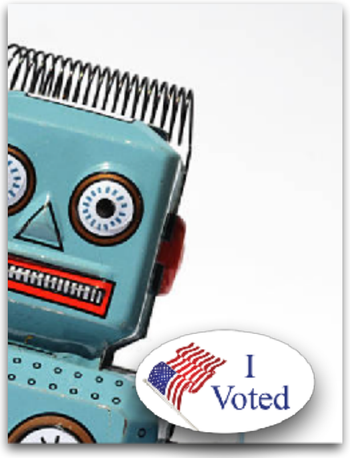
Modeling Democracy

Lecture 1 - **Intro and voting systems**



Moon Duchin, Winter 2026 — Mon Jan 5

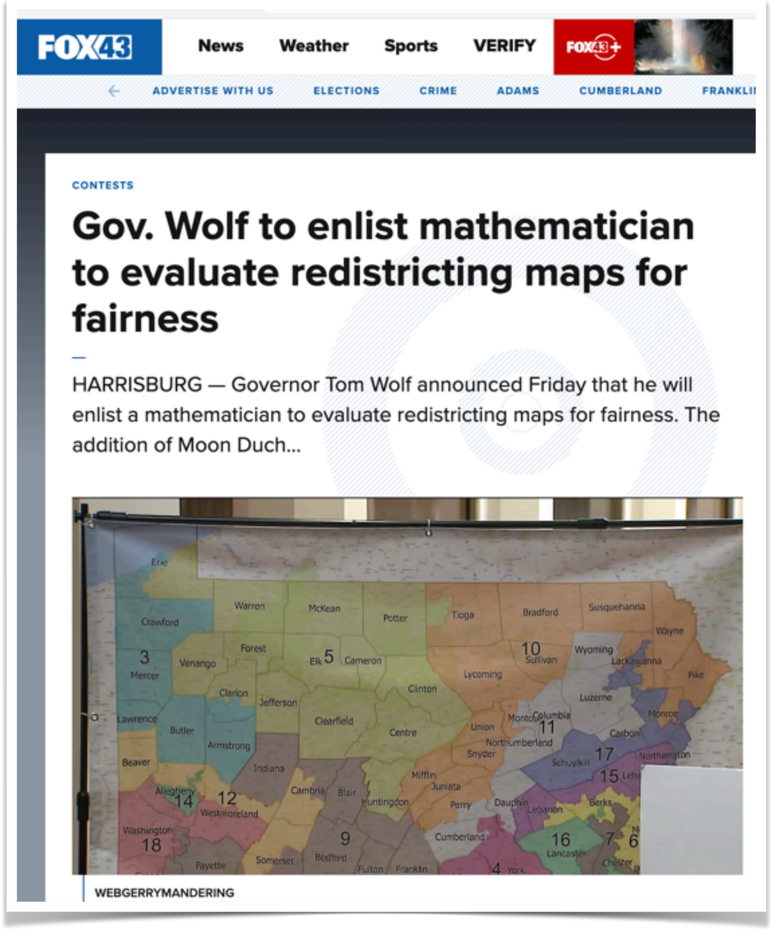
Welcome, course basics

- I'm new(ish) to Chicago. Still figuring out lots of stuff.
- This course is designed for PhD students from across the university. Masters student? Undergrad? Let's chat.
- Our TA is **Alec Sun** (CS PhD student working in econ/CS).

- There will be problem sets and solution sets.
- Each of you *can* give a short presentation (10-20 mins). It is great practice!
- Final projects optional. Last year, two final projects turned into publications.
- **Community norms**: respect, integrity, generosity. Speak in class. Work together on problems. Strong preference for no AI use, but if you use it, tell me how.

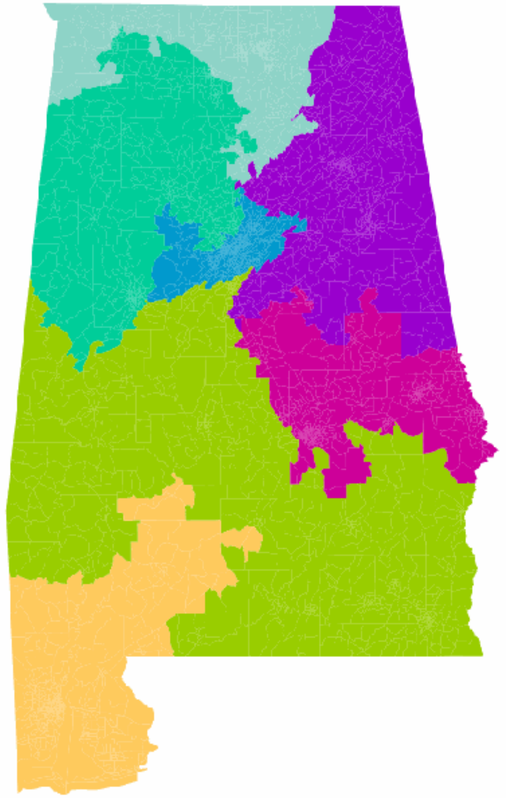
My background

- Math and women's studies — strong interest in philosophy, history of science, STS, civil rights, law
- Math expertise in Teichmüller theory, geometric group theory, dynamical systems
- Post-tenure pivot to studying elections and redistricting, "COMSOC" (computational social choice)
- Expert witness in redistricting
- Consulting for governments and watchdog groups
- Research for democracy reform organizers



