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Subject - Design and Analysis of Algorithm (TCS 505)

Assignment-1

① Asymtotic Notation

Asymtotic Notations are used to tell the complexity of an input algorithm when the input is very large.

The main idea of asymptotic analysis is to have a measure of the efficiency of algorithms that does not depend on the machine.

Following asymtotic notations are used -

① Big O notation
- Big O Notation defines the upper bound of an algorithm, it bound a function only from above
  For ex - Insertion sort $O(n^2)$ (worst case)

② Θ Notation
- The Θ notation bounds a function from above and below, so it defines exact asymtotic behaviour
  For ex - Expression : $2n^2 + n + 1$
  Time complexity = $\Theta(n^2)$

③ Ω Notation
- The Ω notation provides the lower bound of an algorithm
  For ex - Selection sort $\Omega(n^2)$ (best case)

④ little-o Notation
- The little-o notation provides the upper bound of an algorithm, but it is not a tight bound
  For ex Expression : $n + 2$
  Time complexity = $o(n^2)$

⑤ little-ω notation
- The little-ω notation provides the lower bound of an algorithm
  For ex: Expression - $n^2 + 3n + 1$
  Time complexity = $\omega(n)$

(2)

```
for (i=1 to n)
{
    i = i*2
}
```

Time complexity : $O(\log n)$

(3)

$T(n) = \{ 3T(n-1)$ if $n>0$, otherwise $2)$

$T(n) = 3T(n-1)$ -①

$T(n-1) = 3T(n-2)$ -② $T(n-2) = 3(T(n-3))$

Subs. $T(n-1)$ in ①

$T(n) = 3^2(T(n-2))$ -③

Subs $T(n-2)$ in ③

$T(n) = 3^3(T(n-3))$

∴ Time complexity $= O(3^n)$

(4) $T(n) = \{ 2T(n-1) - 1$ if $n>0$, otherwise $2)$

$T(n) = 2T(n-1) - 1$

$= 2(2T(n-2) - 1) - 1$

$= 2^2 T(n-2) - 2 - 1$

$= 2^3 (T(n-3) - 2^2 - 2^1 - 2^0$

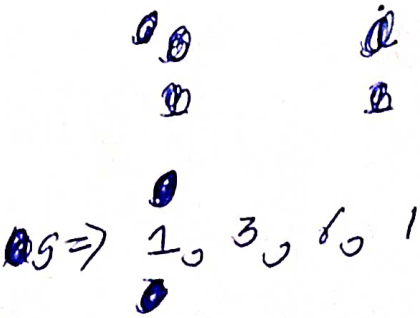
∴ $T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} \text{ ——— } - 2^0$

$T(n) = 1$

$= O(1)$

(5)

```
int i=1, s=1;
while (s <= n) {
    ++i; s = s+i;
    printf("#");
}
```



s ⇒ 1, 3, 6, 10

In general

$\frac{k(k+1)}{2}$

$\Rightarrow \frac{k^2+k}{2} > n$

∴ $TC = O(\sqrt{n})$

⑥
```
void function(int n){
    int i, count = 0;
    for( i =0 ; i*i <=n ; ++i)   - O(√n)
        count ++;                —  O(1)
}
```

TC = $O(\sqrt{n})$

⑦
```
void function (int n){
    int i, j, k, count = 0;
    for( i = n/2; i<=n; i++)              — O(n)
        for(j =1; j<=n; j = j*2)          — O(log n)
            for(k=1; k<=n; k=k*2)         - O(log n)
                ++ count;
```

TC = $O(n (\log n)^2)$

⑧
```
function (int n){
    if(n ==1) return;
    for(i=1 to n){                 —  O(n)
        for(j=1 to n){             - O(n)
            printf("*");
        }
    }
    function (n-3);                - O(n)
}
```

TC = $O(n^3)$

⑨
```
void function(int n){                  — O(n)
    for(i=1 to n){
        for(j=1; j<=n; j =j+i)
            printf("*");
    }
}
```

TC = $O(n \log_2 n)$

(10) We have, n^k a^n

$k \geq 1$ $a > 1$

Let $k = a = 2$

$\Rightarrow n^2$ 2^n

So we can say $n^2 = O(2^n)$

$n^k = O(a^n)$

(11)
```
void fun(int n){
    int j=1, i=0;
    while(i<n){
        i+=j;
        j++;
    }
}
```

T.C. = $O(\sqrt{n})$

(Reference Question-05)

(12) Recurrence Relation for recursive fibonacci series

$T(n) = T(n-1) + T(n-2) + 1$

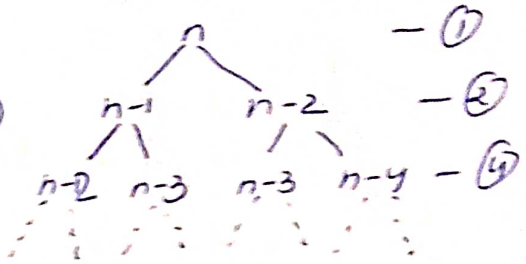
↳ $1, 2, 4, \ldots$ —— 2^n (It is GP)

Here $a = 1$ $r = 2$

$\therefore TC = \left(\frac{1(2^{n+1}-1)}{2-1}\right)$

$= 2^{n+1} - 1$

$TC = O(2^n \cdot 2) = O(2^n)$



Space complexity = $O(n)$

- The space complexity is $O(n)$ because recursive calls do the functio take up space in stack.

