HYPOTHESES TESTING WITH DUMMY VARIABLES

Research question

- Teach for America is a federally-funded program to train non-education majors to be teachers through an intensive program after students graduate with a non-teaching degree. They are often placed in low-income schools that experience teacher shortages.

- We are interested in whether the program is effective with regards to teacher performance in the classroom. Do Teach for America fellows generate better student performance than regular teachers?

- We will compare TFA fellows to other teachers with regular education degrees, and we will control for suburban (typically high-income) and urban (typically low-income) school environments.

Group means as a table:

(group means)	Suburban	Urban
Regular Teachers	75	57
Teach for America	75	66

Average math score in each group
measured in percentiles

Raw data:

| | Suburban Schools | | Urban Schools | |
math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

TFA

REG

TFA

REG

10 Teach for America teaching fellows (tfa)
10 regular teachers (reg)

11 teachers in suburban schools (sub)
9 teachers in urban schools (urb)

DV: Math Scores (percentile)

10 Teach for America teaching fellows (tfa) ⟷ 11 teachers in suburban schools (sub)
10 regular teachers (reg)   9 teachers in urban schools (urb)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

DV: Math Scores (percentile)

Average performance
SUBURBAN SCHOOLS:

$$\frac{(11)(75)}{11} = 75$$

Average performance
URBAN SCHOOLS:

$$\frac{(6)(66) + (3)(57)}{9} = 63$$

Kids do better in suburban schools

10 Teach for America teaching fellows (tfa) ↔ 11 teachers in suburban schools (sub)
10 regular teachers (reg) — 9 teachers in urban schools (urb)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

DV: Math Scores (percentile)

Average performance
TFA INSTRUCTORS:

$$\frac{(4)(75) + (6)(66)}{10} = 69.6$$

Average performance
REGULAR TEACHERS:

$$\frac{(7)(75) + (3)(57)}{10} = 69.6$$

Is Teach for America Effective???

No performance differences.

Do we trust these results? Could they be biased?

10 Teach for America teaching fellows (tfa)   11 teachers in suburban schools (sub)
10 regular teachers (reg)   ←——————→   9 teachers in urban schools (urb)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

Average performance
TFA INSTRUCTORS IN SUBURBAN SCHOOLS:
$$\frac{(4)(75)}{4} = 75$$

Average performance
REGULAR TEACHERS IN SUBURBAN SCHOOLS:

$$\frac{(7)(75)}{7} = 75$$

Performance of both teacher types
is identical in suburban schools

DV: Math Scores (percentile)

10 Teach for America teaching fellows (tfa)
10 regular teachers (reg)
⟷
11 teachers in suburban schools (sub)
9 teachers in urban schools (urb)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

↑
DV: Math Scores (percentile)

Teach for America is effective training
for teachers in urban schools.

Average performance
TFA INSTRUCTORS IN SUBURBAN SCHOOLS:
$$\frac{(6)(66)}{6} = 66$$ ←

Average performance
REGULAR TEACHERS IN URBAN SCHOOLS:
$$\frac{(3)(57)}{3} = 57$$ ←

9-point
performance
difference in
urban schools
for teacher
types!

Question:

10 Teach for America teaching fellows (tfa)
10 regular teachers (reg) ⟷ 11 teachers in suburban schools (sub)
9 teachers in urban schools (urb)

math	Suburban Schools		Urban Schools		
	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg	
75	1	0	0	0	TFA
75	1	0	0	0	
75	1	0	0	0	
75	1	0	0	0	
75	0	1	0	0	REG
75	0	1	0	0	
75	0	1	0	0	
75	0	1	0	0	
75	0	1	0	0	
75	0	1	0	0	
75	0	1	0	0	
66	0	0	1	0	TFA
66	0	0	1	0	
66	0	0	1	0	
66	0	0	1	0	
66	0	0	1	0	
66	0	0	1	0	
57	0	0	0	1	REG
57	0	0	0	1	
57	0	0	0	1	

DV: Math Scores (percentile)

Average performance TFA INSTRUCTORS:

$$\frac{(4)(75) + (6)(66)}{10} = 69.6$$

Average performance REGULAR TEACHERS:

$$\frac{(7)(75) + (3)(57)}{10} = 69.6$$

Why do we have no performance difference when comparing teaching programs directly?

