# VARIETIES OF THE COUNTERFACTUAL

*Jesse Lecy*

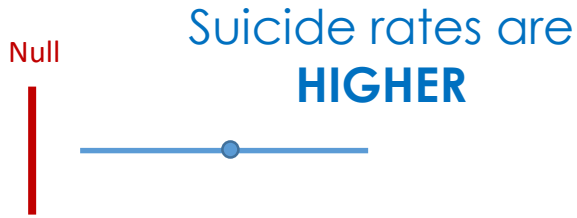
# THE THREE TYPES OF COUNTERFACTUALS

**Pre-Post with comparison group**

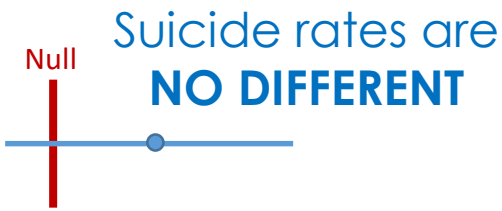
**Post-test only design**

**Reflexive design (pre-post only)**

Assume you have selected a reasonable comparison.




**Null Hypothesis:**
Population Average




**Null Hypothesis:**
All HS Students
OR All Californians

**Null Hypothesis:**
All Suburban HS Students

# You have created your groups, tested for equivalence and attrition bias, decided on the type of measure you will use in the study (ITT or TOT).

**Figure 4.3 Steps in Randomized Assignment to Treatment**

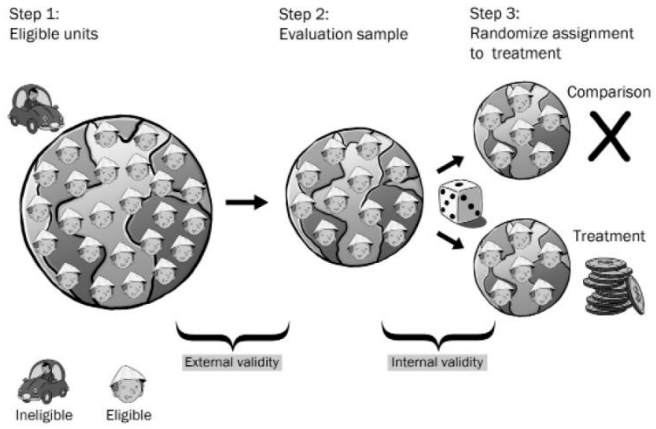



**TABLE 2**
**Background Characteristics of Students in Treatment and Control Groups**
**(Total numbers of cases in parentheses)**

| Characteristic | All students in the study: Choice students | Control students | p value[a] | All students with scores three or four years after application: Choice students | Control students | p value[a] |
|---|---|---|---|---|---|---|
| Math scores before application | 39.7 (264) | 39.3 (173) | .81 | 40.0 (61) | 40.6 (33) | .86 |
| Reading scores before application | 38.9 (266) | 39.4 (176) | .74 | 42.1 (60) | 39.2 (33) | .35 |
| Family income | 10,860 (423) | 12,010 (127) | .14 | 10,850 (143) | 11,170 (25) | .84 |
| Mothers' education 3 = some college 4 = college degree | 4.2 (423) | 3.9 (127) | .04 | 4.1 (144) | 3.8 (29) | .15 |
| Percent married parents | 24 (424) | 30 (132) | .17 | 23 (145) | 38 (29) | .11 |
| Parents' time with children 1 = 1–2 hours/week 2 = 3–4 hours/week 3 = 5 or more | 1.9 (420) | 1.8 (130) | .37 | 1.9 (140) | 1.7 (27) | .26 |
| Parents' education expectations of children 4 = college 5 = graduate school | 4.2 (422) | 4.2 (129) | .85 | 4.2 (142) | 3.7 (27) | .01 |

a. The tests of significance are suggestive of the equivalence of the two groups. Technically, tests of significance should be done at each point of random assignment, but the number of cases at each point is too few for such tests to be meaningful.

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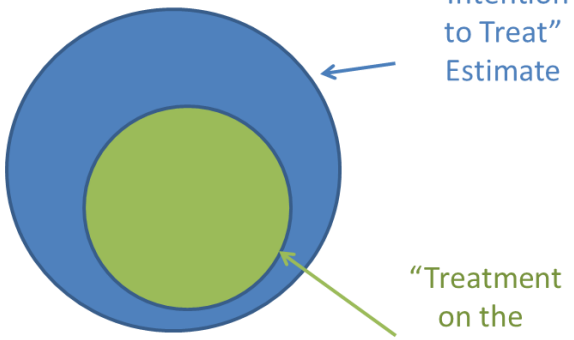


# Now how do we actually calculate program impact?

$$Y = b_0 control + b_1 treat + b_2 time + b_3 controls + e$$

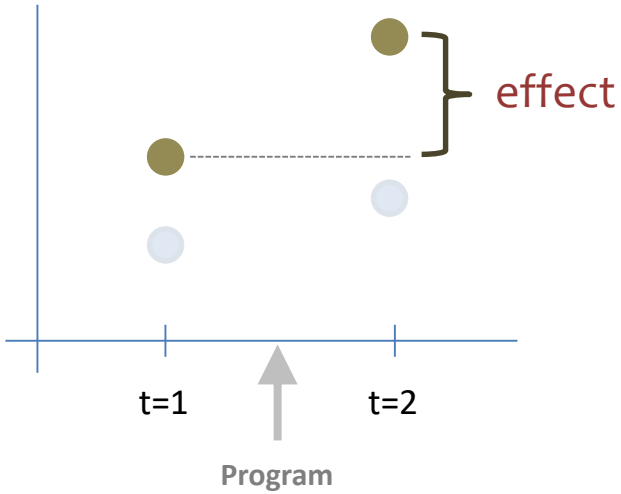
$b_?$ = program impact?

# THE THREE TYPES OF COUNTERFACTUAL ESTIMATES

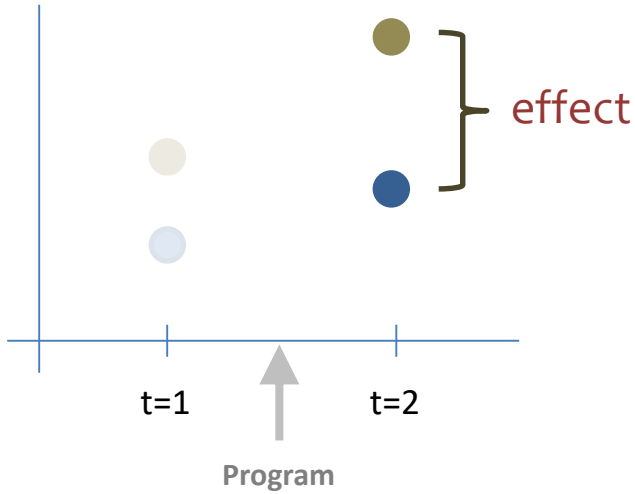
# 3 VALID COUNTERFACTUALS:

Each variety of counterfactual has a different formula for the program effect estimate.

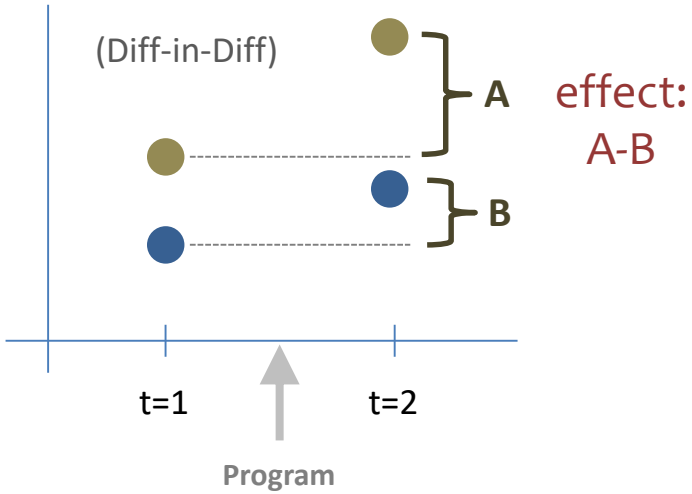
**PRE-POST REFLEXIVE**
Estimator
effect = T2-T1




**POST-ONLY**
Estimator
effect = T2-C2




**PRE-POST W COMPARISON**
Estimator
effect = (T2-T1) – (C2-C1)




Treatment Groups, **T1**=before, **T2**=post-program measure

Control Groups, **C1**=before, **C2**=post-program measure

# 3 VALID COUNTERFACTUALS:

Each variety of counterfactual has a different formula for the program effect estimate.

**PRE-POST REFLEXIVE**
Estimator
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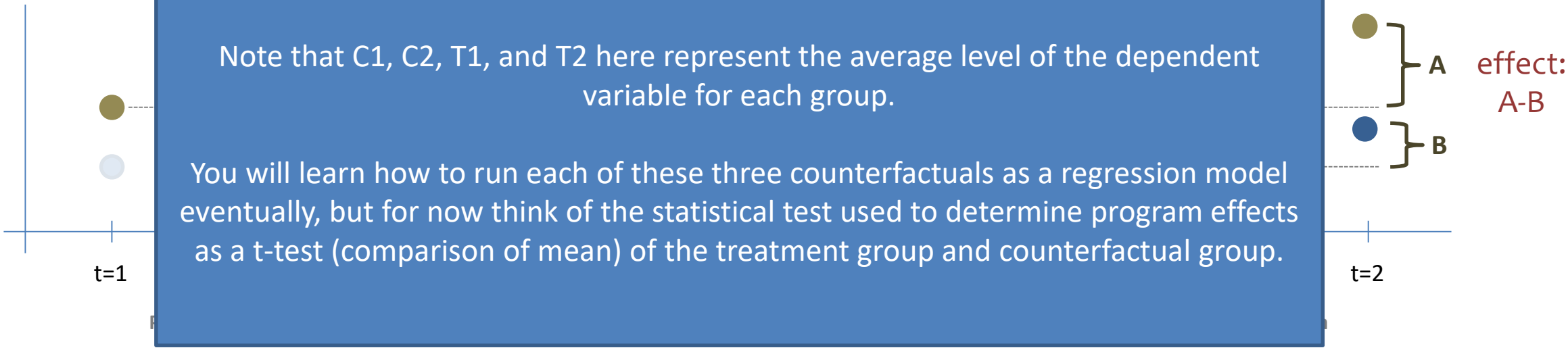
**POST-ONLY**
Estimator
effect = T2-C2

**PRE-POST W COMPARISON**
Estimator
effect = (T2-T1) – (C2-C1)




Note that C1, C2, T1, and T2 here represent the average level of the dependent variable for each group.

