# Lecture 11. Oligopoly and Game Theory

BTM210, KAIST

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Spring 2025

# Topics Covered in This Lecture

Oligopoly: Stackelberg, Bertrand, and Price Competition

Gaming and Strategic Decisions

Repeated Games

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# The Stackelberg Model (First Mover Advantage)

- Stackelberg: Two firms compete and one of the firms can set its output first.
- Cf. Cournot: two firms set their output simultaneously.
- Questions:
  - Is it advantageous to go first?
  - How much will each firm produce?
- An example
  - Zero marginal cost
  - Market demand: $P = 30 - Q = 30 - Q_1 - Q_2$
  - Firm 1 sets its output first by considering how Firm 2 will react.
  - Firm 2, after observing Firm 1' s output, makes its output decision.

- Firm2 makes its output decision after Firm 1 by taking Firm 1' s output $Q_1$ as fixed. Thus, Firm $2'$s reaction curve is given by its Cournot reaction curve

$$Q_2 = 15 - Q_1/2$$

- Firm 1 chooses $Q_1$ so that its marginal revenue equals its marginal cost of zero.

$$\begin{aligned} R_1 &= PQ_1 = (30 - Q_1 - Q_2)Q_1 = (30 - Q1 - 15 + Q_1/2)Q_1 \\ &= 15Q_1 - Q_1^2/2 \\ MR_1 &= 15 - Q_1 = MC_1 = 0 \end{aligned}$$

- Solution: $Q_1 = 15, \quad Q_2 = 7.5, \quad P = 7.5$
- Going first gives Firm 1 an advantage: Firm 1 produces twice as much as Firm 2 and makes twice as much profit.

- Going first gives Firm 1 an advantage.
    - Firm 1 produces twice as much as Firm 2 and makes twice as much profit.
- Why?
    - No matter what Firm 2 does, Firm 1′s output will be large.
    - To maximize profit, Firm 2 must take Firm 1′s large output level as given and set a low level of output for itself.
    - If Firm 2 produced a large level of output, it would drive price down and both firms would lose money.
- Cournot vs. Stackelberg
    - For an industry composed of roughly similar firms, none of which has a strong leadership position, the Cournot model is probably the more appropriate.
    - Some industries are dominated by a large firm that usually takes the lead in setting price. Then the Stackelberg model may be more realistic.

# The Bertrand Model (Price Competition with Homogeneous Products)

- Bertrand: Two firms simultaneously choose prices instead of quantities.
- Cf. Cournot: Two firms choose quantities.
- An example
  - Market demand: $P = 30 - Q = 30 - Q_1 - Q_2$
  - Marginal cost: $MC_1 = MC_2 = \$3$
  - Note that because the good is homogeneous, consumers will purchase only from the lowest-price seller.
- Solution: A Nash equilibrium
  - Both firms set price equal to marginal cost: $P_1 = P_2 = \$3$.
  - $Q = 27, \quad Q_1 = Q_2 = 13.5$
  - A Nash equilibrium: No firm wants to raise or lower its price.

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# Price Competition with Differentiated Products

- Oligopolistic markets often have at least some degree of product differentiation.
- Market shares are determined not just by prices, but also by differences in the design, performance, and durability of each firm' s product.
- In such cases, it is natural for firms to compete by choosing "prices" rather than quantities.
- An example
  - Each of two duopolists has fixed costs of $20 but zero variable costs.
  - Both firms set their prices "at the same time."
  - They face the same demand curves:

$$\text{Firm } 1'\text{s demand:} \quad Q_1 = 12 - 2P_1 + P2$$

$$\text{Firm } 2'\text{s demand:} \quad Q_2 = 12 - 2P_2 + P1$$

- Firm 1′s reaction curve

$$\max_{P_1} \Pi_1 = P_1 Q_1 - 20 = 12P_1 - 2P_1^2 + P_1 P_2 - 20$$

$$\frac{d\Pi_1}{dP_1} = 12 + P_2 - 4P_1 = 0 \quad \Rightarrow \quad P_1 = 3 + P_2/4$$

