Network Theory and Political Science

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The Oxford Handbook of Political Networks
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Print Publication Date: Sep 2017 Subject: Political Science, Political Theory
Online Publication Date: Oct 2016 DOI: 10.1093/oxfordhb/9780190228217.013.12

Abstract and Keywords

This chapter considers the role of network theory in the study of political phenomena, the analytical theoretical basis of network analysis as applied in political science. Using the concepts of centrality, community, and connectivity, it discusses the relationship between the primitives of network theory and their relationship to empirical measurement of political networks. The chapter then discusses one of the most active areas of work on network theory in political science, models of network formation, and offers some concluding thoughts about future directions of network theory in political science. We argue that the deeper theorizing about political networks will complement and improve empirical scholarship on the role of networks in politics.

Keywords: centrality, community, connectivity, empirical measurement, network formation

NETWORK theory is a large tent: from early contributions such as Heider (1946), Harary and Norman (1953), Cartwright and Harary (1956), Davis (1963), and Granovetter (1973) (to list only a few), who worked in a sociological tradition with deep connections to social psychology, the field of "network theory" has grown to encompass all of the social sciences (Borgatti et al., 2009). The fundamental allure of network theory is its ability to describe and analyze the structure of connections between units (individuals, firms, states, parties, etc.) in a rigorous but flexible way. Given that most political phenomena involve collective action and thus interactions between multiple individuals, it is unsurprising that network theory is an active area of research in political science.[1] As indicated by the contents of this handbook, much of the research on political networks to date has been empirically motivated.

In this chapter we consider the role of network theory in the study of political phenomena, by which we mean the analytical theoretical basis of network analysis, as applied in political science.[2] Because the body of theoretical work to date on political networks is much smaller than its empirical complement, our chapter begins with a selection of network concepts around which we frame our discussion of the role of network theory in political science. For reasons of space and focus, we do not discuss applied models in which networks play a role.[3] Partly because there are relatively few such models to date, their commonality with this chapter generally begins and ends with the fact that they include a role for a network.

The main challenge of theoretical research on political networks is that network structures represent in a very literal sense the connective tissue that binds individual decision making and group behavior. Methodological individualism—the belief that the best units of analysis, whenever feasible, are individuals' decisions—is arguably the predominant paradigm in modern, empirical political science research, but the idea that groups may exert one or more forms of independent influences is also at least intuitively relevant for many political phenomena of interest.[4] We discuss the basic building (p. 148) blocks of network theory (nodes, edges, graphs) and their relationship to each other and empirical measurement in the second section.

Reliably measuring classic concepts such as culture, identity, influence, and power—not to mention then attempting to gauge the causal influence of such factors on individual behavior—ultimately requires a theory that can identify these concepts and isolate them from other environmental and individually determined factors. Network theory is particularly well-suited for precisely defining measures of group-based or structurally based concepts. In the third section we consider what we term "measurability" issues that one must confront when attempting to import classical network concepts into the study of political phenomena.

Of course many politically relevant networks are endogenously created from individuals' decisions about with whom to associate. In such situations, an important set of questions arises regarding *why* we observe the network structures that we do. Indeed, many such questions are relevant to identifying the causal impact of network structure. For example, *homophily* (the proclivity of individuals to interact more frequently with other individuals like themselves) is a fundamental issue confronting scholars attempting to identify the independent causal effects of a network. We consider a popular and flexible class of models of network formation in the fourth section.

A roadmap.

While network theory is a common framework, the questions that it can be, and has been, applied to are numerous and varied—sometimes so varied as to obscure the common framework linking them. As many scholars in various fields have argued, the idea of a unifying *theory* of networks is both attractive and notoriously slippery; instead of there being a theory of networks, network concepts serve as unifying features of multiple theories in different fields (e.g., Salancik, 1995; Borgatti and Halgin, 2011). Thus in the first section we describe three such concepts: *centrality, community*, and *connectivity*. Applications of centrality rely at least partially on one or more means of ranking the actors within a network in terms of something akin to "power" or "influence." Community, on the other hand, considers the question of how the network indicates differences and commonalities between the actors. Concepts of connectivity link centrality and community by differentiating pairs of actors on the basis of their positions within the network.

In the second section we introduce the basic building blocks of network theory (nodes, edges, and graphs) and discuss the measurement role of these building blocks. Deciding what are (and are not) nodes and edges in a political network is the most fundamental theoretical action an analyst must take. We then use those basic building blocks to discuss how political networks are measured and characterized in the third section. Our discussion of measurement is structured around the concepts of centrality and community, and our focus is on the multiple ways that each of these two concepts have operationalized and employed. We then turn in the fourth section to one of the most active areas of work on network theory in political science: network formation. Finally, we offer some concluding thoughts in the fifth section. With this roadmap in hand, we now discuss the related concepts of centrality and community.

(p. 149) Three Concepts from Network Theory: Centrality, Community, and Connectivity

Focusing our attention on two loose categories of network theory–based analysis allows us to speak more directly to the role of formal theory in the study of political networks. This is because, at least in the colloquial sense of the term "theory," analyses falling into either of the two categories are derived from a common theoretical basis. The residual vagueness within each of these categories of analysis illuminates the importance of rigorous, formal linkage between the network theory–based tools and the analyst's foundational theory of how the actors in the network behave.

Such a linkage is fundamental to the validity of any attempts to infer or extrapolate from the results of the empirical analysis. That is, our point is valid regardless of what paradigm of behavior the analyst chooses to work in. For example, our argument is independent of whether the analyst's theory is best described as a "rational choice," "agent-based," "behavioral," or "sociological" model. Rather, every network theory–based empirical tool is based upon multiple assumptions about what types of behaviors it will and will not distinguish between. That is, because all but the most trivial of networks are very high dimensional objects, any practical empirical analysis must aggregate, and hence partially lose some of, the information present in the actual data set. Formal theories of political networks provide the ability to precisely track exactly what information is, and what information isn't, lost in this operation (Patty and Penn, 2015).