You will learn how to run each of these three counterfactuals as a regression model eventually, but for now think of the statistical test used to determine program effects as a t-test (comparison of mean) of the treatment group and counterfactual group.

Treatment Groups, **T1**=before, **T2**=post-program measure

Control Groups, **C1**=before, **C2**=post-program measure

# Recall from Unit on Dummy Variable Models

(basic set-up for a comparison of group means in regression)

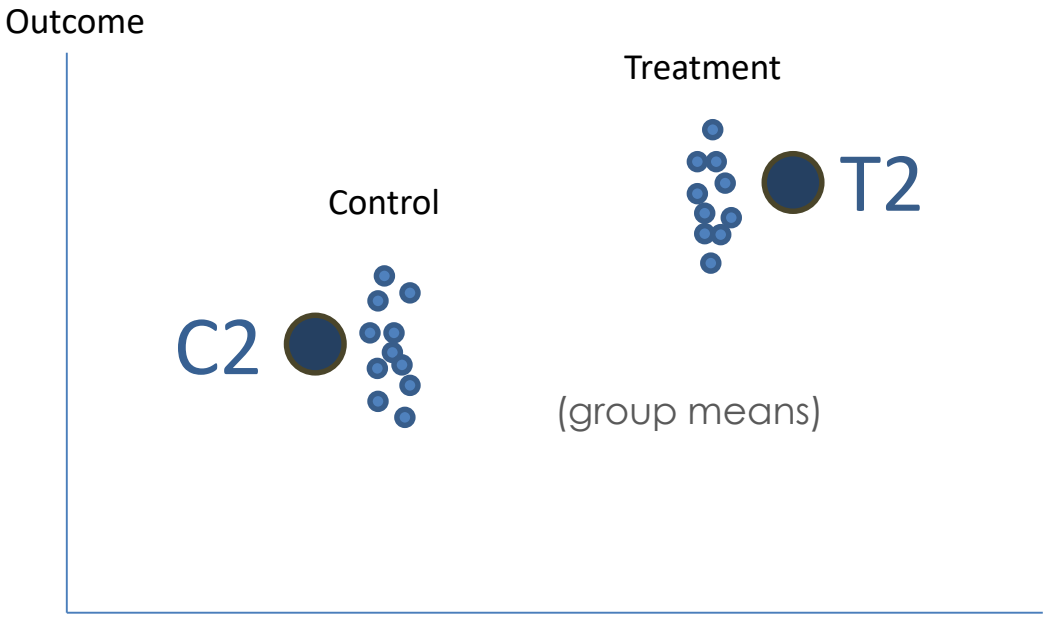
$b_0$ will measure Y-bar of the omitted group C2

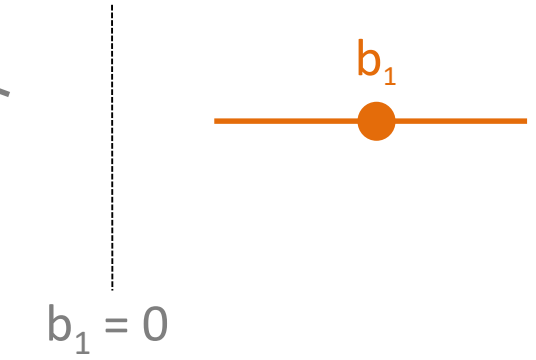
$b_0 + b_1$ represents the Y-bar of T2

$$Y = b_0 + b_1(\text{TreatDummy}) + e$$

$$b_1 = \text{MEAN}_{\text{treat}} - \text{MEAN}_{\text{control}}$$

$$b_1 = \text{T2} - \text{C2}$$

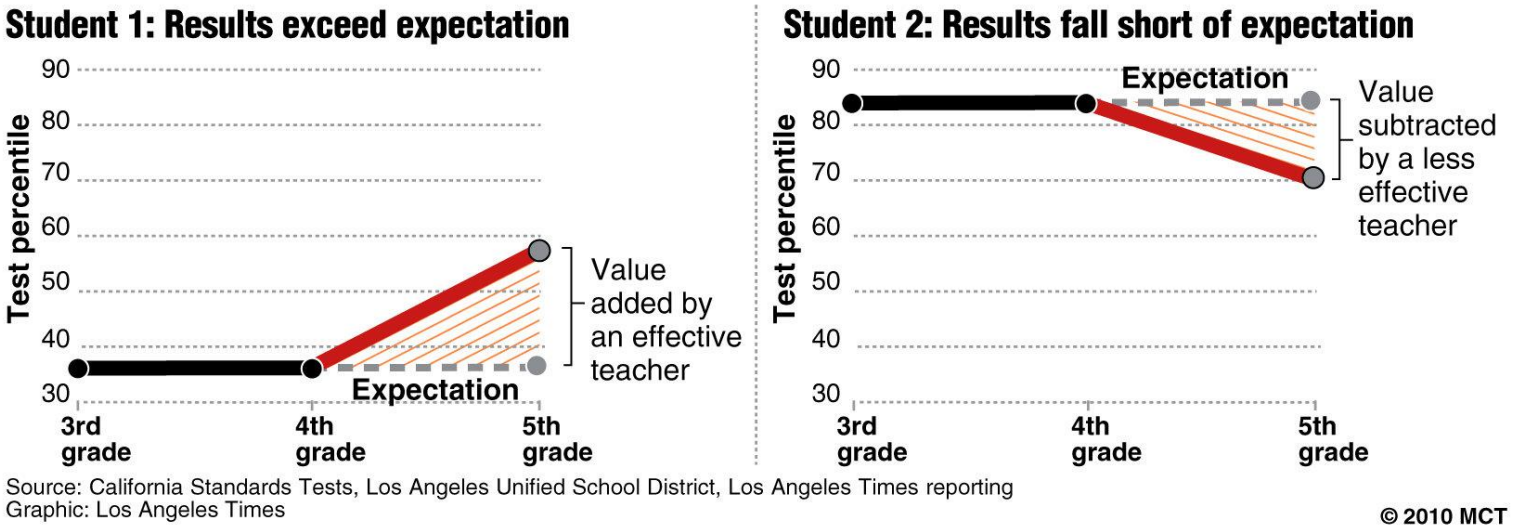



The default hypothesis test in the regression then uses $b_1$ to **test for a meaningful difference between T2 and C2**

# What is 'value added'?

*"Value added" rates teachers based on their students' progress on standardized tests year after year. The difference between a student's expected growth and actual performance is the "value" a teacher adds or subtracts during the year.*




Source: California Standards Tests, Los Angeles Unified School District, Los Angeles Times reporting
Graphic: Los Angeles Times

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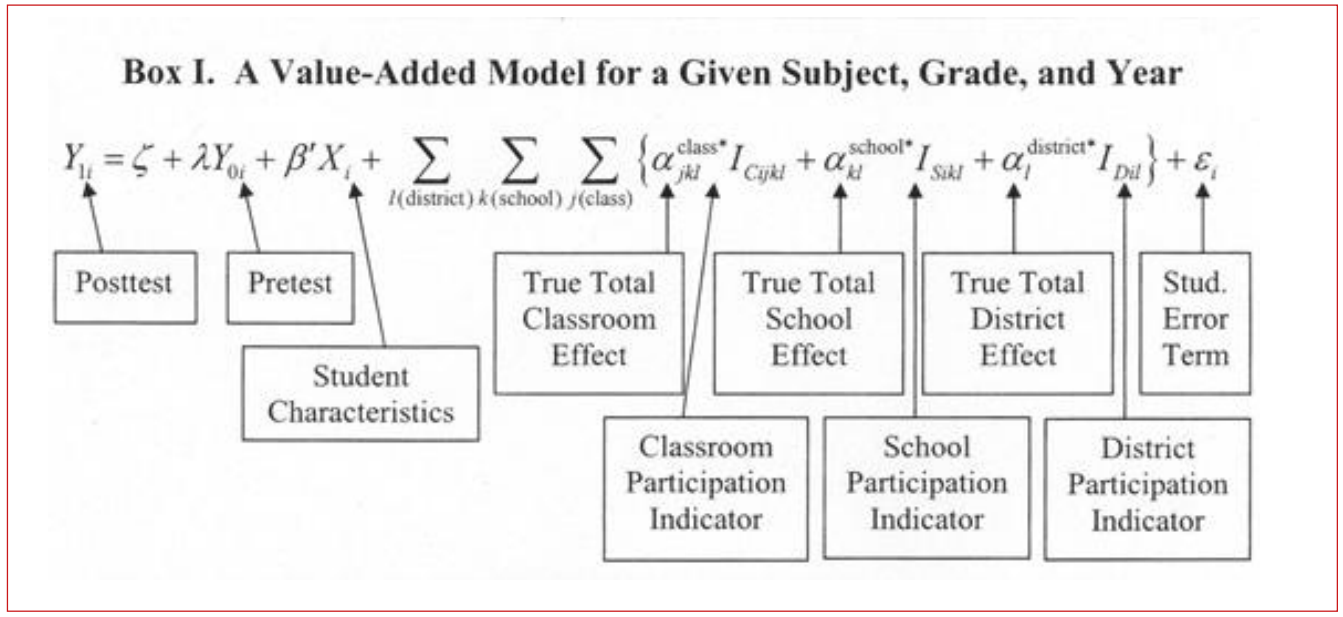


