Lecture 5. Producer Theory

BTM210, KAIST

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Topics Covered in This Lecture

Producer's Problem

Firms and Production Technologies

Production Technologies in the Short Run

Production Technologies in the Long Run

The Cost of Production

Exercises

Understanding Supply Curves

- We've learned how to construct demand curves.
- Consumer's choice $\rightarrow$ Demand curves
  - Solving consumers' utility maximization problems
  - Finding the optimal consumption bundle
  - Diminishing marginal utility $\rightarrow$ downward-sloping demand curves
- A method similar to the one we used when deriving the demand curve.
- Producer's choice $\rightarrow$ Supply curves
  - Solving producers' profit maximization problems
  - Finding the optimal input factors
  - Increasing marginal costs $\rightarrow$ upward-sloping supply curves

Consumer's Problem

- A consumer or household aims to maximize utility by choosing the optimal consumption bundle, given a limited income
- Trade-off
  - Gaining utility (from consuming goods and services)
  - By spending income (to purchase goods and services)
- Modeling preferences using utility functions

- Given income I,
- Choose consumption bundle to maximize the utility function U

$$\max_{F,C} U(F, C)$$

$$s.t. \quad P_F F + P_C C \leq I$$

Lagrangian Multiplier Method (Consumer's Problem)

- Lagrangian

$$\mathcal{L} = U(F, C) + \lambda(I - P_F F - P_C C)$$

- First order conditions

$$\begin{aligned} F: &\quad MU_F - \lambda P_F = 0 \\ C: &\quad MU_C - \lambda P_C = 0 \\ \lambda: &\quad I - P_F F - P_C C = 0 \end{aligned}$$

- Interior solution: Equalizing marginal utility per dollar spent on each good

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C} \quad \text{with} \quad I = P_F F + P_C C$$

Producer's Problem

- A producer or firm aims to maximize profits(Π) by choosing the optimal input factors, capital(K) and labor(L), given the quantity demanded(q)
- Trade-off
  - Earning profits (by producing goods and services)
  - With incurring costs (to employ input factors)
- Use production functions to model production technologies

- Given quantity demanded q or revenue Pq,
- Choose input factors to maximize profits or minimize costs

$$\max_{K,L} \underbrace{\Pi(K,L)}_{\text{Profit}} = \underbrace{Pq}_{\text{Revenue}} - \underbrace{rK}_{\text{Capital cost}} - \underbrace{wL}_{\text{Labor cost}}$$

$$s.t. \quad \underbrace{F(K,L)}_{\text{Production function}} \geq \underbrace{q}_{\text{Quantity demanded}}$$

- Two input factors: Capital(K) and Labor(L)
- Their prices: Rental rate(r) and Wage(w)

Lagrangian Multiplier Method (Producer's Problem)

- Lagrangian

$$\mathcal{L} = Pq - rK - wL + \lambda(F(K, L) - q)$$

- First order conditions

$$\begin{aligned} K: &\quad MP_K - \lambda r = 0 \\ L: &\quad MP_L - \lambda w = 0 \\ \lambda: &\quad F(K, L) - q = 0 \end{aligned}$$

- Interior solution: Equalizing marginal product per dollar spent on each input factor

$$\frac{MP_K}{r} = \frac{MP_L}{w} \quad \text{with} \quad q = F(K, L)$$

Firms Are Producers in the Modern Economy

- Why do firms exist?
  - Firms exist because they allow goods and services to be produced far more efficiently than would be possible without them.
  - Firms offer a means of coordination that would be missing if workers operated independently.
  - A relatively new invention emerged only in the latter part of the 19th century.
- What do firms do?
  - Firms take inputs and turn them into outputs (or goods and services).
  - Inputs or factors of production
    - Capital: land, buildings, equipment, machinery, ...
    - Labor: dayworkers, engineers, managers, ...
    - Intermediate goods or materials: water, electricity, gas, plastic, eggs, ...