Centrality

Perhaps the most common application of network theory in political science is to measure an actor's *centrality* within the group. Centrality has a long history within network theory, particularly as developed within sociology. The degree to which one actor is connected with other actors is broadly accepted—albeit with different interpretations and under varying guises—as one of the principal determinants of an actor's "influence" within a network. We defer a more detailed discussion of centrality measures theories (or, perhaps, arguments) linking centrality with influence, but it is useful to describe a few of the common premises of these theories. We revisit these arguments, and some of the accompanying measures of centrality below.

- *The more people you know, the better.* A fundamental measure of influence is how many people you "know" (or alternatively, how many people "know you"). There are many ways to measure how many people you know (or how many people know you), because there are a variety of ways to define indirect knowledge. As we discuss below, the more complicated measures of centrality along these lines attempt (p. 150) to account for the centralities of the people you know, because arguably people who know important people are more important themselves.
- *The more people you connect, the better.* An important family of centrality measures is derived from the notion of how many actors are connected *by* a particular individual.[5] The basic idea of these, the most widely used of which is known as *betweenness centrality*, is that the centrality of an actor is proportional to the degree that the network structure is more connected (or efficient at transmitting information) with the actor present in the network than it would be if the actor were removed from it.

Community

While a network in a sense presumes a group structure, a powerful application of network theory is detecting subgroups, or “communities,” among the actors in the network. *Community detection* is accomplished using various tools, each based on different notions of why or how individuals form groups. A weakness of many of the tools is a lack of rigorous theoretical underpinnings. Jackson (2008) describes the situation as follows:

> [R]esearchers generally start with a simple algorithm for partitioning the nodes of a network, based on some heuristic, without a firm foundation in terms of defining what communities are, how they influence network formation, or why this algorithm is a natural way for uncovering them. Thus communities have tended to be defined as whatever the algorithms find rather than deriving the algorithms based on a well-defined notion of community.[6]

While community detection has many applications in political science,[7] it has not yet been widely implemented. Part of the reason for this is that these techniques are relatively new, made possible only by the fall in price of computational resources. A related (and more interesting) reason for this is that the proper notion of “community,” not to mention how to detect it empirically, is an important and challenging theoretical problem that is (or should be) largely application driven. We return to these issues below and now turn to the concept of connectivity.

Connectivity

As discussed above, notions of community attempt to partition actors in a network into subgroups, and notions of centrality attempt to rank these actors according to how well connected they are. These two concepts are related to each other through the concept of *connectivity*, which is a dyadic (or higher order) phenomenon. For example, very central actors will typically be connected to many others, and actors within the same community will tend to be better (or more closely) connected than will actors in different communities. Before discussing the building blocks of network theory, we briefly discuss the concept of connectivity.

(p. 151) When considering two actors within a network, we can ask a number of key questions regarding how they are connected to each other. Three examples of such questions are listed here:

1. Are the actors connected to each other at all—does there exist a *path* of edges between the two actors?[8]
2. How long is the shortest path that connects the two actors?
3. How many independent paths are there between the two actors?

An actor that is connected to many other actors by short paths will both have a high centrality score under most centrality measures and be likely to be in lots of other actors' communities, however defined. Thus, an actor's own "connectivity" is clearly an element of both measures of centrality and notions of community. However, a related, complementary question is how important a given actor is (or would be) to establishing, or strengthening the, *connectivity between other pairs of actors*. A high-profile example of such a notion is that of a *bridge*: edges (or nodes) that, if they were removed from the network, would leave two or more actors no longer connected by any path.[9]

Bridges are very important in network theory whenever one is concerned with the global spread (or "contagion") of influence/action/information, and so forth throughout a network. Space precludes a fuller treatment of this topic, but it is worth mentioning precisely because of its potential applications in political science. Important and obvious examples of topics in which the connectivity of a network—and the sensitivity of this connectivity to "minor" alterations of the network—is of central concern include various security topics such as infiltrating terrorist and criminal networks (Krebs, 2002; Morselli et al., 2007), defending infrastructure networks from attack (Comfort and Haase, 2006; Lewis, 2014), and monitoring and controlling communicable diseases (Christakis and Fowler, 2010; Liu et al., 2012). Less obvious, but no less interesting, topics include the structural integrity of formal and informal organizations (Krackhardt, 1990; Fowler, 2006; Scholz et al., 2008; Grossmann and Dominguez, 2009; Berardo and Sholz, 2010; Leifeld and Schneider, 2012), strategic lobbying (Skinner et al., 2012), and monitoring and/or controlling informational flows in society (Enemark et al., 2014; Song and Eveland Jr., 2015).

The Basic Building Blocks of Network Theory

A "political network" is simply a network that is somehow relevant to political outcomes, and the basics of any network are usually described with the tools of what we term "classical network theory." The theoretical foundation of classical network theory in the social sciences is *graph theory*,[10] which essentially describes and characterizes (p. 152) how *nodes* (or *vertices*) are connected by *edges*. A *graph*, then, is any collection of nodes, N, and a collection of edges, E, where each edge connects two distinct nodes.[11] Thus, an edge is typically written $e = (n_1, n_2)$, where n_1 and n_2 are distinct nodes in N, and e represents a connection "from" node n_1 "to" node n_2.[12] By distinguishing between the first and second nodes in an edge, this definition describes what are known as *directed* graphs. For any positive integer n, the set of all directed graphs on n nodes is denoted by G_n. This is the set of all possible networks between n actors. As discussed in the third section, it is important to remember from an empirical standpoint that within the confines of classical network theory, this set represents the entire space of "structures" that one can differentiate between.

