Lecture 6. Producer Theory II

BTM210, KAIST

Duksang Cho (KDI)

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Topics Covered in This Lecture

Cost in the Short Run

Cost in the Long Run

Example: Firm's Problem with a Cobb-Douglas Production Functions

Supply Curves in a Competitive Market

Exercises

Cost in the Short Run

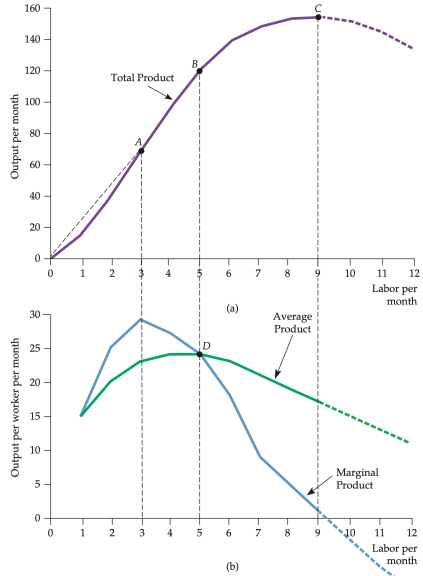
- Diminishing MP_L and increasing MC
  - Labor is the only variable input in the short run.
  - A firm must hire more labor to produce more output.
  - MP_L decreases with (the rate of) output.
  - MC increase with (the rate of) output.

$$MC = \frac{\Delta TC}{\Delta q} = \frac{\Delta VC}{\Delta q} = \frac{w\Delta L}{\Delta q} = \frac{w}{\Delta q/\Delta L} = \frac{w}{MP_L}$$

  - A low MP_L means that a large amount of additional labor is needed $\rightarrow$ a high MC
  - A high MP_L means that the labor requirement is low $\rightarrow$ a low MC

FIGURE 6.1
PRODUCTION WITH ONE VARIABLE INPUT

The total output curve in **(a)** shows the output produced for different amounts of labor input. The average and marginal products in **(b)** can be obtained (using the data in Table 6.1) from the total product curve. At point *A* in **(a),** with 3 units of labor, the marginal product is 29 because the tangent to the total product curve has a slope of 29. The average product of labor, however, is 23, which is the slope of the line from the origin to point *A*. Also, the marginal product of labor reaches its maximum at this point. At point *B*, with 5 units of labor, the marginal product of labor has dropped to 24 and is equal to the average product of labor. Thus, in **(b),** the average and marginal product curves intersect (at point *D*). Note that when the marginal product curve is above the average product, the average product is increasing. When the labor input is greater than 5 units, the marginal product is below the average product, so the average product is falling. Once the labor input exceeds 9 units, the marginal product becomes negative, so that total output falls as more labor is added.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 6.1

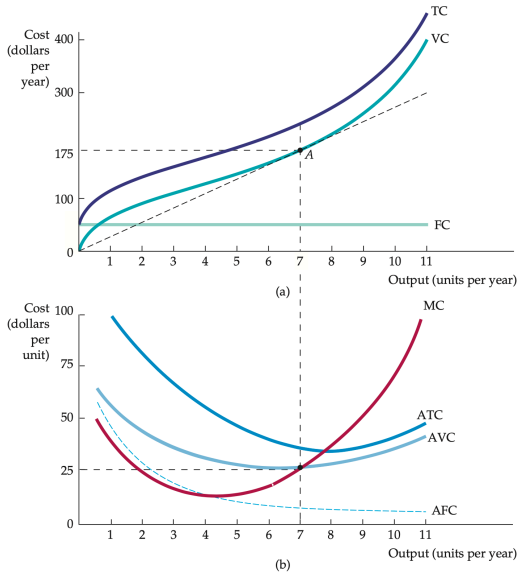


FIGURE 7.1
COST CURVES FOR A FIRM

In **(a)** total cost TC is the vertical sum of fixed cost FC and variable cost VC. In **(b)** average total cost ATC is the sum of average variable cost AVC and average fixed cost AFC. Marginal cost MC crosses the average variable cost and average total cost curves at their minimum points.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.1

Firm's Decision in the Short Run

- The firm's choice of output level depends on marginal and average cost.
- When demand fluctuates in the short run, knowledge of short-run costs is particularly important.
  - If the firm is currently producing output at which MC is sharply increasing,
  - And if demand may increase not in the short run, but in the future,
  - Then, the firm might want to expand production capacity to avoid higher costs, holding the amount of production in the short run.
  - Note that MP_L can increase with K and that MC can decrease with K.

