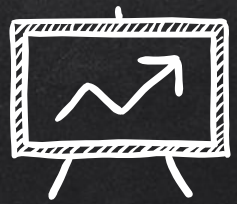


Capital Market

Class 2: Time value of money

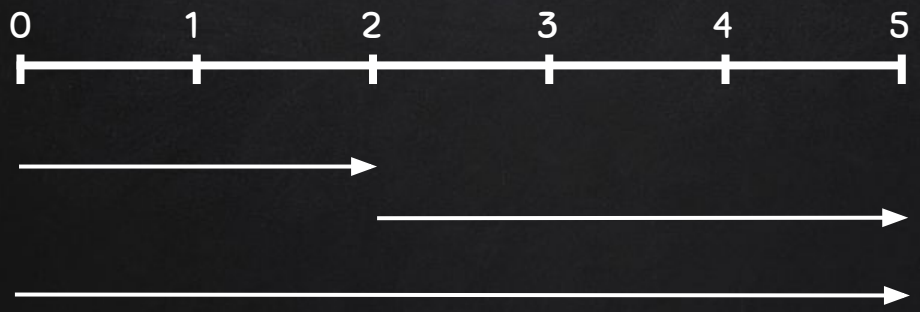
Time value of money

- ✘ The value of money is a function of time
- ✘ Exercise

Time value of money

- ✘ The value of money is a function of time
- ✘ Exercise
- ✘ Most basic form: $V_t = f(t)$

PV = f(0), FV=f(t), or FV = PV * g(t), or PV = FV * g'(t)



Time value of money

- ✘ $FV = PV * (1 + r)^n$

  r: return in each compounding period, n: number of compounding periods

Time value of money

- ✘ $FV = PV * (1 + r)^n$

  r: return in each compounding period, n: number of compounding periods

- ✘ Example 1:

  PV = $100, annual return at 12%, annual compounding, calculate FV at year 2 and 5

$FV_2 = 100 * (1 + 0.12)^2 = 125.44$

$FV_5 = FV_2 * (1 + 0.12)^3 = 176.23$

$FV_5 = 100 * (1 + 0.12)^5 = 176.23$

Time value of money

- ✘ $FV = PV * (1 + r)^n$

  r: return in each compounding period, n: number of compounding periods

- ✘ Example 2:

  PV = $100, annual return at 12%, monthly compounding, calculate FV at year 2 and 5

$FV_2 = 100 * (1 + 0.01)^{24} = 126.97$

$FV_5 = FV_2 * (1 + 0.01)^{36} = 181.67$

$FV_5 = 100 * (1 + 0.01)^{60} = 181.67$

Time value of money

- ✘ $FV = PV * (1 + r/x)^{nx}$

  r: annual return, x: number of compounding periods per year, n: number of years

- ✘ FV = PV * exp(nr) when continuous compounding
- ✘ Example 3:

  PV = $100, annual return at 12%, continuous compounding, calculate FV at year 2 and 5