The Basics of Measurement: Nodes and Edges

Graph theory is abstract and thus agnostic about what nodes and edges represent, but the decision about what they represent is the foundation of any theory of *political* networks. For example, in international relations, nodes are often nations, and edges are interactions such as trade or conflict. In political behavior, nodes are often individuals, and edges denote relationships such as trust or communication. In legislative studies, nodes are often legislators, and edges represent interactions such as cosponsoring legislation or serving on a common committee. In judicial politics, nodes might be written opinions, and edges represent citations. Even in the simplest of studies, the possible assignments of edges and nodes are at least seemingly unbounded.

This vastness becomes even more pronounced once one thinks more broadly and realizes that nodes and/or edges can be quite generally defined. For example, if one is studying legislative politics, one could represent each legislator and each piece of legislation as a separate node, so that there are two "types" of nodes. In such a representation, one can identify sponsorship of a bill by a legislator with an edge. Similarly, one could extend the set of edges to include a second "type" that exists between a legislator and a bill if that legislator voted in favor of the bill. While there are many examples of political networks, we refer to the nodes as individuals, or "actors," but this is only to keep the language simple.

Analysis: Within or Across Networks?

Many scholars have attempted to identify various "network effects" in political settings, and the methodological challenges of such attempts are both well-known and an area of active study (e.g., Fowler et al., 2011; Gentzkow and Shapiro, 2011; Noel and Nyhan, 2011; Klofstad et al., 2013; Campbell, 2013). While we refer the reader to other chapters (e.g., GROSS AND JANSA, ETC.) for discussions of many of the measurement, estimation, and inference challenges inherent to research on political networks, it is useful to note that some network theory concepts compare units such as nodes *within* a network, some (p. 153) allow comparison of two or more different networks, and some can be applied to either type of analysis. It is important to keep this variety in mind when employing network theory concepts: if a concept makes sense only at the network level,[13] then the theoretical underpinnings of a causal linkage between it and individual behaviors is necessarily different than the linkage between such behaviors and a concept that is measured at the node level (as are the concepts of centrality and community discussed in this chapter). Now we turn to what is arguably the most fundamental theoretical challenge facing a social scientist attempting to leverage network theory in empirical research: measuring and characterizing the structure of a network.

Measuring Networks

Measuring, or equivalently, representing networks is an important topic and presents two fundamental (and opposed) challenges for the applied researcher or analyst. The first of these challenges emanates from the fact that networks are “messy,” as we describe below. This messiness precipitates the need for measures that impose structure on this messy set. The second challenge is that within the framework of classical network theory, the set of networks can differentiate between connections (i.e., edges) only in the most coarse way. We now discuss each of these challenges in turn.

The Messiness of Networks

We refer to networks as “messy” because, unlike real numbers, they do not have a natural ordering. The set of networks is qualitatively large in the sense that while one can enumerate every possible network structure for a given set of actors, there is no unambiguously correct way to organize or categorize them. Accordingly, a large body of theoretical work on networks focuses on creating functions that organize them, a limited selection of which we discuss in more detail below. Any of these functions eliminates some of the messiness of the relevant set of networks and accordingly loses some of the information contained in the networks’ structures (e.g., Patty and Penn, 2015).

This loss of information is necessary in much applied work on networks, because the analyst is typically interested in (one or more) “network effects” that are essentially unidimensional. Just a few examples are Ward (2006) (examining the effect of several network centrality measures on nations’ successful pursuit of sustainability practices); Victor and Ringe (2009) (exploring the correlations between legislators’ connectedness and centrality measures and other characteristics such as seniority, race, and gender in the US House of Representatives); and Pelc (2014) (utilizing Katz centrality to measure the value assigned to a case as precedent in subsequent disputes in the World Trade Organization). In these applications, an interpretable and portable measure of a node’s “location” in the relevant network is needed to advance to empirical analysis. To do this, it is impractical to “shove a network into the regression.” These approaches all utilize some form of data reduction.

Many data reduction approaches to measuring and characterizing networks implicitly rely on some notion of *structural equivalence*, due to White (1970). Structural equivalence is an identification concept that posits that a network’s “effect” on any of its embedded actors should be measurable with respect to the (local) structure of the network within which that actor is embedded. That is, *ceteris paribus*, if two actors in a network are identical in all individual respects *and* their local neighborhoods within the network are equivalent, then they should behave/choose in identical ways.[14] There is some understandable ambiguity (or perhaps flexibility) in this notion, inherited from the idea of “neighborhood.” For example, suppose the two actors are directly connected to similar sets of other individuals, but the actors to whom these “first-order connections” are connected (i.e., the actors’ “second-order connections”) differ significantly between the two actors: Are the actors structurally equivalent to each other?

Recent work is increasingly bringing multiple aspects of network theory to bear on applied questions. For example, Patty et al. (2013) combine community detection and a new, axiomatically derived scoring algorithm (Schnakenberg and Penn, 2014) to provide estimates of the weight assigned by the US Supreme Court to its own precedents.

Of course, “how big is one’s neighborhood?” is a theoretical question, based on a consideration of the phenomena of interest. For example, does interaction between the actors occur in a sequential, or a simultaneous, fashion? If interaction is sequential, how many rounds of interaction occur?[15] The proper answers to these questions depend on the realities at hand: applied network theory is necessarily data-informed theory.

The Coarseness of (Classical) Networks

Classical network theory is based on a binary representation of connections. Between any two actors, an edge either exists or it doesn’t. While the notion of a weighted graph—where each edge e is assigned a number, w_e [16]—obviously relaxes this constraint to some degree, this relaxation is not complete, because it assumes that the edges can be ordered by this weight.[17] For reasons of space, we restrict attention to classical, unweighted networks, and now proceed to consider some measures of centrality that are employed in the analysis of political networks.

