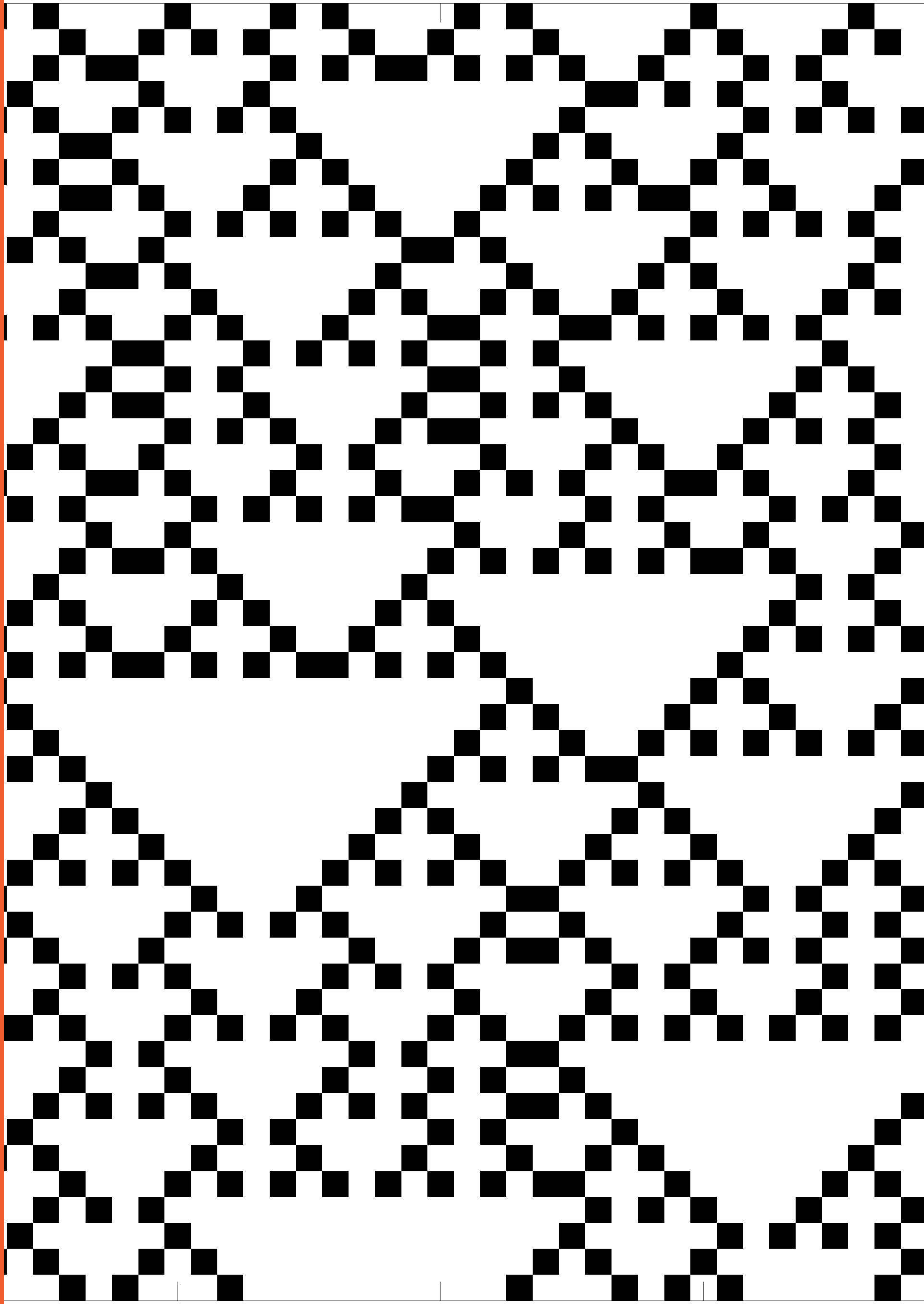
Lecture 6 – Information dynamics – part I



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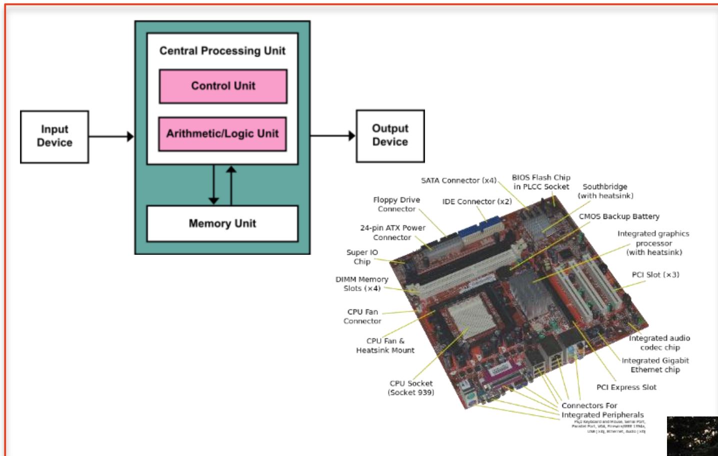
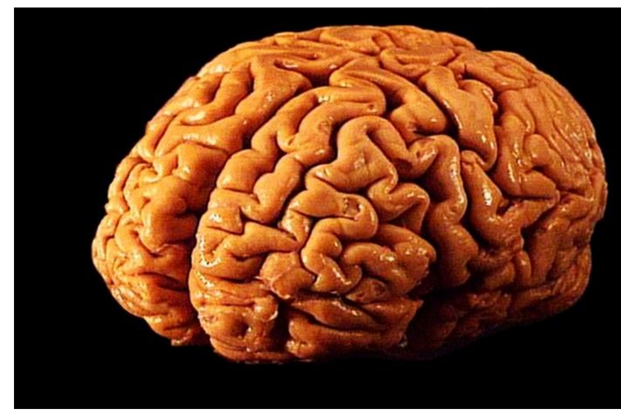
Information dynamics Part I: session outcomes

- Understand philosophy behind information dynamics approach for analysing information processing in complex systems.
- Apply JIDT using AutoAnalyser and extensions of code it produces to analyse information storage in complex systems data sets.

- Primary references:
  - J.T. Lizier, "JIDT: An information-theoretic toolkit for studying the dynamics of complex systems", Frontiers in Robotics and AI, 1:11, 2014; appendix A.2 and A.3
  - J.T. Lizier, "*The local information dynamics of distributed computation in complex systems*", Springer: Berlin/Heidelberg, 2013; chapter 3, 4
  - Bossomaier, Barnett, Harré, Lizier, "An Introduction to Transfer Entropy: Information Flow in Complex Systems", Springer, Cham, 2016; chapter 4 (sections 4.1-4.3); section 5.1

Using Turing machines, is computation easy to spot?





- For each image, consider whether the system is computing.
  - If so: What is it computing? How is it computing that: what are inputs/outputs/information? How are they manipulated?

Can we fit biological computation into dominant computer science paradigm of computation?

– What do you think?
– What would we look for?