Key Idea: Modeling Production Technology

- A production function $F(K, L)$
  - Indicates the highest output(q) that a firm can produce for every specified combination of inputs, capital(K) and labor(L).

$$q = F(K, L)$$

- Note that
  - Production functions describe what is technically feasible when the firm operates efficiently.
  - We can expect that profit-seeking firms will not waste resources.
  - Inputs and outputs are flows.
  - The reference to time matters: short run vs. long run

Production Technologies in the Short Run vs. in the Long Run

- Adjusting inputs takes time for firms.
- In the short run,
  - There is at least one factor that cannot be varied.
  - Such a factor is called a fixed input.
  - Labor is variable, capital is fixed.
- In the long run,
  - Firms can adjust all input factors.
  - Both labor and capital are variable.
- Note that
  - There is no specific time period that separates the short run from the long run.
  - One must distinguish them on a case-by-case basis.

Average and Marginal Product in the Short Run

- Average product of labor (AP_L)
  - The output per unit of labor input
  - The slope of the line drawn from the origin

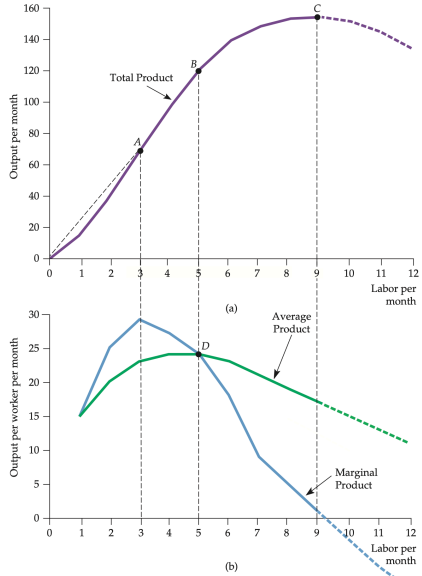
$$AP_L = \frac{\text{Output}}{\text{Labor input}} = \frac{q}{L} = \frac{F(L)}{L}$$

- Marginal Product of labor (MP_L)
  - Additional output produced as the labor input is increased by 1 unit
  - The slope of the tangent line

$$MP_L = \frac{\text{Change in output}}{\text{Change in labor input}} = \frac{\Delta q}{\Delta L} = \frac{dq}{dL} = \frac{dF(L)}{dL}$$

FIGURE 6.1
PRODUCTION WITH ONE VARIABLE INPUT

The total output curve in **(a)** shows the output produced for different amounts of labor input. The average and marginal products in **(b)** can be obtained (using the data in Table 6.1) from the total product curve. At point *A* in **(a),** with 3 units of labor, the marginal product is 29 because the tangent to the total product curve has a slope of 29. The average product of labor, however, is 23, which is the slope of the line from the origin to point *A*. Also, the marginal product of labor reaches its maximum at this point. At point *B*, with 5 units of labor, the marginal product of labor has dropped to 24 and is equal to the average product of labor. Thus, in **(b),** the average and marginal product curves intersect (at point *D*). Note that when the marginal product curve is above the average product, the average product is increasing. When the labor input is greater than 5 units, the marginal product is below the average product, so the average product is falling. Once the labor input exceeds 9 units, the marginal product becomes negative, so that total output falls as more labor is added.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 6.1

Why the Marginal Product Curve to Rise and Fall in the Short Run?

- Based on observations.
- Think of a car manufacturing plant where the amount of equipment or machinery is fixed in the short run.
- Too few workers cannot operate the factory efficiently.
  - MP_L can increase with labor.
- An appropriate number of workers can operate the factory efficiently.
  - MP_L can be at its highest.
- Too many workers can rather interfere with the efficient operation of the factory.
  - MP_L can decrease with labor.
  - The law of diminishing marginal returns

The Law of Diminishing Marginal Returns

- The law of diminishing marginal returns states that
  - As the use of an input increases with other inputs fixed,
  - The resulting additions to output will eventually decrease.
- Diminishing marginal returns results from limitations on the use of other fixed inputs (e.g., machinery)
- Note that
  - Given a production technology, the law of diminishing marginal returns can be applied.
  - Over time, technological improvements may allow the entire total product curve to shift upward, so that more output can be produced with the same inputs.

