# Lecture 4. Uncertainty

BTM210, KAIST

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# Topics Covered in This Lecture

Decision under Uncertainty

Revealed Preferences

Exercises

Decision under Uncertainty

Revealed Preferences

Exercises

# Uncertainty

- So far, we have assumed that every variable is known and realized with certainty.
  - "I want to consume 4 units of food and 2 units of clothing given income and prices."
- However, many real world problems involve uncertainty or risk.
  - Prices of goods can change tomorrow.
  - The item I did not buy today might be sold out tomorrow.
  - Future incomes are uncertain.
- How should we take these uncertainties into account when making consumption or investment decisions?
  1. Describing risk
  2. Define Preferences toward risk

# Describing risk

- Vocabulary
  - Probability: Likelihood that a given outcome will occur
  - Expected value: Probability-weighted average of the payoffs associated with all possible outcomes
  - Payoff: Value associated with a possible outcome
  - Variability: Extent to which possible outcomes of an uncertain event differ
  - Deviation: Difference between expected payoff and actual payoff
  - Variance, standard deviation, ...
- Interpretation of probability in economics
  - Objective probability: The frequency with which certain events tend to occur
  - Subjective probability: The perception that an outcome will occur

# Expected Payoff

- Expected payoff: Probability-weighted average of payoffs
  - Payoff: $X_1, X_2, ..., X_n$
  - Probability: $P_1, P_2, ..., P_n$
  - Expected payoff:

$$E[X] = P_1X_1 + P_2X_2 + ... + P_nX_n = \sum_{i \in 1,2,\ldots,n} P_iX_i$$

- e.g., The value of an offshore oil exploration company
  - Success: a payoff of \$40 per share with probability $0.25$
  - Failure: a payoff of \$20 per share with probability $0.75$

$$E[\text{payoff per share}] = 0.25 * \$40 + 0.75 * \$20 = \$25$$

# Example 1

| TABLE 5.1 INCOME FROM SALES JOBS | OUTCOME 1 | | OUTCOME 2 | | |
|---|---|---|---|---|---|
| | PROBABILITY | INCOME ($) | PROBABILITY | INCOME ($) | EXPECTED INCOME ($) |
| Job 1: Commission | .5 | 2000 | .5 | 1000 | 1500 |
| Job 2: Fixed Salary | .99 | 1510 | .01 | 510 | 1500 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 5.1 Income from Sales Jobs

- Mean, variance, and standard deviation
  - Job 1: Expected payoff $= \$1500$, Variance $= 250,000$, S.D. $= 500$
  - Job 2: Expected payoff $= \$1500$, Variance $= 9,900$, S.D. $= 99.5$

# Example 1 Modified

| TABLE 5.4 | INCOMES FROM SALES JOBS—MODIFIED ($) | | | | | |
|---|---|---|---|---|---|---|
| | OUTCOME 1 | DEVIATION SQUARED | OUTCOME 2 | DEVIATION SQUARED | EXPECTED INCOME | STANDARD DEVIATION |
| Job 1 | 2100 | 250,000 | 1100 | 250,000 | 1600 | 500 |
| Job 2 | 1510 | 100 | 510 | 980,100 | 1500 | 99.50 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 5.4 Income from Sales Jobs - Modified

- If Job 1 can earn \$100 more for both outcome 1 and 2 with certainty,
  - Job 1: Expected payoff $= \$1600$, Variance $= 250,000$, S.D. $= 500$
  - Job 2: Expected payoff $= \$1500$, Variance $= 9,900$, S.D. $= 99.5$
- Which job do you prefer?

# Example 1

- Job 1 is riskier than Job 2.
  - The spread of possible payoffs for Job 1 is greater than the spread for Job 2.