10 Teach for America teaching fellows (tfa) ⟷ 11 teachers in suburban schools (sub)
10 regular teachers (reg) 9 teachers in urban schools (urb)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

DV: Math Scores (percentile)

But we find a 9-point difference here?

Average performance
TFA INSTRUCTORS IN SUBURBAN SCHOOLS:
$$\frac{(6)(66)}{6} = 66$$

Average performance
REGULAR TEACHERS IN URBAN SCHOOLS:
$$\frac{(3)(57)}{3} = 57$$

10 Teach for America teaching fellows (tfa)   11 teachers in suburban schools (sub)
10 regular teachers (reg)   ⟷   9 teachers in urban schools (urb)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

Notice the selection process:

Teach for America instructors more likely to teach in URBAN schools (6 out of 10)

Regular instructors are more likely to select SUBURBAN schools (7 out of 10)

DV: Math Scores (percentile)

10 Teach for America teaching fellows (tfa) ⟷ 11 teachers in suburban schools (sub)
10 regular teachers (reg)   9 teachers in urban schools (urb)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

↑

DV: Math Scores (percentile)

Average performance
TFA INSTRUCTORS:

$$\frac{(4)(75) + (6)(66)}{10} = 69.6$$

Average performance
REGULAR TEACHERS:

$$\frac{(7)(75) + (3)(57)}{10} = 69.6$$

So comparing teaching programs without controlling for differences in teaching environments leads to bias.

We incorrectly conclude the TFA training program is **NOT** working when **IT IS**, but just in urban schools.

Specification: Dummy Variable Design Matrix

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	1	0	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
75	0	1	0	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
66	0	0	1	0
57	0	0	0	1
57	0	0	0	1
57	0	0	0	1

Average performance
TFA IN SUBURBAN SCHOOLS:

REGULAR TEACHERS IN SUBURBAN SCHOOLS:

TFA IN URBAN SCHOOLS:

REGULAR TEACHERS IN URBAN SCHOOLS:

↑

DV: Math Scores (percentile)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg	D1+D2+D3+D4	Intercept
75	1	0	0	0	1	1
75	1	0	0	0	1	1
75	1	0	0	0	1	1
75	1	0	0	0	1	1
75	0	1	0	0	1	1
75	0	1	0	0	1	1
75	0	1	0	0	1	1
75	0	1	0	0	1	1
75	0	1	0	0	1	1
75	0	1	0	0	1	1
75	0	1	0	0	1	1
66	0	0	1	0	1	1
66	0	0	1	0	1	1
66	0	0	1	0	1	1
66	0	0	1	0	1	1
66	0	0	1	0	1	1
66	0	0	1	0	1	1
57	0	0	0	1	1	1
57	0	0	0	1	1	1
57	0	0	0	1	1	1

This is a fully-interacted design matrix where there is exactly one dummy variable for each group.

Since the linear combination of all dummy variables would give us a columns of 1's, we cannot run this model with an intercept due to perfect multi-collinearity (you cannot include two identical variables in a model – the statistics program will automatically drop one.

(group means)	Suburban	Urban
Regular Teachers	75	57
Teach for America	75	66

(design matrix)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	**1**	0	0	0
75	0	**1**	0	0
66	0	0	**1**	0
57	0	0	0	**1**

(75) (75) (66) (57)

$$math = b_1 \cdot d.sub.tfa + b_2 \cdot d.sub.reg + b_3 \cdot d.urb.tfa + b_4 \cdot d.urb.reg$$

Each coefficient represents a separate group mean. Note, there is no intercept!

(design matrix)

math	intercept	d.sub	d.reg	d.sub.reg
75	**1**	**1**	0	0
75	**1**	**1**	**1**	**1**
66	**1**	0	0	0
57	**1**	0	**1**	0

(66) (9)   (-9)   (9)

$$math = b_0 + b_1 \cdot d.sub + b_2 \cdot d.reg + b_3 \cdot d.sub.reg$$

The groups are now additive.