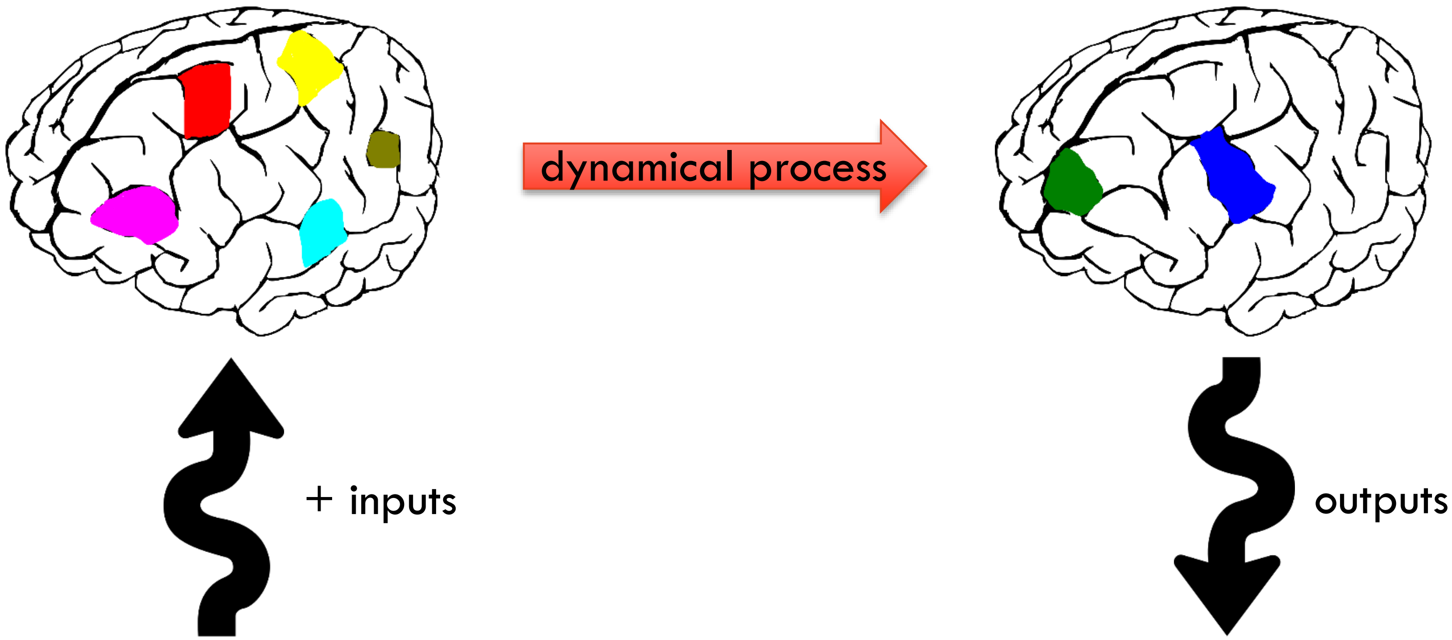
– Mitchell:

	Computer science	Biological computation
What plays the role of information in the system?	Digital static tape	Analog states, patterns distributed in space and time. Gathered via statistical sampling
How is the information communicated and processed?	Deterministic, serial, error-free centralised rules	Decentralised, parallel, local, fine-grained stochastic interactions. Randomness utilised.
How does the information acquire function/purpose/meaning?	(Human) designer	Natural selection

M. Mitchell, "*Complexity: A guided tour*", New York: Oxford University Press, 2009 – chapter 12

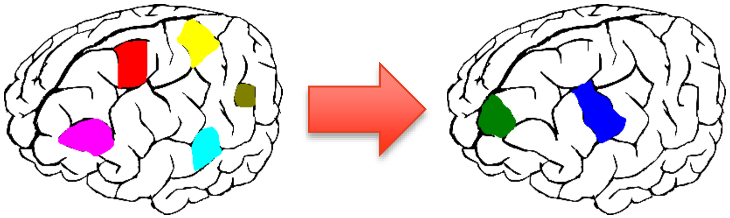
Biological computation: we need a new perspective

– Mitchell: "*Language of dynamical systems may be more useful than language of computation.*"



M. Mitchell, "*Introduction to Complexity*", Lecture 7

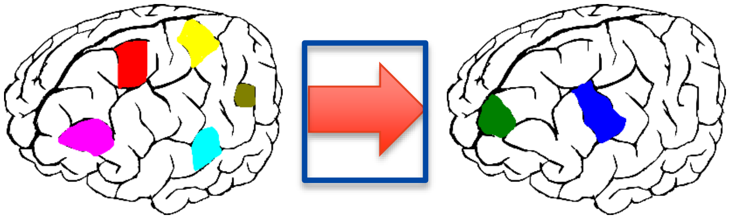
Biological computation: dynamical systems perspective



- What is happening in bio- and bio-inspired information processing?
  - It's distributed, unlike a Turing machine
  - It's ongoing, unlike a Turing machine
    - Intrinsic computation, or information processing doesn't necessarily finish
  - How can we describe it in computational or informational terms?
    - Information storage, transfer and modification
    - Easy to identify (elements performing) these operations on information in a traditional PC, not so easy in biological computation

M. Wibral, J.T. Lizier and V. Priesemann, "*Bits from Brains for Biologically-inspired Computing*", Frontiers in Robotics and AI, vol. 2, 5, 2015

Information dynamics and computation



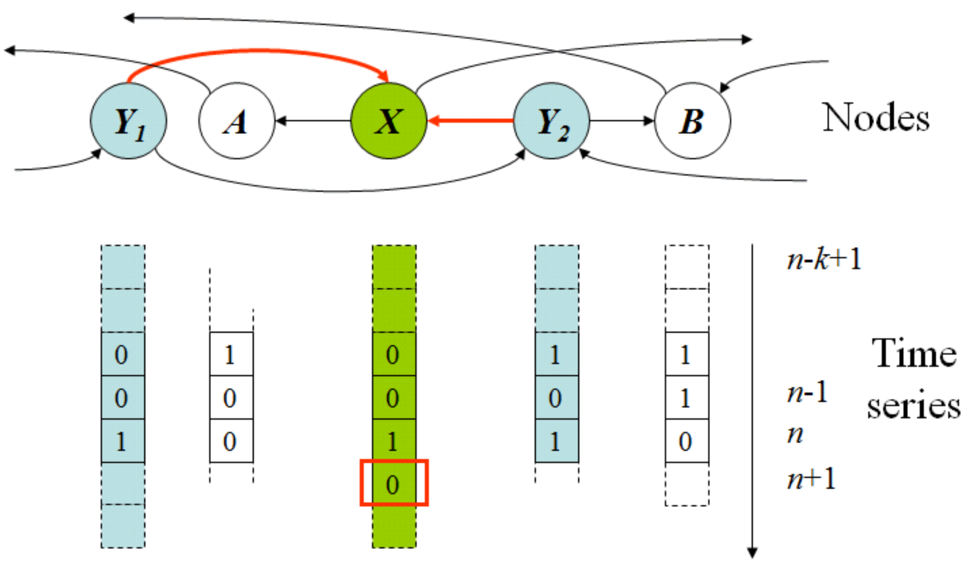
- We *talk* about computation as:
  - Memory
  - Signalling
  - Processing
- Distributed computation is any process that involves these features:
  - Information processing in the brain
  - Time evolution of cellular automata
  - Gene regulatory networks computing cell behaviours
  - Flocks computing their collective heading
  - Ant colonies computing the most efficient routes to food
  - The universe is computing its own future!