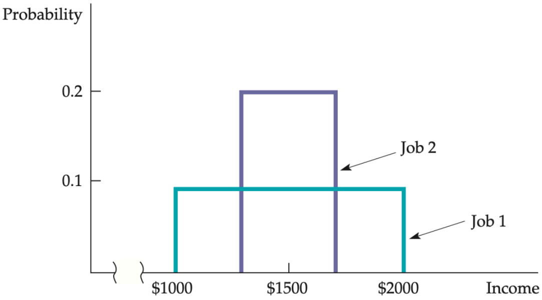



**FIGURE 5.1**
**OUTCOME PROBABILITIES FOR TWO JOBS**

The distribution of payoffs associated with Job 1 has a greater spread and a greater standard deviation than the distribution of payoffs associated with Job 2. Both distributions are flat because all outcomes are equally likely.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.1 Outcome Probabilities for Two Jobs

# Expected Utility

- An economic agent
  - Obtains his utility from choosing among risky alternatives
  - Does not care his expected income per se,
  - But cares his "expected utility", which is the weighted average of the utilities associated with all possible outcomes
  - Chooses the optimal risky alternative that gives the maximum expected utility.

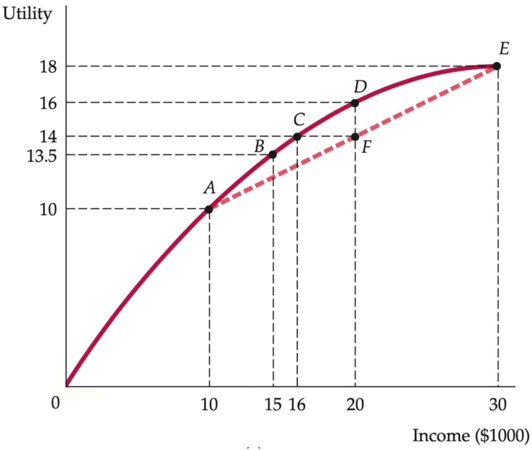
$$\max_{I \in \mathcal{I}} \mathbb{E}[U(I)] = P_1 U(I_1) + P_2 U(I_2) + ... + P_n U(I_n)$$

- Note that the utility of income can be interpreted as the indirect optimized utility function derived from the consumer's constrained optimization problem.

$$\max_{X,Y} U(X, Y) \text{ given } I \Rightarrow U^*(I)$$

# Preferences Toward risk

- In many cases, marginal utility is diminishing and people dislike risk.
- How to capture these characteristics together?
  - A concave utility function



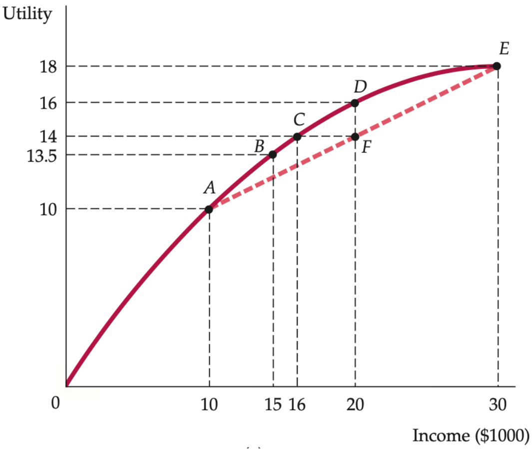

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.3a

# Three Types of Risk Preferences

- People differ in their willingness to bear risk.
- Risk averse
    - Preferring a certain income to a risky income with the same expected value
    - Captured by a concave utility function
- Risk neutral
    - Being indifferent between a certain income and an uncertain income with the same expected value
    - Captured by a linear utility function
- Risk loving
    - Preferring a risky income to a certain income with the same expected value
    - Captured by a convex utility function