$b_0 + b_1 = 66 + 9 = \mathbf{75}$ (suburban TFA)

$b_0 + b_1 + b_2 + b_3 = 66 + 9 - 9 + 9 = \mathbf{75}$ (suburban regular)

$b_0 = \mathbf{66}$ (urban TFA – **reference group**)

$b_0 + b_2 = 66 - 9 = \mathbf{57}$ (urban regular)

(group means)	Suburban	Urban
Regular Teachers	75	57
Teach for America	75	66

No matter which groups you omit, you can always recover the group means. You just multiply all coefficients by the appropriate row in the design matrix.

(design matrix)

math	intercept	d.urb	d.tfa	d.urb.tfa
75	1	0	1	0
75	1	0	0	0
66	1	1	1	1
57	1	1	0	0

$(75)\quad(-18)\qquad(0)\qquad(9)$

$$math = b_0 + b_1 \cdot d.urb + b_2 \cdot d.tfa + b_3 \cdot d.urb.tfa$$

$b_0 + b_2 = 75 + 0 = \mathbf{75}$ (suburban TFA)

$b_0 = \mathbf{75}$ (suburban regular – **reference group**)

$b_0 + b_1 = 75 - 18 + 9 = \mathbf{66}$ (urban TFA)

$b_0 + b_2 = 75 - 18 = \mathbf{57}$ (urban regular)

(design matrix)

math	intercept	d.sub	d.reg	d.sub.reg
75	1	1	0	0
75	1	1	1	1
66	1	0	0	0
57	1	0	1	0

$(66)\quad(9)\qquad(-9)\qquad(9)$

$$math = b_0 + b_1 \cdot d.sub + b_2 \cdot d.reg + b_3 \cdot d.sub.reg$$

$b_0 + b_1 = 66 + 9 = \mathbf{75}$ (suburban TFA)

$b_0 + b_1 + b_2 + b_3 = 66 + 9 - 9 + 9 = \mathbf{75}$ (suburban regular)

$b_0 = \mathbf{66}$ (urban TFA - **reference group**)

$b_0 + b_2 = 66 - 9 = \mathbf{57}$ (urban regular)

Hypothesis-Testing

(group means)	Suburban	Urban
Regular Teachers	75	57
Teach for America	75	66

(design matrix)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	0	1	0	0
66	0	0	1	0
57	0	0	0	1

$$(75) \qquad (75) \qquad (66) \qquad (57)$$
$$\text{math} = b_1 \cdot \text{d.sub.tfa} + b_2 \cdot \text{d.sub.reg} + b_3 \cdot \text{d.urb.tfa} + b_4 \cdot \text{d.urb.reg}$$

If this is the most intuitive way to get group means, why don't
we run this regression model?

(group means)	Suburban	Urban
Regular Teachers	75	57
Teach for America	75	66

(design matrix)

math	d.sub.tfa	d.sub.reg	d.urb.tfa	d.urb.reg
75	1	0	0	0
75	0	1	0	0
66	0	0	1	0
57	0	0	0	1

test: 75=0 ?   test: 57=0 ?

(75)   (75)   (66)   (57)

$$math = b_1 \cdot d.sub.tfa + b_2 \cdot d.sub.reg + b_3 \cdot d.urb.tfa + b_4 \cdot d.urb.reg$$

test: 75=0 ?   test: 66=0 ?
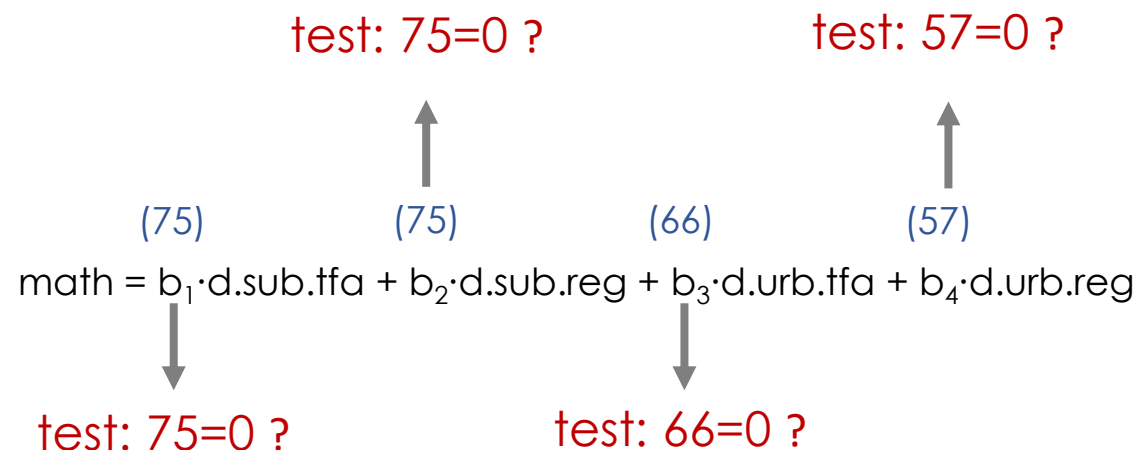
Our research question is whether teachers trained in the Teach for America (TFA) program perform better than teachers trained in the regular program? None of the tests for significance of coefficients b_0 to b_3 in this regression reflect meaningful tests. We already know the group means will not be zero!