Information dynamics and computation

- We *talk* about computation as:
  - Memory
  - Signalling
  - Processing
- Idea: Quantify computation via:
  - Information storage
  - Information transfer
  - Information modification
- Distributed computation is any process that involves these features:
  - Information processing in the brain
  - Time evolution of cellular automata
  - Gene regulatory networks computing cell behaviours
  - Flocks computing their collective heading
  - Ant colonies computing the most efficient routes to food
  - The universe is computing its own future!
- General idea: by quantifying intrinsic computation in the language it is normally described in, we can understand how nature computes and why it is complex.

Information dynamics

– Key question: how is the next state of a variable in a complex system computed?



Complex system as a multivariate time-series of states

Q: Where does the information in x_{n+1} come from, and how can we measure it?
(Where might we look?)

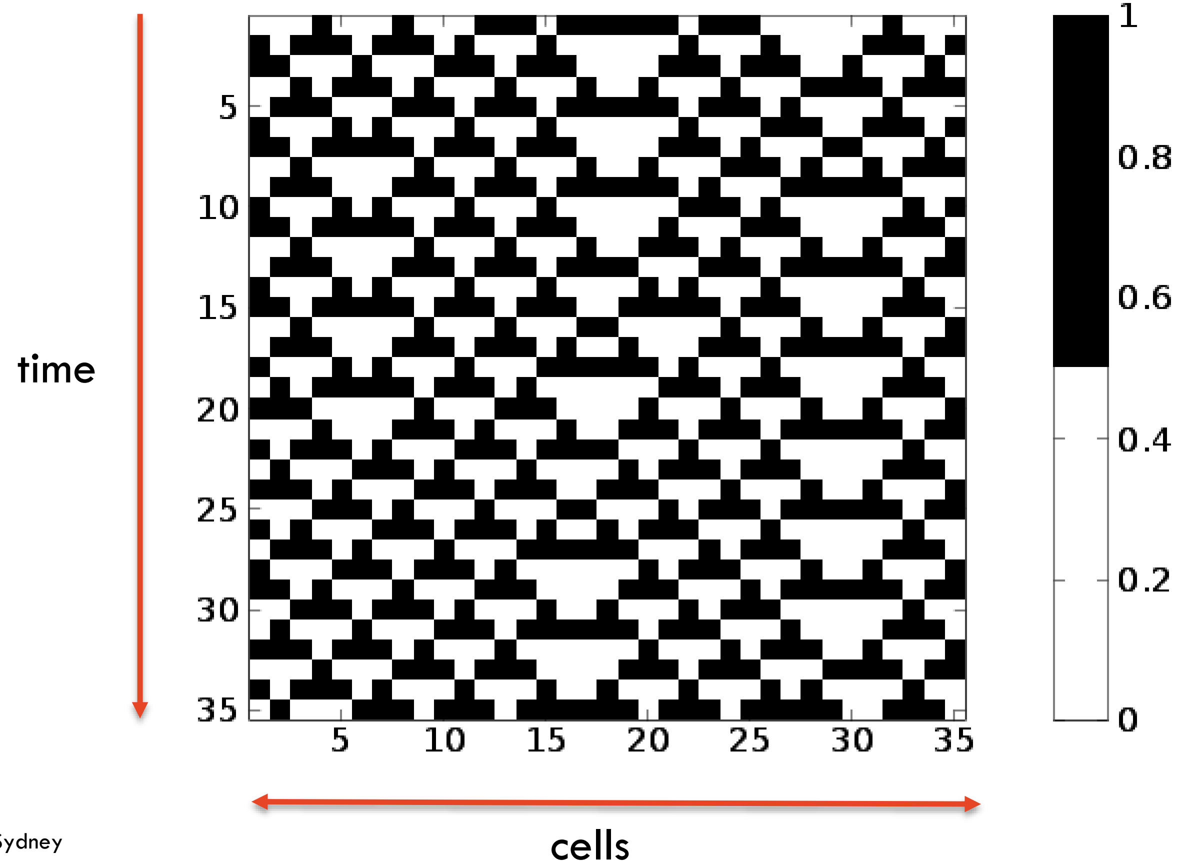
Q: Can we model the information processing in X in terms of:

- how much information was stored?
- how much was transferred?

Q: Can we partition them, do they overlap? etc.

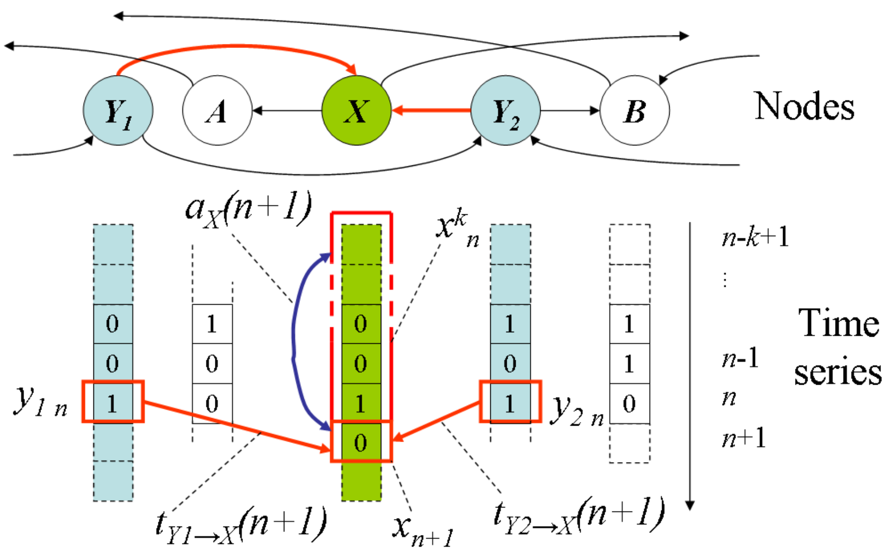
What kinds of multivariate time series could we analyse?

- How did we model behaviour in Scissors-Paper-Rock?
- How can we characterise the updates in cellular automata in terms of operations on information?



Information dynamics

– Studies computation of the next state of a target variable in terms of information storage, transfer and modification:



The measures examine:

- State updates of a target variable;
- Dynamics of the measures in space and time.

J.T. Lizier, "*The local information dynamics of distributed computation in complex systems*", Springer: Berlin/Heidelberg, 2013

Notation

- We consider time-series processes X:
  - Which consist of random variables $\{\ldots X_{n-1}, X_n, X_{n+1}, \ldots\}$;
  - With process realisations $\{\ldots x_{n-1}, x_n, x_{n+1}, \ldots\}$;
  - For countable time indices n.