$$F(L|K) = K^{1/3}L^{2/3}, \quad MP_L = \frac{2}{3}\left(\frac{K}{L}\right)^{1/3}, \quad MC = \frac{w}{MP_L} = \frac{3w}{2}\left(\frac{L}{K}\right)^{1/3}$$

The Price of Capital in the Long Run

- In the long run, firms can adjust inputs, which are fixed in the short run.

$$F(L|A,K) \quad \Rightarrow \quad F(A,K,L)$$

- The price of capital(r)

$$r = \text{Depreciation rate} + \text{Interest rate}$$

  - Unlike labor, large initial expenditures on capital are necessary.
  - We want to express this capital expenditure as a flow.
  - Amortizing the expenditure including any sunk cost of capital by spreading it over the lifetime of the capital (depreciation rate)
  - Accounting for the forgone interest that the firm could have earned by investing the money elsewhere (interest rate)

The Price of Capital in the Long Run

- In a competitive capital market,

$$r = \text{User costs of capital} = \text{Rental rate of capital}$$

- User costs of capital
  - The annual cost of owning and using the capital instead of selling it
  - The sum of the economic depreciation and the interest (or the financial return including risk premium) that could have been earned had the money been invested elsewhere
- Rental rate of capital
  - The cost per year for renting a unit of capital.

Cost in the Long Run

- Isocost line
  - All possible combinations of labor and capital that can be purchased for a given total cost

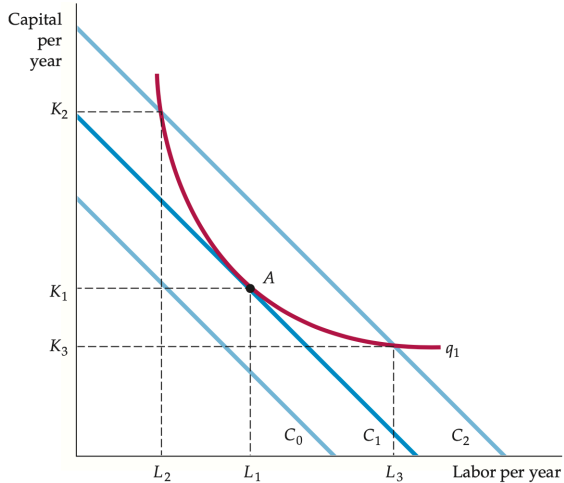
$$C = rK + wL$$

  - Its slope tells us the unit of capital the firm can buy if it gives up a unit of labor, holding the total cost of production

$$K = \frac{C}{r} - \frac{w}{r}L$$

FIGURE 7.3
PRODUCING A GIVEN OUTPUT AT MINIMUM COST

Isocost curves describe the combination of inputs to production that cost the same amount to the firm. Isocost curve C_1 is tangent to isoquant q_1 at A and shows that output q_1 can be produced at minimum cost with labor input L_1 and capital input K_1. Other input combinations—L_2, K_2 and L_3, K_3—yield the same output but at higher cost.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.3

Producer's Problem

- A producer or firm aims to maximize profits(Π) by choosing the optimal input factors, capital(K) and labor(L), given the quantity demanded(q)
- Given quantity demanded q or revenue Pq, choose input factors to minimize costs

$$\max_{K,L} \underbrace{\Pi(K,L)}_{\text{Profit}} = \underbrace{Pq}_{\text{Revenue}} - \underbrace{rK}_{\text{Capital cost}} - \underbrace{wL}_{\text{Labor cost}}$$

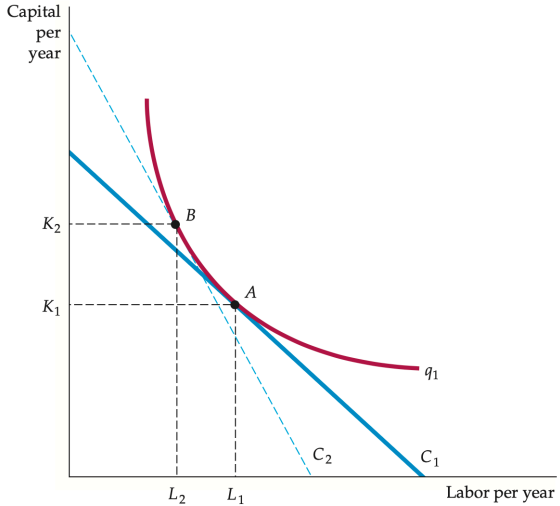
$$s.t. \quad \underbrace{F(K,L)}_{\text{Production function}} \geq \underbrace{q}_{\text{Quantity demanded}}$$