# A Risk-Averse, Concave Utility Function

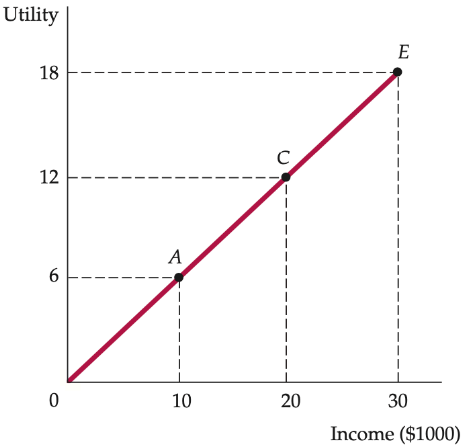
$$\mathbb{E}[U(I)] < U(\mathbb{E}[I])$$




Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.3a

# A Risk-Neutral, Linear Utility Function

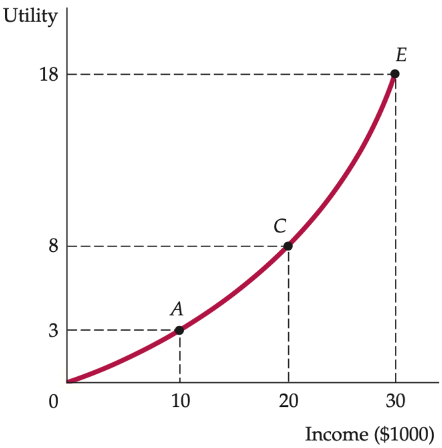
$$\mathbb{E}[U(I)] = U(\mathbb{E}[I])$$




Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.3c

# A Risk-Loving, Convex Utility Function

$$\mathbb{E}[U(I)] > U(\mathbb{E}[I])$$




Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.3b

# Risk Premium

- Risk premium is the maximum amount of money that a risk-averse person will pay to avoid taking a risk.
- Certainty equivalent is the amount of money that a person would consider equally desirable as a risky asset.

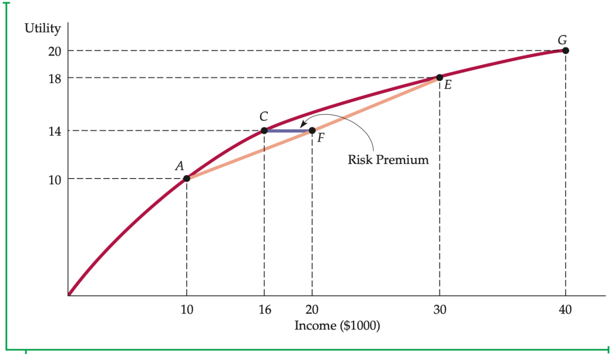



**FIGURE 5.4**
**RISK PREMIUM**

The risk premium, *CF*, measures the amount of income that an individual would give up to leave her indifferent between a risky choice and a certain one. Here, the risk premium is \$4000 because a certain income of \$16,000 (at point *C*) gives her the same expected utility (14) as the uncertain income (a .5 probability of being at point *A* and a .5 probability of being at point *E*) that has an expected value of \$20,000.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.4

# Risk Aversion and Indifference Curves: Graphical Analysis

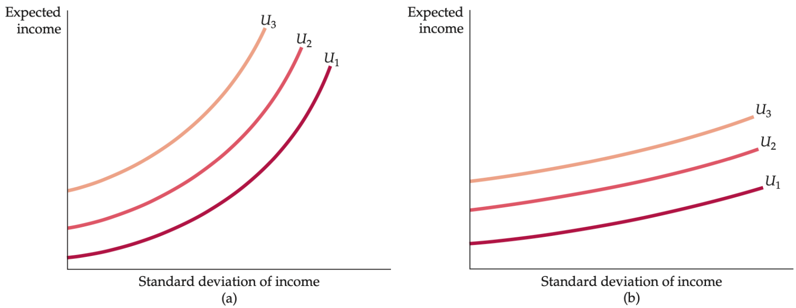



**FIGURE 5.5**
**RISK AVERSION AND INDIFFERENCE CURVES**

Part **(a)** applies to a person who is highly risk averse: An increase in this individual's standard deviation of income requires a large increase in expected income if he or she is to remain equally well off. Part **(b)** applies to a person who is only slightly risk averse: An increase in the standard deviation of income requires only a small increase in expected income if he or she is to remain equally well off.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.5