- Firm 2′s reaction curve

$$P_2 = 3 + P_1/4$$

- A Nash equilibrium

$$P_1 = P_2 = \$4, \quad Q_1 = Q_2 = 8, \quad \Pi_1 = \Pi_2 = \$12$$

- In this example, each firm would be at a distinct disadvantage by moving first.
- Why? Because it gives the firm that moves second an opportunity to undercut slightly and thereby capture a larger market share. (cf. Stackelberg)

- Firm $1'$s reaction curve

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- Firm $2'$s reaction curve

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- A Nash equilibrium

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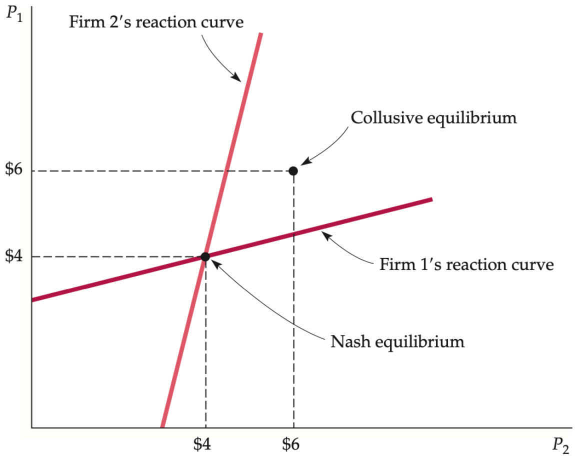


## FIGURE 12.6
## NASH EQUILIBRIUM IN PRICES

Here two firms sell a differentiated product, and each firm's demand depends both on its own price and on its competitor's price. The two firms choose their prices at the same time, each taking its competitor's price as given. Firm 1's reaction curve gives its profit-maximizing price as a function of the price that Firm 2 sets, and similarly for Firm 2. The Nash equilibrium is at the intersection of the two reaction curves: When each firm charges a price of $4, it is doing the best it can given its competitor's price and has no incentive to change price. Also shown is the collusive equilibrium: If the firms cooperatively set price, they will choose $6.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 12.6

Oligopoly: Stackelberg, Bertrand, and Price Competition

Gaming and Strategic Decisions

Repeated Games

# Gaming and Strategic Decision-Making

- There are many questions about market structure and firm behavior that we have not yet addressed.
  - Why do firms tend to collude in some markets and to compete aggressively in others?
  - How do some firms manage to deter entry by potential competitors?
  - How should firms make pricing decisions when demand or cost conditions are changing or new competitors are entering the market?
- In this chapter,
  - Explains some key aspects of game theory
  - Use game theory to answer some questions
  - Shows how it can be used to understand how markets evolve and operate
  - Show how managers should think about the strategic decisions they continually face

- A **game** is any situation in which players make strategic decisions, i.e., decisions that take into account each other's actions and responses.
  - Firms competing with each other by setting prices
  - A group of consumers bidding against each other at an auction for a work of art
- Strategic decisions result in **payoffs** to the players.
  - Payoff is the outcomes that generate rewards or benefits.
- An objective of game theory is to determine the optimal **strategy** for each player.
  - A strategy is a rule or plan of action for playing the game.
  - The optimal strategy for a player is the one that maximizes the expected payoff.
- The central question we seek to answer through game theory:
  - If I believe that my competitors are rational and act to maximize their own payoffs,
  - How should I take their behavior into account when making my decisions?

# Non-Cooperative vs. Cooperative Games

- In a cooperative game,
  - Players can negotiate binding contracts that allow them to plan joint strategies.
  - e.g., Two firms negotiating a joint investment to develop a new technology
- In a non-cooperative game,
  - Negotiation and enforcement of binding contracts are not possible.
  - e.g., Two competing firms take each other' s likely behavior into account when independently setting their prices
- The fundamental difference between cooperative and non-cooperative games lies in the contracting possibilities.
- **Key point**: It is essential to understand your opponent's point of view and to deduce his or her likely responses to your actions.

# Example: A Thousand Won Auction Game

- Game description
  - A KRW 1,000 bill is auctioned.
  - Highest bidder wins KRW 1,000 and pays their bid.
  - Second-highest bidder must also pay their bid **but receives nothing**.
  - Bids must increase by at least KRW 100.
- Example
  - Player A bids KRW 200, Player B bids KRW 300.
  - Player A risks losing KRW 200, so bids KRW 400.
  - If the auction ends, player A wins the auction, pays KRW 400, and earns KRW 1,000.
  - Player B pays KRW 300 and receives nothing.
- Let's play the game. Please raise your hand if you'd like to volunteer. Only the first two will get to join the game!