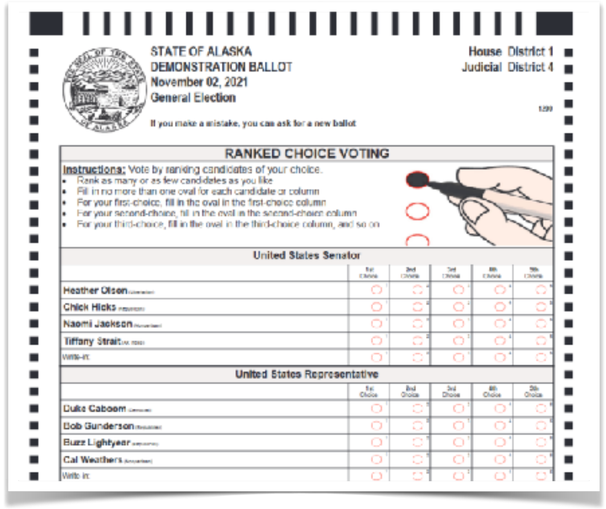
My research

- I found “the literature” surprisingly weak or gappy in several key areas...
    - ★ redistricting baselines
    - ★ fairness criteria, fairness metrics
    - ★ statistical models of voting
- ...and this course will start with the classics and then build up to ideas in those directions. I want to get you ready to contribute.



so let’s begin

Voting rules


STATE OF ALASKA
DEMONSTRATION BALLOT
November 02, 2021
General Election

House District 1
Judicial District 4

If you make a mistake, you can ask for a new ballot

RANKED CHOICE VOTING

Instructions: Vote by ranking candidates of your choice.
- Rank as many or as few candidates as you like
- Fill in no more than one oval for each candidate or column
- For your first-choice, fill in the oval in the first-choice column
- For your second-choice, fill in the oval in the second-choice column
- For your third-choice, fill in the oval in the third-choice column, and so on

United States Senator

Heather Olsen
Chick Hicks
Naomi Jackson
Tiffany Strait
Write-in:

United States Representative

Duke Caboom
Bob Gunderson
Buzz Lightyear
Cal Weathers
Write-in:

- A **ballot** can be one of several types
  - ranking
  - approval (set), possibly limited
  - scores, possibly limited/structured
- A **preference profile** is a collection of the ballots of a group
- A **voting rule** (or election mechanism or social choice function) is a function from profiles to outcomes — single-winner or set, possibly ranking. Classically, deterministic.

Rankings-based voting rules

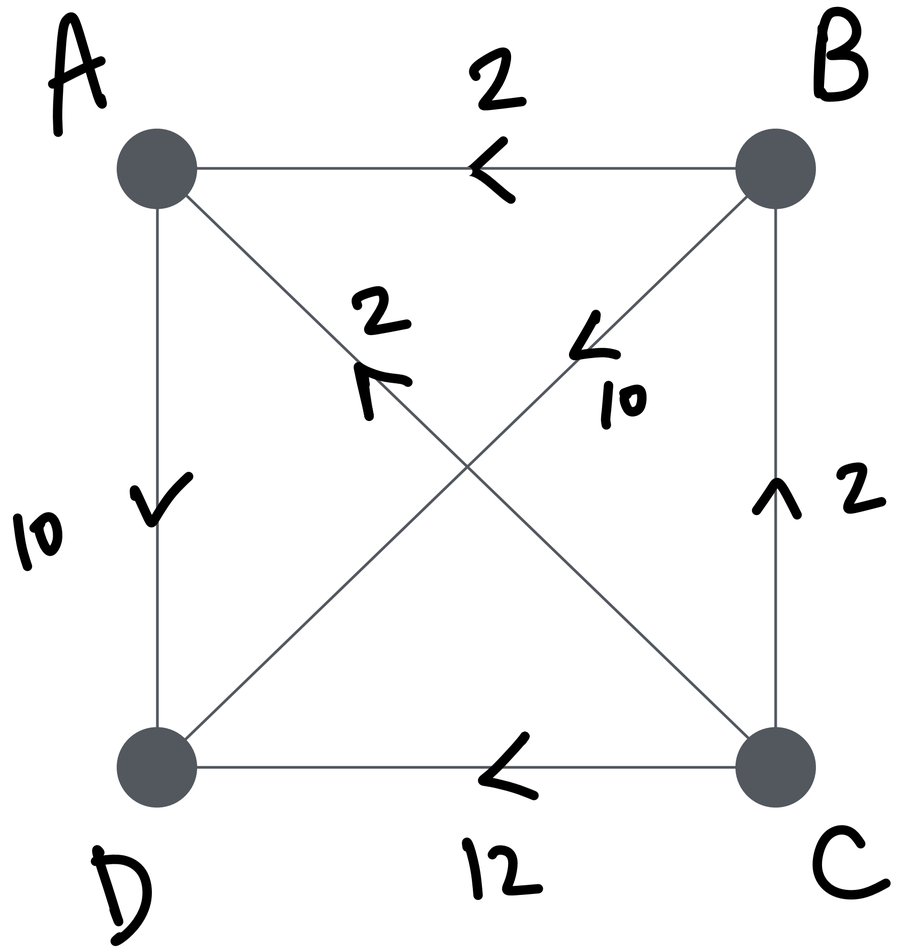
Plurality	Positional (Borda)	Sequential
Pairwise (Lhull)	Condo-Borda	Top-Two
IRV	Coombs	STV
Beatpath (Schulze)	Smith	Smithified rules
Dodgson	Kemeny	“Secondality”