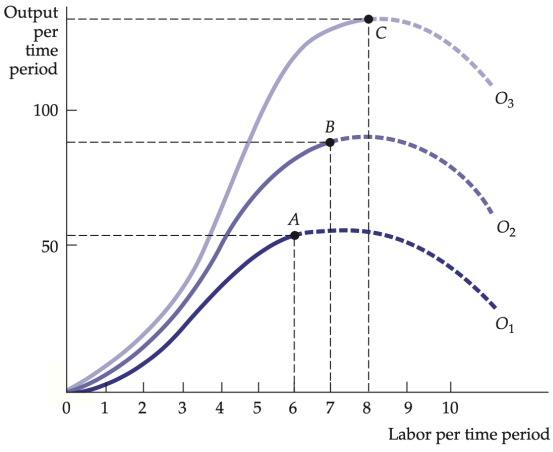


FIGURE 6.2
THE EFFECT OF TECHNOLOGICAL IMPROVEMENT

Labor productivity (output per unit of labor) can increase if there are improvements in technology, even though any given production process exhibits diminishing returns to labor. As we move from point A on curve O_1 to B on curve O_2 to C on curve O_3 over time, labor productivity increases.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 6.2

Production in the Long Run

- In the long run, firms can adjust both labor and capital.
- An "isoquant" is a curve showing all possible combinations of inputs that yield the same output.

TABLE 6.4 PRODUCTION WITH TWO VARIABLE INPUTS

	LABOR INPUT				
CAPITAL INPUT	1	2	3	4	5
1	20	40	55	65	(75)
2	40	60	(75)	85	90
3	55	(75)	90	100	105
4	65	85	100	110	115
5	(75)	90	105	115	120

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Table 6.4

Isoquant

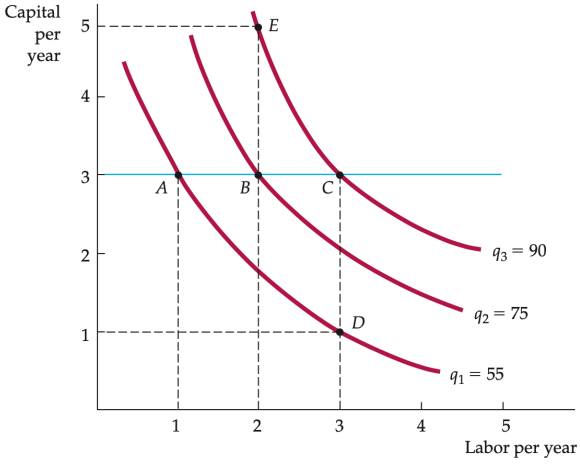


FIGURE 6.5
PRODUCTION WITH TWO VARIABLE INPUTS

Production isoquants show the various combinations of inputs necessary for the firm to produce a given output. A set of isoquants, or *isoquant map*, describes the firm's production function. Output increases as we move from isoquant q_1 (at which 55 units per year are produced at points such as *A* and *D*), to isoquant q_2 (75 units per year at points such as *B*), and to isoquant q_3 (90 units per year at points such as *C* and *E*).

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 6.5

Diminishing Marginal Returns and Diminishing MRTS

- Diminishing marginal returns hold both in the long and short run.
- Adding one factor while holding the other factor constant leads to lower and lower incremental output.
  - The isoquant becomes steeper as more capital is added in place of labor.
  - The isoquant becomes flatter when labor is added in place of capital.
- Marginal rate of technical substitution (MRTS) is
  - The slope of an isoquant indicating how the quantity of one input can be traded off against the quantity of the other, while output is held constant.
  - Convex or diminishing as we move down along an isoquant.
- Diminishing MRTS tells us that the productivity of any one input is limited.