It makes it easier to calculate group means, but makes it impossible to answer our research question based upon regression results.

ALL POSSIBLE TESTS (CONTRASTS)

(group means)	Suburban	Urban
Regular Teachers	A	B
Teach for America	C	D
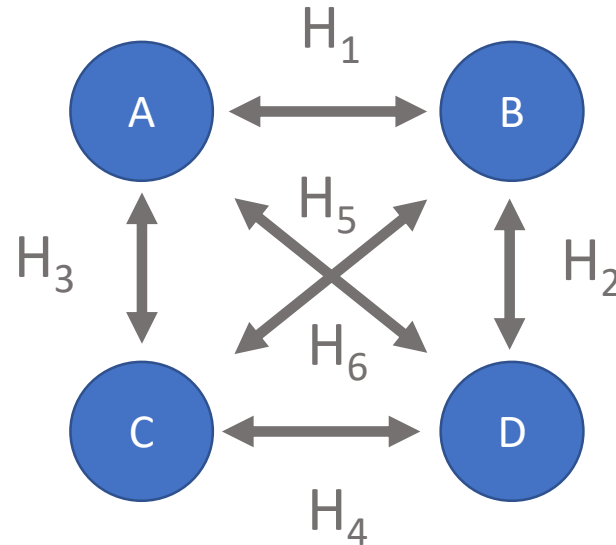
H_1 : A = B ? Do regular teachers perform differently in urban and suburban schools?

H_2 : B = D ? Do regular and TFA teachers perform different in urban schools?

H_3 : A = C ? Do regular and TFA teachers perform different in suburban schools?

etc…

(group means)	Suburban	Urban
Regular Teachers	75	57
Teach for America	75	66

(66) **(9)** **(-9)** **(9)**

$math = b_0 + b_1 \cdot d.sub + b_2 \cdot d.reg + b_3 \cdot d.sub.reg$

The groups are now additive.

(design matrix)

math	intercept	d.sub	d.reg	d.sub.reg
75	1	1	0	0
75	1	1	1	1
66	1	0	0	0
57	1	0	1	0

$b_0 + b_1 = 66 + 9 = \mathbf{75}$ (suburban TFA)

$b_0 + b_1 + b_2 + b_3 = 66 + 9 - 9 + 9 = \mathbf{75}$ (suburban regular)

$b_0 = \mathbf{66}$ (urban TFA – **reference group**)

$b_0 + b_2 = 66 - 9 = \mathbf{57}$ (urban regular)

$b_0 = b_0 + b_1$
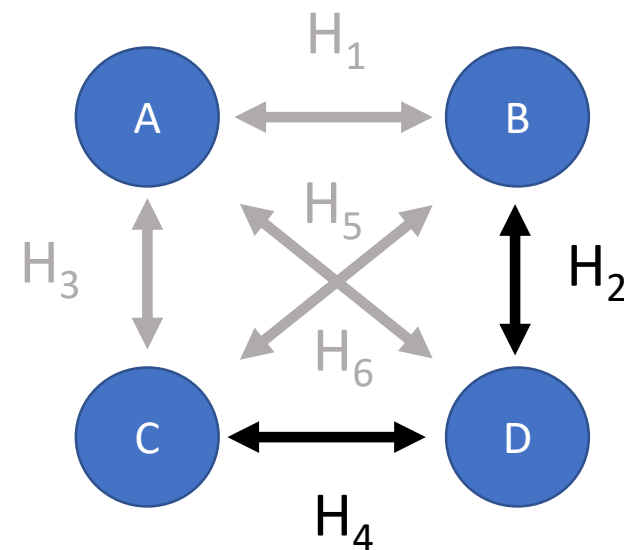$0 = b_1$ (suburban TFA different than urban TFA?)

