Capital Market

Class 3: Fixed income

Interest, return and yield

- ✘ Interest can be viewed as a type of return, but return (yield) doesn't need to be interest
  - Interest implies lending/borrowing
  - You can earn return (yield) without lending/borrowing

$$r = FV / PV - 1$$

Usually annualized for better idea and comparison

Interest, return and yield

- ✘ Interest can be viewed as a type of return, but return (yield) doesn't need to be interest
- ✘ Exercise, simple return calculation
  - Example 1: saving account $100, 5% interest

$$FV_n = 100 * (1 + 5\%)^n$$

$$100 = FV_n / (1 + r)^n$$

$$r = 5\%$$

Interest, return and yield

- ✘ Interest can be viewed as a type of return, but return (yield) doesn't need to be interest
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  - Example 2: purchased AAPL stock at $100, year 1 end at $105, year 2 end at $81, year 5 end at $200

r1 = 105/100 - 1 = 5%, annualized r = 5%

r2 = 81/100 - 1 = -19%, annualized r = -10%

r5 = 200/100 - 1 = 100%, annualized r = 14.87% (72 rule)

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r2 = 81/100 - 1 = -19%, annualized r = -10%

r5 = 200/100 - 1 = 100%, annualized r = 14.87% (72 rule)

Q in mind: So far we looked at single time point return calculation, but what about a stream of cash inflow and outflows?
A: Discounted cash flow

Interest, return and yield

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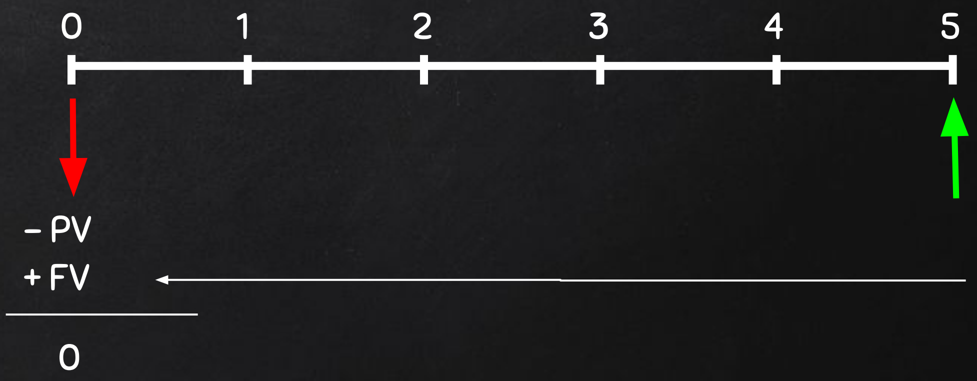
$100 = FV_n / (1 + r)^n$

$r = 5\%$

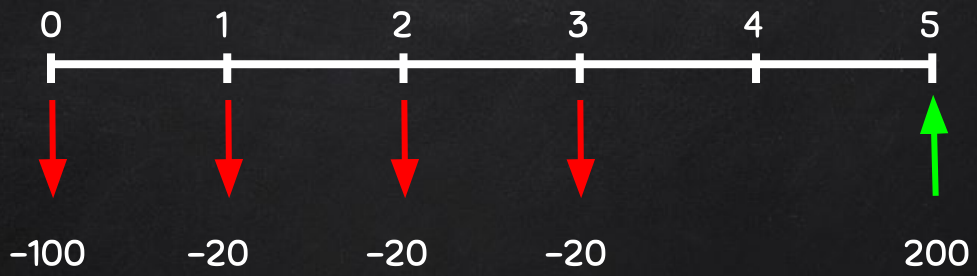


$- PV + FV \text{ (discounted)} = 0$

$- PV + FV_n / (1 + r)^n = 0$

Interest, return and yield

- ✘ Interest can be viewed as a type of return, but return (yield) doesn't need to be interest
- ✘ Exercise, simple return calculation
  - Example 3: made initial business investment of \$100, next 3 years \$20 each, sold business at year 5 for \$200



$$(-100) + (-20)/(1+r) + (-20)/(1+r)^2 + (-20)/(1+r)^3 + 200/(1+r)^5 = 0$$

$$r = 5.35\%$$

Interest, return and yield

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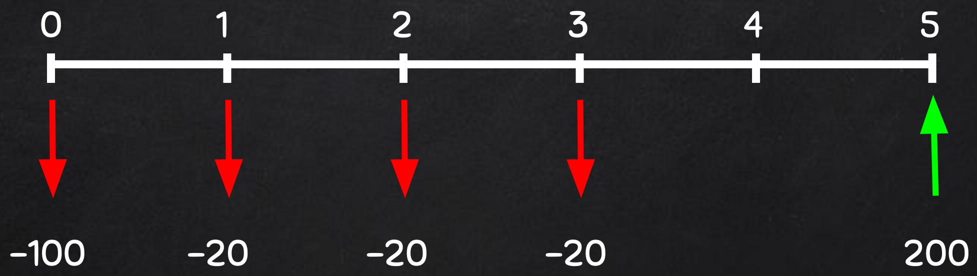

Q in mind: But what about time points in between time windows?

$$(-100) + (-20)/(1+r) + (-20)/(1+r)^2 + (-20)/(1+r)^3 + 200/(1+r)^5 = 0$$

$r = 5.35\%$

Fixed income overview

- Fixed income broadly refers to those types of investment security that pay investors fixed interest or dividend payments until its maturity date. At maturity, investors are repaid the principal amount they had invested.

  -- Investopedia

- In essence, fixed income products are borrowing/lending products
- Fixed amount (coupon/principal payment), fixed schedule (pre-determined dates)

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- Fixed amount (coupon/principal payment), fixed schedule (pre-determined dates)
- Nothing is really fixed ...