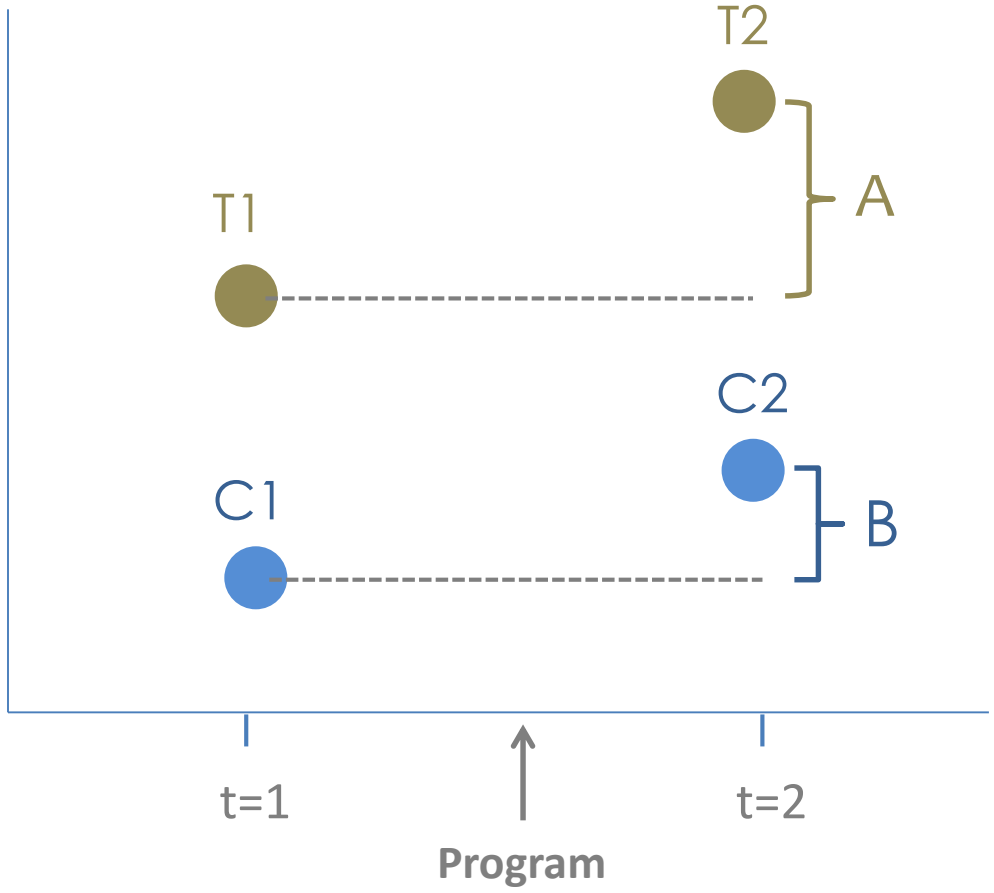

When we have lots of groups, lots of time periods, and lots of control variables at different levels our models will eventually look like this.

But not this semester.

# Difference-in-Difference Estimator

**Pre-Post w Comparison**
Estimator

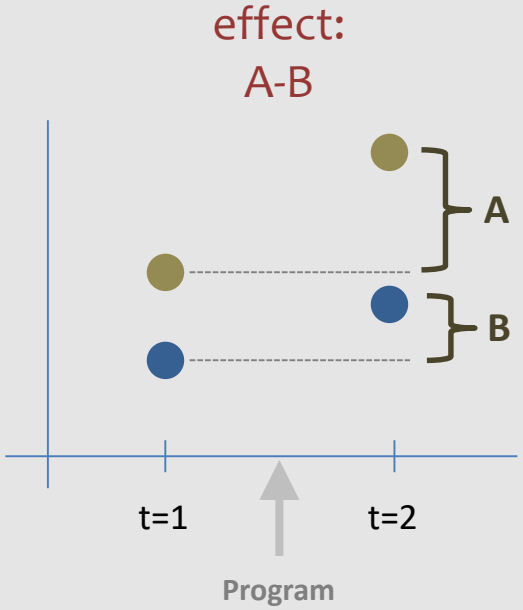
effect = **(T2-T1) – (C2-C1)**




effect:
A-B
(Diff-in-Diff)

Accounts for initial difference in groups and secular trends

# Difference-in-Difference Estimator

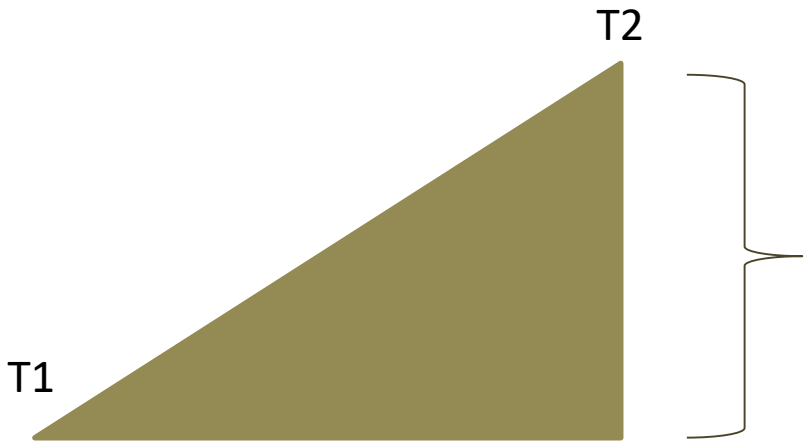



## Pre-Post w Comparison

$$\text{effect} = (T2\text{-}T1) - (C2\text{-}C1)$$

↑ Gains during the program period by treatment group

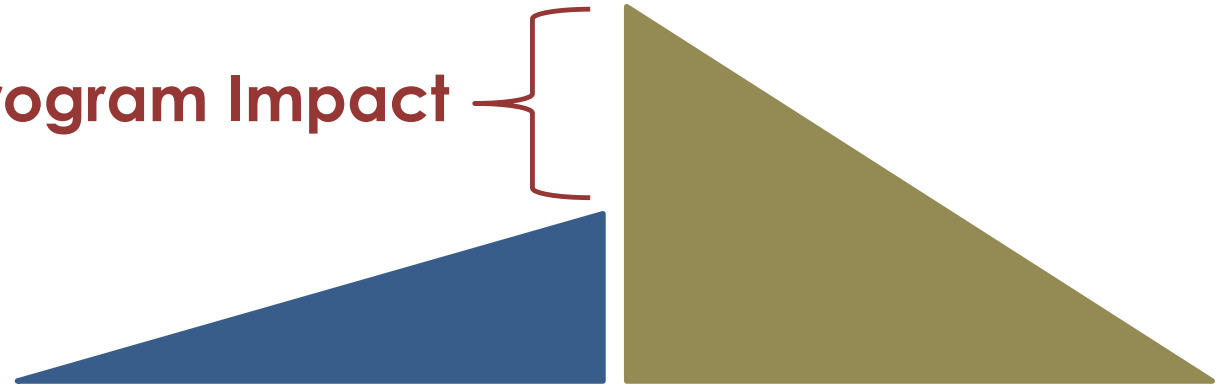
↑ Gains independent of the program (secular trends)



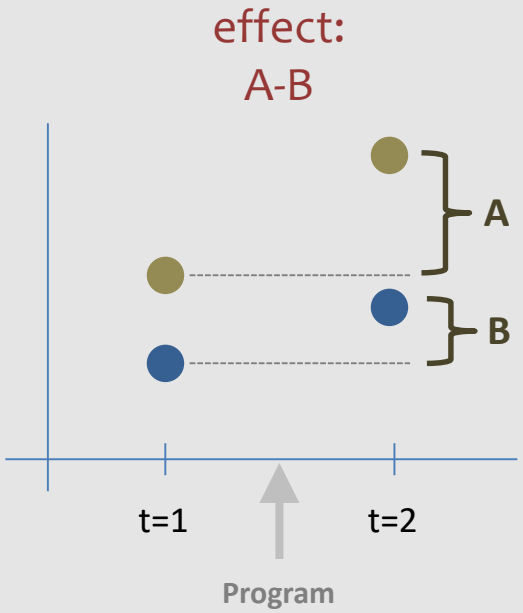

T2 – T1 =
Total Gains in
Treatment Group

C2 – C1 =
Total Gains in
Control Group
(trend)

(T2-T1)-(C2-C1) = **Program Impact**



# Difference-in-Difference Estimator



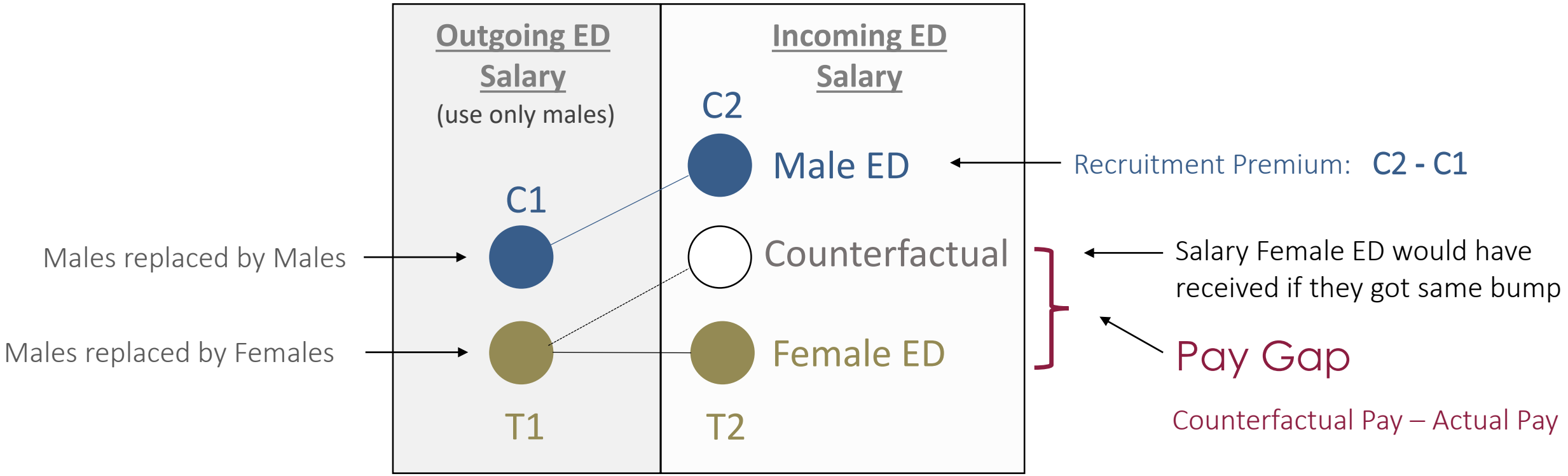

effect = (T2-T1) – (C2-C1)

Two important things to note:

(1) The groups prior to the treatment are not identical. The difference in difference is robust because it does not require equivalent treatment and “control” (comparison) groups.