MRS vs. MRTS

- The marginal rate of substitution (MRS) describes how consumers substitute among two goods while holding the level of satisfaction constant.
- The MRS of food(F) for clothing(C) is the maximum amount of clothing(C) that a person is willing to give up to obtain one additional unit of food(F).

$$MRS_{FC} = -\left.\frac{dC}{dF}\right|_{U=\bar{U}} = \frac{MU_F}{MU_C}$$

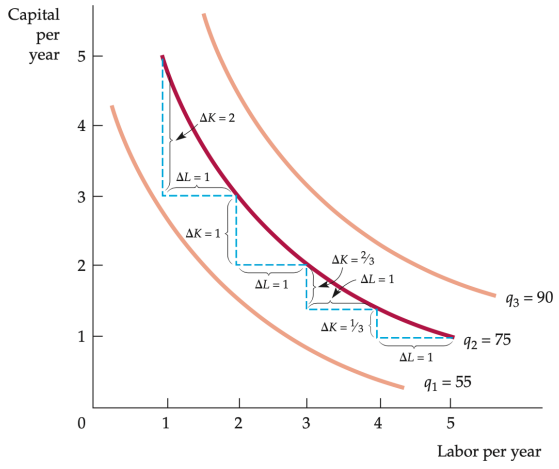
- The marginal rate of technical substitution (MRTS) describes how firms substitute among two input factors while holding the level of production constant.
- The MRTS of labor(L) for capital(K) is the amount of capital(K) that can be reduced when one extra unit of labor(L) is used.

$$MRTS_{LK} = -\left.\frac{dK}{dL}\right|_{q=\bar{q}} = \frac{MP_L}{MP_K}$$

MRTS

FIGURE 6.6
MARGINAL RATE OF TECHNICAL SUBSTITUTION

Like indifference curves, isoquants are downward sloping and convex. The slope of the isoquant at any point measures the marginal rate of technical substitution—the ability of the firm to replace capital with labor while maintaining the same level of output. On isoquant q_2, the MRTS falls from 2 to 1 to 2/3 to 1/3.

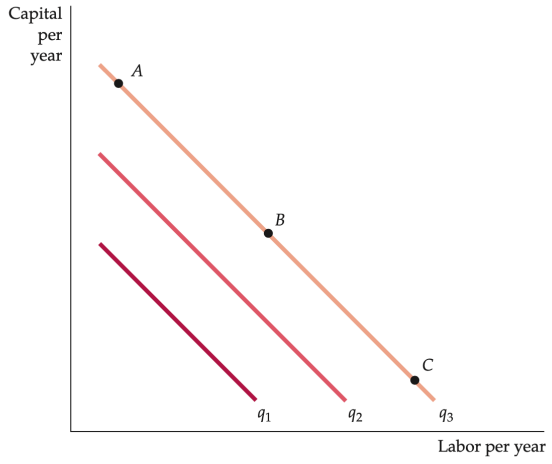


Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 6.6

Linear Production Function: $F(K, L) = K + L$

FIGURE 6.7
ISOQUANTS WHEN INPUTS ARE PERFECT SUBSTITUTES

When the isoquants are straight lines, the MRTS is constant. Thus the rate at which capital and labor can be substituted for each other is the same no matter what level of inputs is being used. Points *A*, *B*, and *C* represent three different capital-labor combinations that generate the same output q_3.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 6.7

Leontief Production Function: $F(K, L) = \min\{K, L\}$

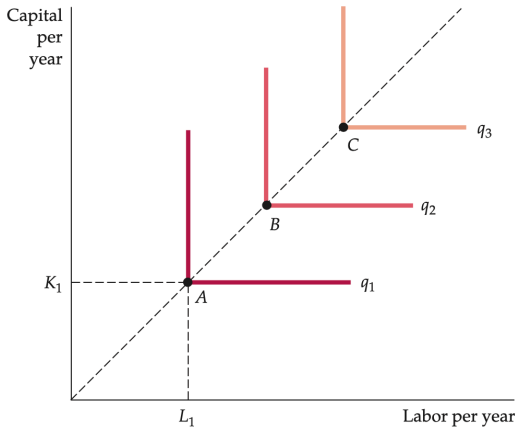


FIGURE 6.8
FIXED-PROPORTIONS PRODUCTION FUNCTION

When the isoquants are L-shaped, only one combination of labor and capital can be used to produce a given output (as at point A on isoquant q_1, point B on isoquant q_2, and point C on isoquant q_3). Adding more labor alone does not increase output, nor does adding more capital alone.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 6.8