A Menagerie of Measures: Centrality, Power, and Influence

The notion of centrality is one of the most commonly applied concepts in the study of political networks. This is partly because centrality is often loosely equated with power, (p. 155) an equating that is often justified by the presumption that “links” are desirable or at the very least denote some form of influence.

Unsurprisingly, there are numerous measures of centrality. In line with the focus of this chapter, we discuss some of the more commonly employed measures, but omit a detailed accounting of how they have been used in the empirical study of political networks. Instead, we describe several of the measures so as to illustrate the differences in their theoretical foundations. These foundations have been explored by many scholars (e.g., Freeman, 1979; Bonacich, 1987; Friedkin, 1991; Monsuur and Storcken, 2004; Borgatti and Everett, 2006; and Boldi and Vigna, 2014). Space constraints require our discussion to be simply an introduction to the theoretical connections and distinctions between these measures, so we refer the interested reader to the literature cited here and throughout the chapter for more details.

Degree Centrality.

The most fundamental, and most commonly used, measure of centrality is known as *degree centrality*. There are two notions of degree centrality in a directed network: *in-degree* and *out-degree*. In-degree is the number of (other) nodes *from* which an edge extends *to* the node in question, and out-degree is the number of nodes *to* which an edge extends *from* the node in question. The relevance of each these measures of centrality is obvious: a person’s in-degree measures how many people “talk to” him or her, and a person’s out-degree measures how many “listen to” him or her. Neither in-degree nor out-degree is inherently more relevant than the other, because the definition of "speaker" and "listener" is arbitrary. Nonetheless, the fact that "speaker" and "listener" can represent very different roles is fundamental.

Degree centrality is both incredibly easy to compute and simple to understand. These virtues come at significant costs, of course. In particular, degree centrality is a strictly "local" measure of an actor's centrality. In particular, an actor's degree centrality is completely insensitive to the structure of the network among other actors. That is, whether two other actors *j* and *k* are connected does not affect the degree centrality of a third actor *i*.[18]

Eigenvector Centrality.

Almost as obvious as the relevance of degree centrality is its principal weakness: neither notion of degree centrality is responsive to the identities of the other nodes to which a node is connected. This weakness is partially addressed by the notion of *eigenvector centrality*, which is due to Bonacich (1972) (see also Bonacich, 1987, 2007). The basic idea of eigenvector centrality is that the centrality of an actor is positively associated with the centralities of the actors to whom the actor is connected. In other words, eigenvector centrality is *self-referential*—in particular, "important people know important people." More formally, this is one example of a *spectral measure* (e.g., Vigna, 2009), a class of measures that includes several other prominent measures of centrality, including the PageRank algorithm (Page et al., 1999), the hyperlink-induced topic search (HITS) measure (Kleinberg, 1999), Katz centrality (Katz, 1953), and the Kendall-Wei method (Wei, 1952; Kendall, 1955). Though these measures are increasingly being employed in the empirical analysis of networks (e.g., Fowler and (p. 156) Jeon, 2008; Gray and Potter, 2012), their theoretical foundations in the context of political networks remain unexplored.[19]

Betweenness Centrality.

Both degree centrality and eigenvector centrality measure how many actors a given actor is connected to. Betweenness centrality (Freeman, 1977) considers *how many actors are connected to each other by a given actor*. Thus, betweenness centrality partially captures the notion that an actor's centrality is based on how many other actors "need" that actor to communicate or interact with them. An important characteristic of betweenness centrality is that one actor's betweenness centrality can be sensitive to connections between *other* pairs of actors—something that distinguishes this concept from degree centrality. Because of these properties of the measure, it has been used in empirical research on influence in informal policy and issue networks (e.g., Scholz et al., 2008; Skinner et al., 2012).

Closeness Centrality.

Closeness centrality (first defined by Bavelas, 1948, 1950) measures, in essence, how many “steps” it would take to get from one actor to every other actor. It is a particularly useful notion of centrality in informational networks, because longer paths require more time to traverse and involve more risk of information loss. Accordingly, it has been employed in some empirical studies of political networks (e.g., Koger et al., 2009). A problem with closeness centrality is that it is ambiguous how to best define it for “unconnected” networks in which there exists some pair of actors for which there is no path of edges connecting the two actors. A second weakness of the measure, shared with degree centrality, is that it assigns all actors equal weight. That is, unlike eigenvector centrality, closeness centrality is not self-referential. That said, closeness centrality does capture more of the structure of the network than does degree centrality.

A Melting Pot of Measures.

Each of the measures described above has intuitive appeal, but to be clear, the theoretical foundations in political networks are frequently murky. After all, regardless of the centrality measure one chooses, it is unclear in what types of situations one would seek to maximize one’s own centrality when forming a network. For example, focusing on eigenvector centrality for a moment, it is clear that being connected with other “well-connected people” can be advantageous in some situations (e.g., situations involving the exchange of goods or information), undesirable in others (e.g., situations involving the exchange of disease), or ambiguous from an a priori perspective (e.g., situations involving violence or coercion).

Part of this murkiness is due to theoretical incompleteness; a model of the preferences and incentives of, and actions available to, the actors within the network is needed to more precisely adjudicate what the various centrality measures “mean” in the context in which they are being employed. But even with a fully specified theory of the actors’ behaviors, the panoply of centrality measures is indicative of a fundamental ambiguity in the notion of centrality that mirrors that of the concept of power (e.g., Bachrach and Baratz, 1962): the different measures vary because centrality is inherently a multifaceted (p. 157) concept. Working through the primitives of the situation being examined is a good first step toward understanding the relative strengths and weaknesses of each of the various measures.