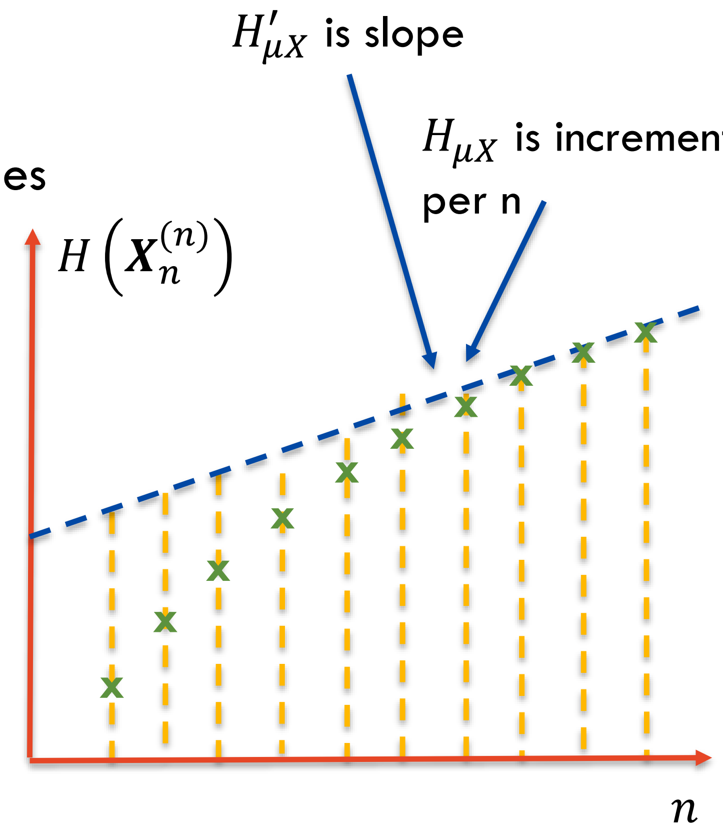
  - Denote consecutive block vector: $X_n^{(k)} = \{X_{n-k+1}, \ldots, X_{n-1}, X_n\}$
  - which has realisations $x_n^{(k)} = \{x_{n-k+1}, \ldots, x_{n-1}, x_n\}$

- Formally, we ask: "where does the information in a random variable X_{n+1} come from, in terms of other variables Y_m, Z_m, etc. for $m \leq n$?"

J.T. Lizier, "*The local information dynamics of distributed computation in complex systems*", Springer: Berlin/Heidelberg, 2013

Entropy rate

- Historically, entropy rate was first consideration here:
  1. Measures limiting rate at which block entropies scale with block length:
  $$H'_{\mu X} = \lim_{n\to\infty} \frac{1}{n} H(X_1, X_2, \dots, X_n)$$
  $$H'_{\mu X} = \lim_{n\to\infty} \frac{1}{n} H\left(\boldsymbol{X}_n^{(n)}\right)$$
  2. Measures uncertainty of next R.V. given past of process:
  $$H_{\mu X} = \lim_{n\to\infty} \frac{1}{n} H(X_n | X_1, X_2, \dots, X_{n-1})$$
  $$H_{\mu X} = \lim_{n\to\infty} H\left(X_n | \boldsymbol{X}_{n-1}^{(n-1)}\right)$$
- $H'_{\mu X} = H_{\mu X}$ for stationary processes



- Implication is that we're using past of the process as first informative source, and asking how much uncertainty remains.

T. M. Cover and J. A. Thomas. "Elements of Information Theory". Wiley-Interscience, New York, 1991. Section 4.2. *Note: primes are reversed in our notation!*
J. P. Crutchfield and D. P. Feldman, "Regularities unseen, randomness observed: Levels of entropy convergence", Chaos 13, 25 (2003).

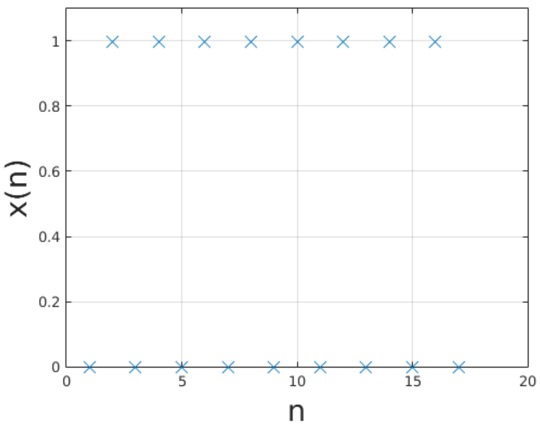
Information storage

- How much information from the past of the variable helps us predict its next state?

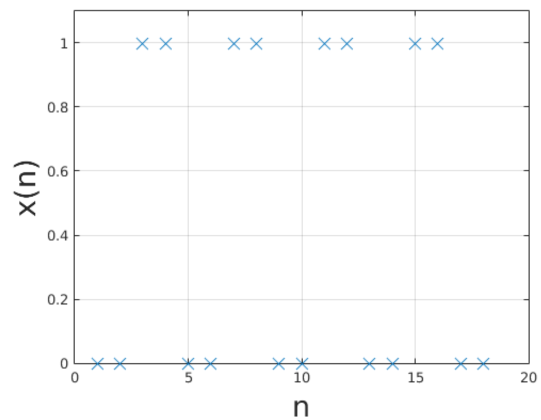
- Or, in modelling the dynamics of the variable, how much information storage would we include in that model by accounting for the past influence of that variable?

Information storage – using our intuition

- In each example:
  - Try to predict the next value of the variable.
  - What assumptions did you make?
  - Where specifically did you take the information from to make that prediction?
  - How much information did you get from the past?



(a)



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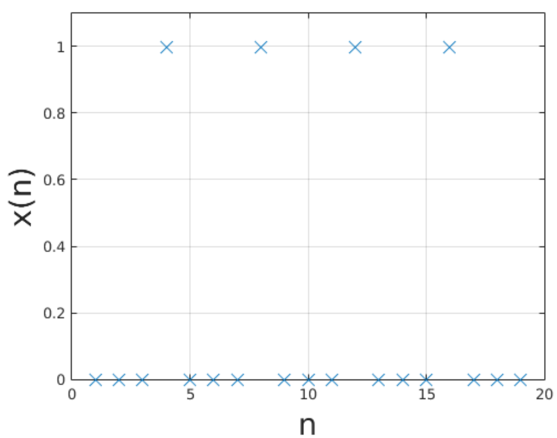

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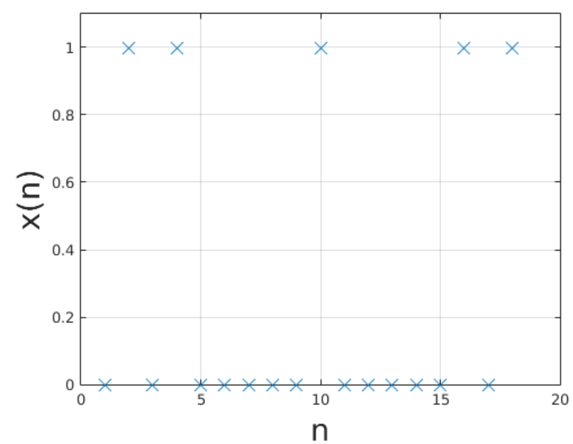
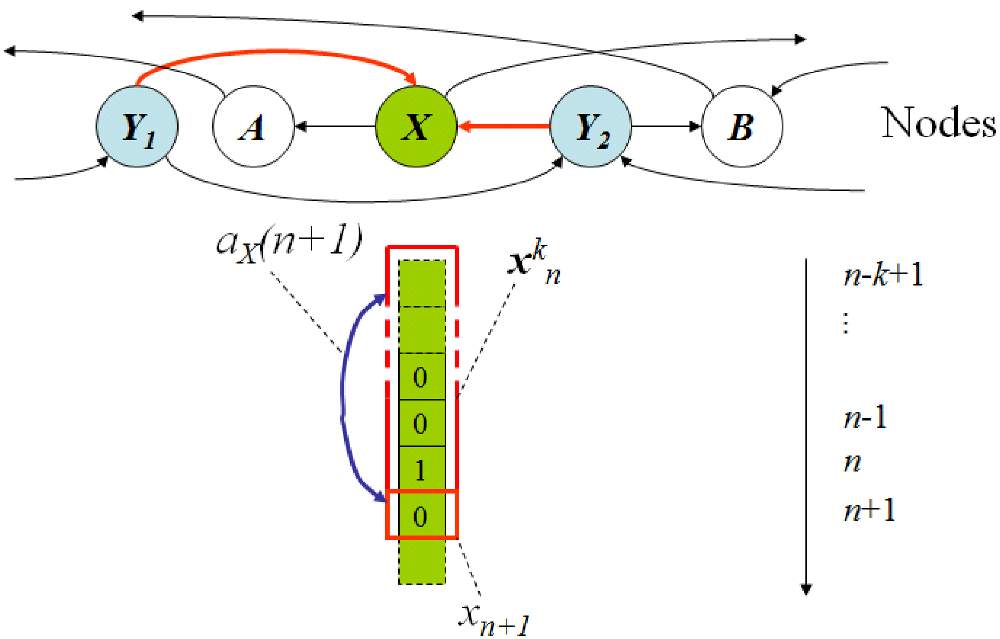