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- Why bidding escalates?
  - Player A bids KRW 200, Player B bids KRW 300.
  - Player A risks losing KRW 200, so bids KRW 400.
  - Escalation continues: bids rise to KRW 1,000 and beyond.
  - Players try to minimize loss by bidding more, even irrationally.
- Nash equilibrium
  - No one bids.
  - Rational players foresee potential losses.
  - If everyone expects escalation and loss, no one enters the auction.
  - Thus, not bidding is the optimal mutual strategy.
- Reality vs Theory
  - In practice, players bid to avoid losses, leading to irrational escalation.
  - Psychological and strategic traps override rational analysis.

# Dominant Startegies

- How can we decide on the best strategy for playing a game?
  - We need something to help us determine how the rational behavior of each player will lead to an equilibrium solution.
  - Some strategies may be successful if competitors make certain choices but fail if they make other choices.
  - Other strategies, however, may be successful regardless of what competitors do.
- **A dominant strategy** is an optimal strategy no matter what an opponent does.
- When every player has a dominant strategy, we call the outcome of the game an **equilibrium in dominant strategies**.
- These games are straightforward to analyze because each player's optimal strategy can be determined without worrying about the actions of the other players.

# Advertising Game

- Firms A and B sell competing products.
- They are deciding whether to undertake advertising campaigns.
- Each firm will be affected by its competitor's decision.
- The payoff matrix is as follows: the first number in each cell is the payoff to A and the second is the payoff to B.

**TABLE 13.1 PAYOFF MATRIX FOR ADVERTISING GAME**

| | | FIRM B | |
|---|---|---|---|
| | | **ADVERTISE** | **DON'T ADVERTISE** |
| **FIRM A** | **ADVERTISE** | 10, 5 | 15, 0 |
| | **DON'T ADVERTISE** | 6, 8 | 10, 2 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.1

# Modified Advertising Game

- Let's change our advertising example slightly.
- The payoff matrix in Table 13.2 is the same as in Table 13.1 except for the bottom right-hand corner: If neither firm advertises, Firm B will again earn a profit of 2, but Firm A will earn a profit of 20.
- Now Firm A has no dominant strategy.

**TABLE 13.2 MODIFIED ADVERTISING GAME**

| | | FIRM B | |
|---|---|---|---|
| | | **ADVERTISE** | **DON'T ADVERTISE** |
| **FIRM A** | **ADVERTISE** | 10, 5 | 15, 0 |
| | **DON'T ADVERTISE** | 6, 8 | 20, 2 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.2

# The Nash Equilibrium

- Dominant strategies are stable, but in many games, one or more players do not have a dominant strategy.
- A Nash equilibrium is a set of strategies (or actions) such that each player is doing the best it can given the actions of its opponents.
- Because each player has no incentive to deviate from its Nash strategy, the strategies are stable.
- Note that a dominant strategy equilibrium is a special case of a Nash equilibrium.

| | |
|---|---|
| *Dominant Strategies:* | I'm doing the best I can *no matter what you do*.<br>You're doing the best you can *no matter what I do*. |
| *Nash Equilibrium:* | I'm doing the best I can *given what you are doing*.<br>You're doing the best you can given what I am doing. |

# Product Choice Problem with Multiple Nash Equilibria

- Two breakfast cereal companies face a market in which two new variations of cereal can be successfully introduced
- Each firm has the resources to introduce only one new product.
- Each variation is introduced by only one firm.
- Two Nash equilibria exist.

**TABLE 13.3 PRODUCT CHOICE PROBLEM**

| | | FIRM 2 | |
|---|---|---|---|
| | | **CRISPY** | **SWEET** |
| **FIRM 1** | **CRISPY** | −5, −5 | 10, 10 |
| | **SWEET** | 10, 10 | −5, −5 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.3

# The Beach Location Game

- You(Y) and a competitor(C) plan to sell soft drinks on a beach this summer.
- The beach is 200 yards long, and sunbathers are spread evenly across its length.
- You and your competitor sell the same soft drinks at the same prices, so customers will walk to the closest vendor.
- Where on the beach will you locate, and where do you think your competitor will locate?
- Nash equilibrium
  - the only Nash equilibrium calls for both you and your competitor to locate at the same spot in the center of the beach.
- A variety of similar examples: Presidential candidates typically move close to the center as they define their political positions.