From a purely theoretical angle, work should continue on characterizing what it is that the various centrality measures are actually measuring. Deriving *axiomatic foundations* of existing measures—that is, determining the general properties of the various measures, as well as possibly deriving new ones from desirable properties—will allow a precise and general characterization that can be used to more clearly and reliably choose which measure to use in different applications. In general, we concur with Boldi and Vigna (2014), who describe the value of axiomatic development as follows:

> [E]ven if one does not believe [the axioms] are really so discriminative or necessary, they provide a very specific, formal, provable piece of information about a centrality measure that is much more precise than folklore intuitions such as "this centrality is really correlated to indegree" or "this centrality is really fooled by cliques."[20]

The use of centrality measures (and indeed, many network theory concepts) in empirical work is currently (at least in appearance) based on ad hoc matching of algorithm with theoretical concept or construct. This is a classic issue of measurement: the analyst is typically attempting to ferret out a latent construct from messy and coarse empirical data. This is a difficult job, and it will be rare for a perfect measure to exist. But the current state of theoretical work offers little help in this challenge. Hopefully this will not always be the case. We now turn to a related, but distinct, empirical challenge: measuring and identifying groups within a network.

A Cornucopia of Classifications: Communities, Cliques, and Groups

Whereas centrality is generally equated with power in the study of political networks, the notion of over whom one has power—that is, something like a "group"—is left unaddressed by the centrality measures discussed previously. The problem of identifying the (sub)group structure within a network is one of *classification*, assigning each node to one (or occasionally, more than one) set of nodes. It almost goes without saying that there are many applications for estimating the group structure within a political network.

Cohesion and Structural Equivalence.

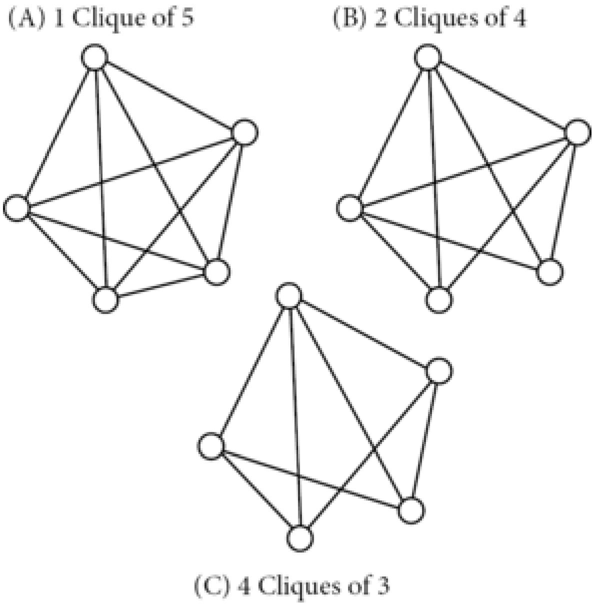
Two theoretical notions commonly relied on in deriving measures of groups are structural equivalence, mentioned previously, and *cohesion* (e.g., Burt, 1978, 1987; Friedkin, 1984; White and Harary, 2001; Moody and White, 2003). Just as with centrality, there are a variety of formal definitions of each of these concepts. Loosely put, a method of grouping nodes is based on the principle **(p. 158)** of cohesion, to the degree that two nodes are more likely to be grouped together if there are more paths between them. A grouping method is based on the principle of structural equivalence, insofar as it is more likely to group two nodes together if the nodes occupy similar positions in the network. To make the distinction between these concepts clear, note that two nodes might be grouped together under the principle of structural equivalence even if there exists no path between them, but this would not be the case under a cohesion-based method.

Cliques.

The strongest notion of a subgroup within a network, and the heart of cohesion-based measures of groups, is that of a *clique* (Luce and Perry, 1949): a maximally completely connected subgraph within the network or a group of nodes that are directly connected with each other such that there is no other node to which all of the members of the group are each connected as well.[21] Figure 6.1 illustrates this concept.

The notion of a clique is useful as a baseline theoretical standpoint for two reasons. First, a clique is clearly a well-defined group: if a clique within a network is not a group, then it is not clear what structure within the network would constitute a group.[22] Second, the notion can serve as the basis for imputing or estimating latent group structures. This is because the clique is a generalization of the idea of *triadic closure* (Heider, 1946; Granovetter, 1973), a sociological concept that posits that a collection of three individuals will rarely possess exactly two connections.

That said, while representing a clear theoretical baseline, figure 6.1 illustrates why the notion of a clique is arguably too demanding. From a theoretical standpoint, cliques are "fragile," and as a result, large cliques (e.g., containing more than three or four members) are typically rare in empirical networks. Comparing graph (A) in figure 6.1 with graph (B), the removal of a single edge not only destroyed the clique of five nodes, but created two (overlapping) cliques of four nodes. Perhaps more disturbingly from a fragility (p. 159) perspective, removal of a second edge, as in graph (C), destroys both cliques of four and results in four overlapping cliques of three nodes.



Click to view larger

Figure 6.1 Illustration of a Clique and Its Fragility.

In addition to their fragility, cliques are arguably of limited interest from a social science perspective because all cliques have identical "internal" structure: every member of any clique is connected to every other member of that clique. Accordingly, using clique as the definition of a "group" leaves nothing to differentiate between groups other than their sizes. Unless one believes that there is only a single dimension along which groups vary, this reality implies that from a networks perspective, the set of groups is richer, and hence larger, than the set of cliques. Because of these weaknesses of the notion of a clique, various relaxations of the notion have been proposed. We now discuss a few of these.

Generalizing Cliques: Clans and Plexes.