# Risk Can Be Reduced

- Diversification
  - Allocating resources to a variety of activities whose outcomes are not closely related or negatively correlated
- Insurance
  - if the cost of insurance is equal to the expected loss, risk-averse people will buy enough insurance to recover fully from any financial losses they might suffer
- Law of large numbers
  - Insurance companies rely on the law of large numbers by operating on a large scale.
  - When insurance premium is equal to the expected payout, we say the insurance is actuarially fair.
- Information

# Investment Decision

- Why do risk-averse people invest in the stock market?
- Because of the trade-off between risk and return.

| TABLE 5.8 | INVESTMENTS – RISK AND RETURN (1926–2014) | | |
|---|---|---|---|
| | AVERAGE RATE OF RETURN (%) | AVERAGE REAL RATE OF RETURN (%) | RISK (STANDARD DEVIATION) |
| **Common stocks (S&P 500)** | 12.1 | 8.8 | 20.1 |
| **Long-term corporate bonds** | 6.4 | 3.3 | 8.4 |
| **U.S. Treasury bills** | 3.5 | 0.5 | 3.1 |

Source: © 2015 Morningstar, Inc. All Rights Reserved. Reproduced with permission.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 5.8

# Understanding the Demand for Risky Assets

- Vocabulary
  - "Asset" is something that provides a flow of money or services to its owner.
    - Risky asset provides an uncertain flow of money or services to its owner
    - Risk-free asset provides a flow of money or services that is known with certainty
  - "Return" is the total monetary flow of an asset as a fraction of its price.
    - Real return is the nominal return on an asset less the rate of inflation
    - Expected return is the return that an asset should earn on average
  - "Price of risk" is the extra risk that an investor must incur to enjoy a higher expected return.

# An Investor's Optimal Portfolio Problem

- An investor wants to invest his savings $S$ in two assets: a risk-free bond with return $R_f$ and a risky stock with expected return $R_m$ and standard deviation $\sigma_m$.
  - How much money the investor should put in each asset?
  - Or what fraction of his savings should be invested in the risky asset?
- If he invests $b$ fraction of his savings in the risky asset
  - The expected rate of return on his portfolio

$$R_p = (1 - b)R_f + bR_m$$

  - The standard deviation of his portfolio

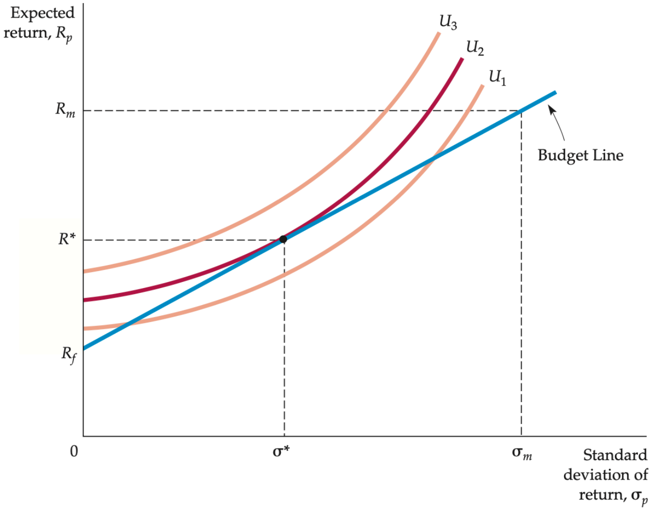
$$\sigma_p = b\sigma_m$$

# The Expected Rate of Return and Its Standard Deviation

$$R_p = R_f + b(R_m - R_f) = R_f + \frac{(R_m - R_f)}{\sigma_m}\sigma_p$$

- The expected rate of return on the portfolio $R_p$ increases as the standard deviation of that return $\sigma_p$ increases.
- Trade-off between $R_p$ and $\sigma_p$ drawing a budget line on the $R_p - \sigma_p$ plane.
- The slope of this budget line is the price of risk, because it tells us how much extra risk an investor must incur to enjoy a higher expected return.

