Lecture 3. Demand Theory

BTM210, KAIST

Duksang Cho (KDI)

Spring 2025

Topics Covered in This Lecture

Demand Curves

Substitution and Income Effects

Applications: Consumer Surplus and Network Externality

The Method of Lagrange Multipliers

What If the Price of a Good Changes?

- A demand curve shows the relationship between a good's price and the quantity demanded.

$$Q_D = Q_D(P)$$

- How to construct it?
- We assume that aggregate demand curves inherits the characteristics of individual demand curves. (individual demand curves $\rightarrow$ aggregate demand curves)
  - Step 1. A mathematical/Numerical example
  - Step 2. Graphical analysis
  - Step 3. Economic intuition

A Numerical Example

- A consumer maximizes her utility given her budget constraint

$$\max_{C,F\geq 0} C^{1/2}F^{1/2} \quad s.t. \quad P_C C + P_F F = I$$

- Necessary conditions

$$MRS_{FC} = \frac{MU_F}{MU_C} = \frac{P_F}{P_C} \quad \Rightarrow \quad P_F F = P_C C \quad \Rightarrow \quad F^* = \frac{I}{2P_F}$$

- As the price of a good increases($\Delta P_F > 0$), its demand decreases($\Delta F^* < 0$).
- The individual demand curve slopes downward.

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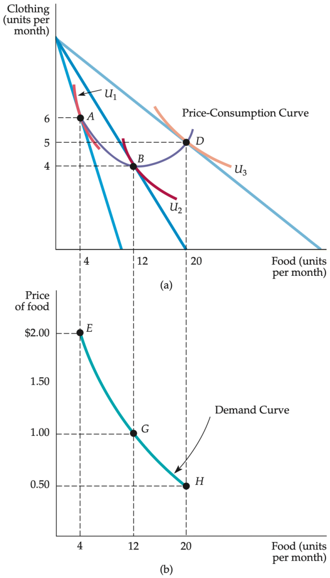
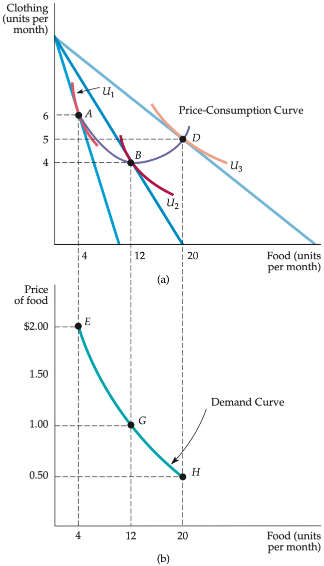


FIGURE 4.1
EFFECT OF PRICE CHANGES

A reduction in the price of food, with income and the price of clothing fixed, causes this consumer to choose a different market basket. In **(a)**, the baskets that maximize utility for various prices of food (point *A*, \$2; *B*, \$1; *D*, \$0.50) trace out the price-consumption curve. Part **(b)** gives the demand curve, which relates the price of food to the quantity demanded. (Points *E*, *G*, and *H* correspond to points *A*, *B*, and *D*, respectively).

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.1 Effect of Price Changes

Graphical Analysis

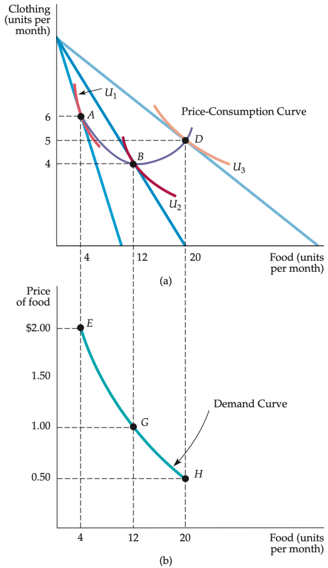


- Top panel: A ”price-consumption curve” traces the utility-maximizing combinations of goods.
- Bottom panel: An ”individual demand curve” relates the quantity of a good that a single consumer will buy to the price of that good.
- As the food price increases($\Delta P_F > 0$)
  - The level of utility falls($U_3 > U_2 > U_1$)
  - The marginal rate of substitution(MRS_{FC}) increases.

$$\frac{P_F}{P_C} = MRS_{FC}$$

  - Assumption: Income and other prices are fixed.

