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Section - ML

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Subject - Design & Analysis of Algo.

TCS 505

### Tutorial

1

$$T(n) = 3T(n/2) + n^2$$

Here,  $a = 3$        $b = 2$

$$c = \log_2 3 = 1.58$$

$$n^2 > n^{1.58}$$

$$T.C. = \Theta(n^2)$$

2

$$T(n) = 4T(n/2) + n^2$$

$$c = \log_2 4 = 2$$

$$n^2 = n^2$$

$$T.C. = \Theta(n^2 \log n)$$

3

$$T(n) = T(n/2) + 2^n$$

Since  $2^n$  is a polynomial

$\therefore$  Master's Theorem does not apply

4

$$T(n) = 2^n T(n/2) + n^n$$

$\therefore$  This relation can't be solved using master's method

$$5 \quad T(n) = 16 T(n/4) + n$$

$$c = \log_4 16 = 2$$

$$n^2 > n$$

$$\boxed{T(n) = \Theta(n^2)}$$

$$6 \quad T(n) = 2 T(n/2) + n \log n$$

By extended euclidean algorithm

$$T(n) = a T(n/b) + \Theta(n^k \log^p n)$$

$$T(n) = \Theta(n^{\log_2 2} \log^2 n)$$

$$\boxed{T(n) = \Theta(n \log^2 n)}$$

$$7 \quad T(n) = 2 T(n/2) + n \log^7 n$$

By extended euclidean algorithm

$$T(n) = \Theta(n^{\log_2 2} \log^7 n)$$

$$\boxed{T(n) = \Theta(n \log^7 n)}$$

$$8 \quad T(n) = 2 T(n/4) + n^{0.51}$$

$$c = \log_4 2 = 0.5$$

$$n^{0.5} < n^{0.51}$$

$$\boxed{T(n) = \Theta(n^{0.51})}$$

9

$$T(n) = 0.5 T(n/2) + n^{-1}$$

Since  $a < 1$

Master theorem does not apply here.

10

$$T(n) = 16 T(n/4) + n!$$

$$c = \log_4 16 = 2$$

$$n^2 < n!$$

$$T(n) = \Theta(n!)$$

11

$$T(n) = 4 (T(n/2)) + \log n$$

By extended masters theorem

$$T(n) = a T(n/b) + O(n^c \log^k n)$$

$$T(n) = \Theta(n^{\log_2 4})$$

$$T(n) = \Theta(n^2)$$

12

$$T(n) = \sqrt{n} T(n/2) + \log n$$

Master theorem does not apply here  
as 'a' here is not constant

13

$$T(n) = 3 T(n/2) + n$$

$$c = \log_2 3 = 1.58$$

$$T(n) = \Theta(n^{1.58})$$

14

$$T(n) = 3T(n/3) + \sqrt{n}$$

$$c = \log_3 3 = 1$$

$$n > f(n)$$

$$T(n) = \Theta(n)$$

15

$$T(n) = 4T(n/2) + cn$$

$$(\log_2 4) n^2 > n$$

$$T(n) = \Theta(n^2)$$

16

$$T(n) = 3T(n/4) + n \log n$$

By extended master theorem

$$T(n) = \Theta(n^c \log^p n)$$

$$T(n) = \Theta(n \log n)$$

17

$$T(n) = 3T(n/3) + n/2$$

$$c = \log_3 3 = 1$$

$$n^1 = n$$

$$T(n) = \Theta(n \log n)$$

18

$$T(n) = 6T(n/3) + n^2 \log n$$

By extended master theorem

$$T(n) = \Theta(n^c \log^p n)$$

$$= \Theta(n^2 \log n)$$



19

$$T(n) = 4T(n/2) + n \log^{-1} n$$

By extended master theorem

$$T(n) = aT(n/b) + O(n^c \log^k n)$$

$$a=4 \quad b=2 \quad c=1$$

$$a > b^c$$

$$T(n) = O(n \log^4 n)$$

$$\boxed{T(n) = O(n^2)}$$

20

$$T(n) = 4T(n/2) - n^2 \log n$$

Here,  $f(n)$  is negative

$\therefore$  Master theorem is not applicable

$$21 \quad T(n) = 7T(n/3) + n^2$$

$$c = \log_3 7 = 1.77$$

$$f(n) > n^c$$

$$\therefore \boxed{T(n) = O(n^2)}$$

$$22 \quad T(n) = T(n/2) + n(2 - \cos n)$$

since,

$$a f(n/b) \leq c f(n) \quad c < 1$$

$\therefore$  Master theorem does not apply here