(d)

Active information storage

– How much information about the next observation X_n of process X can be found in its past state $X_n^{(k)} = \{X_{n-k+1}, \dots, X_{n-1}, X_n\}$?



Active information storage

$$A_X = \lim_{k \to \infty} I(X_n^{(k)}; X_{n+1})$$

$$A_X(k) = I(X_n^{(k)}; X_{n+1})$$

$$A_X = H(X_{n+1}) - H_{\mu X}$$

$$A_X(k) = \left\langle \log_2 \frac{p(x_{n+1} | x_n^{(k)})}{x_{n+1}} \right\rangle$$

$$a_X(k) = \log_2 \frac{p(x_{n+1} | x_n^{(k)})}{x_{n+1}}$$

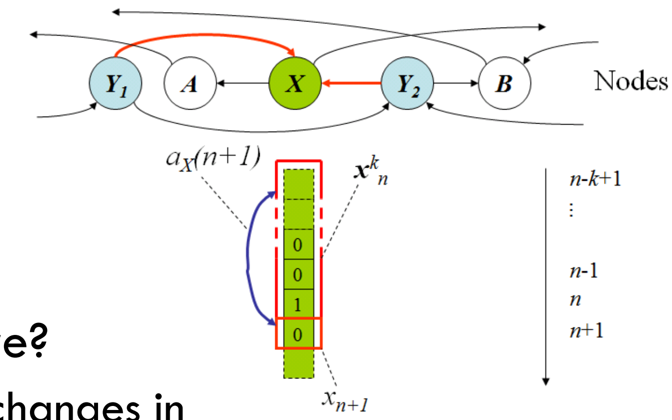
– AIS: Average information from past state that is in use in predicting the next value

– Local AIS: information from a specific past state in use in predicting specific next value

J.T. Lizier, M. Prokopenko, & A.Y. Zomaya, "*Local measures of information storage in complex distributed computation*", Information Sciences 208, 39 (2012)

J.T. Lizier, "*JIDT: An information-theoretic toolkit for studying the dynamics of complex systems*", Frontiers in Robotics and AI, 1:11, 2014; appendix A.2 and A.3

Active information storage: interpretations

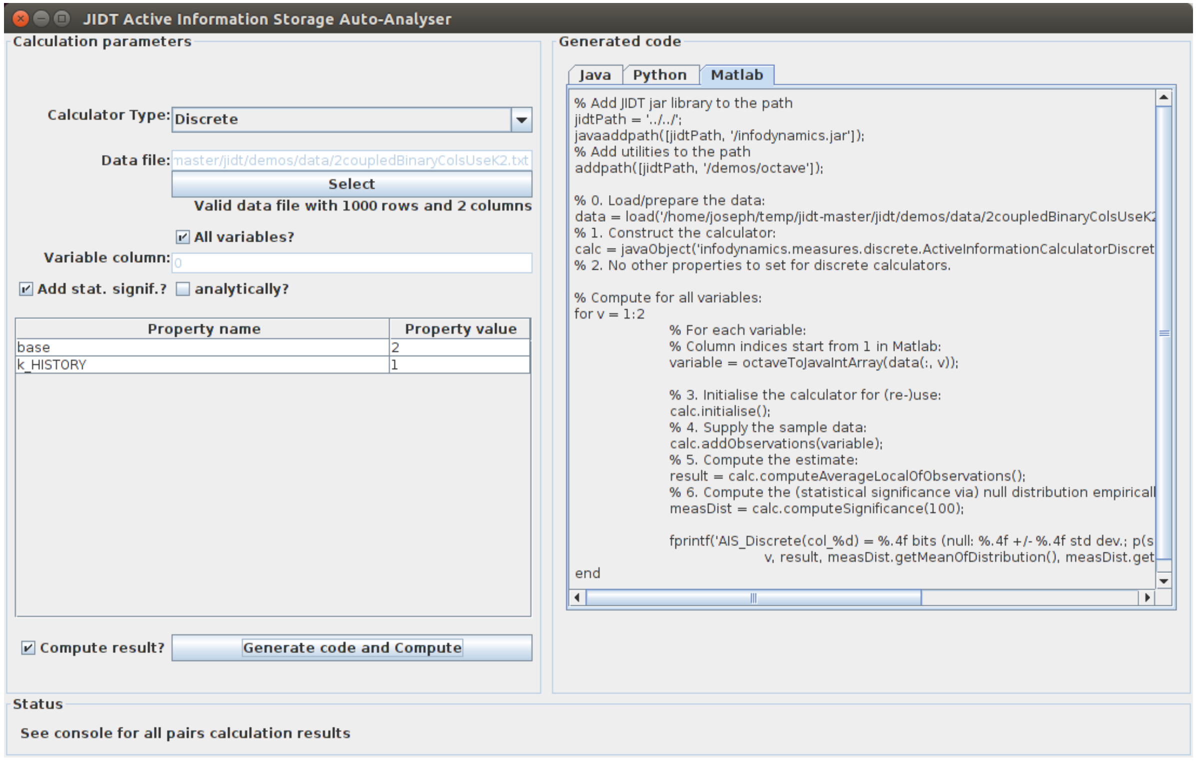


- Captures total memory and nonlinear effects
  - Autocorrelation just linear component from each past value separately.
- What types of information storage does A_X capture?
  - Active storage in dynamics (as opposed to passive changes in underlying structure);
  - Internal (causally) stored information;
  - Distributed information storage via feedback and feedforward loops, i.e. recurrent connections (network effects);
  - Input-driven storage: Patterns in input dynamics driving a variable
- All of these are intrinsically *modelled* as information storage to an observer when we account for the information here.

