

HW2 Hint

It is helpful to realize that  $x$  and  $y$  depend not only on  $t$  but on the four parameters  $p = (\alpha, \beta, \gamma, \delta)$ .

Thus it is reasonable to evolve not only  $u=[x, y]$  with time but also the eight variables in the 2x4 matrix:

$$\frac{\partial u}{\partial p} = \begin{pmatrix} \frac{\partial x}{\partial \alpha} & \frac{\partial x}{\partial \beta} & \frac{\partial x}{\partial \gamma} & \frac{\partial x}{\partial \delta} \\ \frac{\partial y}{\partial \alpha} & \frac{\partial y}{\partial \beta} & \frac{\partial y}{\partial \gamma} & \frac{\partial y}{\partial \delta} \end{pmatrix}.$$

Thus we are evolving 10 variables in total. I'm wondering if it matters if we start these eight variables at 0 at  $t=0$  or not?

Here

$$f(u, p, t) = \begin{pmatrix} \alpha x - \beta xy \\ -\gamma y + \delta xy \end{pmatrix}.$$

You will need the Jacobian of  $f$  with respect to  $x$  and  $y$ :

$$\frac{\partial f}{\partial u} = \begin{pmatrix} \alpha - \beta y & -\beta x \\ \delta y & -\gamma + \delta x \end{pmatrix},$$

and also the Jacobian of  $f$  with respect to  $\alpha, \beta, \gamma, \delta$ :

$$\frac{\partial f}{\partial p} = \begin{pmatrix} x & -xy & 0 & 0 \\ 0 & 0 & -y & xy \end{pmatrix}.$$