Returns to Scale

- Returns to scale is the rate at which output increases as inputs are increased proportionately.
  - $F(\alpha K, \alpha L) > \alpha F(K, L)$: Increasing returns to scale (IRS)
  - $F(\alpha K, \alpha L) = \alpha F(K, L)$: Constant returns to scale (CRS)
  - $F(\alpha K, \alpha L) < \alpha F(K, L)$: Decreasing returns to scale (DRS)
- In the long run, with all inputs variable, a firm considers the best way to increase output.
- One way to do so is to change the scale of the operation by increasing all of the inputs to production in proportion.

Increasing Returns to Scale (IRS)

$$F(\alpha K, \alpha L) > \alpha F(K, L)$$

- IRS might arise because the larger scale of operation allows managers and workers to specialize in their tasks and to make use of more sophisticated, large-scale factories and equipment.
- The manufacturing assembly line is an example of increasing returns.
- Given IRS technology, it is economically advantageous to have one large firm producing (at relatively low cost) rather than to have many small firms (at relatively high cost).
- Markets could not be competitive. Regulations may be required to improve welfare.

Constant and Decreasing Returns to Scale (CRS and DRS)

$$F(\alpha K, \alpha L) = \alpha F(K, L)$$

- With CRS, the size of the firm' s operation does not affect the productivity.
- One plant using a particular production process can easily be replicated.

$$F(\alpha K, \alpha L) < \alpha F(K, L)$$

- With DRS, difficulties in organizing and running a large-scale operation can lead to decreased productivity of both labor and capital.
- Communication between workers and managers can become difficult to monitor as the workplace becomes more impersonal.

The Cost of Production

- A firm can produce a certain output
  - With a lot of labor and very little capital,
  - With very little labor and a lot of capital,
  - Or with some other combination of the two.
- Production technology is the key to understand the cost structure of firms.
  - If a firm is productive, it can produce additional output at a lower cost.
  - The higher the marginal product, the lower the marginal cost.
  - In the short run when the amount of capital is fixed, the marginal cost (MC) of a firm is inversely proportional to its marginal product of labor (MP_L).

$$MC = \frac{w}{MP_L}$$

Which Costs Matter?

- What items should be included as part of a firm's cost?
  - Cost obviously includes the wages that a firm pays its workers and the rent that it pays for office space.
  - But what if the firm already owns an office building and doesn't have to pay rent?
  - How should we treat money that the firm spent two or three years ago and can't recover for equipment or for research and development?
- Economic cost matters.
  - Economic cost is the cost of utilizing resources in production.
  - Different from accounting cost that financial accountants measure.
  - Firms consider future benefits and costs, not past expenditures.
  - Economic cost is, in fact, opportunity cost.

Economic Cost = Opportunity Cost

- Opportunity cost is the cost associated with opportunities that are forgone by not putting the firm's resources to their best alternative use.
- (ex.1) Consider a firm that owns a building and therefore pays no rent for office space.
  - The firm could have earned rent on the office space by leasing it to another company.
  - Leasing the office space would mean putting this resource to an alternative use, a use that would have provided the firm with rental income.
  - This forgone rent is the opportunity cost of utilizing the office space.
  - And because the office space is a resource that the firm is utilizing, this opportunity cost is also an economic cost of doing business.

Opportunity Cost

- (ex.2) Consider an owner that manages her own retail toy store and does not pay herself a salary.
  - Had the toy store owner chosen to work elsewhere, she would have been able to find a job that paid $60,000 per year for essentially the same effort.
  - Thus, the opportunity cost of the time she spends working in her toy store business is $60,000.
- (ex.3) Suppose that she acquired an inventory of toys for which she paid $1 million.
  - After the acquisition, she receives an offer from another toy retailer to acquire her inventory for $1.5 million.
  - Then, the opportunity cost of keeping it is $1.5 million, not the $1.0 million.
  - Because if she keeps the inventory for her own use, she would be sacrificing the $1.5 million that she could have received by selling the inventory to another firm.