M. Wibral, J. T. Lizier, S. Vögler, V. Priesemann, and R. Galuske, "*Local active information storage as a tool to understand distributed neural information processing*", Frontiers in Neuroinformatics 8, 1+ (2014).
Zipser, D., Kehoe, B., Littlewort, G., and Fuster, J. (1993), "*A spiking network model of short-term active memory*", Journal of Neuroscience, 13, 8, 3406–3420
Lizier, J. T., Atay, F. M., and Jost, J. (2012), "*Information storage, loop motifs, and clustered structure in complex networks*", Phys Rev E, 86, 2, 026110
Obst, O., Boedecker, J., Schmidt, B., and Asada, M. (2013), "*On active information storage in input-driven systems*", arXiv:1303.5526

Active information storage in JIDT

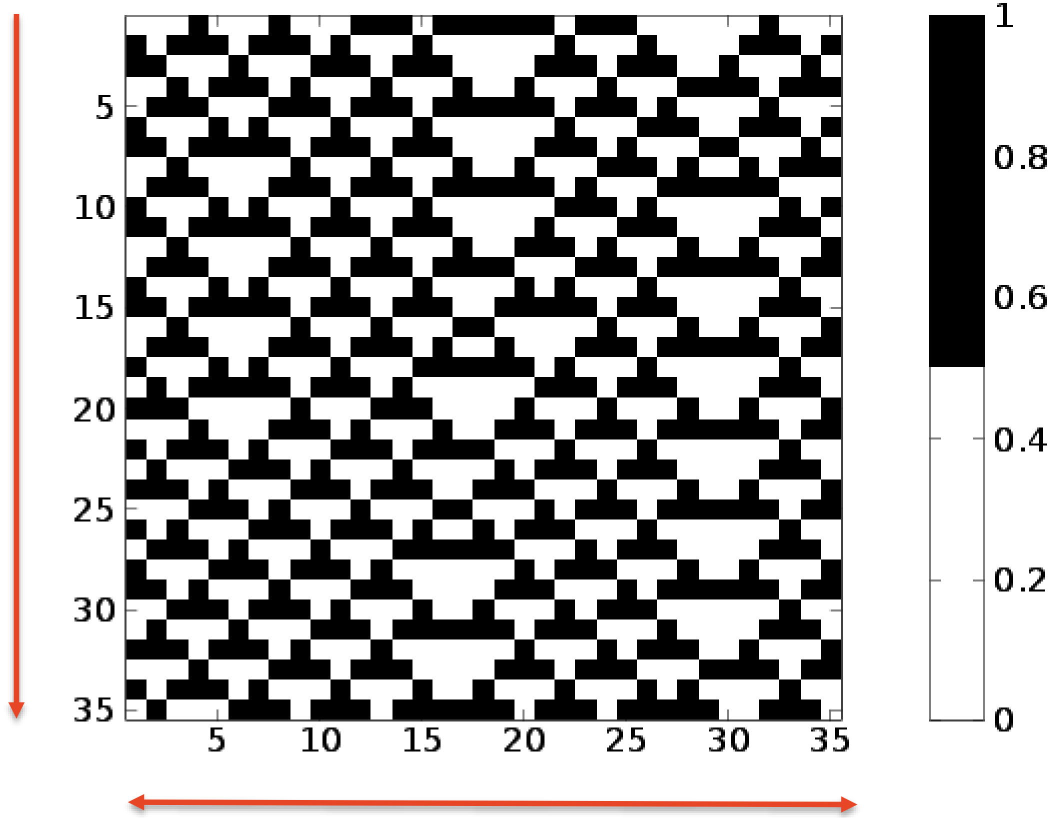
- Start AIS AutoAnalyser
- Notice the important `k_HISTORY` parameter
- Has all types of underlying MI estimators available, same parameters as each and features (e.g. statistical significance, local values)



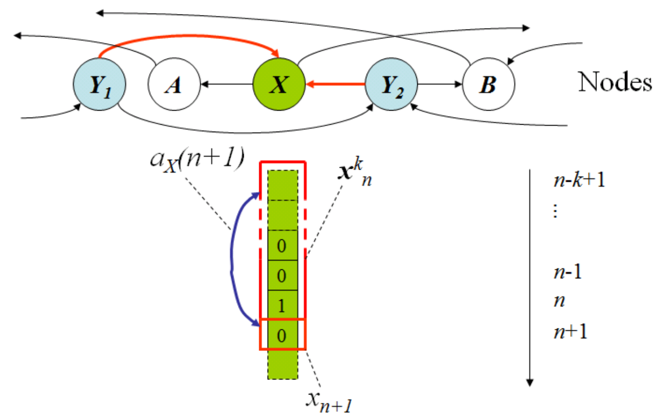
AIS: key question – how to set history length k?

– For example in Elementary Cellular Automata rule 54?

time



AIS: key question – how to set history length k?



– $X_n^{(k)} = \{X_{n-k+1}, \ldots, X_{n-1}, X_n\}$ is a Takens' embedding of the past state of X.
– Want to set embedding for optimal prediction of next value X_{n+1}
  – $p\left(x_{n+1} \middle| x_n^{(k)}, x_{n-k}\right) = p(x_{n+1}|x_n^{(k)})$
– But we have competing concerns:
  – Want k as large as possible to capture all potential memory.
  – But increasing k increases our exposure to undersampling.
– There will be a "sweet spot" in between, either:
  – Where further values from past don't actually contribute, or
  – Where there is not enough data to validate their contribution.

AIS: key question – how to set embedding parameters?

- Can use embedding delay τ also: $X_n^{(k,\tau)} = \{X_{n-(k-1)\tau}, \dots, X_{n-\tau}, X_n\}$

- Option 1: Ragwitz criteria to minimize prediction error
  - Find K nearest neighbours for each $x_n^{(k)}$
  - Find mean of their corresponding x_{n+1}
  - Compute difference to actual x_{n+1}
  - Take mean over all points and minimize w.r.t history length k and embedding delay τ

M. Ragwitz and H. Kantz. "*Markov models from data by simple nonlinear time series predictors in delay embedding spaces*". *Phys. Rev. E*, 5(5):056201+, 2002
M. Wibral, R. Vicente, and M. Lindner. "*Transfer entropy in neuroscience*". In M. Wibral, R. Vicente, and J. T. Lizier, editors, *Directed Information Measures in Neuroscience*, pp. 3-36. Springer, Berlin/Heidelberg, 2014.

AIS: key question – how to set embedding parameters?

- Option 2: Maximize bias-corrected AIS

$$A'_X = A_X - \langle A_X^S \rangle$$

  - (where A_X^S are surrogates, created by destroying the past-next relationship)
  - w.r.t history length k and embedding delay τ

- This means we use more points from history so long as they contribute more information about the next value than the increase in bias due to the higher dimensionality.
- For KSG estimator, bias correction is already built in!