$FV_2 = 100 * exp(2*0.12) = 127.12$

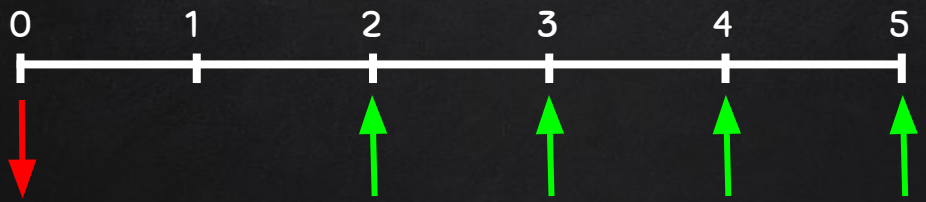
$FV_5 = FV_2 * exp(3*0.12) = 182.21$

$FV_5 = 100 * exp(5*0.12) = 182.21$

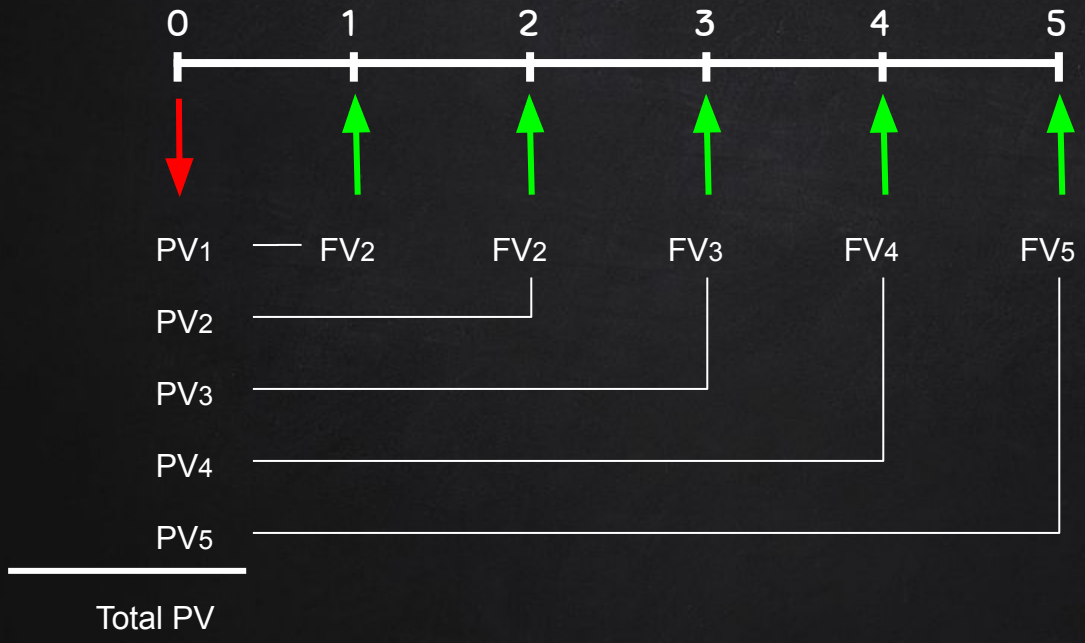
Annuity

- Annuity: An annuity is a contract between you and an insurance company that requires the insurer to make payments to you, either immediately or in the future. You buy an annuity by making either a single payment or a series of payments. Similarly, your payout may come either as one lump-sum payment or as a series of payments over time.