Fixed income overview

- ✘ Fixed income securities come in different flavors
- ✘ Who, when, how long, how much, on what, for what, options?
- ✘ By issuer - Treasury, Agency, Municipal, Corporate
- ✘ By tenor - 1M, 2M, 3M, 6M, 1Y (T-bills), 2Y, 5Y, 7Y, 10Y (T-notes), 20Y, 30Y (T-bonds)
- ✘ By underlying asset - Credit, collateral (CDO, ABS, MBS)
- ✘ By embedded options - early redemption is a call option

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Q in mind: There are so many ways to look at the fixed income securities, is there a single metric to evaluate them from investment perspective?
A: A flawed single metric – credit rating (based on assessment of capability, willingness)

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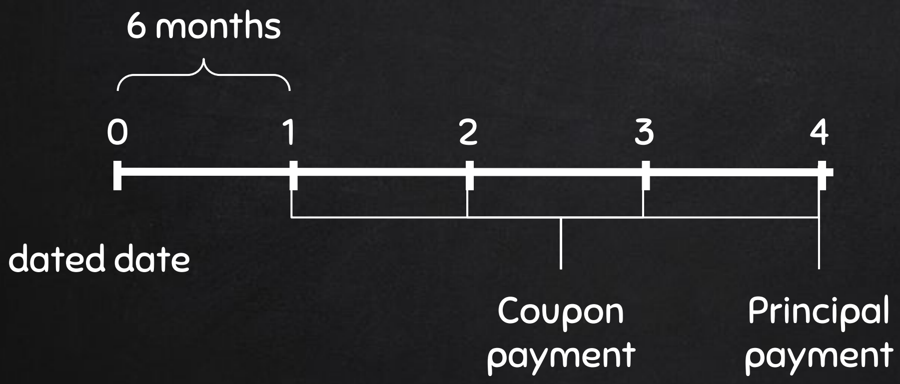
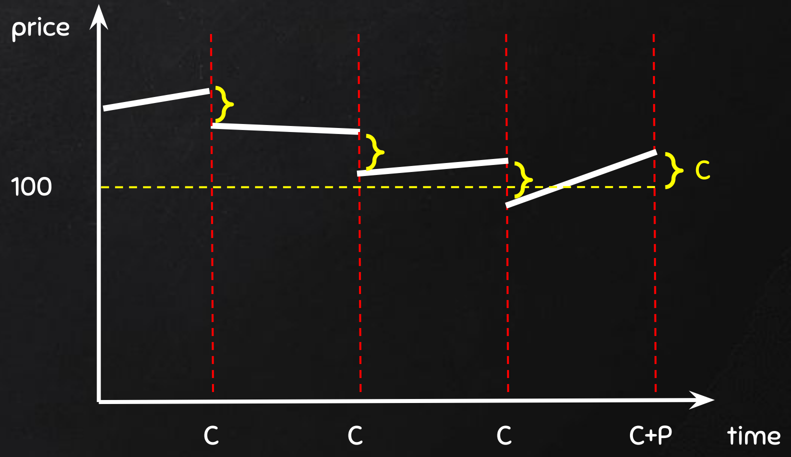
- Fixed amount (coupon/principal payment), fixed schedule (pre-determined dates)
- Nothing is really fixed …
- Amount can be linked to other mechanisms (TIPS, ABS, MBS), schedule can be cut short by early redemption (call option exercise) or even default

Fixed income as an investment

- ✘ Cash flow example: Treasury 2Y note, 1% coupon rate, $1000 notional amount
- ✘ Key dates: auction date, issue date, dated date, maturity date



4 coupon payments of $5, 1 principal payment of $1000



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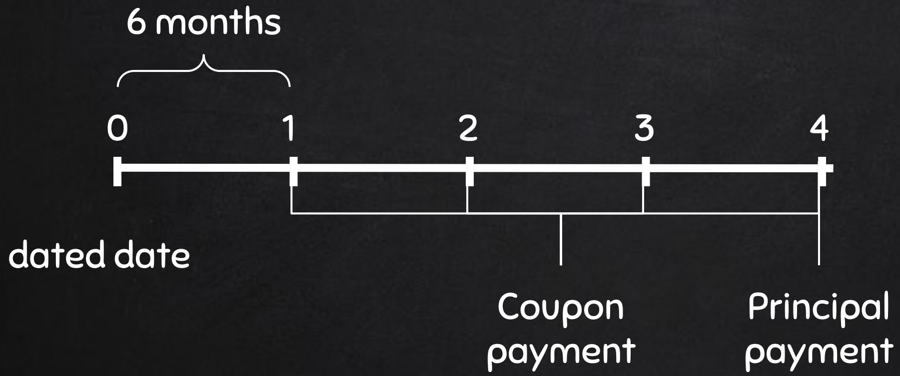
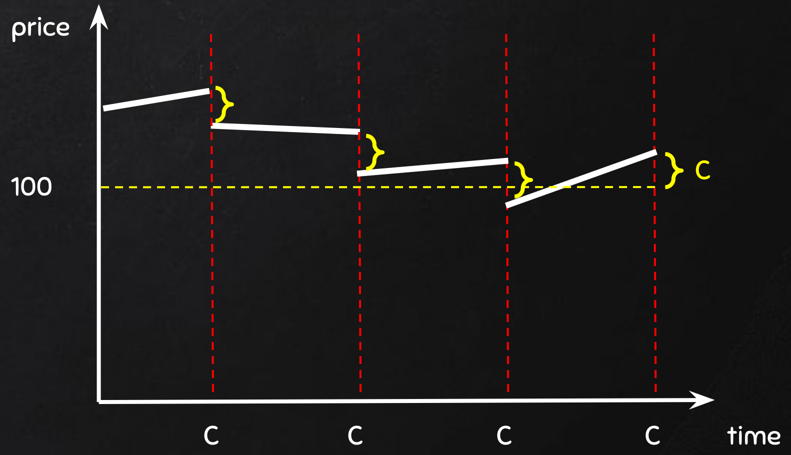


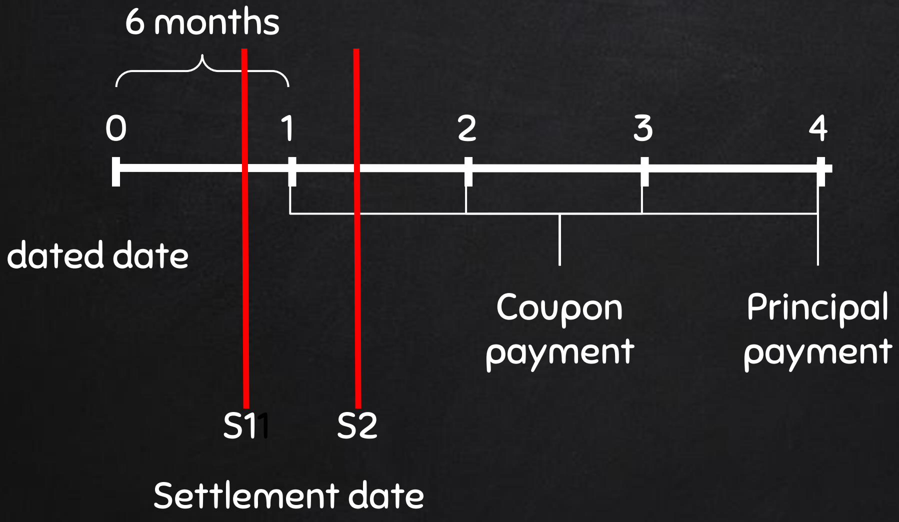
4 coupon payments of $5, 1 principal payment of $1000

Q in mind: what if zero coupon rate?