Graphical Analysis



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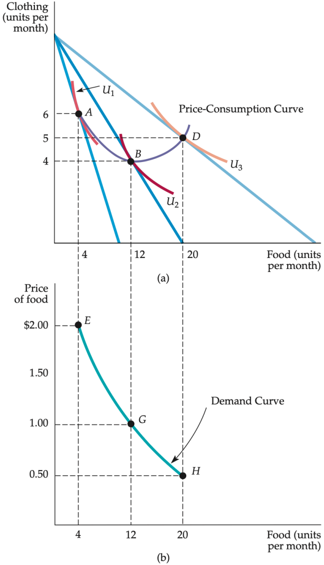

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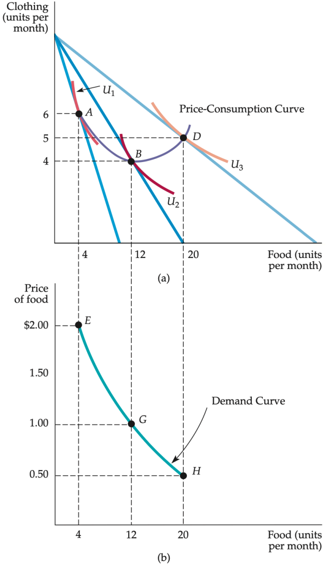

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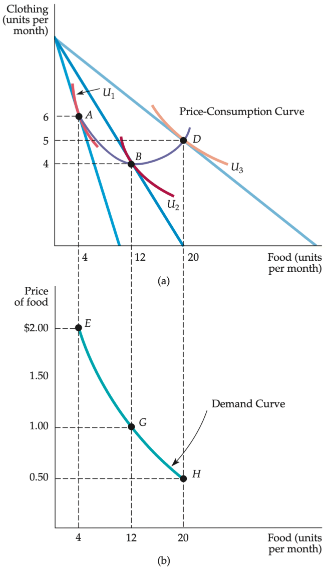

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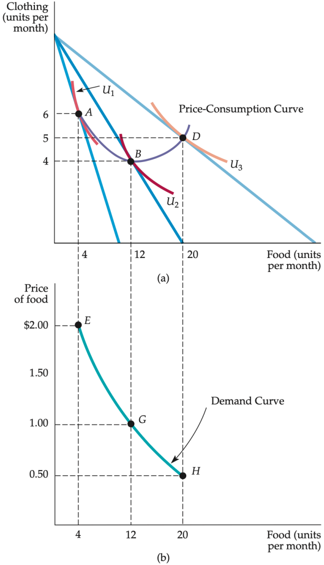

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Economic Intuition

- An individual demand curve tells us how much the consumer would be willing to pay for an additional unit of the good.

$$
\begin{aligned}
\frac{P_F}{P_C} &= MRS_{FC} \\
\therefore P_F &= MRS_{FC} * P_C = \frac{MU_F}{MU_C/P_C} = \frac{MU_F}{\text{Marginal utility of \$1}} \\
&= \text{\$ value of one more unit of F}
\end{aligned}
$$

- $MRS_{FC} * P_C$ is the dollarized value of clothing that a person is willing to give up to obtain an additional unit of food.

Why Are Individual Demand Curves Downward-Sloping?