-- investor.org

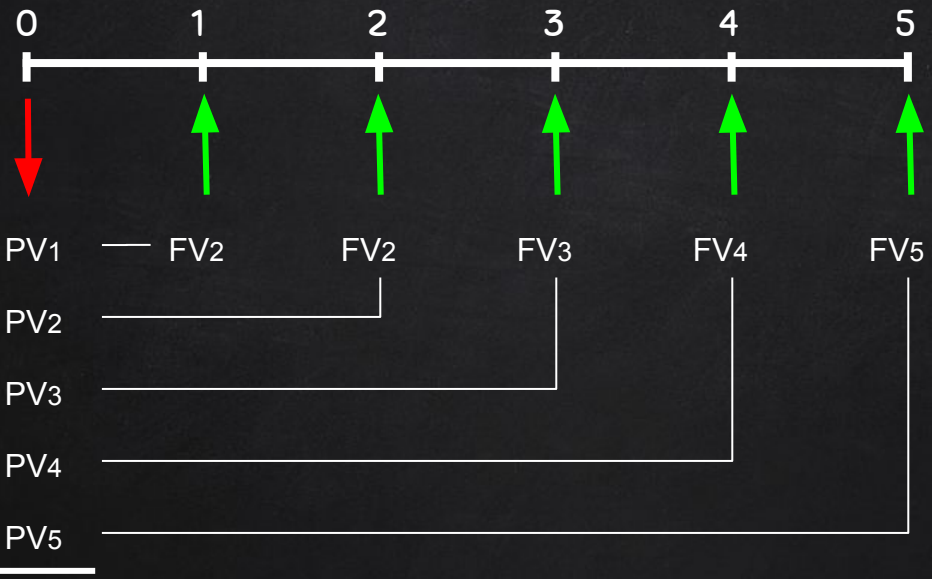


Annuity



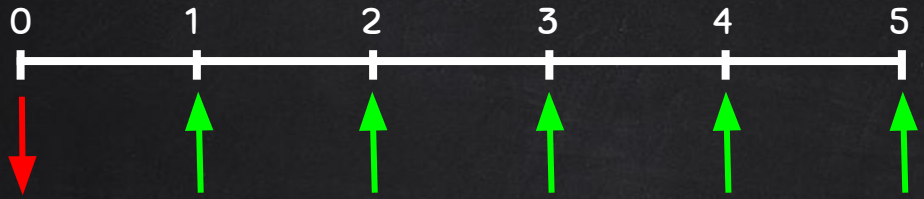
- ✘ Annuity due and ordinary annuity
- ✘ What r should be used?

Annuity



Total PV = Payment * $(1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots)$

Annuity



Total PV = Payment * $(1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \dots)$

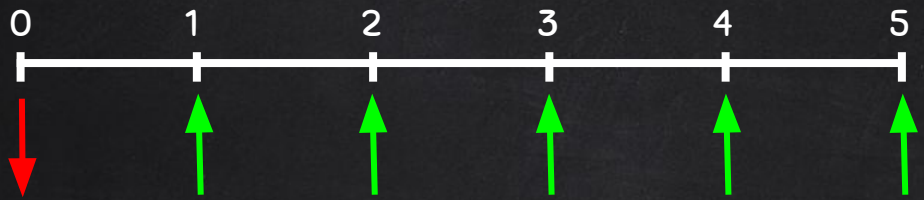
$A = 1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \dots$

$A * (1+r) = 1 + (1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \dots) = 1 + A$

$A = 1/r$

Total PV = Payment / r

Annuity



Total PV = Payment * $(1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \dots)$

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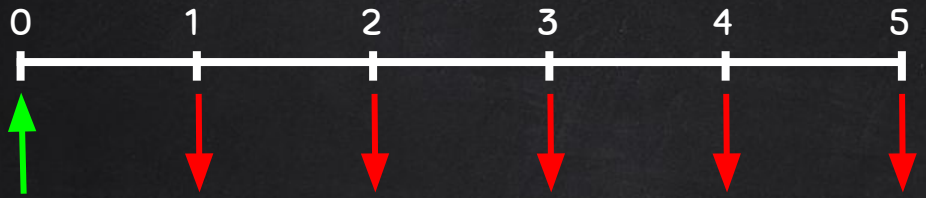
$A * (1+r) = 1 + (1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \dots) = 1 + A$

$A = 1/r$

Total PV = Payment / r

- ✘ Constant payment amount
- ✘ Timing of cash flow
- ✘ What is really r?
- ✘ Can r go negative?

Mortgage



Loan Amount = Monthly Payment * $(1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots + 1/(1+r)^{360})$

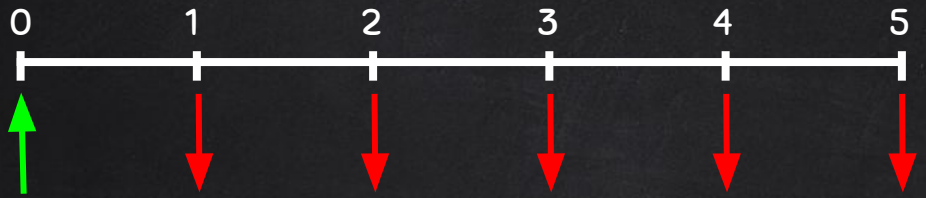
$A = 1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots + 1/(1+r)^{360}$

$A * (1+r) = 1 + (1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots + 1/(1+r)^{359}) = 1 + A - 1/(1+r)^{360}$

$A = (1 - 1/(1+r)^{360})/r$

Loan Amount = Monthly Payment * $(1 - 1/(1+r)^{360})/r$, r = mortgage interest rate / 12