Fixed income as an investment

- Two prices for Treasuries, clean and dirty
- Dirty price = clean price + accrued interest



Q in mind: if dirty price is the true transaction price, why bother to have clean price?
A: 1. No need to remember C date for fair price comparison; 2. Remove the impact by accrued interest (cash); 3. Dissect the true mechanism of market forces

Fixed income as an investment

Current yield:

annual coupon income / current value = 100 * coupon rate / quoted price

Notional amount/Face value/Par value : the final principal payment amount

Market value = Par value * price / 100

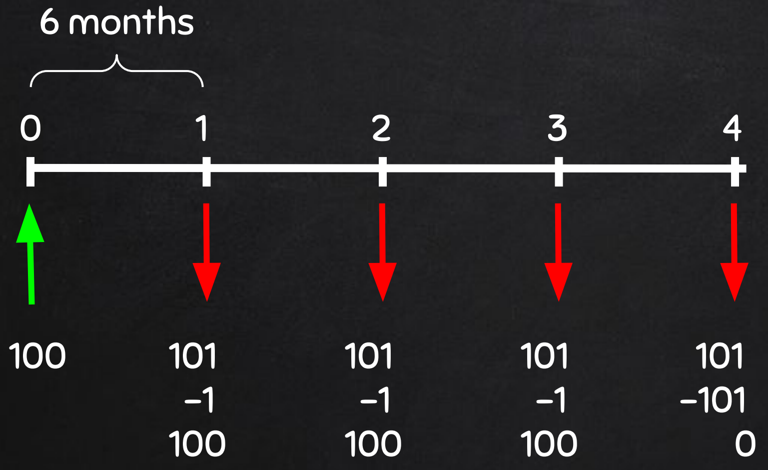
Example: Par value $100K, 2Y-Treasury note, 2% annual coupon rate, bi-annual coupon payment, trading at 98.0

Current yield = 100 * 2% / 98.0 = 2.04%

Fixed income as an investment

- Cash flow again, at par, premium, discount bond

Example: 2Y-Treasury note, 2% annual coupon rate, bi-annual coupon payment



Current yield = 100/price * coupon rate

At par:

price = 100, yield = coupon rate

Premium:

price > 100, yield < coupon rate

Discount:

price < 100, yield > coupon rate

Fixed income as an investment

- ✘ Cash flow again, at par, premium, discount bond

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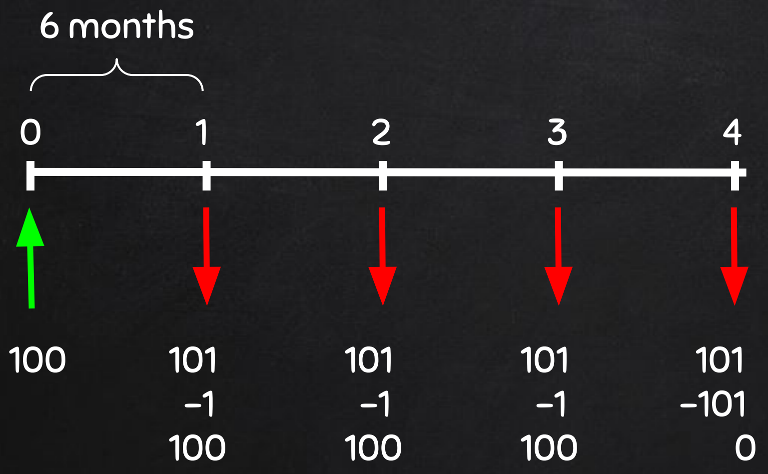


Current yield = 100/price * coupon rate

At par:
price = 100, yield = coupon rate
Premium:
price > 100, yield < coupon rate
Discount:
price < 100, yield > coupon rate

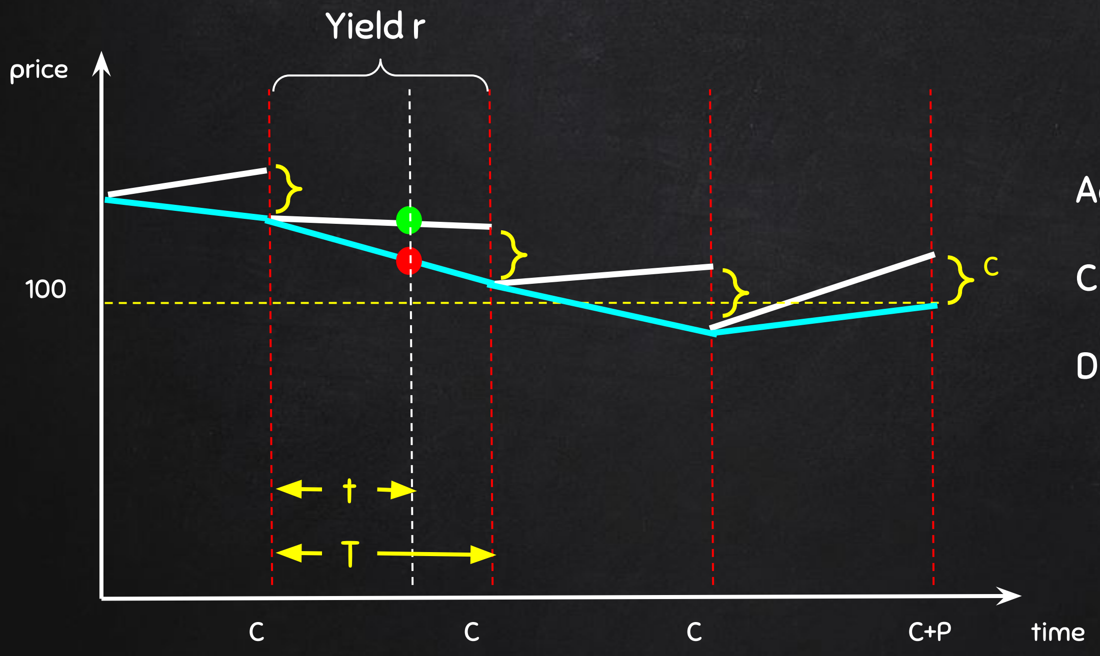
Q in mind: I only received $104, where is the compounding?