- A solution to the consumer's constrained optimization problem

$$\frac{P_F}{P_C} = MRS_{FC} = \frac{MU_F}{MU_C}$$

- Both MRS_{FC} and MU_F decrease with F, which implies that F and P_F are negatively associated.

$$\frac{\partial MRS_{FC}}{\partial F} < 0, \quad \frac{\partial MU_F}{\partial F} < 0$$

- Note that this is the assumption of the rational consumer's preference.
  - Diminishing marginal rate of substitution
  - Diminishing marginal utility

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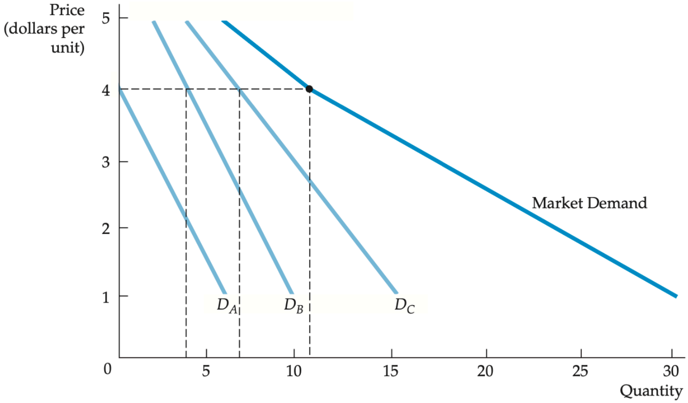
Downward-Sloping Aggregate Demand Curves

- The aggregate(market) demand curve can be obtained by summing all individual demand curves.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.10 Summing to Obtain a Market Demand Curve

What If a Consumer's Income Changes?

- A demand curve shifts to the right as income increases given prices.

$$Q_D = Q_D(P|I)$$

- A numerical example
  - A consumer maximizes her utility given her budget constraint

$$\max_{C,F\geq 0} C^{1/2}F^{1/2} \quad s.t. \quad P_C C + P_F F = I$$

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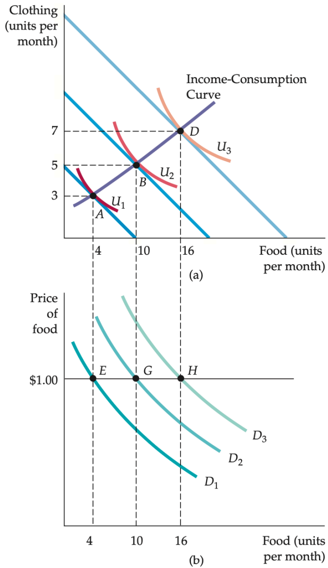
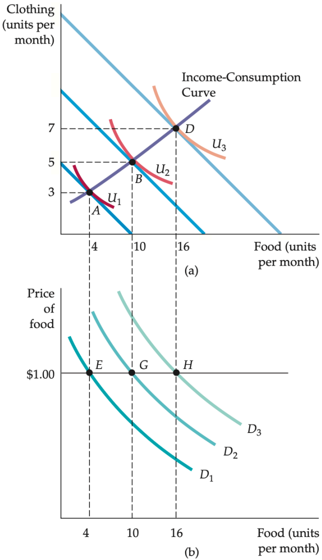


FIGURE 4.2
EFFECT OF INCOME CHANGES

An increase in income, with the prices of all goods fixed, causes consumers to alter their choice of market baskets. In part **(a)**, the baskets that maximize consumer satisfaction for various incomes (point *A*, \$10; *B*, \$20; *D*, \$30) trace out the income-consumption curve. The shift to the right of the demand curve in response to the increases in income is shown in part **(b)**. (Points *E*, *G*, and *H* correspond to points *A*, *B*, and *D*, respectively.)

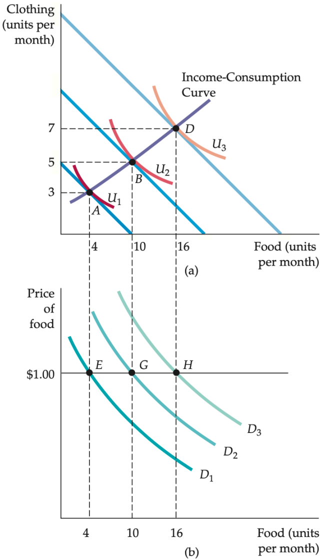
Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.2 Effect of Income Changes

Graphical Analysis



- Top panel: A "income-consumption curve" traces out the utility-maximizing combination bundles associated with every income level.
- Bottom panel: An "individual demand curve" shifts rightward because each demand curve is measured for a particular level of income.
- As the income increases($\Delta I > 0$)
  - The level of utility rises($U_1 < U_2 < U_2$)
  - The budget constraint is relaxed or shifts to the right.
  - Assumption: Prices are fixed.