# Graphical Representation: Indifference Curves and Budget Line

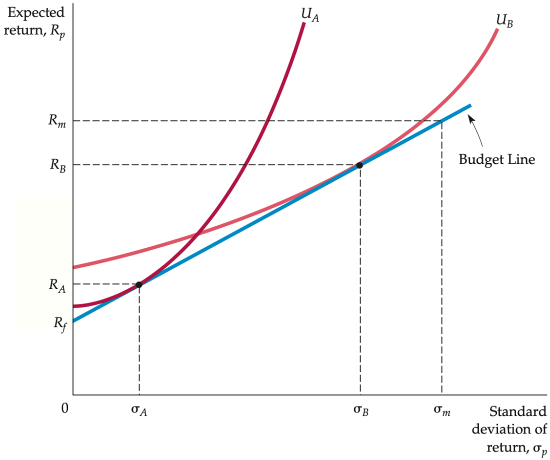



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.6

# Investor's Choice Depends on His or Her Preference

**FIGURE 5.7**
**THE CHOICES OF TWO DIFFERENT INVESTORS**

Investor $A$ is highly risk averse. Because his portfolio will consist mostly of the risk-free asset, his expected return $R_A$ will be only slightly greater than the risk-free return. His risk $\sigma_A$, however, will be small. Investor $B$ is less risk averse. She will invest a large fraction of her funds in stocks. Although the expected return on her portfolio $R_B$ will be larger, it will also be riskier.

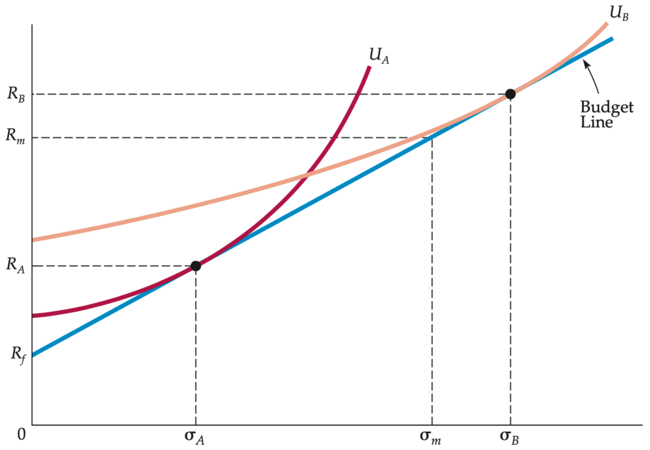



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.7

# Buying Stocks on Margin

**FIGURE 5.8**
**BUYING STOCKS ON MARGIN**

Because Investor *A* is risk averse, his portfolio contains a mixture of stocks and risk-free Treasury bills. Investor *B*, however, has a very low degree of risk aversion. Her indifference curve, $U_B$, is tangent to the budget line at a point where the expected return and standard deviation for her portfolio exceed those for the stock market overall. This implies that she would like to invest *more* than 100 percent of her wealth in the stock market. She does so by buying stocks *on margin*—i.e., by borrowing from a brokerage firm to help finance her investment.




Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 5.8

Decision under Uncertainty

Revealed Preferences

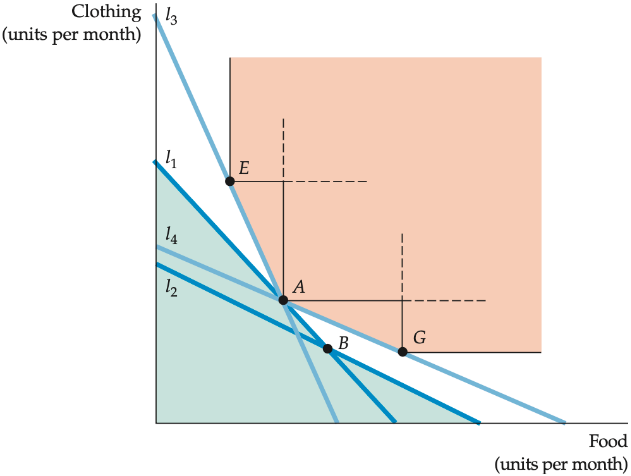
Exercises

# Revealed Preferences

- Given preferences, we can decide the optimal consumption bundle.
- Given choices that a consumer has made, can we determine his or her preferences?
  - Yes, we can if we have information about a sufficient number of choices that have been made when prices and income levels varied.
- Revealed preference theory
  - Is the method of to infer the preferences of individuals given the observed choices.
- How it works?
  - If a consumer chooses one market basket over another,
  - And if the chosen market basket is more expensive than the alternative,
  - Then the consumer must prefer the chosen market basket.