Dictatorship

Plurality	Positional (Borda)	Sequential
Pairwise (Lhull)	Condo-Borda	Top-Two
IRV	Coombs	STV
Beatpath (Schulze)	Smith	Smithified rules
Dodgson	Kemeny	“Secondality”

- **Plurality** — most first-place votes (FPV) wins
- **Positional (Borda)** — voter gives out points by position of ranking (e.g., 3 for 1st, 2 for 2nd, 1 for 3rd)
- **Sequential** — order fixed in advance, candidates face each other pairwise with one advancing based on head-to-head (H2H) comparison

x5	x2	x8	x1
A	D	C	C
B	A	B	D
C	B	A	B
D	C	D	A



B>A by (net) 2

C>B by 2

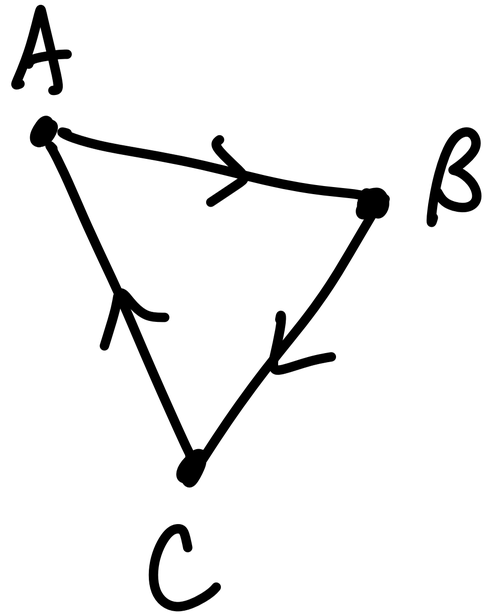
C>A by 2

B>D by 10

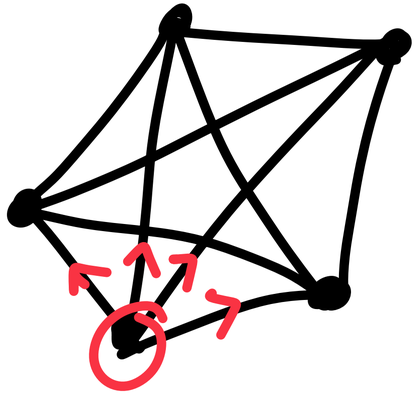
A>D by 10

C>D by 12

n_1	n_2	n_3
A	B	C
B	C	A
C	A	B



Condorcet cycle



Condorcet candidate

Plurality	Positional (Borda)	Sequential
Pairwise (Lhull)	**Condo-Borda**	**Top-Two**
IRV	Coombs	STV
Beatpath (Schulze)	Smith	Smithified rules
Dodgson	Kemeny	“Secondality”

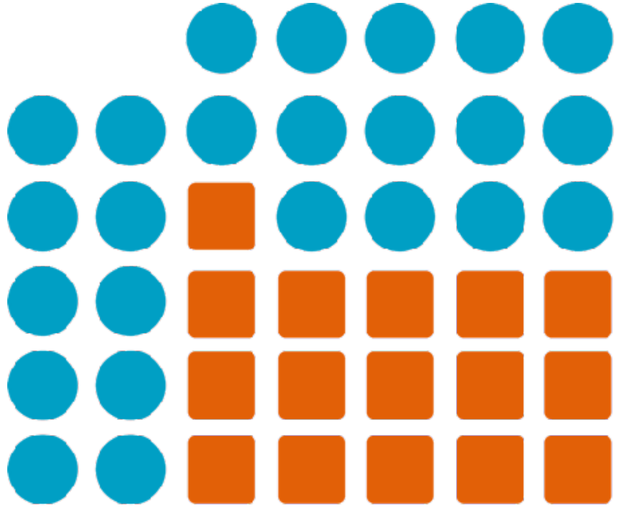
- **Pairwise** — each candidate gets 1 point per H2H win, 1/2 per tie
- **Condo-Borda** — Condorcet candidate if they exist, else Borda winner (a “Frankenstein” rule)
- **Top-Two** — two candidates with most FPV advance to instant runoff, decided by H2H