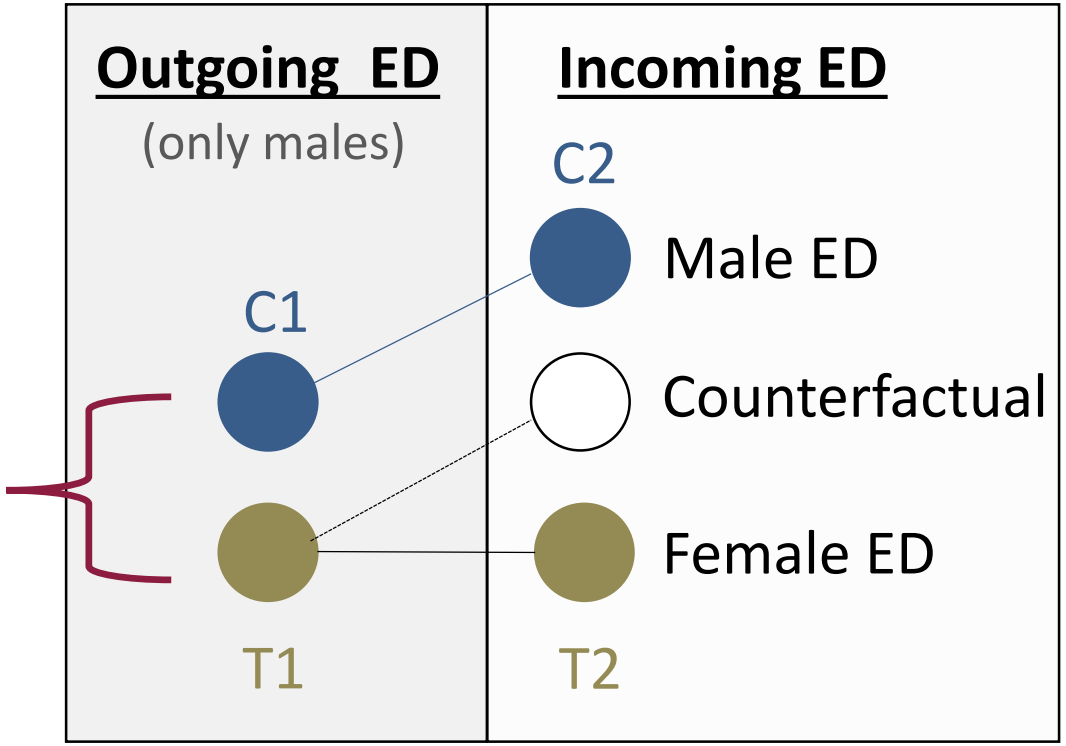
(2) The comparison group measures the changes we would expect to observe in the treatment group over time independent of the treatment. This assumption must be true for the diff-in-diff estimator to be unbiased.

# Example: Estimating the gender pay gap for Executive Directors



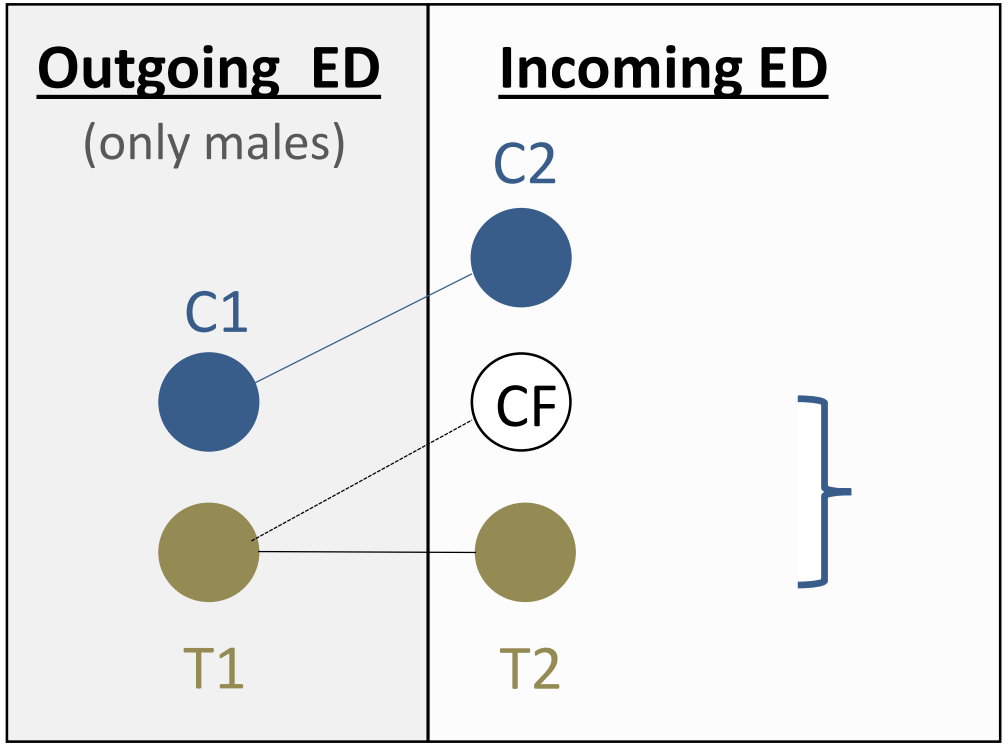

Counterfactual Pay: Baseline Pay (T1) + Typical (male) Recruitment Premium ( C2 – C1 )

Note that in the diff-in-diff the groups do not have to be identical prior to the treatment (new hire)

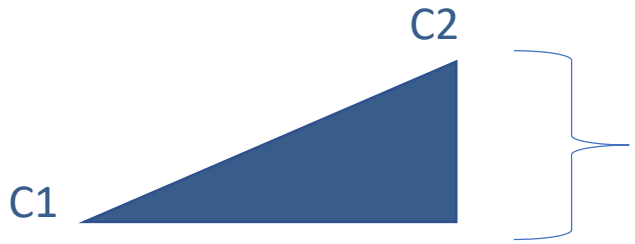



This is the major strength of the estimator.

Also note that we never compare C2 to T2 (the two post-treatment groups)

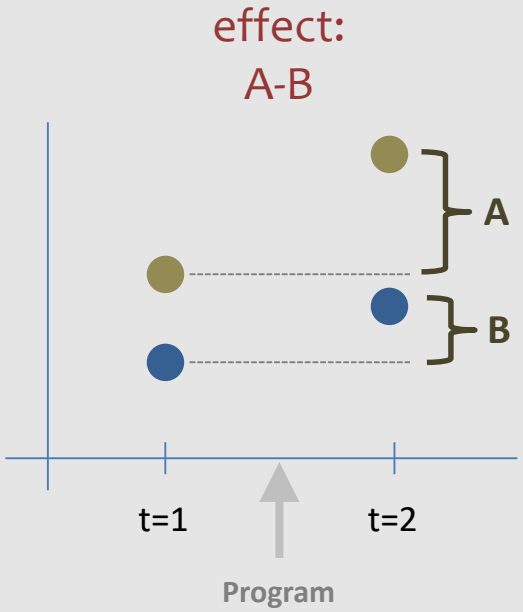



The proper counterfactual is how much would female EDs be getting paid if they were given recruitment premiums as incoming male EDs.



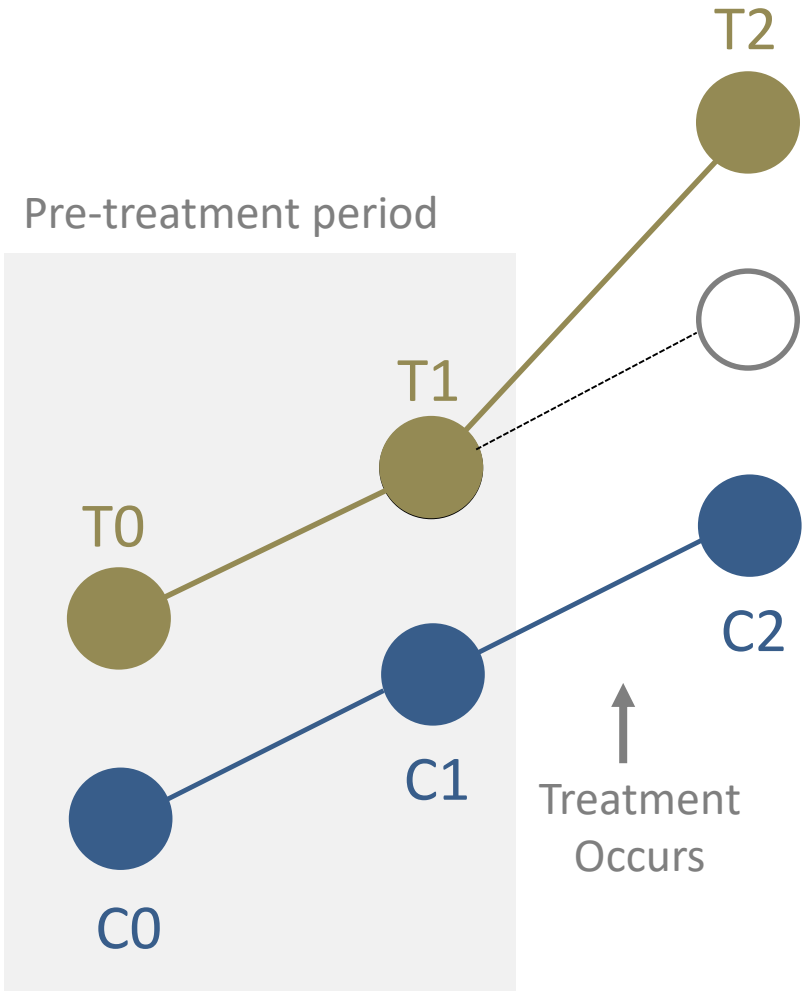

The comparison group is there to capture the trend, which is then used to construct the counterfactual.