- Interior solution: Equalizing marginal product per dollar spent on each input factor

$$\frac{MP_K}{r} = \frac{MP_L}{w} \quad \Leftrightarrow \quad MRTS = \frac{MP_L}{MP_K} = \frac{w}{r}$$

FIGURE 7.4
INPUT SUBSTITUTION WHEN AN INPUT PRICE CHANGES

Facing an isocost curve C_1, the firm produces output q_1 at point A using L_1 units of labor and K_1 units of capital. When the price of labor increases, the isocost curves become steeper. Output q_1 is now produced at point B on isocost curve C_2 by using L_2 units of labor and K_2 units of capital.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.4

Economic Intuition

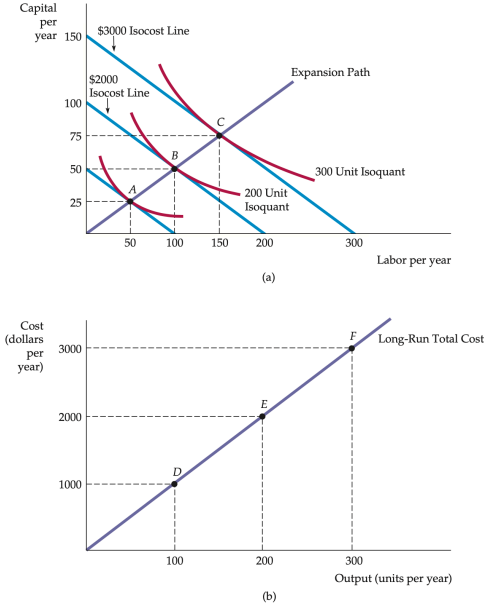
- Equalizing marginal values
  - MP_L/w is the additional output that results from spending an additional dollar for labor.
  - MP_K/r is the additional output that results from spending an additional dollar for capital.
  - Thus, a cost-minimizing firm should choose its quantities of inputs so that the last dollar's worth of any input added to the production process yields the same amount of extra output.

$$\frac{MP_L}{w} > \frac{MP_K}{r} \quad \Rightarrow \quad \Delta L > 0, \Delta K < 0$$

$$\frac{MP_L}{w} < \frac{MP_K}{r} \quad \Rightarrow \quad \Delta L < 0, \Delta K > 0$$

The Expansion Path and Long-Run Costs

- Expansion path
  - The curve passing through the points of tangency between the firm's isocost lines and its isoquants.
  - Describing the combinations of labor and capital that the firm will choose to minimize costs at each output level.
- Note that
  - As long as the use of both labor and capital increases with output, the expansion curve will be upward sloping.
  - We can derive a corresponding long-run total cost curve from the expansion curve.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.6

Short-Run vs. Long-Run Expansion Path

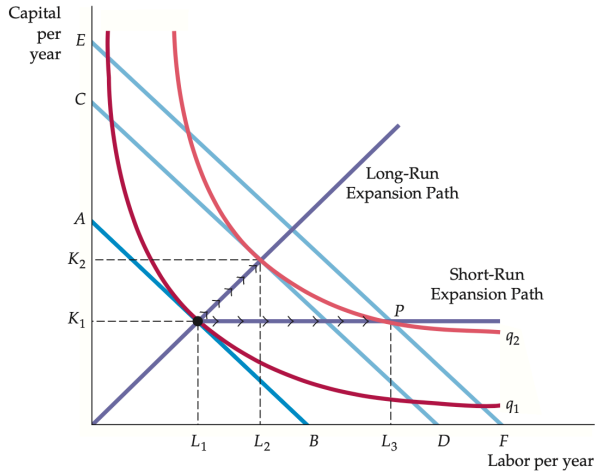


FIGURE 7.8
THE INFLEXIBILITY OF SHORT-RUN PRODUCTION

When a firm operates in the short run, its cost of production may not be minimized because of inflexibility in the use of capital inputs. Output is initially at level q_1. In the short run, output q_2 can be produced only by increasing labor from L_1 to L_3 because capital is fixed at K_1. In the long run, the same output can be produced more cheaply by increasing labor from L_1 to L_2 and capital from K_1 to K_2.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.8

Long-Run Cost Curves

- The long-run average cost curve (LAC) is U-shaped because IRS $\rightarrow$ CRS $\rightarrow$ DRS.