Any systematic generalization of the notion of a clique requires a choice about how to measure how similar to a clique any given subgraph is. We briefly describe three different notions of similarity and how they are used to define groups in a way that is similar to, but more expansive than that of, a clique.[23]

1. *k-cliques.* A clique of n nodes is a (maximal) subgraph in which each pair of nodes is connected by a shortest path of length 1. Thus, one notion of how similar a subgroup is to a clique is the maximum distance between any pair of nodes in the underlying graph (Luce, 1950). To see the nesting of the notion of a clique within that of a k-clique, note that the set of 1-cliques is identical to the set of cliques.
2. *k-clans.* For $k > 1$, a k-clique can induce a subgraph that is itself disconnected. To correct for this possibility, the notion of a *k-clan* refines the set of k-cliques as follows. A set of nodes is a k-clan if and only if it is a k-clique such that each node in the k-clan has a maximal distance of no more than *k using only nodes within the k-clan* (Alba, 1973). Because every k-clan is a k-clique, it follows that a set of nodes is a 1-clan if and only if it is a clique.
3. *k-plexes*. Note that a clique of n nodes has the following property: each of the n nodes possesses a link with each of the other $n-1$ nodes. Accordingly, one family of subgraphs that is "close" to a clique on n nodes is the family of subgraphs on n nodes, such that each of the nodes is connected to at least $n-2$ of the other nodes. More generally, a *k-plex* consisting of n nodes is a subgraph in which each of the n nodes is connected to at least n–k of the other nodes (Seidman and Foster 1978). As with k-cliques and k-clans, a 1-plex is a clique, and vice versa.[24]

Repeating a common refrain in this chapter, there remains much work to be done in terms of linking community-classification concepts such as cliques, clans, and plexes with various political environments. As with the centrality measures described above, the various approaches to community classification capture different dimensions of a "group," so the proper choice will vary across different political networks, and (p. 160) determining what the proper choice(s) are will be aided by explicitly modeling the individual interactions within the network.

While cliques, clans, and plexes are not yet widely used in political network research, a related approach—latent space models—has attracted some attention.[25]

Latent Spaces.

While many community detection algorithms conceive of groups as discrete objects (possibly allowing individuals to have partial, continuous "memberships" in these groups), one can also conceive of actors as being embedded in a space that can then be used to define or infer group structures. These approaches, known as *latent space models* (see Hoff et al., 2002 and Ward et al., 2013 for an application to trade), attempt to describe the nodes as positions in a space such that nodes that are closer to each other in the space are more likely to be connected by an edge.[26]

Communities.

Another notion utilized in group classification is that of a *community* (e.g., Radicchi et al., 2004; Clauset, 2005; Porter et al., 2007; Fortunato, 2010). Generally speaking, whereas clique-based approaches to group classification focus almost exclusively on the internal structure of a group,[27] community-based approaches attempt to incorporate information about both the internal structure and external connections of the group. As Radicchi et al. (2004) describe it, "a community is defined as a subset of nodes within the graph such that connections between the nodes are denser than connections with the rest of the network."[28] This type of approach has attracted some attention in political science (e.g., Zhang et al., 2008), but has to date been developed in largely atheoretical ways, focusing on the computational time and complexity of various algorithms more than on the linkage between the algorithms and the underlying nature of the data being examined.

Different Notions, Different Purposes

Historically, approaches to measurement of network concepts such as centrality and groups have been derived from (often informal) theoretical arguments that, owing to the interdisciplinary interest in and origins of network analysis, come from widely varying starting points. An important task for scholars of political networks is to revisit and reconsider preexisting network measures in order to better link them with the scholars' goals and the theoretical underpinnings of the phenomena being studied. In other words, while it is tempting to pull these ready-made measures off the shelf, this temptation should be resisted at least temporarily. While a "kitchen sink" approach that throws many preexisting measures at a data set might be useful in early explorations of *new* data, the productivity of such an approach is directly tied to the degree to which the scholar is aware of the theoretical foundations of the different measures. Uncovering the foundations (i.e., characterizing the properties) of preexisting measures has recently begun attracting attention (e.g., Borgatti, 2005; Borgatti (p. 161) and Everett, 2006; Borgatti et al., 2006; and Lubell et al., 2012). While there remains much to be done, this is particularly true with respect to linking the properties and foundations of network concepts with the incentives and behaviors within political settings.

Models of Network Formation

Scholars have long realized that to the degree that political networks affect outcomes, the formation of such networks will not be completely random. Accordingly, scholars have examined how political networks are formed and sustained (e.g., Chaharbaghi et al., 2005; Rogowski and Sinclair, 2012; Eveland et al., 2013). Similarly, scholars have started to grapple with the reality that many political and social networks are impossible to directly observe, motivating the development of tools to detect or estimate such networks (e.g., Cranmer and Desmarais, 2011; Heaney, 2014; Box-Steffensmeier and Christenson, 2014, 2015). Linking these two strands of work, of course, is the need for a theory of how networks form. This arguably represents the area of work in political networks that is currently experiencing the most active and fertile mixing of theoretical and empirical insights. In this section we discuss the two most prevalent approaches to modeling network formation, random graph theory and exponential random graph models.

Random Graph Theory

Given the unstructured nature of most sets of networks, obtaining and describing general theoretical results about networks was arguably made possible only by the conceptualization of the "random graph" (Erdős and Réyni, 1959). Random graph theory allows one to establish well-defined likelihoods of encountering any given feature in a network. Accordingly, it serves as the theoretical baseline of inference (i.e., the canonical "null hypothesis") when addressing questions about how networks form.

The canonical random graph model is known as the *Poisson random graph model*,[29] which is parameterized by two numbers: the number of nodes, n, and a probability of connection, $p \in (0, 1)$. The model assumes that p represents the probability of any pair of distinct nodes being connected by an edge, and that the realization of any given edge $e = (n_i, n_j)$ is independent of whether any other edge, $e' = (n_k, n_m)$ is realized. This model then induces a probability distribution over the set of all graphs on n nodes, G_n.

From a social science perspective, the Poisson random graph model is best viewed as a starting point. More specifically, for scholars attempting to infer the influence of various (p. 162) factors on the observed structure of political networks, random graph establishes the classic null hypothesis, because the distribution of truly random graphs is arguably the predicted distribution if the network structure is independent of all the factors under consideration.