# Difference-in-Difference Estimator




Parallel lines test:

$$T1-T0 = C1-C0$$




Parallel lines assumption: for the diff-in-diff counterfactual to be valid C2-C1 has to accurately capture the secular trend (gains independent of the treatment). The test of this is comparing rates of change in treatment and comparisons groups prior to the intervention. They should b the same.

$b_0$ will measure Y-bar of the omitted group **T1**

$b_0 + b_1$ represents the Y-bar of **T2**

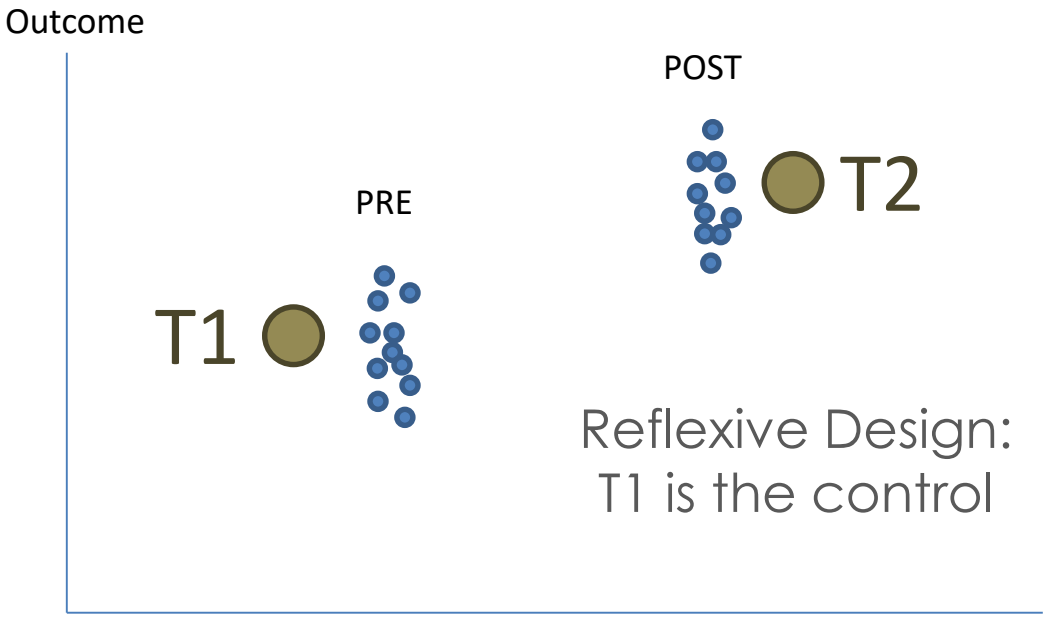
$$Y = b_0 + b_1(\text{TreatDummy}) + e$$

$$b_1 = \text{MEAN}_{\text{treat}} - \text{MEAN}_{\text{control}}$$

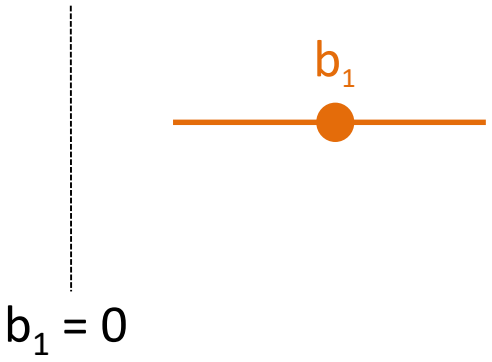
$$b_1 = T2 - T1$$

# In reflexive models T1 is the counterfactual or control group

The default hypothesis test in the regression then uses $\mathbf{b_1}$ to test for a meaningful difference between T2 and T1

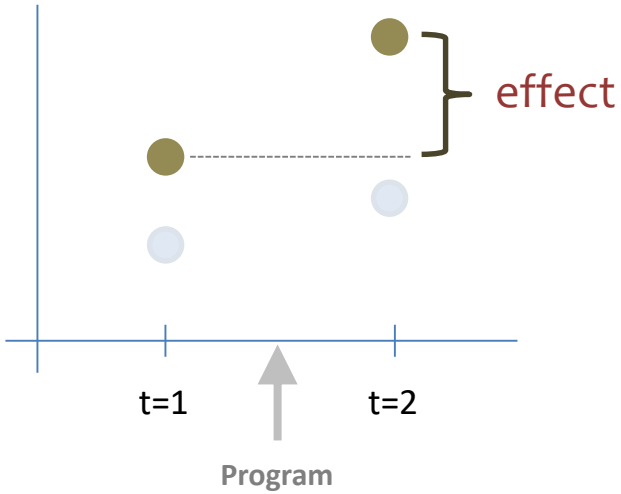



Reflexive Design:
T1 is the control

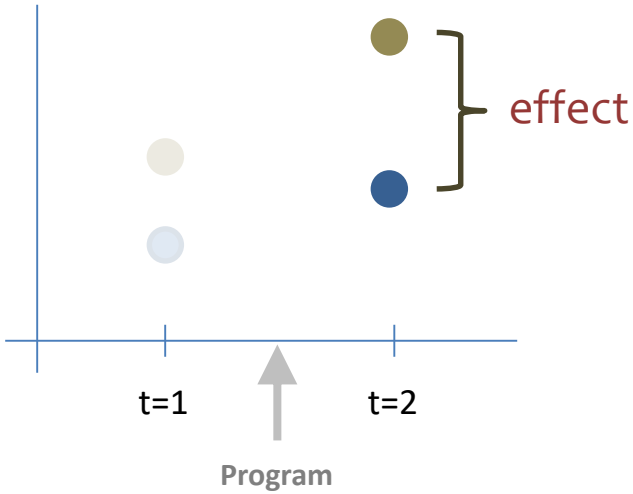


# 2 REMAINING COUNTERFACTUALS:

**PRE-POST REFLEXIVE**
Estimator
effect = T2-T1




**POST-ONLY**
Estimator
effect = T2-C2




**PRE-POST W COMPARISON**
Estimator
effect = (T2-T1) – (C2-C1)

# Validity of the reflexive estimator:

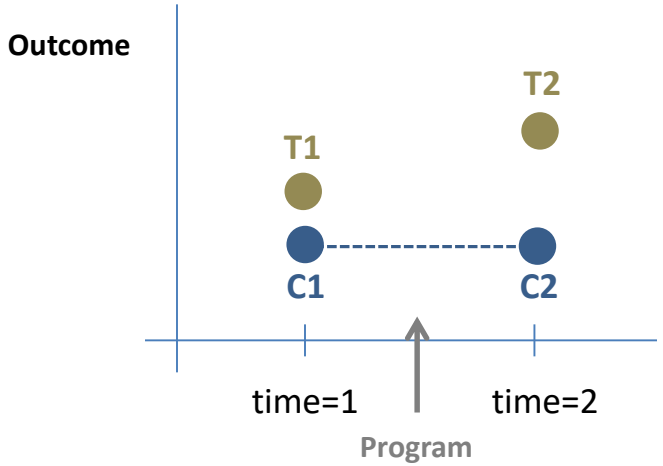
**Reflexive (Pre-Post)**

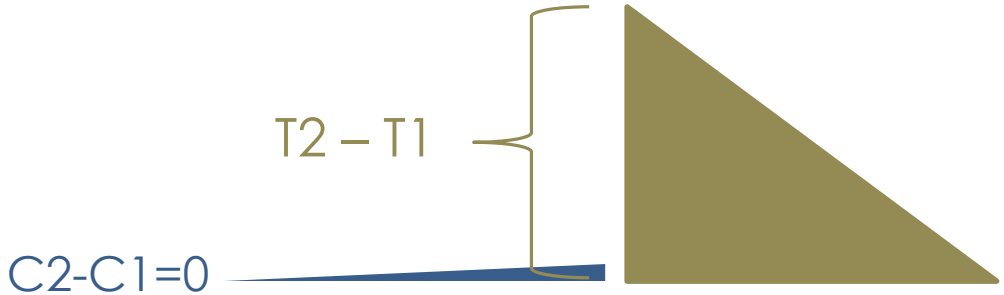
$(T2 - T1) - (C2 - C1) = T2 - T1$

IFF

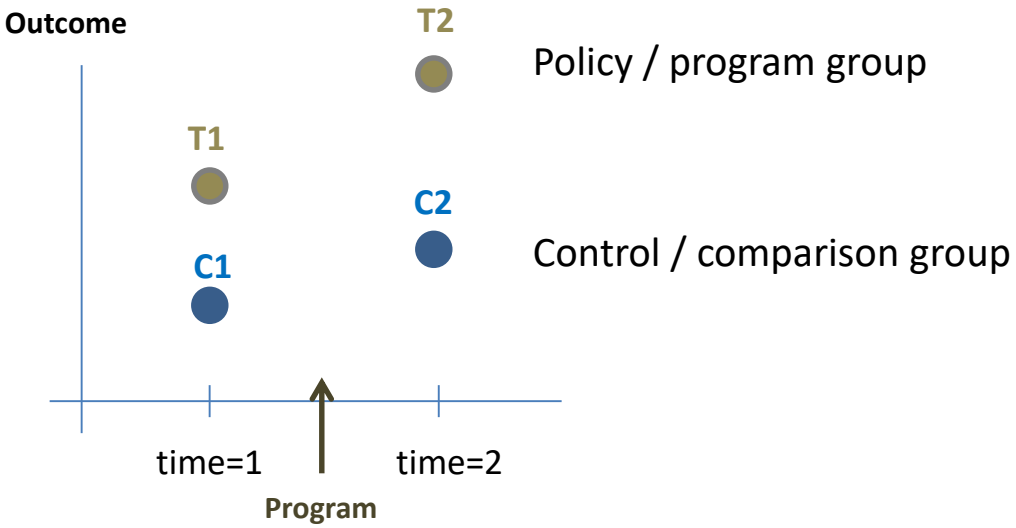
$C2 - C1 = 0$
(no trend)

In some special cases we expect there to be no change independent of the treatment. When this is true the reflexive estimator T2-T1 is appropriate and unbiased.