Plurality	Positional (Borda)	Sequential
Pairwise (Lhull)	Condo-Borda	Top-Two
IRV	**Coombs**	**STV**
Beatpath (Schulze)	Smith	Smithified rules
Dodgson	Kemeny	“Secondality”

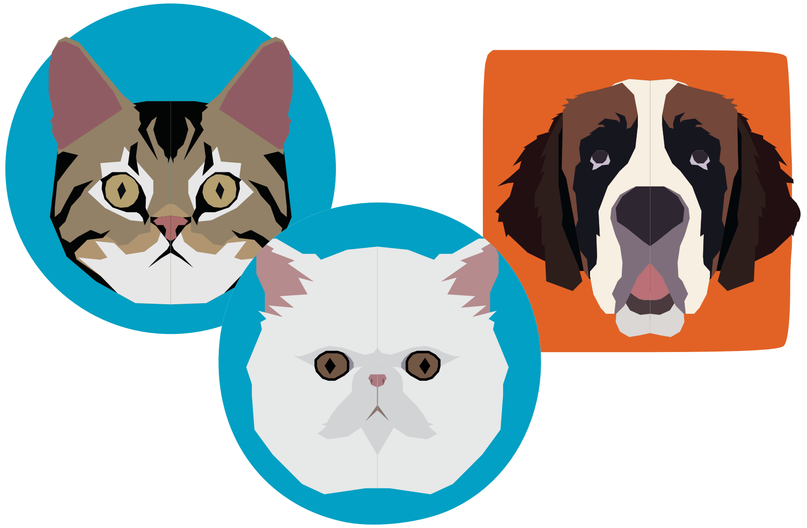
- **IRV (instant runoff voting)** — successively eliminate those with fewest FPV, transfering their votes, until someone has a majority

- **Coombs**— same, but eliminate those with most last-place votes

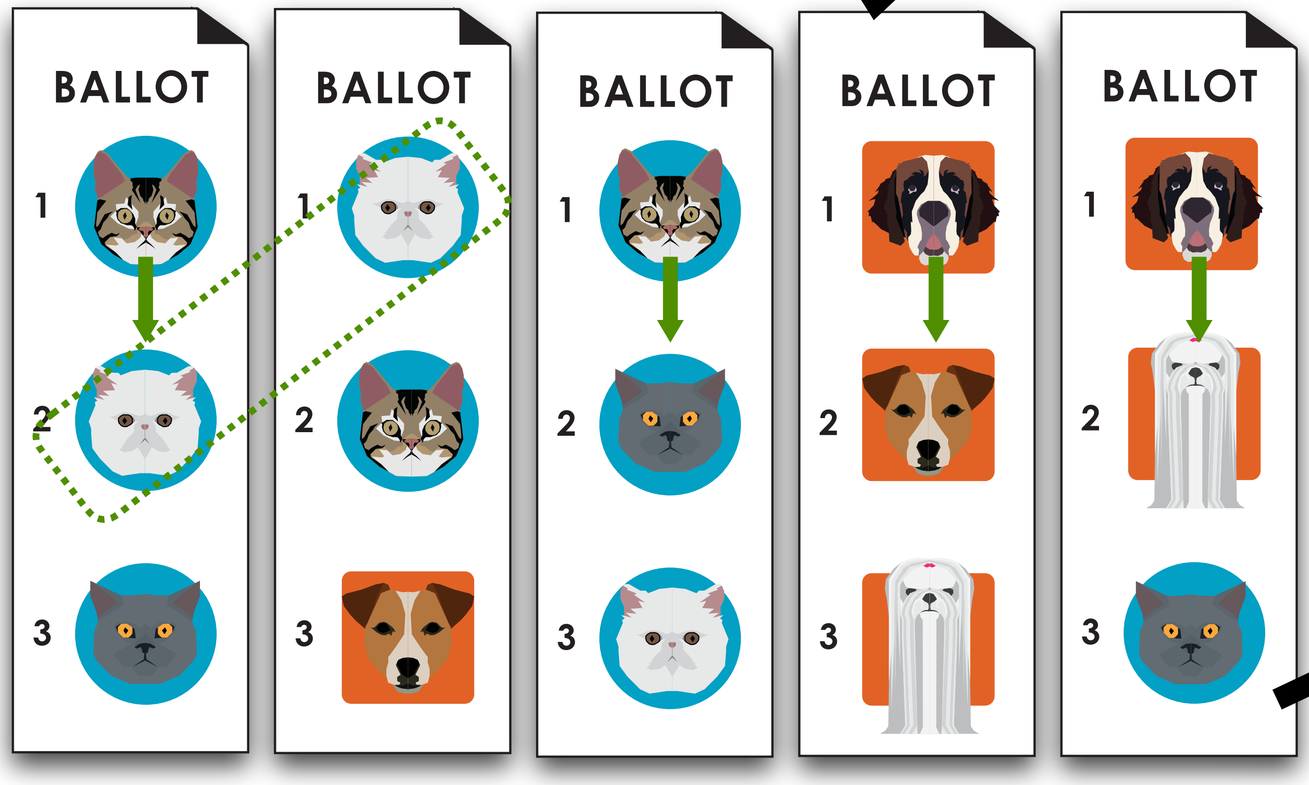
- **STV (single transferable vote)** — set a threshold of election (typically $\frac{1}{k+1}$ or so) and successively elect and eliminate until k are elected