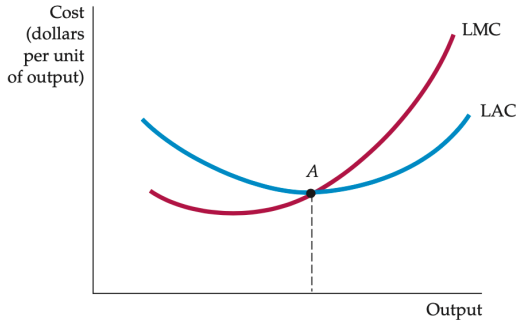


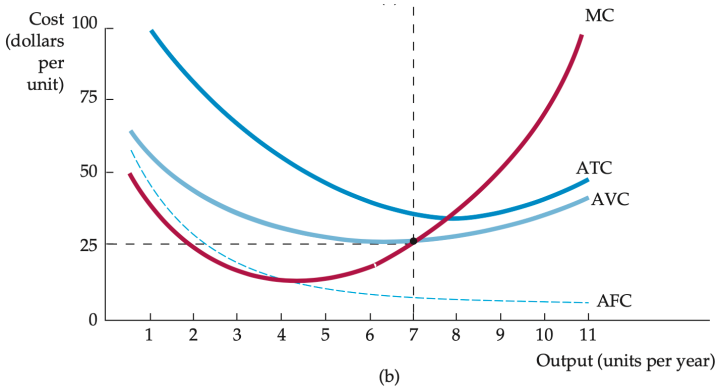
FIGURE 7.9
LONG-RUN AVERAGE AND MARGINAL COST

When a firm is producing at an output at which the long-run average cost LAC is falling, the long-run marginal cost LMC is less than LAC. Conversely, when LAC is increasing, LMC is greater than LAC. The two curves intersect at *A*, where the LAC curve achieves its minimum.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.9

Short-Run Cost Curves

- The short-run average total cost curve (ATC) is also U-shaped.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.1.b

Short-Run vs. Long-Run Cost Curves

- Both short-run and long-run average cost curves are U-shaped for different reasons.
- Short-run cost curves depend on returns to a factor of production.

$$ATC = \frac{FC}{q} + \frac{VC}{q}, \quad MC = \frac{w}{MP_L}$$

- Long-run cost curves depend on economies of scale.

$$\text{Economies of scale:} \quad 2TC(q) > TC(2q) \quad \Leftrightarrow \quad LAC(q) > LAC(2q)$$

$$\text{Diseconomies of scale:} \quad 5 * TC(2q) < TC(10q) \quad \Leftrightarrow \quad LAC(2q) < LAC(10q)$$

Why LAC Decreases and then Increases?

- The long-run average cost(LAC) can decrease in small scale:
  - If the firm operates on a larger scale, workers can "specialize" in the activities.
  - Scale can provide "flexibility". By varying inputs utilized to produce the firm's output, managers can organize the production process more effectively.
  - The firm may be able to acquire some production inputs at lower cost because it is buying them in large quantities and can therefore "negotiate better prices".
- The long-run average cost(LAC) can increase in large scale:
  - Managing a larger firm may become more "complex and inefficient" as the number of tasks increases.
  - The advantages of buying in bulk may have disappeared once certain quantities are reached. At some point, available supplies of "key inputs may be limited", pushing their costs up.

Returns to Scale vs. Economies of Scale

- Increasing returns to scale
  - Output more than doubles when the quantities of all inputs are doubled

$$F(2K, 2L) > 2F(K, L)$$

- Economies of scale
  - A doubling of output requires less than a doubling of cost

$$F(2.1K, 1.7L) = 2F(K, L) \quad w/ \quad 2.1rK + 1.7wL < 2(rK + wL)$$

- Note that
  - The term economies of scale includes increasing returns to scale as a special case.