# "Control" Versus "Comparison" Groups

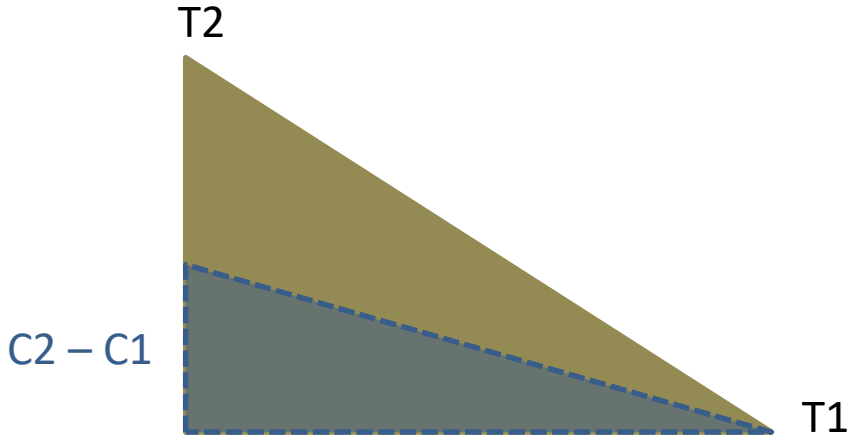



The term "control group" typically refers to a group created through randomization, so they are assumed to be identical. We sometimes use the terminology "comparison group" to differentiate the these cases. The control groups represents the counterfactual, the comparison group does not (it just captures trend in this case).

| Single Difference: | Difference in Difference: |
|---|---|
| Effect = T2 – T1 | Effect = (T2 – T1) – (C2 – C1) |
| | ***trend*** |
| Effect = T2 – C2 | |

The comparison group is necessary to capture any changes that we expect during the study period independent of the treatment group. If we fail to account for those changes we will incorrectly attribute all change to the program and over-state program impact.

**Program Impact?**



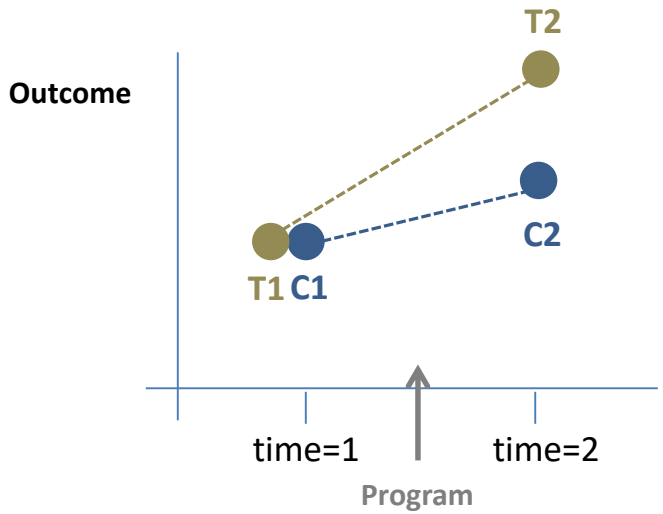
# Validity of the post-test only estimator:

**Post-Only**

$(T2 - T1) - (C2 - C1) = T2 - C2$

IFF

$C1 - T1 = 0$
(equivalent at time 1)




If we have confidence that the two groups are identical prior to the treatment, then mathematically T2-C2 will still account for gains independent of the treatment. This condition is necessary for the post-test only estimator to be unbiased.

In experimental design, this is usually accomplished through randomization or lottery.

Observational students typically use matching models to create equivalent groups.

# Recall from Unit on Dummy Variable Models

(basic set-up for a comparison of group means in regression)

$b_0$ will measure Y-bar of the omitted group C2

$b_0 + b_1$ represents the Y-bar of T2

$$Y = b_0 + b_1(\text{TreatDummy}) + e$$

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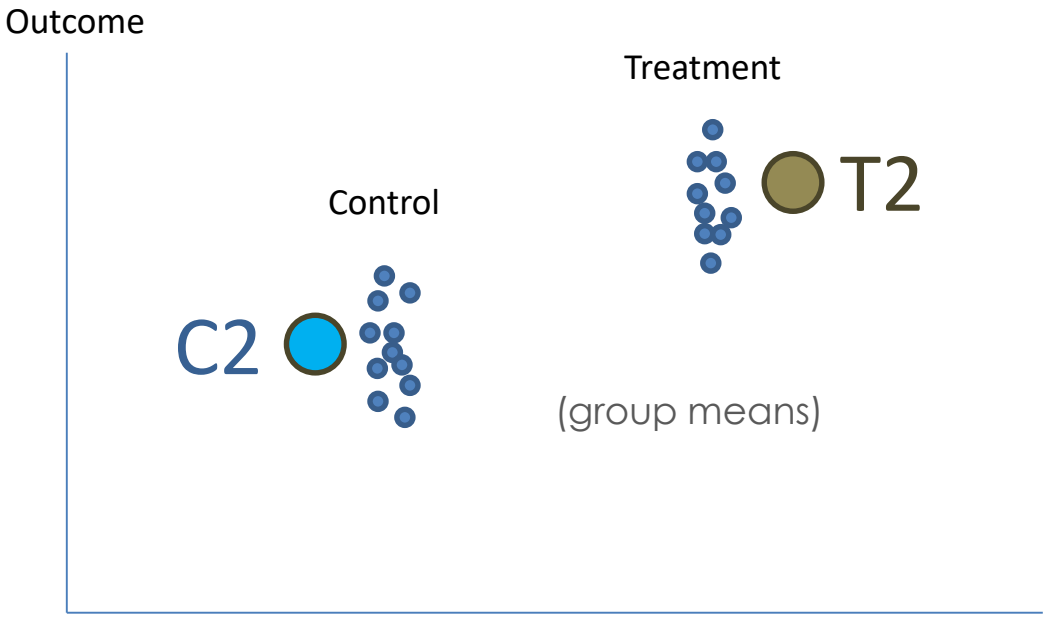
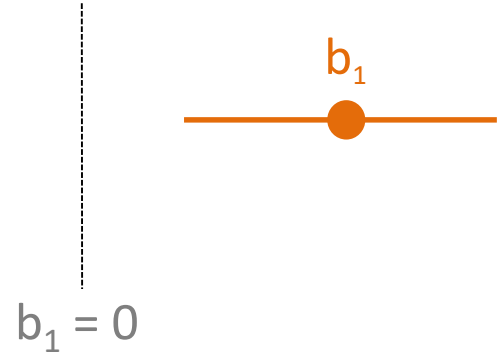
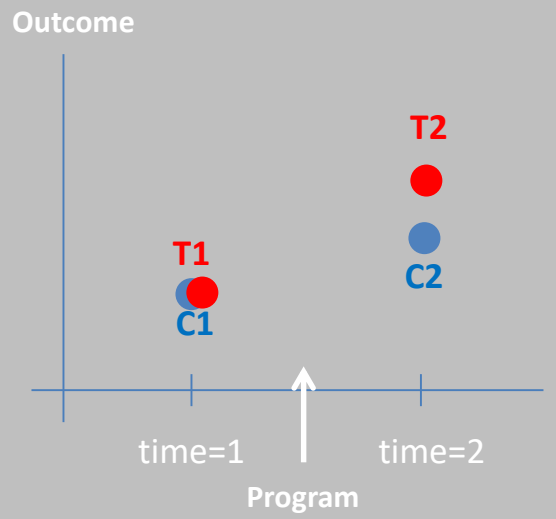
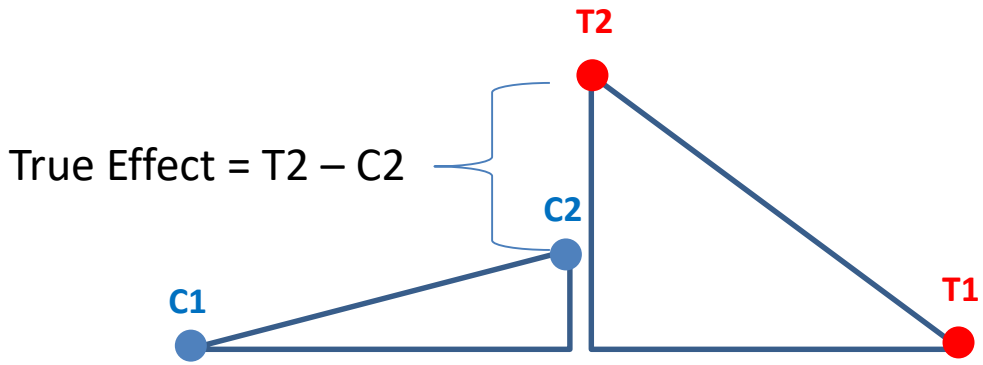



The default hypothesis test in the regression then uses $b_1$ to **test for a meaningful difference between T2 and C2**