Graphical Analysis



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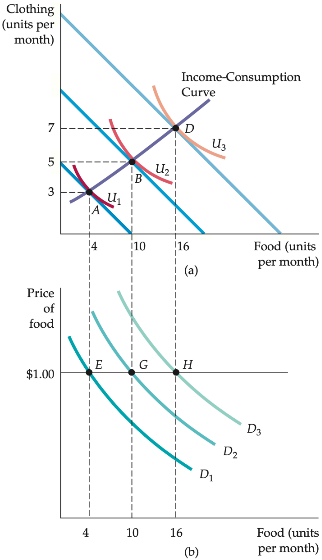

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Income-Quantity Plane: Engel Curves

- Engel curves relate income to the quantity of a good consumed.

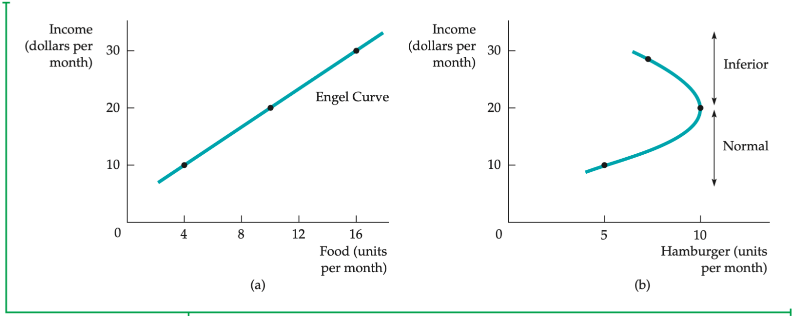


FIGURE 4.4
ENGEL CURVES

Engel curves relate the quantity of a good consumed to income. In **(a)**, food is a normal good and the Engel curve is upward sloping. In **(b)**, however, hamburger is a normal good for income less than $20 per month and an inferior good for income greater than $20 per month.

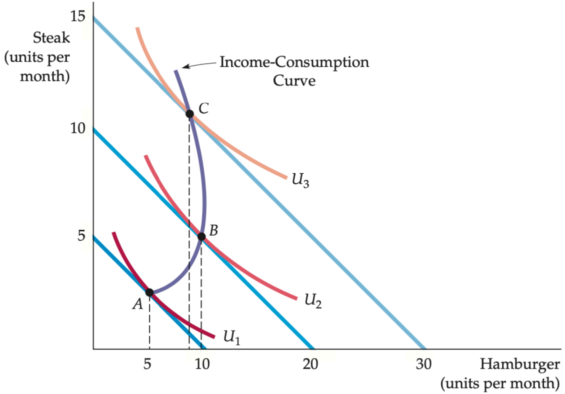
Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.4 Engel Curves

Inferior Goods

- The quantity demanded of an inferior good falls as income rises.

FIGURE 4.3
AN INFERIOR GOOD

An increase in a person's income can lead to less consumption of one of the two goods being purchased. Here, hamburger, though a normal good between *A* and *B*, becomes an inferior good when the income-consumption curve bends backward between *B* and *C*.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.3 An Inferior Good

Decomposing the Effect of a Decrease in the Food Price

$$\Delta P_F < 0 \quad \Rightarrow \quad (\Delta F, \Delta C)?$$

- Consumers will buy more food($\Delta F > 0$) and less clothing($\Delta C < 0$).
  - Because food is cheaper and clothing is relatively more expensive.
- Consumers can purchase more of both food and clothing.
  - Because food is cheaper, they can buy the same consumption bundle for less money.
  - They have money left over for additional purchases, and can buy more.