Fixed income as an investment

Yield to maturity (YTM) : discounted cash flow to match current market price

Q1 in mind: what market price should be used?
Q2 in mind: again, how to discount back to the time point in between coupon dates?

Fixed income as an investment

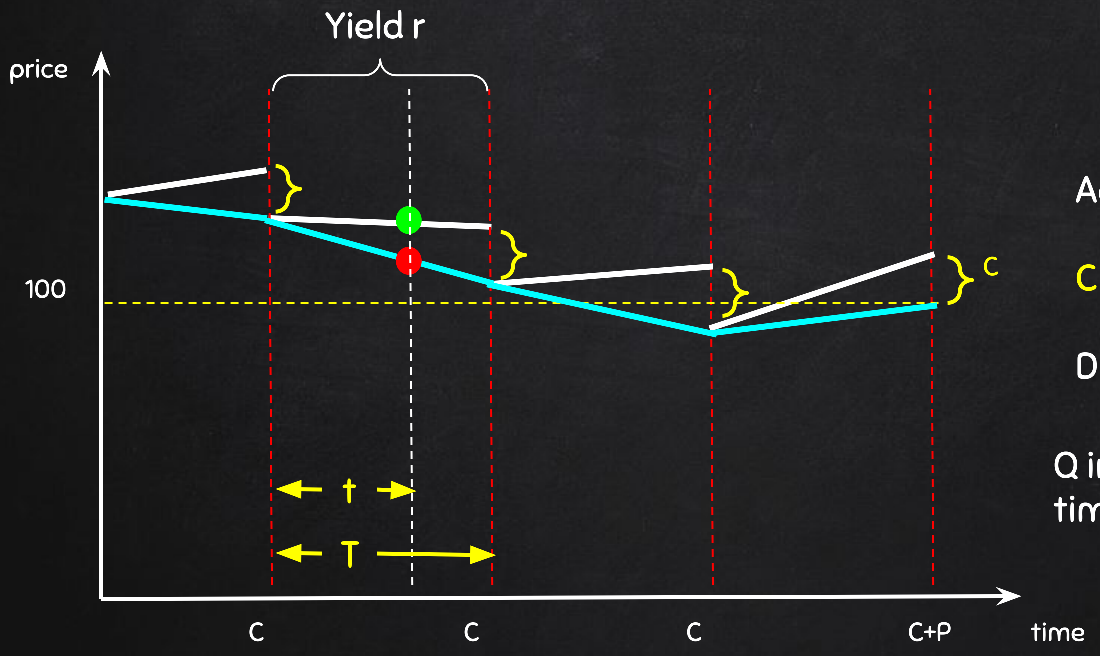


Accrued interest (AI): t/T * C

$CP = C/(1+r) + C/(1+r)^2 + (C+P)/(1+r)^3$

$DP = CP + AI$

Fixed income as an investment



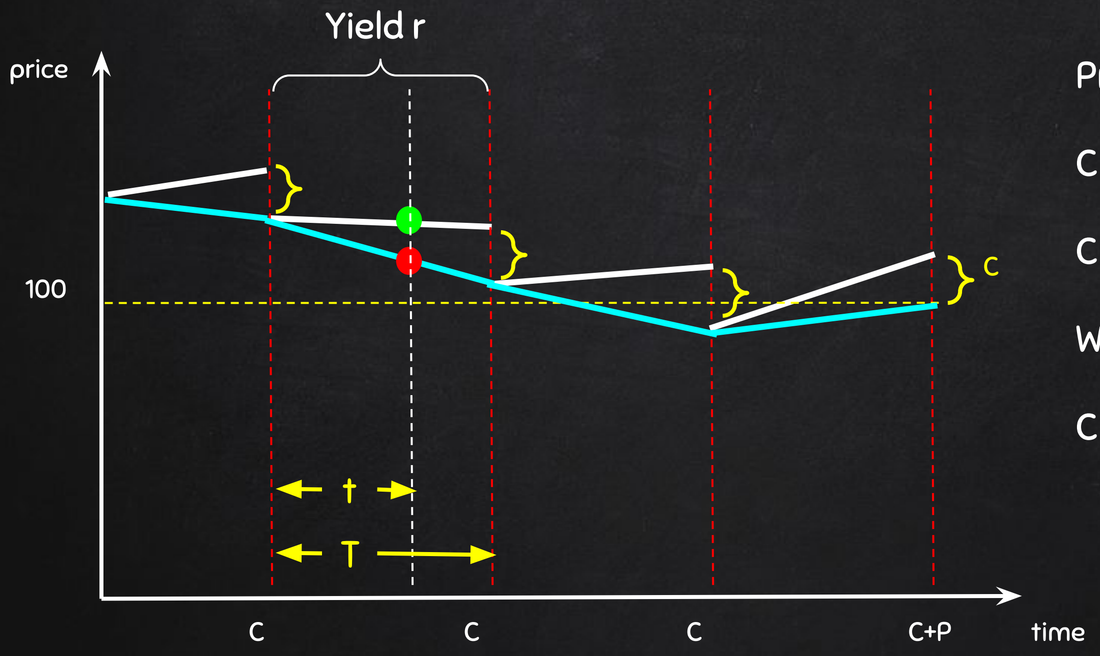
Accrued interest (AI): t/T * C

$CP = C/(1+r) + C/(1+r)^2 + (C+P)/(1+r)^3$

$DP = CP + AI$

Q in mind: Is it fair to treat CP as the starting time point of the compounding window?

Fixed income as an investment



Proposed modification:

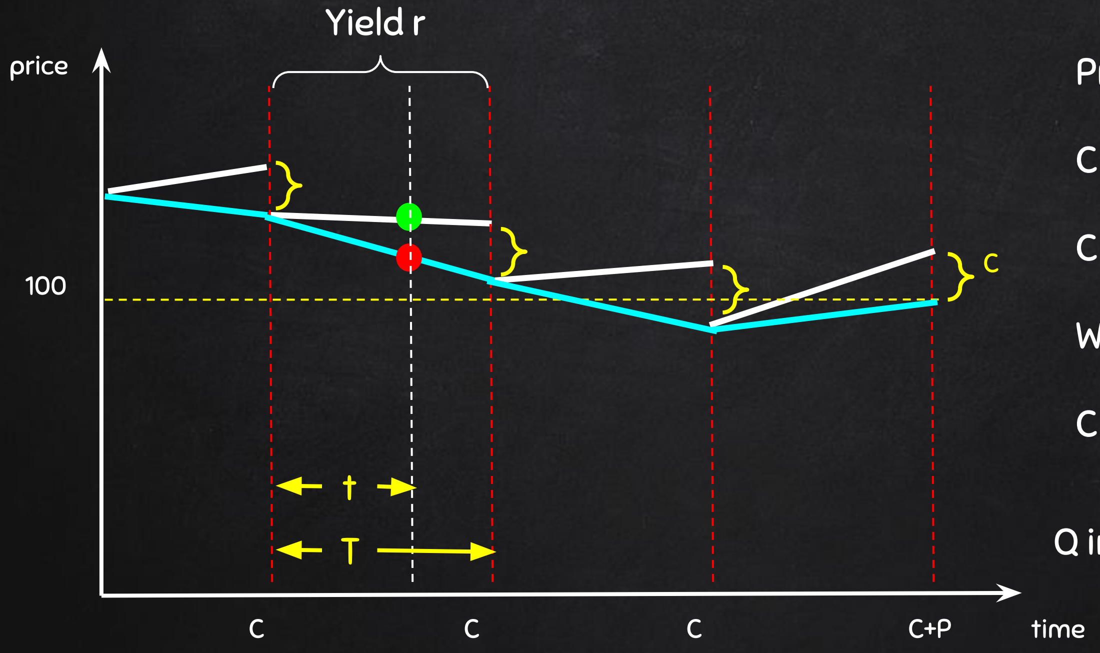
$CP_0 = C/(1+r) + C/(1+r)^2 + (C+P)/(1+r)^3$

$CP_1 = C/(1+r) + (C+P)/(1+r)^2$

Which implies $CP_0 = (C + CP_1) / (1+r)$

$CP_t = CP_0 + (t/T) * (CP_1 - CP_0)$

Fixed income as an investment



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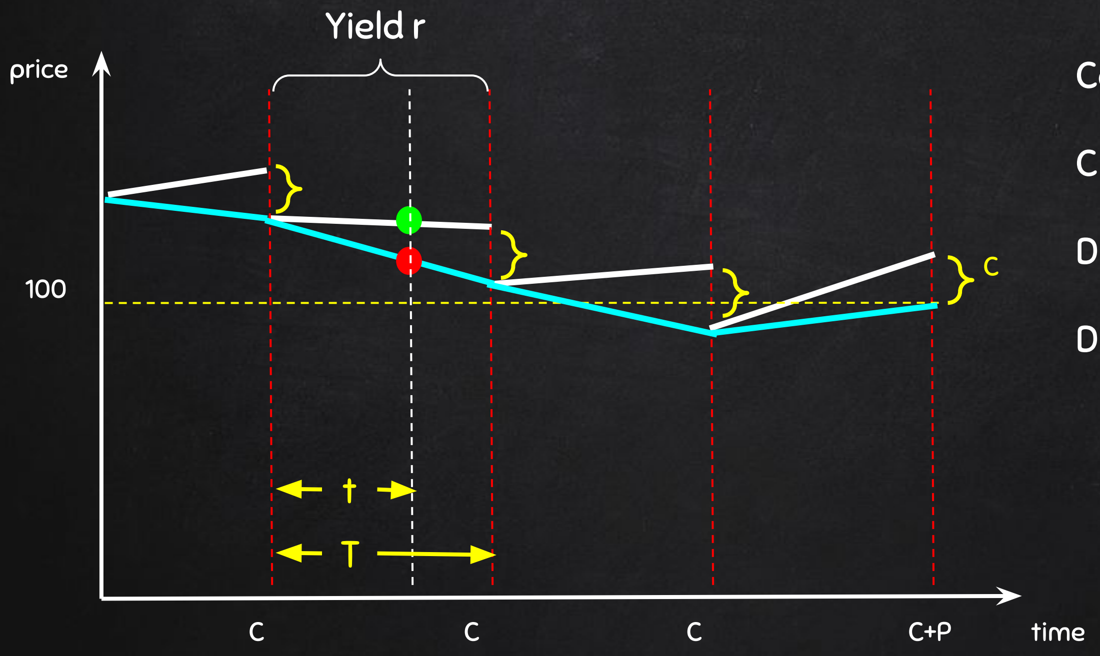
Which implies $CP_0 = (C + CP_1) / (1+r)$

$CP_t = CP_0 + (t/T) * (CP_1 - CP_0)$

Q in mind: Did you say DP gives same r? Why?

Fixed income as an investment



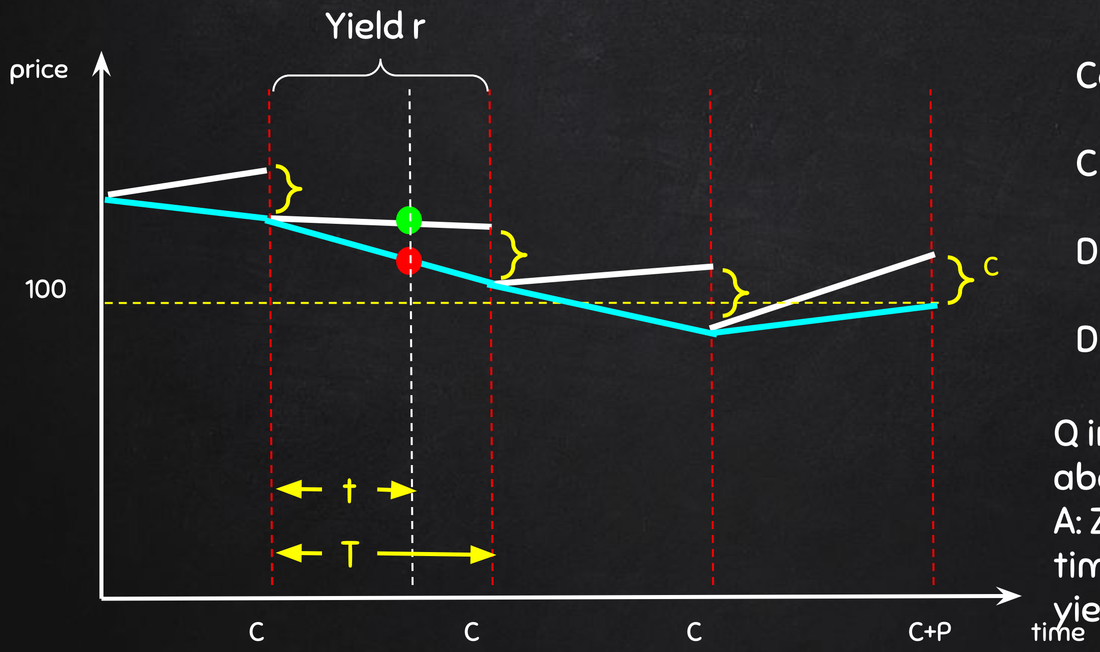
Compare:

$CP_0 = \quad C/(1+r_c) + C/(1+r_c)^2 + (C+P)/(1+r_c)^3$

$DP_0 = C + C/(1+r_d) + C/(1+r_d)^2 + (C+P)/(1+r_d)^3$

$DP_0 = CP_0 + C$

Fixed income as an investment



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$DP_0 = C + C/(1+r_d) + C/(1+r_d)^2 + (C+P)/(1+r_d)^3$

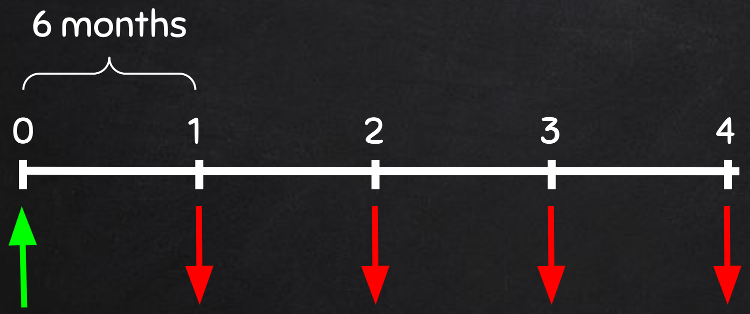
$DP_0 = CP_0 + C$

Q in mind: All seems to make sense, but how about zero-coupon bond?
A: Zero-coupon bond is one time point to one time point discounting situation, the implied yield applies to the time window only.

Fixed income as an investment

- Fixed income sensitivity measure: duration
- Macaulay duration

$$MacD = (PV_1 / PV) * t_1 + (PV_2 / PV) * t_2 + \ldots + (PV_n / PV) * t_n$$



100	1	1	1	101	
	0.5	1	1.5	2	
	0.0050	0.0098	0.0146	1.9412	1.9706

Fixed income as an investment

- Fixed income sensitivity measure: duration
- Macaulay duration

$$MacD = (PV_1 / PV) * t_1 + (PV_2 / PV) * t_2 + \ldots + (PV_n / PV) * t_n$$

- Modified duration

% change of a bond price in response to 100-basis-point (1 percent) change in interest rate

Example: bond price at 130, duration of 10, estimate price due to 20bps rate drop

$$(- 10 * (- 20 / 100) / 100 + 1) * 130 = 132.6$$

Q in mind: Does duration imply linear price change in response to rate change?
A: Duration captures first order rate effect, convexity captures second order rate effect, etc..

Fixed income as an investment

- Treasury curve

Date	1 Mo	2 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr
09/01/20	0.09	0.11	0.12	0.13	0.12	0.13	0.14	0.26	0.46	0.68	1.20	1.43
09/02/20	0.10	0.10	0.12	0.12	0.13	0.14	0.16	0.26	0.45	0.66	1.16	1.38
09/03/20	0.10	0.11	0.11	0.12	0.12	0.13	0.15	0.24	0.43	0.63	1.13	1.34
09/04/20	0.09	0.10	0.11	0.12	0.13	0.14	0.18	0.30	0.50	0.72	1.25	1.46
09/08/20	0.10	0.10	0.13	0.14	0.15	0.14	0.17	0.28	0.47	0.69	1.22	1.43
09/09/20	0.10	0.11	0.12	0.14	0.14	0.14	0.17	0.28	0.48	0.71	1.25	1.45

- How does FED adjust it? Federal fund rate.

Fixed income as an investment

- Credit risk
  - Default risk
  - Downgrade risk
  - Credit spread risk
- Liquidity risk
- Option risk

Fixed income as an investment

- Introducing nominal spread:

  Calculate risky bond yield, r_{risky}

  Calculate benchmark risk-free bond yield, $r_{risk\text{-}free}$

  nominal spread = $r_{risky} - r_{risk\text{-}free}$

Fixed income as an investment

- Forward interest rate

  Implied future interest based today's spot interest rate

- Example:

  Today's saving account interest rate, 1 year @ 2%, 2 years @ 2.3%, 3 years @ 2.5%.

  What does this tell you about 1 year interest rate in 1 year, and in 2 years?

  $(1 + 2.3\%)^2 = (1 + 2\%) * (1 + {}_1f_1)$, ${}_1f_1 = 2.6\%$

  $(1 + 2.5\%)^3 = (1+2.3\%)^2 * (1 + {}_1f_2)$, ${}_1f_2 = 2.9\%$

Fixed income as an investment

- ✘ Major limitation of Treasury curve
  - Each Treasury security is using its own yield to discount, even for same time point
  - Is there such a measure of true discount factor for different future time points?

	0	1	2	3	4	5	6	7	8	9
6 Mo		$1+r_{6\text{-Mo}}$								
1 Yr		$1+r_{1\text{-Yr}}$	$(1+r_{1\text{-Yr}})^2$							
2 Yr		$1+r_{2\text{-Yr}}$	$(1+r_{2\text{-Yr}})^2$	$(1+r_{2\text{-Yr}})^3$	$(1+r_{2\text{-Yr}})^4$					
Spot Curve		$1+f_1$	$(1+f_2)^2$	$(1+f_3)^3$	$(1+f_4)^4$	$(1+f_5)^5$	$(1+f_6)^6$	$(1+f_7)^7$	$(1+f_8)^8$	$(1+f_9)^9$