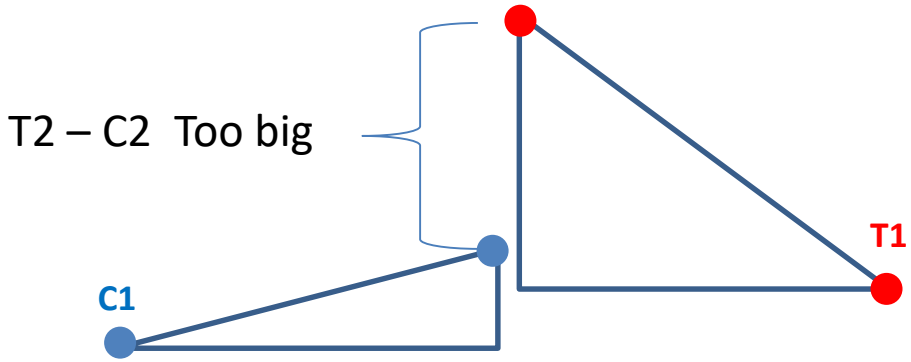
# Post-Test Only Measure



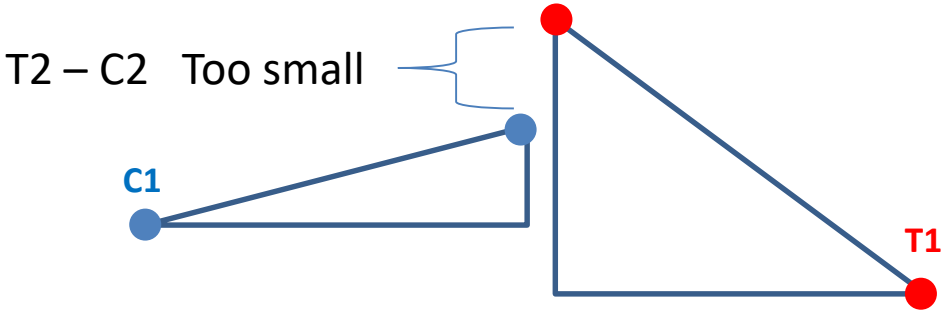

If C1 = T1

Measured effect accurately represents program impact




C1 < T1

Measured effect overstates program impact

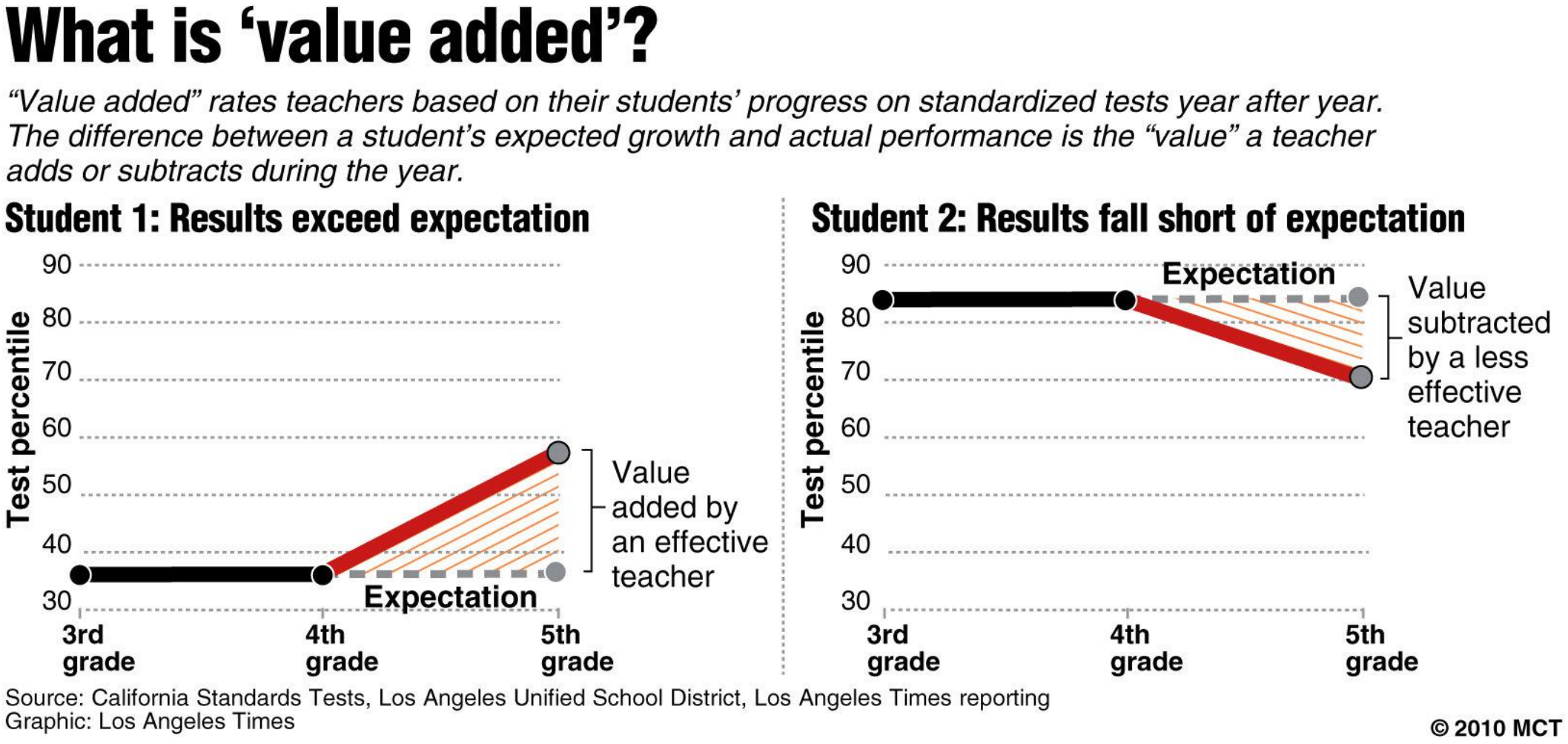



C1>T1

Measured effect understates program impact

# EXAMPLES

# Example from education policy – which estimator is this?

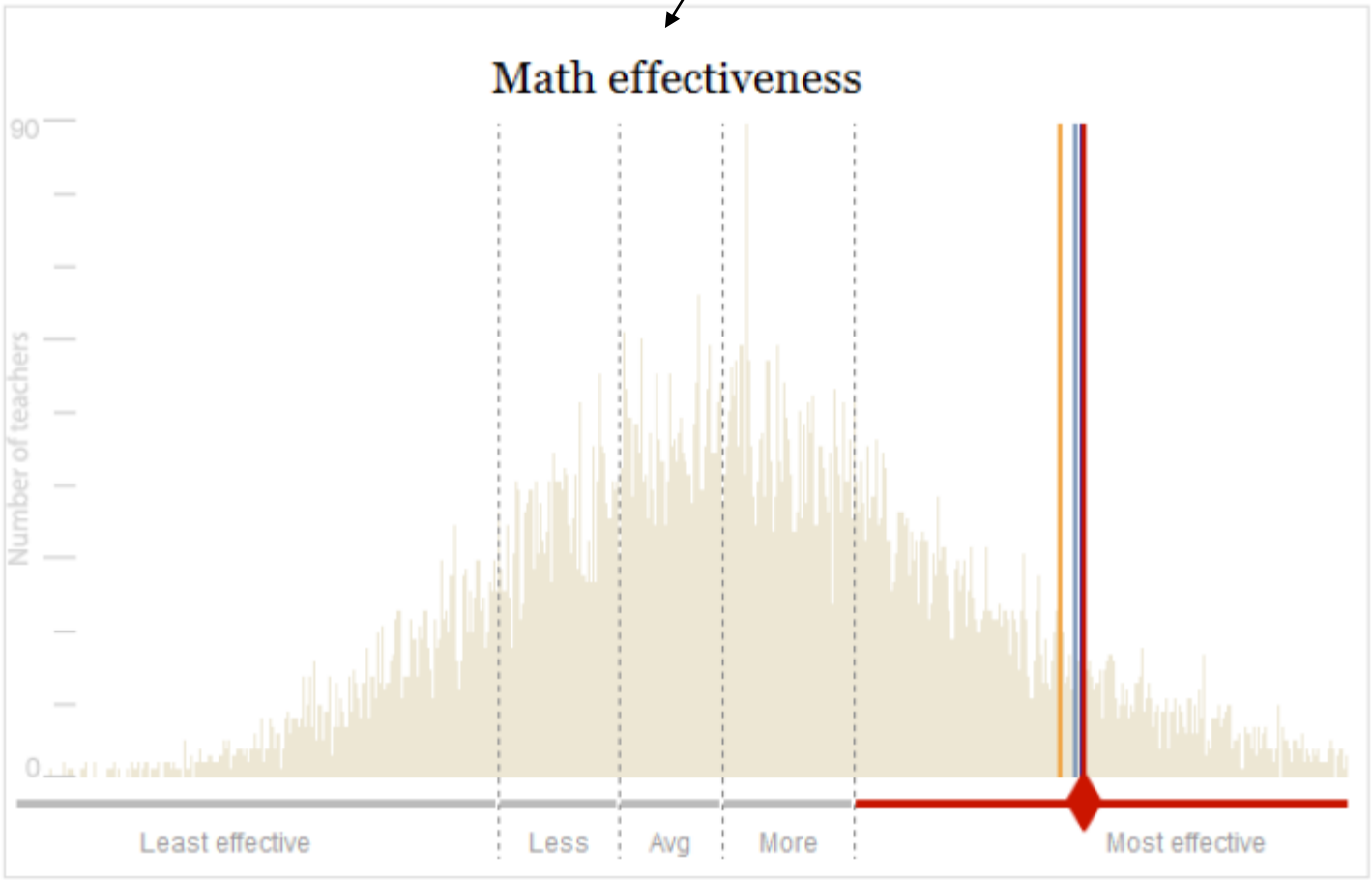


# COMPARING TEACHER "EFFECTS"

Change in student performance from the beginning of the academic year to the end of the academic year (measured in percentile units).

**Reflexive measure** because our notion of "impact" comes from comparing performance of the same students over time, not comparing one classroom to another at a point in time.

Note, because we use percentiles for performance (tests will be different at the end of each grade so it is hard to compare raw scores) there is an element of a difference-in-difference design. But percentiles require that the average change across all groups (the secular trend or expectation of gain independent of the treatment) has to be zero across all groups because of the math. But if it is a sub-population there could be a trend, in which case the diff-in-diff might be appropriate.



# REVISIT THE MICRO-FINANCE EXAMPLE

# Microfinance example of bias from selection INTO a study group

Number of each "type" of person in the study

| | NOT Entrepreneurial | Entrepreneurial |
|---|---|---|
| No Loan | 30 | 15 |
| Takes a Loan | 20 | 35 |

Average weekly income after loan period

| | NOT Entrepreneurial | Entrepreneur |
|---|---|---|
| No Loan | $10 | $20 |
| Takes a Loan | $10 | $20 |

Income not impacted by the loan

You are more likely to take a loan if you know you are good at business

Takes Loan?