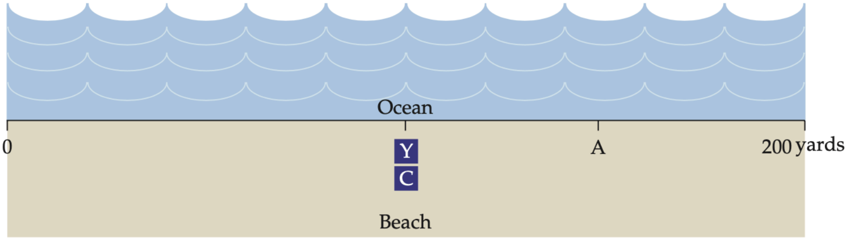


**FIGURE 13.1**
**BEACH LOCATION GAME**

You (*Y*) and a competitor (*C*) plan to sell soft drinks on a beach. If sunbathers are spread evenly across the beach and will walk to the closest vendor, the two of you will locate next to each other at the center of the beach. This is the only Nash equilibrium. If your competitor located at point *A*, you would want to move until you were just to the left, where you could capture three-fourths of all sales. But your competitor would then want to move back to the center, and you would do the same.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 13.1

# Maximin Strategies

- The concept of a Nash equilibrium relies heavily on individual rationality.
- Each player's choice of strategy depends not only on its own rationality, but also on the rationality of its opponent.
- This can be a limitation. What if Firm 1 is unsure about what Firm 2 will do?
- **Maximin strategy** maximizes the minimum gain that can be earned.

**TABLE 13.4 MAXIMIN STRATEGY**

| | | FIRM 2 | |
|---|---|---|---|
| | | DON'T INVEST | INVEST |
| FIRM 1 | DON'T INVEST | 0, 0 | −10, 10 |
| | INVEST | −100, 0 | 20, 10 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.4

# Maximizing the Expected Payoff

- If Firm 1 can assign probabilities to each feasible action for Firm 2,
- It could use a strategy that maximizes its expected payoff.
- Suppose that Firm 1 thinks that there is only a 10-percent chance that Firm 2 will not invest.
- In that case, Firm 1's expected payoff from investing is $(.1)(-100) + (.9)(20) = 8$.
- Its expected payoff if it doesn't invest is $(.1)(0) + (.9)(-10) = -9$.
- In this case, Firm 1 should invest.

# Prisoners' Dilemma

- The ideal outcome is one in which neither prisoner confesses, so that both get two years in prison.
- Confessing, however, is a dominant strategy for each prisoner.
- It yields a higher payoff regardless of the strategy of the other prisoner.
- Dominant strategies are also maximin strategies.

**TABLE 13.5 PRISONERS' DILEMMA**

| | | PRISONER B | |
|---|---|---|---|
| | | CONFESS | DON'T CONFESS |
| PRISONER A | CONFESS | −5, −5 | −1, −10 |
| | DON'T CONFESS | −10, −1 | −2, −2 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.5

# Competition vs Collusion: The Prisoner′s Dilemma

- Why don't firms cooperate without explicitly colluding?
- Why not just set that price and hope your competitor will do the same?
- The problem is that your competitor probably won't choose to set price at the collusive level.
- Because your competitor would do better by choosing a lower price, even if it knew that you were going to set price at the collusive level.
- The prisoners' dilemma illustrates the problem faced by oligopolistic firms.
- Price competition

$$\text{Firm } 1'\text{s demand:} \quad Q_1 = 12 - 2P_1 + P2$$

$$\text{Firm } 2'\text{s demand:} \quad Q_2 = 12 - 2P_2 + P1$$

# Mixed Strategies

- In all of the games that we have examined so far, we have considered strategies in which players make a specific choice or take a specific action.
- Strategies of this kind are called **pure strategies**.
- There are games, however, in which a pure strategy is not the best way to play.
- **Mixed strategies**
  - Players make random choices among two or more possible actions, based on sets of chosen probabilities.
  - Although there is no Nash equilibrium in pure strategies, there is a Nash equilibrium in mixed strategies.