Garland, J., James, R.G., Bradley, E. "*Leveraging information storage to select forecast-optimal parameters for delay-coordinate reconstructions*". Physical Review E 2016, 93, 022221+

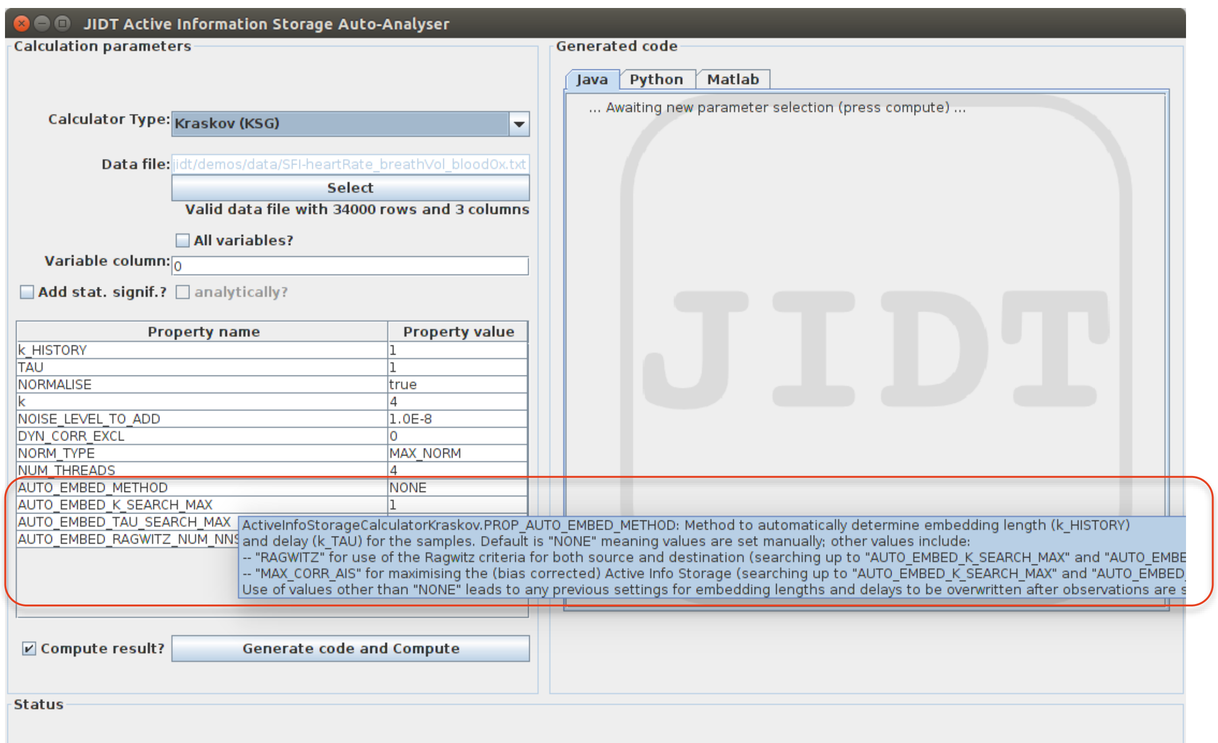
AIS: key question – how to set embedding parameters?

- Option 3: Non-uniform embedding
  - Incrementally select points from past which make a statistically significant contribution beyond the points already selected.
  - Not yet implemented in JIDT but is in our higher-level IDTxl toolbox

L. Faes, G. Nollo, and A. Porta, "*Information-based detection of nonlinear Granger causality in multivariate processes via a nonuniform embedding technique*", Physical Review E 83, 051112+ (2011)

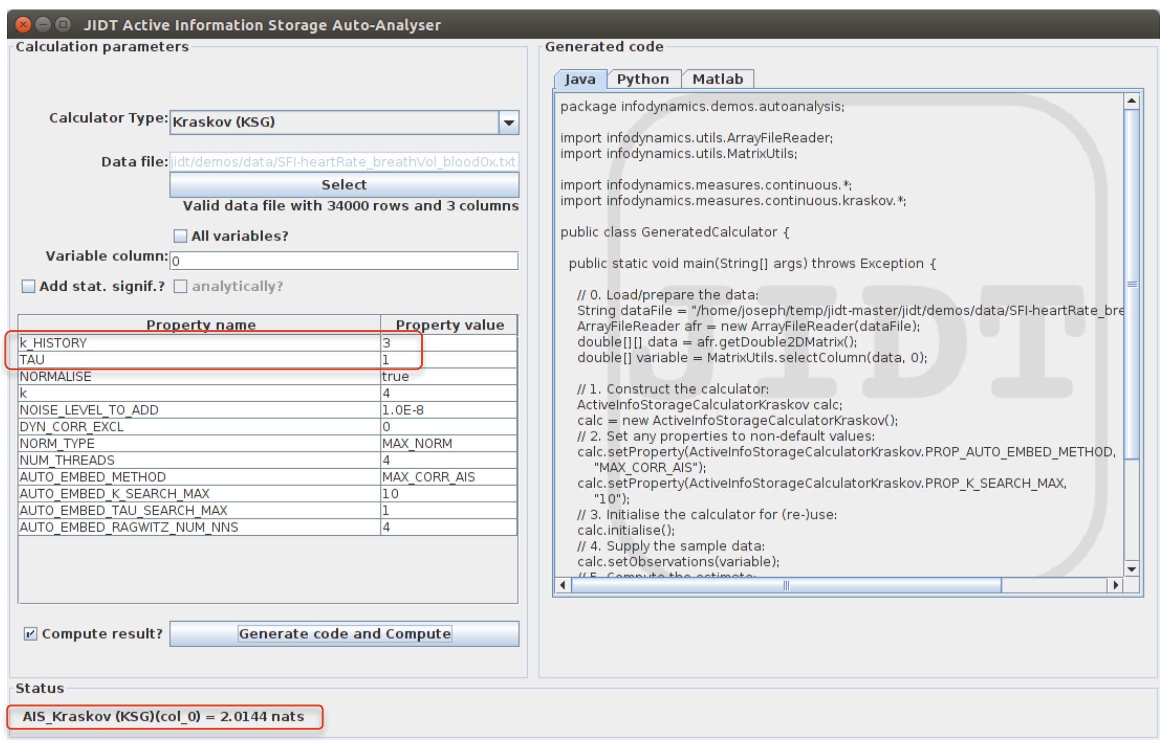
AIS: setting embedding parameters in JIDT

- Select KSG estimator
- Hover on `AUTO_EMBED_METHOD` property to see options:
  - `NONE`: set `k_HISTORY` and `TAU` manually.
  - `RAGWITZ`: optimal parameters to minimise prediction error scanned up to `AUTO_EMBED_K_SEARCH_MAX` and `AUTO_EMBED_TAU_SEARCH_MAX`
  - `MAX_CORR_AIS`: optimal parameters to max. bias-corrected AIS scanned up to `AUTO_EMBED_K_SEARCH_MAX` and `AUTO_EMBED_TAU_SEARCH_MAX`