40% dog lovers
60% cat lovers

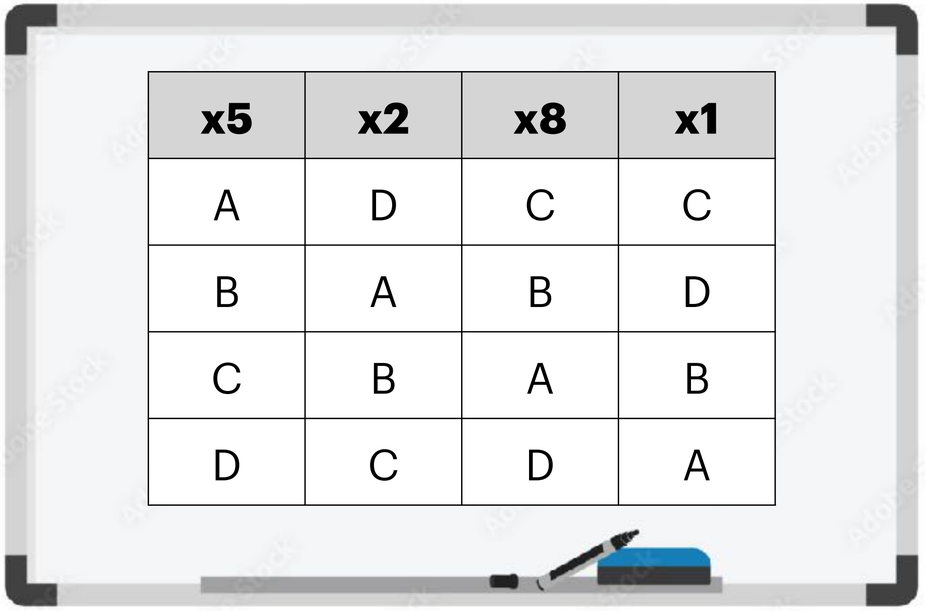


winner set



votes transfer from those elected or eliminated

Chalkboard vs Real World



x5	x2	x8	x1
A	D	C	C
B	A	B	D
C	B	A	B
D	C	D	A

$$\mathscr{W}_{\text{plu}} = \{C\} \quad \text{or} \quad W_{\text{plu}} = C$$

Real Minneapolis Mayoral election, 2013

x3864	x3309	x3031	x2502	x2212	x1831	x1479	x1445	x1343	x1305
Andrew	Hodges	Hodges	Andrew	Hodges	Hodges	Hodges	Hodges	Winton	Andrew
	Andrew	Samuels	Hodges		Samuels	Andrew	Andrew		Hodges
	Samuels	Andrew	Samuels		Cherry homes	Cherry homes			Cherry homes

Real Minneapolis Mayoral election, 2013

x3864	x3309	x3031	x2502	x2212	x1831	x1479	x1445	x1343	x1305
Andrew	Hodges	Hodges	Andrew	Hodges	Hodges	Hodges	Hodges	Winton	Andrew
	Andrew	Samuels	Hodges		Samuels	Andrew	Andrew		Hodges
	Samuels	Andrew	Samuels		Cherry homes	Cherry homes			Cherry homes

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Andrew	Hodges	Hodges	Andrew	Hodges	Hodges	Hodges	Hodges	Winton	Andrew
	Andrew	Samuels	Hodges		Samuels	Andrew	Andrew		Hodges
	Samuels	Andrew	Samuels		Cherry homes	Cherry homes			Cherry homes

These **top ten ballots** account for 22,321 out of 85,922 votes cast — about 1/4 of the profile.