Cost-Output Elasticity

- Economies of scale are often measured in terms of a cost-output elasticity, E_C.
- E_C is a % change in the cost of production resulting from a 1% increase in output.

$$E_C = \frac{\Delta C/C}{\Delta q/q} = \frac{\Delta C/\Delta q}{C/q} = MC/AC$$

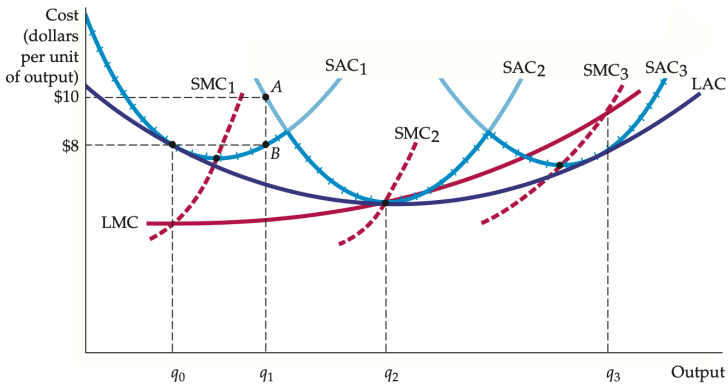
- $E_C < 1 \quad \Leftrightarrow \quad MC < AC \Leftrightarrow \quad AC$ decreases with outputs: Economies of scale
- $E_C > 1 \quad \Leftrightarrow \quad MC > AC \Leftrightarrow \quad AC$ increases with outputs: Diseconomies of scale

The Relationship between Short-Run and long-Run Cost

- The LAC is the envelope of the SAC. (LMC is not the envelope of the SMC.)

FIGURE 7.11
LONG-RUN COST WITH ECONOMIES AND DISECONOMIES OF SCALE

The long-run average cost curve LAC is the envelope of the short-run average cost curves SAC_1, SAC_2, and SAC_3. With economies and diseconomies of scale, the minimum points of the short-run average cost curves do not lie on the long-run average cost curve.



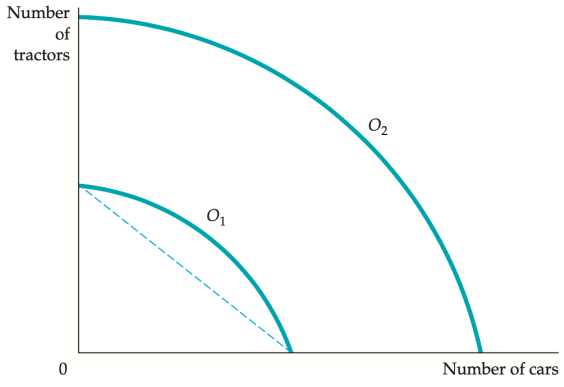
Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.11

Product Transformation Curves

- A production transformation curve shows the various combinations of two different outputs (products) that can be produced with a given set of inputs.

FIGURE 7.12
PRODUCT TRANSFORMATION CURVE

The product transformation curve describes the different combinations of two outputs that can be produced with a fixed amount of production inputs. The product transformation curves O_1 and O_2 are bowed out (or concave) because there are economies of scope in production.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.12

Economies of Scope

- Economies of scope is the situation such that
  - The joint output of a single firm is greater than the output that could be achieved by two different firms each producing a single product.
  - A single firm can produce multiple products with less costs than multiple firms each producing a single product.
- The degree of economies of scope (SC)

$$SC = \frac{C(q_1) + C(q_2) - C(q_1, q_2)}{C(q_1, q_2)}$$

  - $SC > 0$: Economies of scope
  - $SC < 0$: Diseconomies of scope

The Learning Curve

- A firm can learn over time as cumulative output increases. $(F(K, L) \rightarrow F(K, hL))$
- A learning curve describes the relationship between a firm's cumulative output and the amount of inputs needed to produce each unit of output.

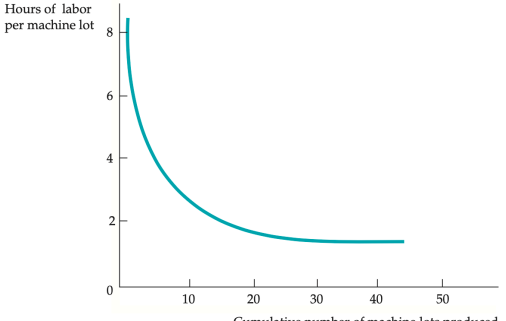


FIGURE 7.13
THE LEARNING CURVE

A firm's production cost may fall over time as managers and workers become more experienced and more effective at using the available plant and equipment. The learning curve shows the extent to which hours of labor needed per unit of output fall as the cumulative output increases.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.13