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Two Channels

- The price effect, which refers to the change in the quantity demanded of a good due to price changes, can be decomposed into (1) the substitution effect and (2) the income effect.

1. Substitution effect
   - The change in the quantity demanded of a good when its relative price changes, while holding the utility level constant.
   - Explains why consumers tend to buy more of a good that has become cheaper and less of a good that has become relatively more expensive.
2. Income effect
   - The change in the quantity demanded of a good due to a change in the consumer's real income (purchasing power), while holding relative prices constant.

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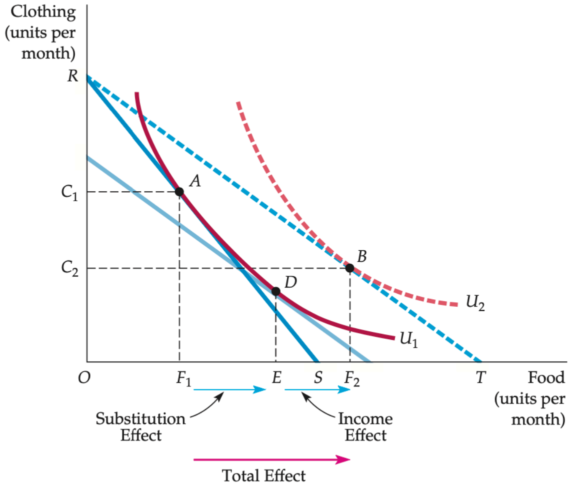
Substitution and Income Effects

FIGURE 4.6
INCOME AND SUBSTITUTION EFFECTS: NORMAL GOOD

A decrease in the price of food has both an income effect and a substitution effect. The consumer is initially at A, on budget line RS. When the price of food falls, consumption increases by F_1F_2 as the consumer moves to B. The substitution effect F_1E (associated with a move from A to D) changes the relative prices of food and clothing but keeps real income (satisfaction) constant. The income effect EF_2 (associated with a move from D to B) keeps relative prices constant but increases purchasing power. Food is a normal good because the income effect EF_2 is positive.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.6 Income and Substitution Effects: Normal Good

Inferior Good: A Good That Has a Negative Income Effect

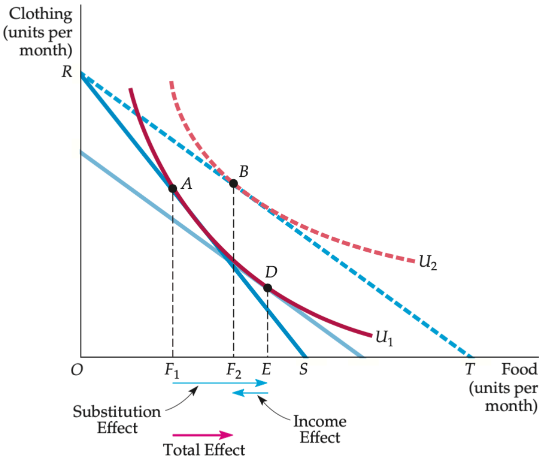


FIGURE 4.7

INCOME AND SUBSTITUTION EFFECTS: INFERIOR GOOD

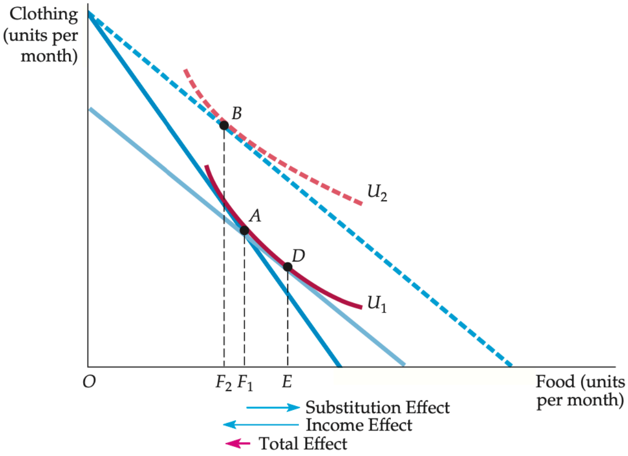
The consumer is initially at A on budget line RS. With a decrease in the price of food, the consumer moves to B. The resulting change in food purchased can be broken down into a substitution effect, F_1E (associated with a move from A to D), and an income effect, EF_2 (associated with a move from D to B). In this case, food is an inferior good because the income effect is negative. However, because the substitution effect exceeds the income effect, the decrease in the price of food leads to an increase in the quantity of food demanded.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.7 Income and Substitution Effects: Inferior Good