In fact, in this format, the full table would have 7084 columns, where columns 2999-7084 (or 4086 ballots, more than half of the types cast) have exactly **one vote each**.

(There were **35 candidates**, including Abdul "The Rock" Rahaman, Ole Savior, and Captain Jack Sparrow.)

x3864	x3309	x3031	x2502	x2212	x1831	x1479	x1445	x1343	x1305
4.87%	4.17%	3.82%	3.15%	2.79%	2.31%	1.87%	1.82%	1.69%	1.65%
And	Hod	Hod	And	Hod	Hod	Hod	Hod	Win	And
	And	Sam	Hod		Sam	And	And		Hod
	Sam	And	Sam		Che	Che			Che

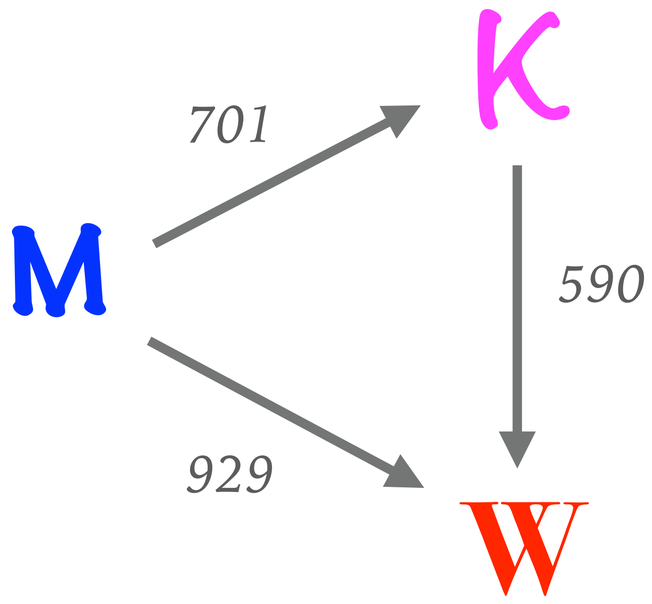
The Minneapolis *outcome*, however, was very stable.

But sometimes "the system votes" in the real world!

Actual Burlington, VT mayoral race (2009)

767	1332	455	2043	371	568	1513	495	1289
M	M	M	K	K	K	W	W	W
W	K		M	W		M	K	
K	W		W	M		K	M	

2554 2982 3297



what are the goals of representative democracy?

Athenian democracy: sortition (government by the random)

Hobbes, Leviathan (1651): "A commonwealth is said to be instituted, when a multitude of men do agree, and covenant, every one with every one, that to whatsoever man, or assembly of men, shall be given by the major part, the right to present the person of them all, that is to say, to be their representative; every one... shall authorize all the actions and judgments, of that man, or assembly of men, in the same manner, as if they were his own."

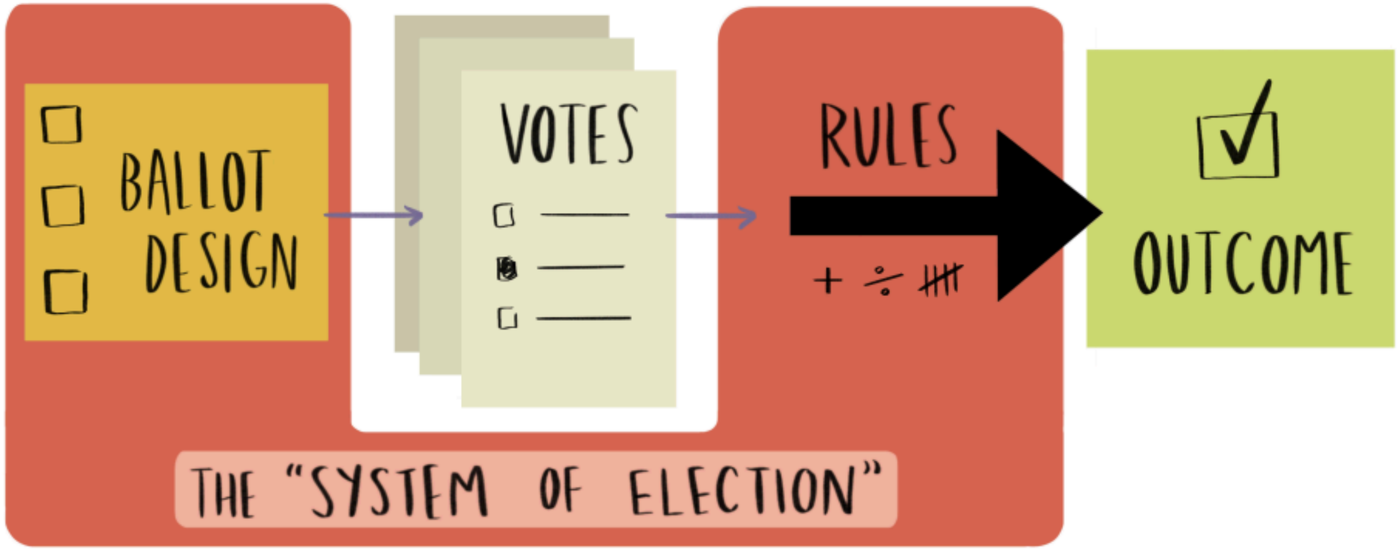
Raymond Poincaré: "most exact possible image of the country"