Mortgage



Loan Amount = Monthly Payment * $(1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots + 1/(1+r)^{360})$

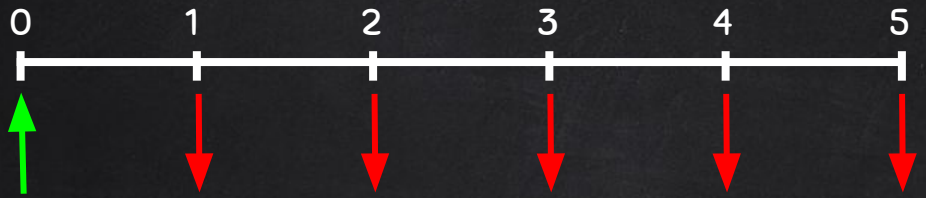
$A = 1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots + 1/(1+r)^{360}$

$A * (1+r) = 1 + (1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots + 1/(1+r)^{359}) = 1 + A - 1/(1+r)^{360}$

$A = (1 - 1/(1+r)^{360})/r$

Loan Amount = Monthly Payment * $(1 - 1/(1+r)^{360})/r$, r = mortgage interest rate / 12
Monthly Payment is only the principal + interest payment, excluding property tax and homeowner insurance
How to solve for the interest rate?

Mortgage



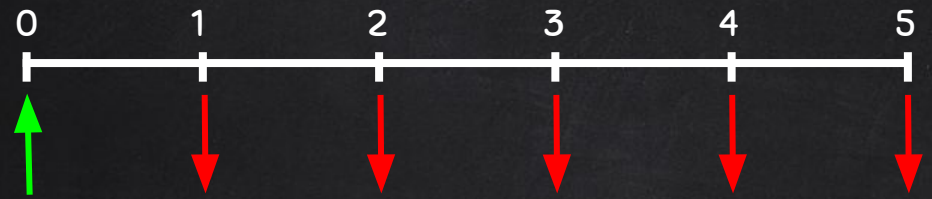
Interest rate is NOT your true borrowing cost, APR is.

APR: Annual percentage rate

APR is the annual cost of a loan to a borrower — including fees. Like an interest rate, the APR is expressed as a percentage. Unlike an interest rate, however, it includes other charges or fees such as mortgage insurance, most closing costs, discount points and loan origination fees.

-- BOA

Mortgage



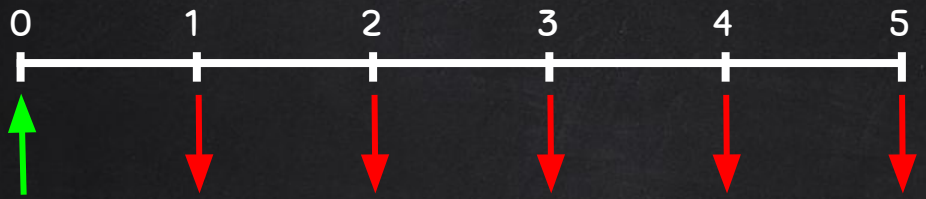
How to calculate your mortgage APR?

Loan Amount = Monthly Payment * $(1/(1+r) + 1/(1+r)^2 + 1/(1+r)^3 + 1/(1+r)^4 + 1/(1+r)^5 + \ldots + 1/(1+r)^{360})$

$$A = (1 - 1/(1+r)^{360})/r$$

1. Loan Amount + origination cost, mortgage interest rate to solve Monthly Payment
2. Loan Amount and Monthly Payment to solve APR

MORTGAGE



Exercise:

1. calc_mth_pmt(borrowing_amt, mortgage_rt, years=30)
2. calc_borrowing_amt(mth_pmt, mortgage_rt, years=30)
3. calc_mortgage_rt(borrowing_amt, mth_pmt, years=30)
4. calc_closing_cost(borrowing_amt, mortgage_rt, apr, years=30)
5. calc_apr(borrowing_amt, mortgage_rt, closing_cost, years=30)