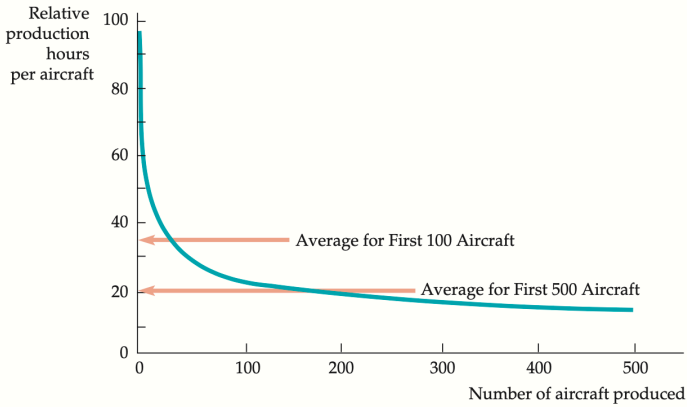


FIGURE 7.15
LEARNING CURVE FOR AIRBUS INDUSTRIE

The learning curve relates the labor requirement per aircraft to the cumulative number of aircraft produced. As the production process becomes better organized and workers gain familiarity with their jobs, labor requirements fall dramatically.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 7.15

A Cobb-Douglas Production Function

$$F(K, L) = AK^{\alpha}L^{\beta}, \quad \alpha, \beta \in (0, 1)$$

- Returns to scale
  - $\alpha + \beta > 1$: IRS
  - $\alpha + \beta = 1$: CRS
  - $\alpha + \beta < 1$: DRS
- Marginal productivity

$$MP_K = \frac{\partial F}{\partial K} = \alpha A K^{\alpha-1} L^{\beta} = \alpha \frac{F}{K}$$

$$MP_L = \frac{\partial F}{\partial L} = \beta A K^{\alpha} L^{\beta-1} = \beta \frac{F}{L}$$

Optimal Inputs

- FOCs

$$\frac{w}{r} = MRTS = \frac{MP_L}{MP_K} = \frac{\beta F/L}{\alpha F/K} = \frac{\beta K}{\alpha L}$$

- The expansion path

$$L = \frac{\beta r K}{\alpha w}$$

- Put the above equation into the production function

$$q = F(K, L) = AK^{\alpha}\left(\frac{\beta r K}{\alpha w}\right)^{\beta} = A\left(\frac{\beta r}{\alpha w}\right)^{\beta} K^{\alpha+\beta}$$

- The optimal inputs given q

$$K = \left(\frac{\alpha w}{\beta r}\right)^{\frac{\beta}{\alpha+\beta}}\left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}, \quad L = \left(\frac{\beta r}{\alpha w}\right)^{\frac{\alpha}{\alpha+\beta}}\left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

Cost Functions

- Total cost (TC)

$$TC = rK + wL = \left\{ r\left(\frac{\alpha w}{\beta r}\right)^{\frac{\beta}{\alpha+\beta}} + w\left(\frac{\beta r}{\alpha w}\right)^{\frac{\alpha}{\alpha+\beta}} \right\} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

$$= r^{\frac{\alpha}{\alpha+\beta}} w^{\frac{\beta}{\alpha+\beta}} \left\{ \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\alpha+\beta}} + \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} \right\} \left(\frac{q}{A}\right)^{\frac{1}{\alpha+\beta}}$$

- Average cost (AC)

$$AC = \frac{TC}{q} \propto q^{\frac{1-\alpha-\beta}{\alpha+\beta}}$$

  - If $\alpha + \beta > 1$, AC decreases with $q \Rightarrow$ IRS
  - If $\alpha + \beta = 1$, TC is constant $\Rightarrow$ CRS
  - If $\alpha + \beta < 1$, AC increases with $q \Rightarrow$ DRS

Firm's Problem in a Competitive Market

- A firm chooses its quantity supplied q to maximize its profit with the optimized input functions, $K^*(q)$ and $L^*(q)$, given P in the competitive market.

$$\max_q \underbrace{\Pi(q)}_{\text{Profit}} = \underbrace{Pq}_{\text{Revenue}} - \underbrace{(rK^*(q) + wL^*(q))}_{\text{Cost}}$$