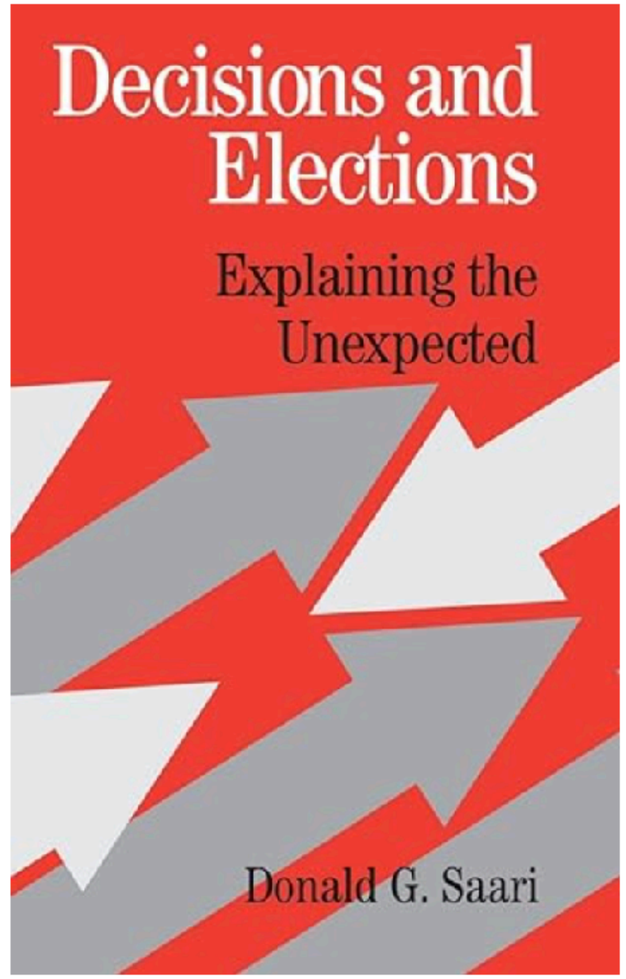
John Adams (1776): "The principal difficulty lies, and the greatest care should be employed, in constituting this representative assembly. It should be in miniature an exact portrait of the people at large. It should think, feel, reason, and act like them."

John Adams (1776) — fuller quote

In a Community consisting of large Numbers, inhabiting an extensive Country, it is not possible that the whole Should assemble, to make Laws. The most natural Substitute for an Assembly of the whole, is a Delegation of Power, from the Many, to a few of the most wise and virtuous. In the first Place then establish Rules for the Choice of Representatives: Agree upon the Number of Persons who shall have the Privilege of choosing one. As the Representative Assembly, should be an exact Portrait, in Miniature, of the People at large, as it should think, feel, reason and act like them...

REPRESENTATIVE DEMOCRACY, TO A MATHEMATICIAN





prove the properties of various systems

...TO A LANGUAGE PHILOSOPHER

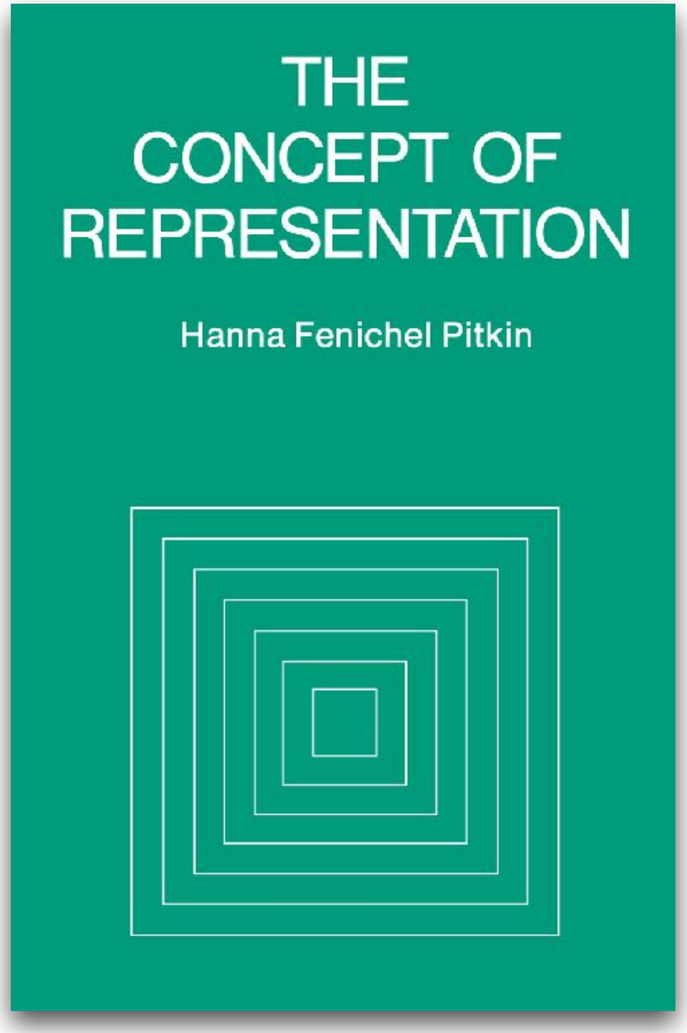
Should the legislature be a portrait, reflection, image of the people?