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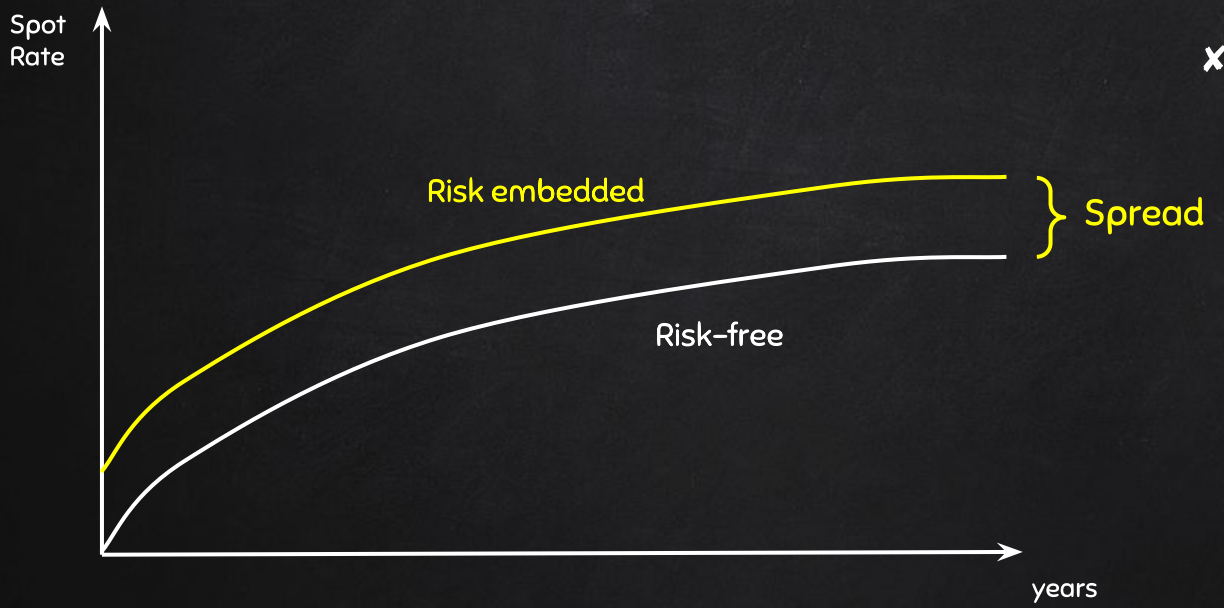
Spot Curve		$1+f_1$	$(1+f_2)^2$	$(1+f_3)^3$	$(1+f_4)^4$	$(1+f_5)^5$	$(1+f_6)^6$	$(1+f_7)^7$	$(1+f_8)^8$	$(1+f_9)^9$

IMPORTANT: When setting up the equation to solve for spot rate, make sure the cash flow is matching the corresponding ON-THE-RUN bond cash flow structure, with C+P at maturity.

Fixed income as an investment



- Introducing Z-spread for risky bond:

  Discount risky bond CF to find PV

  PV > P due to risks

  Introduce spread until PV = P

  This spread is Z-spread

Fixed income as an investment

- Introducing OAS (option adjusted spread):

  So to fairly compare spread across bonds, without the complication of embedded options

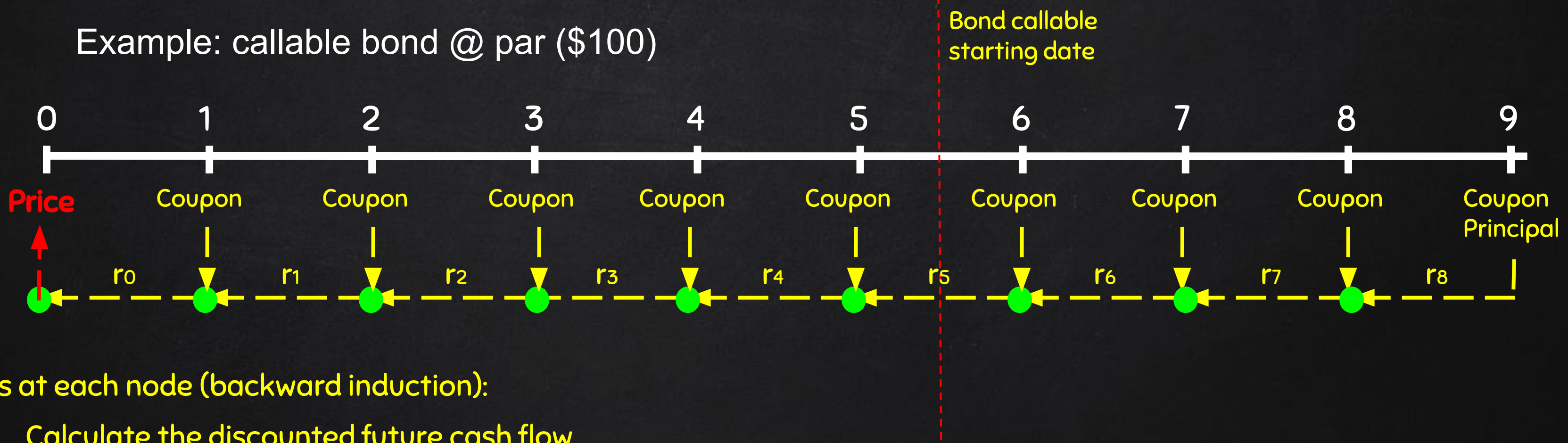
  Challenge: we are introducing both embedded option and volatility at the same time

  Solution: like we design experiments, one change at a time

Fixed income as an investment

- ✘ Re-introducing Z-spread for option-embedded bond

  Example: callable bond @ par ($100)



Steps at each node (backward induction):

1. Calculate the discounted future cash flow.
2. Cap it at the call price ($100) -- Skip this step when before call schedule
3. Keep moving backward

Fixed income as an investment

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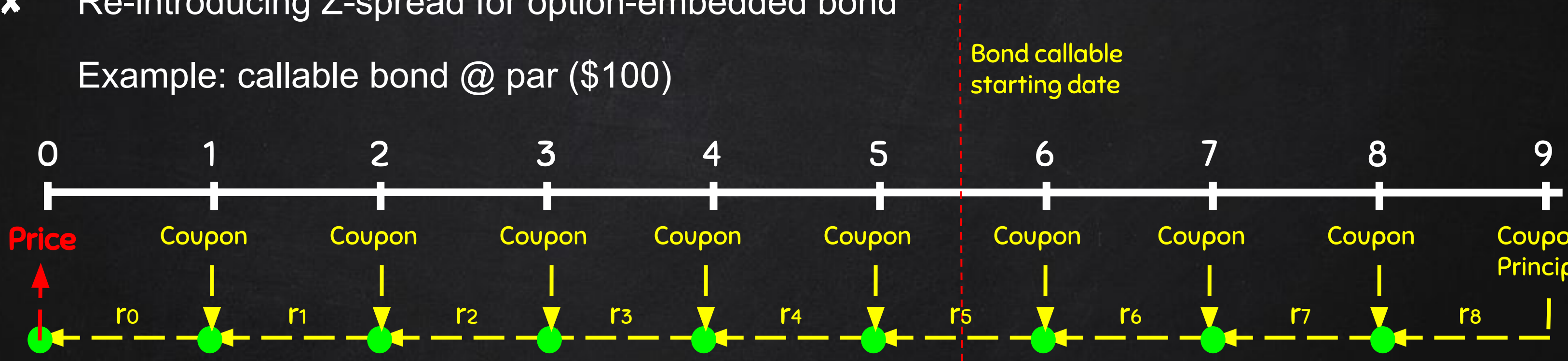