$b_0 = b_0 + b_2$
$0 = b_2$ (urban regular different than urban TFA?)

$b_0 + b_1 + b_2 = b_0 + b_1 + b_2 + b_3$
$0 = b_3$
   (suburban **x** regular   different than   suburban + regular   ?)

Each specification creates a set of hypotheses tests.

We can never test all hypotheses with a single model, but we get several tests from one.

(group means)	Suburban	Urban
Regular Teachers	A	B
Teach for America	C	D

(design matrix)

math	intercept	d.sub	d.reg	d.sub.reg
75	1	1	0	0
75	1	1	1	1
66	1	0	0	0
57	1	0	1	0

(66) (9)   (-9)   (9)

$$math = b_0 + b_1 \cdot d.sub + b_2 \cdot d.reg + b_3 \cdot d.sub.reg$$

$b_0 + b_1 = 66 + 9 = \mathbf{75}$ (suburban TFA)

$b_0 + b_1 + b_2 + b_3 = 66 + 9 - 9 + 9 = \mathbf{75}$ (suburban regular)

$b_0 = \mathbf{66}$ (urban TFA – **reference group**)

$b_0 + b_2 = 66 - 9 = \mathbf{57}$ (urban regular)

$b_0 = b_0 + b_1$
$0 = b_1$ (suburban TFA different than urban TFA?)

$b_0 = b_0 + b_2$
$0 = b_2$ (urban regular different than urban TFA?)

$b_0 + b_1 + b_2 = b_0 + b_1 + b_2 + b_3$
$0 = b_3$

    (suburban **x** regular   different than   suburban + regular   ?)

(group means)	Suburban	Urban
Regular Teachers	A	B
Teach for America	C	D

(design matrix)

math	intercept	d.urb	d.tfa	d.urb.tfa
75	1	0	1	0
75	1	0	0	0
66	1	1	1	1
57	1	1	0	0

$$(75) \quad (-18) \qquad (0) \qquad (9)$$
$$\text{math} = b_0 + b_1 \cdot \text{d.urb} + b_2 \cdot \text{d.tfa} + b_3 \cdot \text{d.urb.tfa}$$

$b_0 + b_2 = 75 + 0 = \mathbf{75}$ (suburban TFA)

$b_0 = \mathbf{75}$ (suburban regular – **reference group**)

$b_0 + b_1 = 75 - 18 + 9 = \mathbf{66}$ (urban TFA)
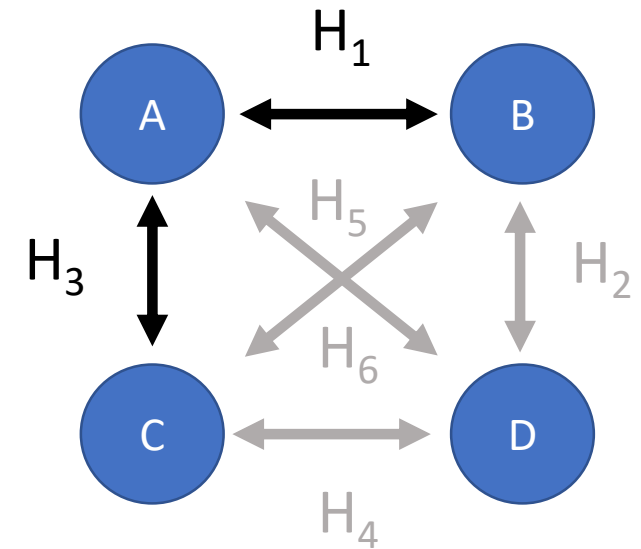
$b_0 + b_2 = 75 - 18 = \mathbf{57}$ (urban regular)

$b_0 = b_0 + b_1$
$0 = b_1$ (suburban regular different than urban regular?)