Sunk Cost

- Sunk cost is an expenditure that has been made and cannot be recovered.
- After a sunk cost incurred, it should always be ignored when making future economic decisions.
- Because a sunk cost cannot be recovered, it should not influence the firm' s decisions.
- (ex) Imagine you bought a movie ticket for $10.
  - After watching the first 30 minutes, you realize the movie is terrible, and you're not enjoying it at all.
  - Then, the $10 you paid for the ticket is a sunk cost, because it is already spent and cannot be recovered.
  - You should decide whether to stay or leave based only on whether you' ll enjoy the rest of the movie, not because you've already spent the money.

Vocabulary

- Total cost (TC): Total economic cost of production, consisting of fixed and variable costs
- Fixed cost (FC): A cost that does not vary with the level of output and that can be eliminated only by going out of business.
  - Fixed costs affect the firm's decisions looking forward, whereas sunk costs do not.
- Variable cost (VC): A cost that varies as output varies
- Amortization: Policy of treating a one-time expenditure as an annual cost spread out over some number of years
  - Amortizing large capital expenditures and treating them as ongoing fixed costs can simplify the economic analysis of a firm's operation.

Average and Marginal Costs

- Average total cost or average cost (ATC or AC) is the firm's total cost divided by its level of output, TC/q.
- Average fixed cost (AFC) is the fixed cost divided by the level of output, FC/q.
- Average variable cost (AVC) is variable cost divided by the level of output, VC/q.

$$TC = FC + VC \quad \Rightarrow \quad ATC = AFC + AVC$$

- Marginal cost (MC) is the increase in cost for producing one extra unit of output.
  - Fixed cost (FC) does not change as the firm's level of output changes.
  - Thus, marginal cost (MC) is equal to the increase in variable cost (VC) or the increase in total cost (TC) that results from an extra unit of output.

$$\Delta TC = \cancel{\Delta FC} + \Delta VC \quad \Rightarrow \quad MC = \frac{\Delta TC}{\Delta q} = \frac{\Delta VC}{\Delta q}$$

Exercises

1. What is a production function?
   - A production function represents how inputs are transformed into outputs by a firm.
   - It describes the maximum output that a firm can produce.
2. Why does production eventually experience diminishing marginal returns to labor in the short run?
   - MP_L will eventually diminish because there will be at least one fixed factor of production, such as capital.
   - As more and more labor is used along with a fixed amount of capital, there is less and less capital for each worker to use, and the productivity of additional workers necessarily declines.

Exercises

3. You are an employer seeking to fill a vacant position on an assembly line. Are you more concerned with AP_L or MP_L for the last person hired?
   - You should be concerned with the marginal product of the last worker hired, because the marginal product measures the effect on output of hiring another worker.
   - MP_L determines the additional revenue generated by hiring another worker, which should then be compared to the cost of hiring the additional worker.
4. What does "MRTS of labor for capital is 4" mean?
   - Capital can be reduced by 4 units as labor is increased by one unit, and output will remain the same.

Exercises

5. For each of the following examples, draw a representative isoquant. What can you say about the marginal rate of technical substitution in each case?
   - A firm can hire only full-time employees to produce its output, or it can hire some combination of full-time and part-time employees. For each full-time worker let go, the firm must hire an increasing number of temporary employees to maintain the same level of output.
   - A firm finds that it can always trade two units of labor for one unit of capital and still keep output constant.
   - A firm requires exactly two full-time workers to operate each piece of machinery in the factory.

Exercises

6. Do the following functions exhibit increasing, constant, or decreasing returns to scale? What happens to the marginal product of each individual factor as that factor is increased and the other factor held constant?
   - $q = 3L + 2K$
   - $q = (2L + 2K)^{1/2}$
   - $q = L^{1/2}K^{1/2}$
7. Please explain whether the following statements are true or false.
   - If the owner of a business pays himself no salary, then the accounting cost is zero, but the economic cost is positive.
   - A firm that has positive accounting profit does not necessarily have positive economic profit.