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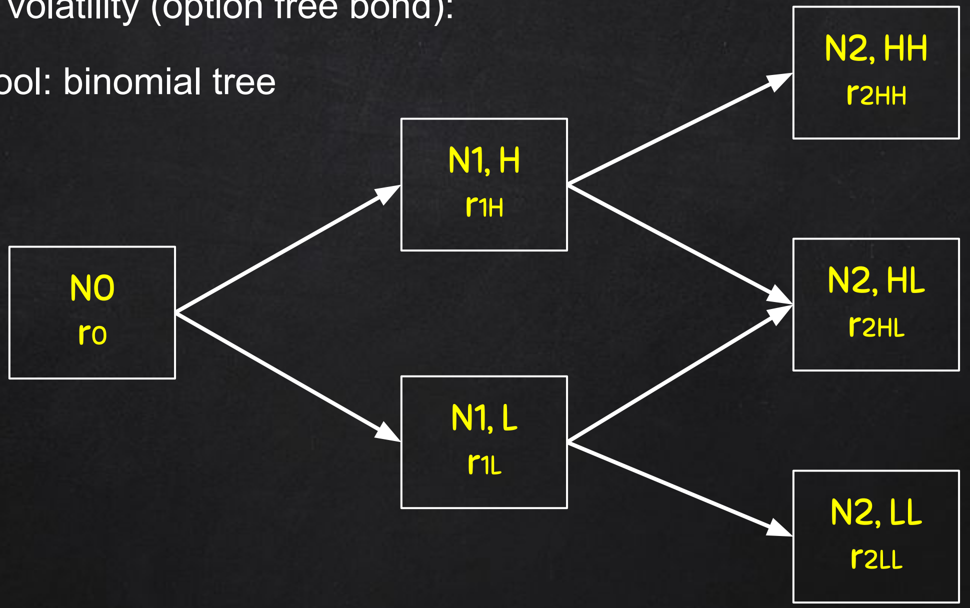
Q in mind: What discount rate to use?

A: The forward interest rate implied by the spot rate curve

Fixed income as an investment

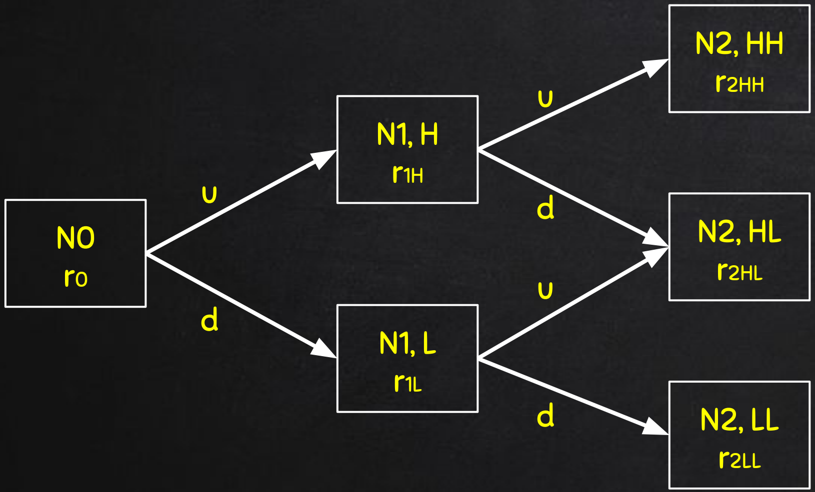
- Introducing volatility (option free bond):

  Analytical tool: binomial tree



Fixed income as an investment

- Binomial tree calibration: volatility and forward rate constraint



Volatility constraint:

$r(u) / r(d)$ = constant, which is dictated by volatility

Forward rate constraint:

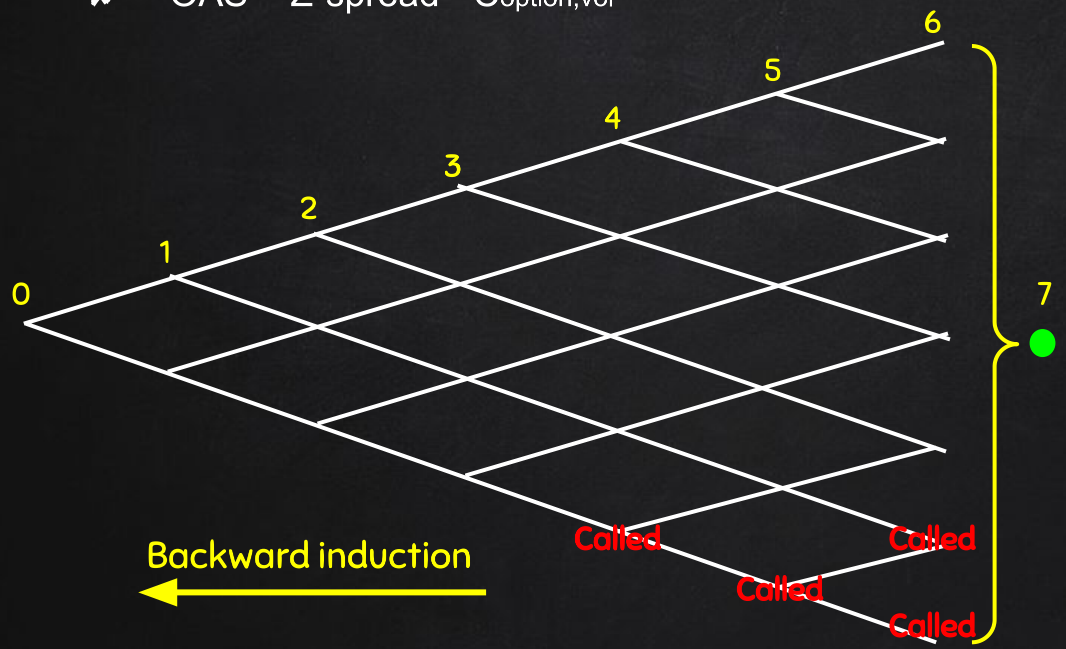
$0.5 * 1/(1+r_{1H}) + 0.5 * 1/(1+r_{1L}) = r_1$

$0.25 * 1/(1+r_{2HH}) + 0.5 * 1/(1+r_{2HL}) + 0.25 * 1/(1+r_{2LL}) = r_2$

r1 and r2 are the corresponding forward rate

Fixed income as an investment

✘ $\text{OAS} = \text{Z-spread} - C_{\text{option,vol}}$



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