# Matching Pennies

- Each player chooses heads or tails and the two players reveal their coins at the same time.
- If the coins match (i.e., both are heads or both are tails), Player A wins and receives a dollar from Player B.
- If the coins do not match, Player B wins and receives a dollar from Player A.

**TABLE 13.6 MATCHING PENNIES**

| | | PLAYER B | |
|---|---|---|---|
| | | HEADS | TAILS |
| PLAYER A | HEADS | 1, −1 | −1, 1 |
| | TAILS | −1, 1 | 1, −1 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.6

# Some Remarks

- Some games (such as "Matching Pennies" ) do not have any Nash equilibria in pure strategies.
- It can be shown, however, that once we allow for mixed strategies, every game has at least one Nash equilibrium.
- Mixed strategies, therefore, provide solutions to games when pure strategies fail.
- Whether solutions involving mixed strategies are reasonable will depend on the particular game and players.

# The Battle of the Sexes

- Jim and Joan would like to spend Saturday night together.
- Jim would most prefer to go to the opera with Joan, but prefers watching mud wrestling with Joan to going to the opera alone, and similarly for Joan.
- Two Nash equilibria in pure strategies
- An equilibrium in mixed strategies

**TABLE 13.7 THE BATTLE OF THE SEXES**

| | | JIM | |
|---|---|---|---|
| | | **WRESTLING** | **OPERA** |
| **JOAN** | **WRESTLING** | 2, 1 | 0, 0 |
| | **OPERA** | 0, 0 | 1, 2 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.7

Oligopoly: Stackelberg, Bertrand, and Price Competition

Gaming and Strategic Decisions

Repeated Games

# Repeated Games

- Firms often find themselves in a prisoners' dilemma when making output or pricing decisions.
- Can firms find a way out of this dilemma, so that oligopolistic coordination and cooperation (whether explicit or implicit) could prevail?
- Although some prisoners may have only one opportunity in life to confess or not, most firms set output and price over and over again.
- In real life, firms play repeated games.
  - Actions are taken and payoffs received over and over again.
  - Strategies can become more complex.
  - With each repetition of the prisoners′ dilemma, each firm can develop a reputation about its own behavior and can study the behavior of its competitors.

# Tit-for-Tat Strategy

- A player responds in kind to an opponent's previous play, cooperating with cooperative opponents and retaliating against uncooperative ones.
  - I start out and maintain a high price so long as you continue to "cooperate".
  - As soon as you lower your price, I follow suit and lower mine.
  - If you later decide to cooperate and raise your price, I raise my price as well.

**TABLE 13.8 PRICING PROBLEM**

| | | FIRM 2 | |
|---|---|---|---|
| | | LOW PRICE | HIGH PRICE |
| FIRM 1 | LOW PRICE | 10, 10 | 100, −50 |
| | HIGH PRICE | −50, 100 | 50, 50 |

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 13.8

# Infinitely Repeated Game

- Suppose the game is infinitely repeated: my competitor and I repeatedly set prices month after month, forever.
- Cooperative behavior (i.e., charging a high price) is then the rational response to a tit-for-tat strategy.
- The cumulative loss of profits that results must outweigh any short-term gain that accrued during the first month of undercutting.
- With infinite repetition of the game, the expected gains from cooperation will outweigh those from undercutting.
- Thus, it is not rational to undercut.
- A tit-for-tat strategy works best on average against all, or almost all, other strategies.

# Finite Number of Repetitions

- Now suppose the game is repeated a finite number of times, N months.
- Then, the only rational outcome is for both of us to charge a low price every month.
- Why?

# Tit-for-Tat in Practice

- Most of us do not expect to live forever.
- Can the tit-for-tat strategy work in practice?
- It can sometimes work and cooperation can prevail.
  - If the end point of the repeated game is unknown, the unraveling argument that begins with a clear expectation of undercutting in the last month no longer applies.
  - My competitor thinks that perhaps I have not figured out that he will undercut me in the last month.
- Just the possibility can make cooperative behavior a good strategy (until near the end) if the time horizon is long enough.