The Giffen Good

FIGURE 4.8
UPWARD-SLOPING DEMAND CURVE: THE GIFFEN GOOD

When food is an inferior good, and when the income effect is large enough to dominate the substitution effect, the demand curve will be upward-sloping. The consumer is initially at point *A*, but, after the price of food falls, moves to *B* and consumes less food. Because the income effect EF_2 is larger than the substitution effect F_1E, the decrease in the price of food leads to a lower quantity of food demanded.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.8 Upward-Sloping Demand Curve: The Giffen Good

Demand Curves

Substitution and Income Effects

Applications: Consumer Surplus and Network Externality

The Method of Lagrange Multipliers

Consumer Surplus

- Consumer surplus measures how much better off individuals are.
- Individual consumer surplus
  - Is the difference between the maximum amount that a consumer is willing to pay for a good and the amount that the consumer actually pays.
  - Is calculated as the total benefit from the consumption of a product, less the total cost of purchasing it.
- Aggregate consumer surplus
  - Is the sum of all individual consumer surpluses
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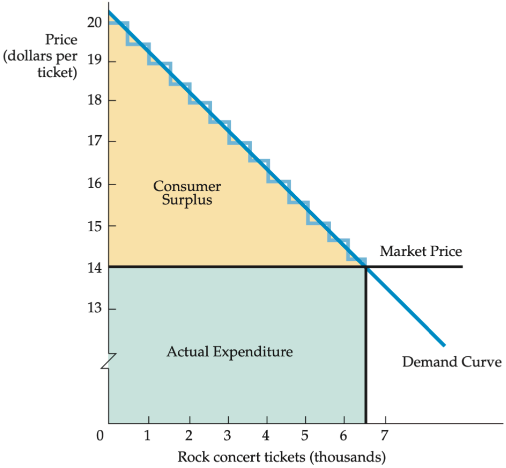

FIGURE 4.15
CONSUMER SURPLUS GENERALIZED

For the market as a whole, consumer surplus is measured by the area under the demand curve and above the line representing the purchase price of the good. Here, the consumer surplus is given by the yellow-shaded triangle and is equal to 1/2 × ($20 − $14) × 6500 = $19,500.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.15 Consumer Surplus Generalized

Network Externality

- Implicit assumptions until now
  - Consumers' demands for a good are independent of one another.
- What if they are not independent?
  - For some goods, one person's demand depends on the demands of other people.
  - We call this situation "network externality."
- Example of network externality
  - Word processing software, SNS apps, congestion
  - Bandwagon effect: Positive network externality in which a consumer wishes to possess a good in part because others do.
  - Snob effect: Negative network externality in which a consumer wishes to own an exclusive or unique good.

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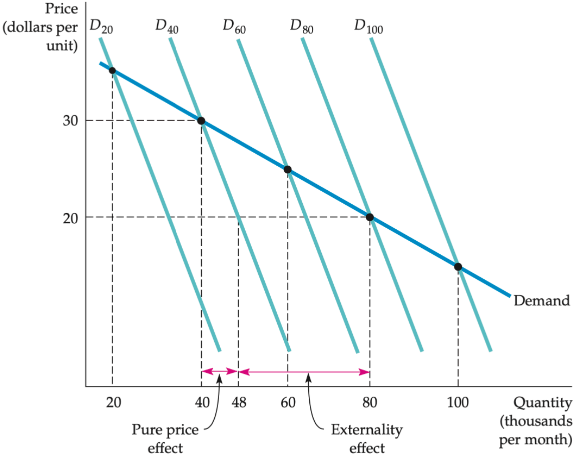
Network Externality

FIGURE 4.17
POSITIVE NETWORK EXTERNALITY

With a positive network externality, the quantity of a good that an individual demands grows in response to the growth of purchases by other individuals. Here, as the price of the product falls from \$30 to \$20, the positive externality causes the demand for the good to shift to the right, from D_{40} to D_{80}.



Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.17 Positive Network Externality

Network Externality

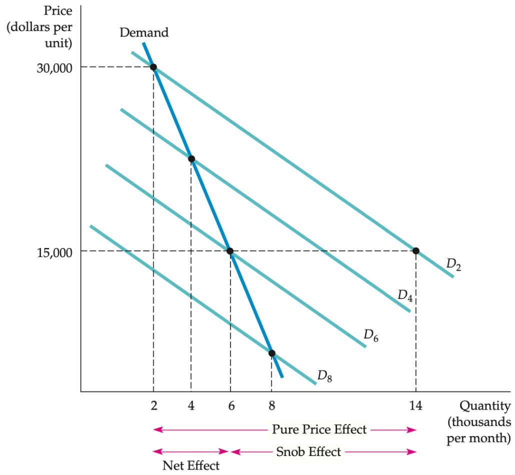


FIGURE 4.18
NEGATIVE NETWORK EXTERNALITY: SNOB EFFECT

The snob effect is a negative network externality in which the quantity of a good that an individual demands falls in response to the growth of purchases by other individuals. Here, as the price falls from \$30,000 to \$15,000 and more people buy the good, the snob effect causes the demand for the good to shift to the left, from D_2 to D_6.

Source: *Microeconomics, 9th ed.* (Pindyck and Rubinfeld, 2018), Figure 4.18 Negative Network Externality

The Method of Lagrange Multipliers

- The method of Lagrange multipliers is a technique that can be used to maximize or minimize a function subject to one or more constraints
- A cookbook approach
  1. State the problem: an objective function and constraints
  $$\max_{x,y} f(x, y) \quad s.t. \quad g(x, y) \geq 0$$
  2. Construct the Lagrangian $(\mathcal{L})$
  $$\mathcal{L}(x, y, \lambda) = f(x, y) + \lambda g(x, y)$$
  3. Compute the first-order conditions $(FOCs)$
  $$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial \mathcal{L}}{\partial y} = \frac{\partial \mathcal{L}}{\partial \lambda} = 0$$
  4. Solve the system of equations consisting of *FOCs* (3 unknown variables, 3 equations)

An Example

- A consumer's utility maximization problem given budget constraints

$$\max_{C,F \geq 0} C^{1/2}F^{1/2} \quad s.t. \quad P_C C + P_F F \leq I$$

- Construct the Lagrangian ($\mathcal{L}$)

$$\mathcal{L} = C^{1/2}F^{1/2} + \lambda(I - P_C C - P_F F)$$

- Compute the first-order conditions ($FOCs$)

$$\frac{1}{2}C^{-1/2}F^{1/2} = \lambda P_C, \quad \frac{1}{2}C^{1/2}F^{-1/2} = \lambda P_F, \quad I = P_C C + P_F F$$

- Solve the system of equations consisting of $FOCs$

$$P_C C = P_F F = \frac{I}{2} \quad \therefore C^* = \frac{I}{2P_C}, \; F^* = \frac{I}{2P_F}$$

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Intuition behind the Example

- The Lagrangian multiplier λ
  - Represents the extra utility generated when the budget constraint is relaxed.
  - Is the extra utility that results from an extra dollar of income.

$$\lambda = \frac{MU_F}{P_F}$$

- The first-oder conditions are equivalent to the following optimization conditions.

$$MRS_{FC} = \frac{P_F}{P_C} \quad \& \quad I = P_C C + P_F F$$

- Economically, the FOCs describe the point
  - On which an indifference curve touches the budget line tangentially
  - Implying the highest utility level the consumer can achieve

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