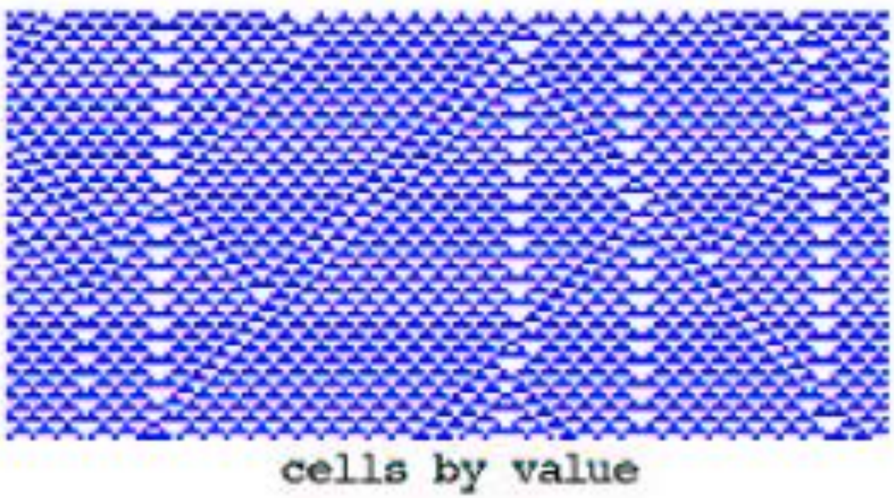


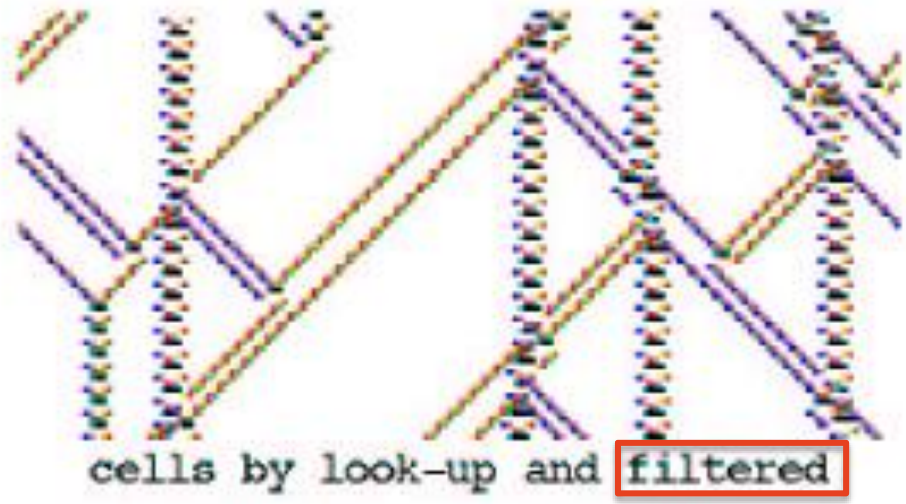
AIS: setting embedding parameters in JIDT

- Select the `SFI-heartRate_breathVol_bloodOx.txt` data set
- Set `AUTO_EMBED_METHOD` to `MAX_CORR_AIS` and `AUTO_EMBED_K_SEARCH_MAX` to 10 and `AUTO_EMBED_TAU_SEARCH_MAX` to 1.
- Click Compute
- The result is returned with optimal parameters shown in `k_HISTORY` and `TAU`. You can retrieve them in code via a `getProperty()` call.



Example: Computational role of emergent structure in CAs





- Emergent structure:
  - Domains, blinkers
  - Particles
    - Gliders, Domain walls
  - Collisions

- Conjectured to represent
  - Information storage
  - Information transfer
    - “
  - Information modification

A. Wuensche, “Classifying cellular automata automatically: Finding gliders, filtering, and relating space-time patterns, attractor basins, and the Z parameter,” Complexity, vol. 4, no. 3, pp. 47–66, 1999. (plus image credit)

C. G. Langton, “Computation at the edge of chaos: phase transitions and emergent computation,” Physica D, vol. 42, no. 1-3, pp. 12–37, 1990.

Example: Computational role of emergent structure in CAs

- Go to tutorial sheet to try out our AIS calculator on CA data:
  - We'll compute appropriate embedding length
  - We'll compute local AIS values and see whether domains and blinkers do indeed have strong information storage values.

Predictive information

– How much information about the future $X_{n+1}^{(k+)} = \{X_{n+1}, X_{n+2}, \dots, X_{n+k}\}$ of process X can be found in its past state $X_n^{(k)} = \{X_{n-k+1}, \dots, X_{n-1}, X_n\}$?

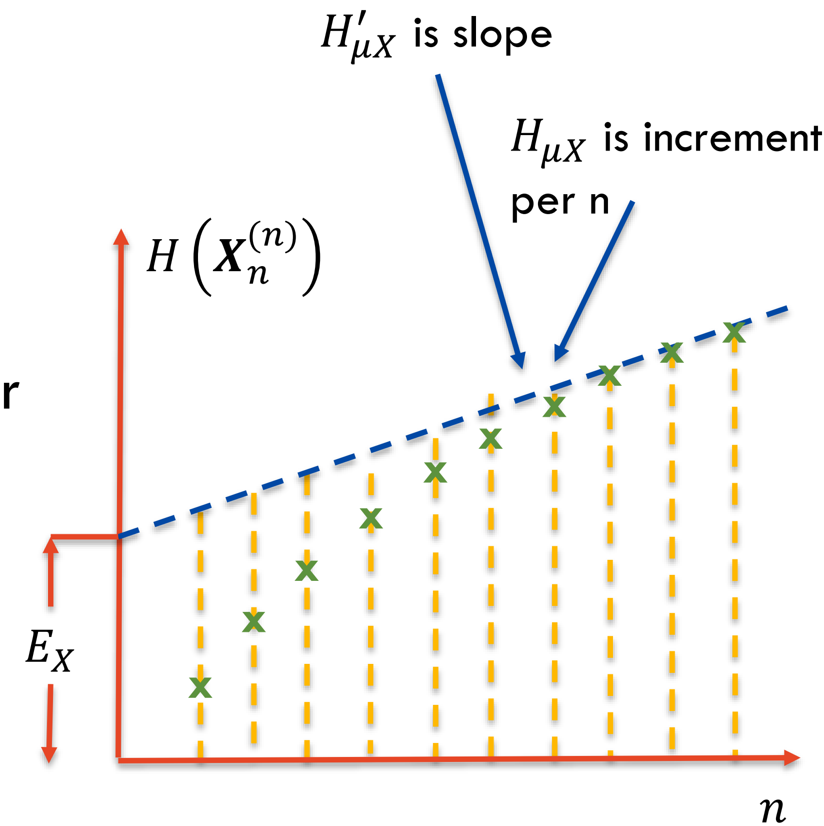
$$E_X = \lim_{k\to\infty} I(X_n^{(k)}; X_{n+1}^{(k+)})$$
$$E_X(k) = I(X_n^{(k)}; X_{n+1}^{(k+)})$$

– Captures *all* of the information stored in the past that is used *at some point* in the future.
  – Contrast to AIS which measures the part of the stored information in use in computing the next value.
  – We're more interested in AIS because it focusses on the computation of the next value and is complementary to information transfer.

W. Bialek, I. Nemenman, and N. Tishby, "*Complexity through nonextensivity*", Physica A 302, 89 (2001)
J. P. Crutchfield and D. P. Feldman, "*Regularities unseen, randomness observed: Levels of entropy convergence*", Chaos 13, 25 (2003).

Predictive information and excess entropy

– Excess entropy quantifies total structure or memory as slowness of the approach of the conditional entropy rate estimates to their limiting value:
– Is equal to predictive information for stationary processes.



P. Grassberger, “*Toward a quantitative theory of self-generated complexity*”, Int. J. Theoretical Physics 25, 907 (1986)
J. P. Crutchfield and D. P. Feldman, “*Regularities unseen, randomness observed: Levels of entropy convergence*”, Chaos 13, 25 (2003).

Information dynamics Part I: summary

- We've looked at the philosophy behind the information dynamics approach for analysing information processing in complex systems.
- In particular, we've focussed on how information storage is characterised.
  - And used JIDT AutoAnalyser and extensions of code to analyse information storage in complex systems data sets.

- *Next lecture*: Information processing in complex systems Part II – information transfer.

Questions