# Example

- Observation
  1. $A$ given $\ell_1$
  2. $B$ given $\ell_2$
  3. $E$ given $\ell_3$
  4. $G$ given $\ell_4$
- With 1 and 2, $A$ is preferred to the green area.
- With 3 and 4, the pink area is preferred to $A$ due to the diminishing MRS assumption.




Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 3.19

Decision under Uncertainty

Revealed Preferences

Exercises

# Exercises

1. Are there economic factors that can affect risk preferences?
   - A wealthy person is more likely to take risks than a moderately well off person, because the wealthy person can better handle losses.
   - People are more likely to take risks when the stakes are low (buying a Lotto ticket) than when stakes are high (purchasing fire insurance).
2. Can you think of a case in which a person might not maximize expected utility?
   - To maximize expected utility, consumer should know each possible outcome that may occur and the probability of each outcome.
   - Sometimes consumers either do not know all possible outcomes and the relevant probabilities, or they have difficulty evaluating low-probability, extreme-payoff events.

# Exercises

3. Suppose that Natasha′s utility function is given by $U(I) = (10I)^{1/2}$, where $I$ represents annual income in thousands of dollars.
   - Is Natasha risk loving, risk neutral, or risk averse?
   - Suppose that Natasha is currently earning an income of $\$40,000 (I = 40)$ and can earn that income next year with certainty. She is offered a chance to take a new job that offers a $0.6$ probability of earning $\$44,100$ and a $0.4$ probability of earning $\$36,100$. Should she take the new job?
   - Would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (Hint: What is the risk premium?)

# Exercises

3. Suppose that Natasha′s utility function is given by $U(I) = (10I)^{1/2}$, where $I$ represents annual income in thousands of dollars.
   - Is Natasha risk loving, risk neutral, or risk averse? (Risk averse)
   - Suppose that Natasha is currently earning an income of $\$40,000 (I = 40)$ and can earn that income next year with certainty. She is offered a chance to take a new job that offers a $0.6$ probability of earning $\$44,100$ and a $0.4$ probability of earning $\$36,100$. Should she take the new job? (Yes, she should.)
   - Would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (Hint: What is the risk premium?) (Her risk premium or willingness to pay for insurance is $\$96$.)

# Exercises

4. Minsu currently has $\$1,000,000$ in cash. He has the following two investment options.
    - Risk-free asset: Guarantees the invested amount without any gain or loss (0% return).
    - Risky asset: The investment outcome depends on two possible scenarios
        - Good outcome (50% probability): Investment value increases by 60%.
        - Bad outcome (50% probability): Investment value decreases by 40%.
    - Minsu's utility function is given by $U(I) = \sqrt{I}$, where $I$ represents the final wealth (in units of 10,000 KRW).

- To maximize Minsu's expected utility, how much should he invest in the risky asset, and how much should he invest in the risk-free asset?

## Exercises

4. Minsu currently has $1,000,000$ KRW in cash. He has the following two investment options.
   - Risk-free asset: Guarantees the invested amount without any gain or loss (0% return).
   - Risky asset: The investment outcome depends on two possible scenarios
     - Good outcome (50% probability): Investment value increases by 60%.
     - Bad outcome (50% probability): Investment value decreases by 40%.
   - Minsu's utility function is given by $U(I) = \sqrt{I}$, where $I$ represents the final wealth (in units of 10,000 KRW).

- To maximize Minsu's expected utility, how much should he invest in the risky asset, and how much should he invest in the risk-free asset? (KRW 833,333 in the risky asset, KRW 166,666 in the risk-free asset)