$F(n) = F(n-1) + F(n-2)$

$F(n-2)$ is called once $F(n-1)$ is finised executing which take up $O(n)$ space

$\therefore SC = O(n)$

(12)

(a) $n(\log n)$

```
for(int i=0; i<n; ++i){
    for(int j=1; j<=n; j*=i){
        count += i;
    }
}
```

(b) n^3

```
for(int i=0; i<n; ++i)
    for(int j=0; j<n; ++j)
        for(int k=0; k<n; ++k)
            count += i;
```

(c) $\log \log n$

```
int function(int n){
    if(n<=2)
        return n;
    else
        return (function(floor(sqrt(n)))+n);
}
```

(13)

$$T(n) = T(n/4) + T(n/2) + cn^2$$

Here, we can assume

$$T(n/2) >= T(n/4)$$

$$\therefore T(n) = 2T(n/2) + cn^2$$

Now, using Master's theorem

$$a = 2 \quad b = 2$$

$$\therefore k = \log_2 2 = 1$$

Here

$$n^k = n^1 = n$$

But $f(n) = n^2$

$$\therefore TC = O(n^2)$$

(15)

```
int fun(int n){
    for(int i=1; i<=n; ++i){          — O(n)
        for(int j =1; j<n; j +=i){    - O(log n)
            //some O(1) task
        }
    }
}
```

TC = $O(n \log n)$

(16)

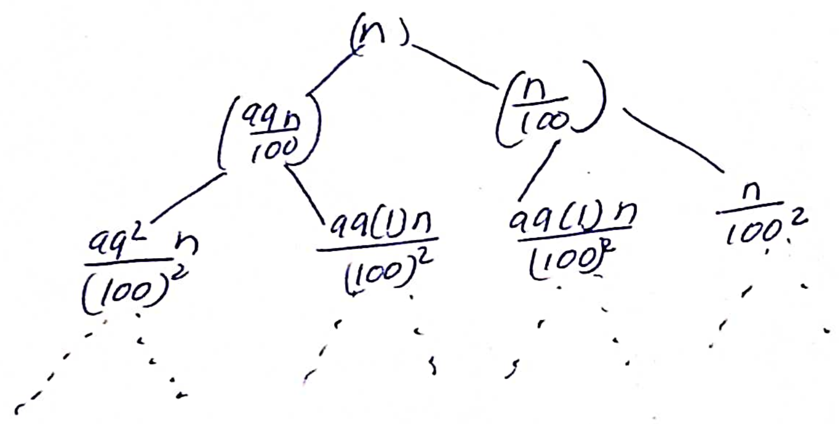
```
for(int i = 2; i<=n; i = pow(i,k)) {
    //some O(1) expression
}
```

TC = $O(\log \log n)$

↳ For any k greater than 1

(17) Recurrence Relation

$$T(n) = T\left(\frac{99n}{100}\right) + T\left(\frac{n}{100}\right)$$



Here, Taking $\left(\frac{99n}{100}\right)$

so TC = $\log_{\frac{100}{99}} n \approx \log_b n$

Since, it is a log complexity here we can see our base is constant therefore it does not matter as compared to n.

(18)

Increasing order

(a) $100 \prec \log\log n \prec \log n \prec \sqrt{n} \prec n \prec \log n! \prec n\log n$
$\prec n^2 \prec 2^n \prec 2^{2n}/4^n \prec n!$

(b) $1 \prec \log\log n \prec \sqrt{\log n} \prec \log n \prec 2\log n \prec \log 2n \prec n$
$\prec 2n \prec 4n \prec \log n! \prec n\log n \prec n^2 \prec 2(2^n) \prec n!$

(c) $96 \prec \log_8 n \prec \log_2 n \prec 5n \prec \log n! \prec n\log_7 n \prec n\log_2 n$
$\prec 8n^2 \prec 7n^3 \prec 8^{2n} \prec n!$

(19) Linear search pseudo code

```
int linear_search (array, key) {
    for (int i = 0  to SIZE)
```

(19) Linear search pseudo code

```
int linear_search ( array, size, key) {
    for ( i ← 0 to size) {
        if ( array[i] = key)
            return i;
    {
}
```

(25) Iterative Insertion sort

```
void insertion_sort (int arr[], int n) {
    int i, temp, j;
    for( i ← 1 to n) {
        temp ← arr[i];
        j ← i-1;
        while (j >= 0 && arr[j] > temp) {
            arr[j+1] ← arr[j];
            j = j -1;
        }
        arr[j+1] ← temp;
    }
}
```

Recursive Insertion Sort

```
void insertion_sort(int arr[], int n){
    if (n <= 1)
        return;
    insertion_sort(arr, n-1);
    int last = arr[n-1];
    int j = n-2;
    while(j >= 0 AND arr[j] > last){
        arr[j+1] = arr[j];
        j--;
    }
    arr[j+1] = last;
}
```

- It considers one input element per iteration and produces a partial solution without considering future element
  ⇒ It is called online sorting

21 & 22

Sorting Algorithm

Algorithm	Best case	Avg. case	worst case	space complexity	stable	Inplace	online
Bubble Sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(1)$	YES	YES	NO
Selection sort	$O(n^2)$	$O(n^2)$	$O(n^2)$	$(O(1))$	NO	YES	NO
Insertion sort	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$	YES	YES	YES

(23) Pseudo Binary Search

```
int binary_search(int arr[], int l, int r, int key){
    if(r >= 1){
        int mid = (l+r)/2;
        if(arr[mid] > x){
            return binary_search(arr, l, mid-1, x);
        }else{
            return binary_search(arr, mid+1, r, x);
        }
    }
    return -1;
}
```

	Linear Search	(Recursive) Binary Search	(Iterative) Binary Search
Time complexity	$O(n)$	$O(\log n)$	$O(\log n)$
Space complexity	$O(1)$	$O(\log n)$	$O(1)$

(24) Recursive Binary Search

Recurrence Relation

$$T(n) = T(n/2) + C$$