Are they to be expert emissaries?

Descriptive or symbolic?

How are they held accountable?

And just whom do they represent?

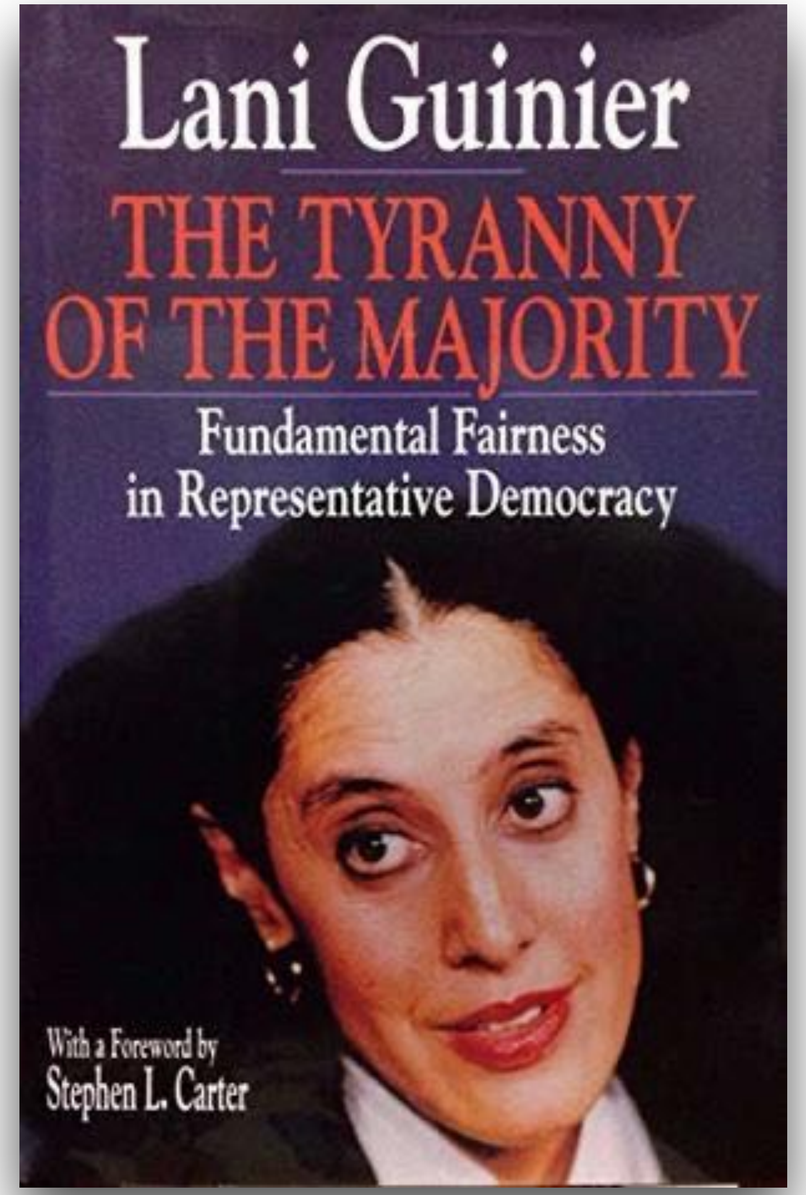


...TO A CIVIL RIGHTS SCHOLAR

"I have always wanted to be a civil rights lawyer. This lifelong ambition is based on a deep-seated commitment to democratic fair play—to playing by the rules as long as the rules are fair.

...To me, fair play means that the rules encourage everyone to play. They should reward those who win, but they must be acceptable to those who lose. The central theme of my academic writing is that **not all rules lead to elemental fair play**. Some even commonplace rules work against it."

A fixed and permanent majority has no checks on its power.



we will try to blend these perspectives.