- FOC

$$\underbrace{\frac{d}{dq}\Pi}_{\text{Marginal profit}} = \underbrace{\frac{d}{dq}(Pq)}_{\text{Marginal revenue}} - \underbrace{\frac{d}{dq}(rK^*(q) + wL^*(q))}_{\text{Marginal cost}} = 0$$

- Supply curves

$$MR(q) = MC(q) \quad \Leftrightarrow \quad P = MC(q)$$

Supply Curve in the Short Run

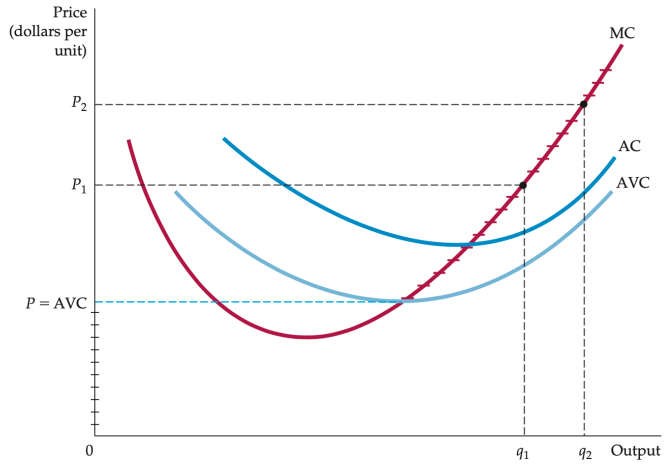


FIGURE 8.6
THE SHORT-RUN SUPPLY CURVE FOR A COMPETITIVE FIRM

In the short run, the firm chooses its output so that marginal cost MC is equal to price as long as the firm covers its average variable cost. The short-run supply curve is given by the crosshatched portion of the marginal cost curve.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 8.6

Supply Curve in the Long Run

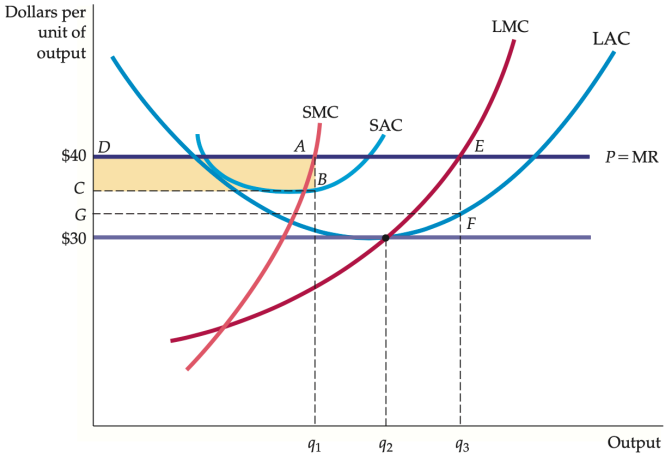


FIGURE 8.13
OUTPUT CHOICE IN THE LONG RUN

The firm maximizes its profit by choosing the output at which price equals long-run marginal cost LMC. In the diagram, the firm increases its profit from *ABCD* to *EFGD* by increasing its output in the long run.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 8.13

Exercises

1. How does a change in the price of one input change the firm's long-run expansion path?
   - The expansion path describes the cost-minimizing combination of inputs that the firm chooses for every output level.
   - The firm substitutes away from the more expensive input toward the cheaper input.
   - Thus, the expansion path bends toward the axis of the now cheaper input.
2. Suppose the economy takes a downturn, and that labor costs fall by 50 percent and are expected to stay at that level for a long time. Show graphically how this change in the relative price of labor and capital affects the firm' s expansion path.
   - Because the expansion path is the set of points where the MRTS is equal to the ratio of prices, as the isocost lines become flatter, the expansion path becomes flatter and moves toward the labor axis.

Exercises

3. Is the firm' s expansion path always a straight line?
   - No.
   - If the optimal capital-labor ratio changes as output is increased, the expansion path is not a straight line.
   - In the short run the expansion path may be horizontal if capital is fixed.
4. Assume that the marginal cost of production is increasing. Can you determine whether the average variable cost is increasing or decreasing? Explain.
   - When marginal cost is increasing, average variable cost can be either increasing or decreasing.
   - AVC is pulled down where MC is below AVC.
   - MC cuts through the minimum point of AVC.
   - AVC begins to rise where MC is above AVC.