$b_0 = b_0 + b_2$
$0 = b_2$ (suburban regular different than suburban TFA?)

$b_0 + b_1 + b_2 = b_0 + b_1 + b_2 + b_3$
$0 = b_3$
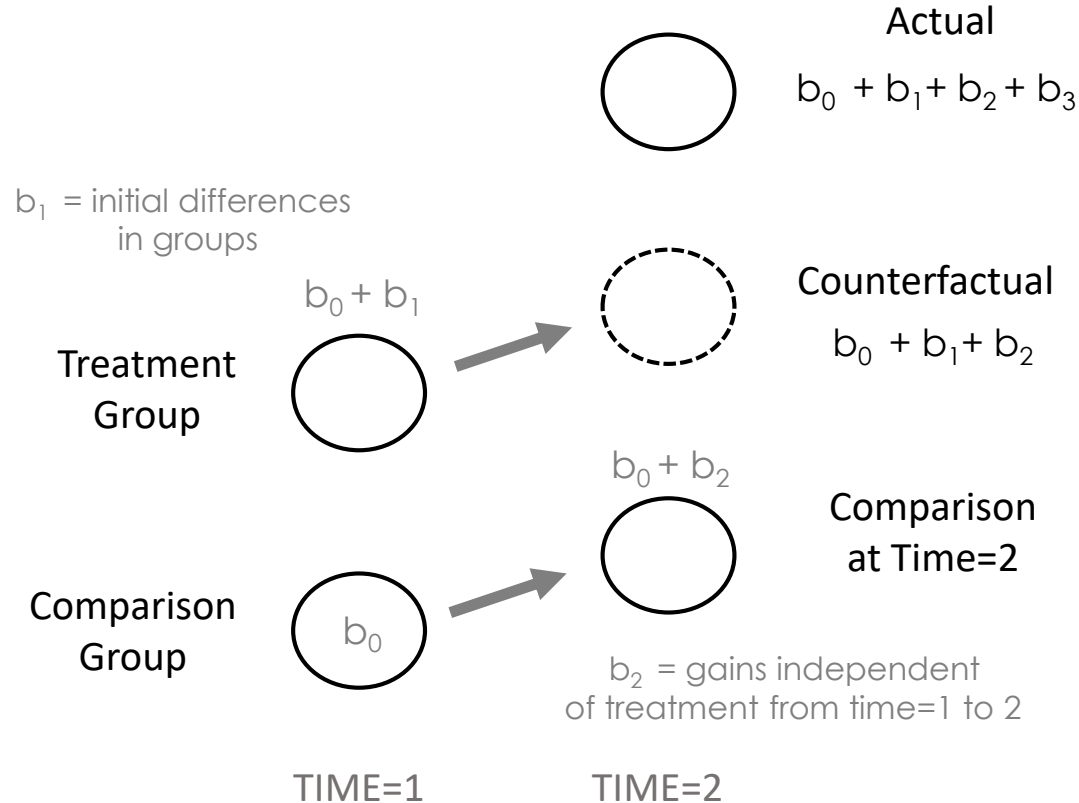
(urban **x** TFA different than urban + TFA ?)

Test for treatment effects in pre-post design with control group:

Does the treatment group improve more than expected ?
(the counterfactual captures the expectation if they have similar gains as control)

outcome $= b_0 + b_1 \cdot d.treat + b_2 \cdot d.time2 + b_3 \cdot d.treat.post$

$b_0 + b_1 + b_2 = b_0 + b_1 + b_2 + b_3$
$0 = b_3$

b_1 = initial differences in groups

Actual
$b_0 + b_1 + b_2 + b_3$

b_3

Counterfactual
$b_0 + b_1 + b_2$

$b_0 + b_1$

Treatment Group

$b_0 + b_2$

Comparison at Time=2

Comparison Group

b_0

b_2 = gains independent of treatment from time=1 to 2

TIME=1   TIME=2

Expected treatment group mean if program is ineffective

Actual observed group mean

$b_0 + b_1 + b_2 = b_0 + b_1 + b_2 + b_3$

0 = b$_3$?

b_3 : Test for whether the treatment was effective – if the group looks different than we would expect

Test for b_3: $b_0 + b_1 + b_2$ = $b_0 + b_1 + b_2 + b_3$