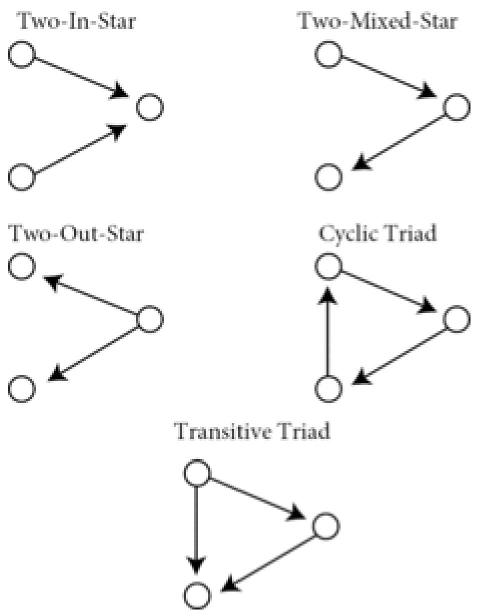
The pure theory of random graphs is only a "starting point," because one might find that an empirical distribution of network structures that differs from "random" is also independent of any of the factors one is studying, but the fact that one does not necessarily know the distribution of "errors" (i.e., factors outside the model) is generally true in statistical analysis of observational data.

Exponential Random Graph Models (ERGMs)

A popular and tractable family of models of network formation is the exponential random graph model, or ERGM. First described by Frank and Strauss (1986), the ERGM approach refers to a category of random graph models that provide a richer parameterization than, and thus subsumes, the Poisson random graph model. While space precludes an in-depth presentation of the details of the ERGM approach,[30] a brief outline of the structure of this approach is useful when considering how it has been, and can be, applied to political networks.

The parameters of an ERGM can be broken down into two basic categories. The first is *structural parameters*, which characterize the probability of different graph structures without reference to the labels/identities of the actors/nodes themselves. Two classic examples of such parameters in a directed graph are (1) the probability, for any randomly drawn pair of nodes *i* and *j*, that *i* will be connected to *j and* that *j* will *not* be connected to *i*; and (2) the probability, for any randomly drawn pair of nodes *i* and *j*, that *i* will be connected to *j and* that *j will* be connected to *i*. The first probability measures the likelihood of asymmetrical relationships in the network, and the second probability measures the likelihood of symmetrical relationships. One can easily extend this to include additional probabilities for the occurrence of more complicated subgraphs such as "two-in-stars," "two-mixed-stars," and so forth, as displayed in figure 6.2.

The parameters in this category are classically "network-focused" and thus apolitical. Nonetheless, they can be used for exploratory analysis. A common example of this is high-order structures, such as triads: mutual connections among three distinct nodes.[31] Because the triad is the simplest example of higher-order interdependence, examining its impact and emergence is arguably the starting point of true *network* analysis. Thus, while the political theory behind these parameters will often be vague, these parameters can be a useful starting point for establishing a reasonable suspicion that a network analysis (and ultimately, network-based theory) is specifically called for in the application at hand. (p. 163)



Click to view larger

Figure 6.2 Some Subgraph Structures.

While the first category differentiates between various network forms, the second category of parameters in an ERGM provides the structure for differential dependence between nodes based on the nodes' characteristics. For example, a social network analysis of cooperative behavior in a partisan legislature might allow the probability of cooperation between copartisans to be different from the probability of cooperation between members of differing parties. Obviously, analogous relationships emerge immediately when considering other political networks: how alliances affect conflict; how resources and factors of production affect trade; how factors such as religion, income, and education affect group membership decisions; and so forth. As always, the principal limitation on the number and complexity of factors that one can include in this category is the amount of data available.

This second category of parameters is arguably the focus of the political science within the political network being examined. Accordingly, it is important to note that these parameters should be derived from an explicit theory. In any event, the decision of which these to include (and how to include them) represents one or more important theoretical choices regarding the phenomenon being studied. In line with the discussion of measures of centrality and group classification, a key need in this area is more work modeling the preferences and behaviors that undergird the formation of political networks, so that explicit linkages between such micro-foundations and statistical models of network formation such as ERGMs can be derived.

(p. 164) Conclusion: One Theory, Many Directions

Network theory is multifaceted and can be applied in many seemingly disparate ways. This is, of course, one of the most powerful appeals of network theory to social scientists. This general applicability represents a challenge as well: the notion of a political network links a menagerie of ancillary concepts and tools, so it is important to remember where these tools come from. Even for the empirically focused researcher, paying attention to “pure network theory” questions is important. Most obviously, such attention helps assure that the assumptions and conditions undergirding the network-based empirical tools being employed are consistent with the researcher’s theoretical explanation. More subtly, when multiple network-based empirical tools are employed, paying attention to the network theory undergirding them helps ensure that these tools are consistent with each other.

We have attempted to provide a quick tour through some of the theoretical challenges and opportunities in the study of political networks. Because of space constraints, we have regretfully omitted much that is of interest. In a sense, we have sought to provide a kind of rough roadmap to some of the more widely applied network theory concepts, such as centrality, community, and connectivity. In so doing, we have attempted to illustrate a few directions in which these concepts can be developed. Broadly speaking, we see two areas of work as being particularly important in the coming years.

First, when empirically studying political networks, we need to spend more time considering *how* the data were generated: What do the nodes and edges represent? Why did we collect (or why are we using) *these* data? What are the incentives of the actors within the network? How could the network be altered? If the network is estimated from behavior, the answer to this question—the ultimate goal of policy prescriptions, for example—depends on the incentives of the actors themselves as well as the arsenal of policy levers that can be feasibly altered.

Second, many network concepts and measures are themselves relatively undertheorized, particularly as applied to political settings. That is, aside from intuition and occasionally some computational experimentation, many of the algorithms and approaches to quantifying and classifying concepts such as centrality, power, community, groups, and similarity have poorly understood theoretical foundations and properties. A key challenge is to develop these foundations and articulate the properties in a fashion that can be readily used by the empirically minded scholar of political networks.

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Notes:

(1.) Recent reviews of applications of network theory in political science range from the general (e.g., Lazer, 2011; Ward et al., 2011) to more substantively focused reviews such as Huckfeldt (2009) (American politics); Siegel (2011a) (comparative politics); Hafner-Burton et al. (2009) (international relations); and Fowler et al. (2007) (judicial politics).

(2.) Due to space constraints, we largely omit any discussion of political networks in other disciplines (primarily sociology). This is not intended as a judgment on the quality of such work, much of which is fascinating. Rather, a proper description of the theoretical challenges and opportunities faced by work in the traditions outside of modern political science would have a different focus due to the fields' varying reliance on concepts more closely associated with political science and economics, such as methodological individualism and revealed preference.

(3.) Some recent examples of such models in political science journals are Volden et al. (2008); Pierskalla (2009); Siegel (2009, 2011b, 2013); Baybeck et al. (2011); and Patty and Penn (2014).

(4.) A few recent thoughtful considerations of these issues from a variety of perspectives are List and Pettit (2011); Peters (2011); List and Spiekermann (2013); and Zahle and Collin (2014).

(5.) The most famous popular example of this concept, of course, is the "six degrees of Kevin Bacon."

(6.) Jackson (2008, 446).

(7.) A few examples are (1) detecting factions within political parties using legislator behaviors, (2) estimating informal coalitions among interest groups using endorsements, (3) examining linkages between agencies using rule-making and advisory panel collaboration, and (4) detecting issue linkages between campaign contributors using campaign donation data.

(8.) In a directed network, one can further ask whether there exists a path from each actor to the other.

(9.) Nearly equivalent is the notion of a *structural hole* (Burt 2009)—an edge that, when present, creates a bridge between two or more actors.

(10.) We provide the barest of descriptions here, due to space constraints. An excellent textbook treatment of these topics designed for social science applications is provided by Jackson (2008).

(11.) Technically, an edge could connect more than two nodes; such a graph is sometimes referred to as a *hypergraph*. Similarly, an edge could connect a node to itself, known as a “self-loop.” These extensions raise interesting possibilities (and a few challenges), but they are currently relatively infrequently employed in the study of political networks, which is why we exclude them from our notion of “classical network theory.”

(12.) It is not uncommon to assign “weights” to various edges in order to distinguish between ties of varying (say) intensities, frequencies, or importance. We revisit this possibility below, but we generally constrain the discussion to the classical case of unweighted graphs.

(13.) Examples of such concepts include density and clustering coefficient (e.g., Newman, 2010, 134 and 199).

(14.) Of course, that might be probabilistically identical.

(15.) This seemingly simple question can become complicated rather quickly in general networks, because one can vary “rules of interaction” such as whether interaction can occur between a given dyad multiple times.

(16.) Weights are typically (but not always) between zero and one.

(17.) Multigraphs—in which two nodes can be connected by multiple connections—are another direction for generalization (not to mention weighted multigraphs), but these have not been commonly employed in political science.

(18.) This point is discussed in more detail in Patty and Penn (2015), who leverage this property of degree centrality to illustrate the value of taking an axiomatic approach to describing and classifying networks and other examples of “big data.”

(19.) For comparisons between spectral measures and other measures of centrality, see Perra and Fortunato (2008).

(20.) Boldi and Vigna (2014, 237).

(21.) For a graph $G = (N, E)$ and any subset of the nodes, $M \subseteq N$, the subgraph induced by M is denoted by G_M and defined as $G_M = (M, E_M)$, where E_M contains exactly those edges $e' \in E$ such that both ends of e' are elements of M. A subgraph G_M is maximally completely connected if it is completely connected (every node in M is connected to every other node in M by an edge in E) and, for any strict superset of M, $M' \supset M$, $G_{M'}$ is not completely connected.

(22.) This presumes, in line with our discussion in the second section, that the choice of how to represent edges between nodes is sensible. In particular, if edges represent more than one type of connection, then one clique in the resulting network might be very different than another in qualitative terms. But this is not a problem with the notion of a clique, it is a problem with the utilized definition of an edge in the network under examination.

(23.) For more on these notions, as well as that of a *k-club*, see Mokken (1979).

(24.) Closely related to the notion of a k-plex is that of a *k-core*, which is a maximal subgraph such that each node is connected to at least k of the other nodes in the subgraph (Seidman, 1983).

(25.) Within the study of political networks, this attention is motivated by at least two facts. First, some kind of latent space model is necessary for the classic visualization of a network as a graph (e.g., Brandes et al., 1999) and, second, the spatial model (e.g., Davis et al., 1970) has a long history in modern political science.

(26.) Thus, latent space models can be loosely thought of as complementary to approaches such as exponential random graph models (ERGMs), discussed below. While approaches like ERGMs take exogenous attributes of the nodes and attempt to estimate the effect of these attributes on the probability of edges between any pair of nodes, the latent space model takes the edges as given and attempts to construct a set of attributes (coordinates in the latent space) that would be most compatible with an ERGM-style approach based on these estimated locations.

(27.) Typically, the only “external” requirement is that the subgraph induced by the group be maximal with respect to the desired internal structure property.

(28.) Radicchi et al. (2004, 2658).

(29.) See Jackson (2008, 9–14) for an accessible treatment of this framework.

(30.) And even if space were not a constraint, there are already multiple primers on the ERGM approach. For example, an excellent introduction to this family of models is provided by Robins et al. (2007), who update the primer offered by Anderson et al. (1999).

(31.) There is one such structure for undirected networks—the triangle—and five for directed networks.

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