The loan appears to have an impact!

$$NO: \quad \frac{30 \cdot \$10 + 15 \cdot \$20}{45} = \$13.33$$

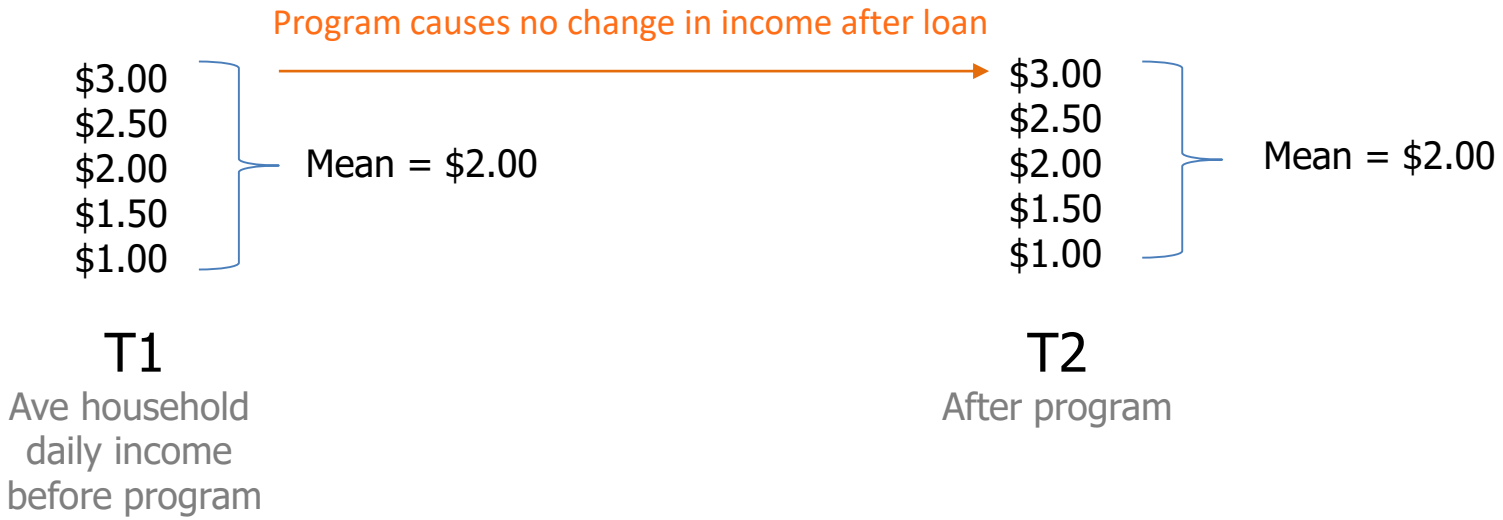
$$YES: \quad \frac{20 \cdot \$10 + 35 \cdot \$20}{55} = \$16.37$$

Which counterfactual estimator are we using here?

Which counterfactual estimator would be unbiased?

# Microfinance example of bias from selection OUT OF a study group

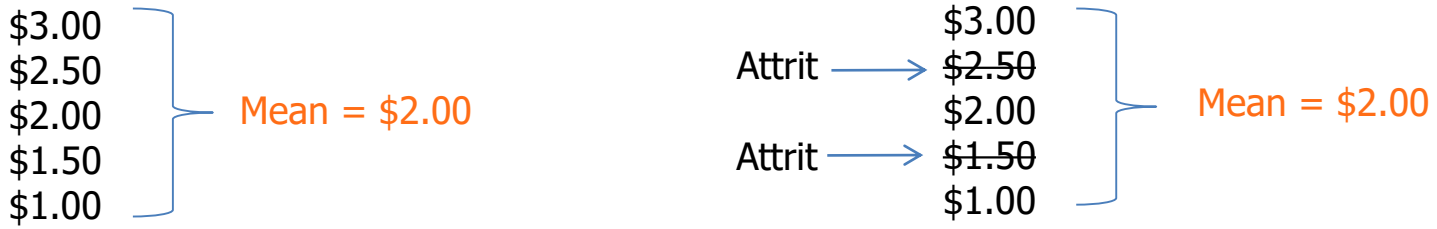
## Reflexive design




$$T2 - T1 = 0$$

causal estimate is unbiased

# Random Attrition Example



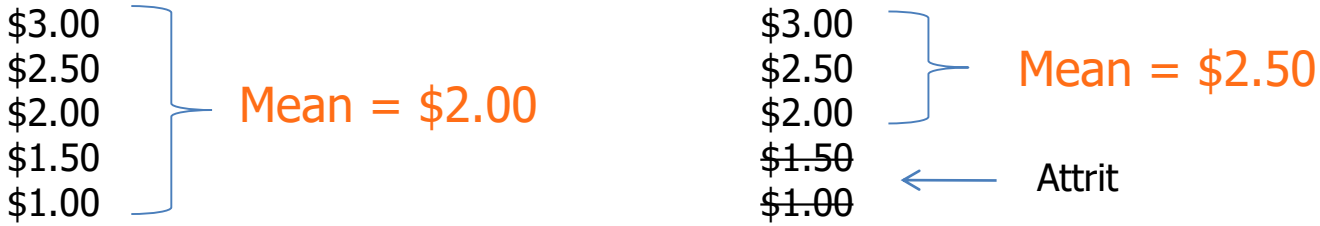

T2 - T1 = 0

Impact study accurately represents program effects

Program is not determined to be effective (no change)

# Which estimator is used here?

## Non-Random Attrition Example




$$T2 - T1 \neq 0$$

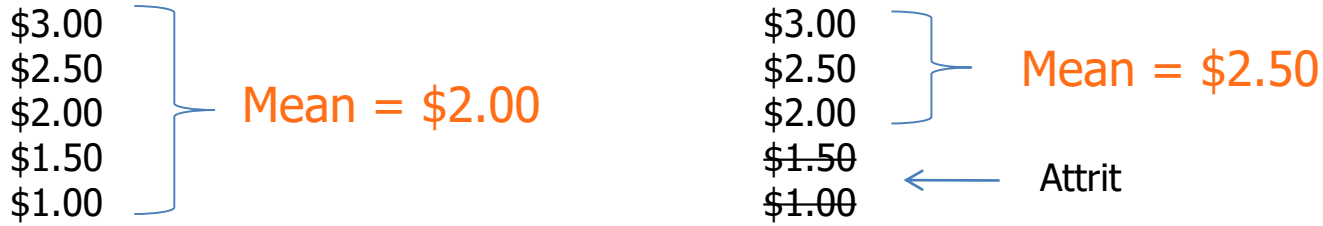
We over-estimate program effects

Program appears to be effective

Which estimator is used here?

**Can we use a different one?**

# Non-Random Attrition Example



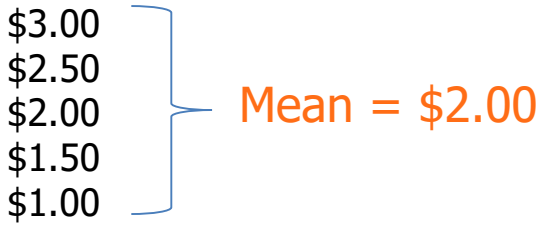

Reflexive → $T2 - T1 \neq 0$

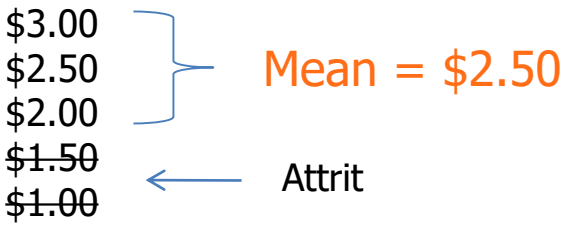
We over-estimate program effects

Program appears to be effective

Which effect measure is used here? Intention to treat, or treatment on the treated?

# Non-Random Attrition Example




$$T2 - T1 \neq 0$$

We over-estimate program effects

Program appears to be effective

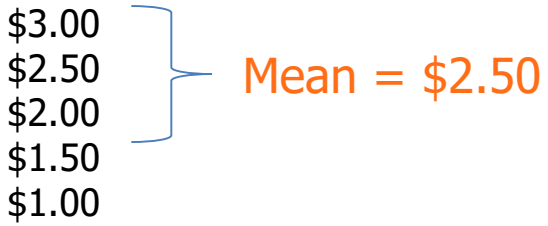
Can we use a better treatment measure?

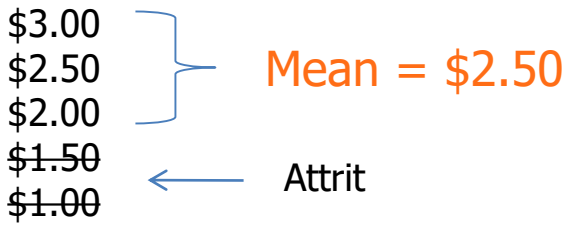
Which effect measure is used here? Intention to treat, or treatment on the treated?

# Non-Random Attrition Example

Remove attriters from T1

T2 – T1 = 0




This is not about ITT vs TOT! To calculate those we need data on both groups in T2. Rather this is about how we use data when attrition is present

# PREVIEW OF CPP 525: ADVANCED REGRESSION

# COUNTERFACTUALS IN PRACTICE: QUASI-EXPERIMENT METHODS

1) **Difference-in-Difference: Effect = (T2-T1) – (C2-C1)**

    1) Difference-in-Difference Analysis
    2) Fixed Effects Models

2) **Reflexive Design: Effect = T2 – T1**

    1) Pre-Post Design with only a Treatment Group
    2) Time Series Regression

3) **Post-Test Only Design: Effect = T2 – C2**

    1) True Experiments (T2-C2)
    2) Post-Test Only Comparison, No Baseline (T2-C2)
    3) Propensity Score Matching
    4) Regression Discontinuity Design

**Box I. A Value-Added Model for a Given Subject, Grade, and Year**

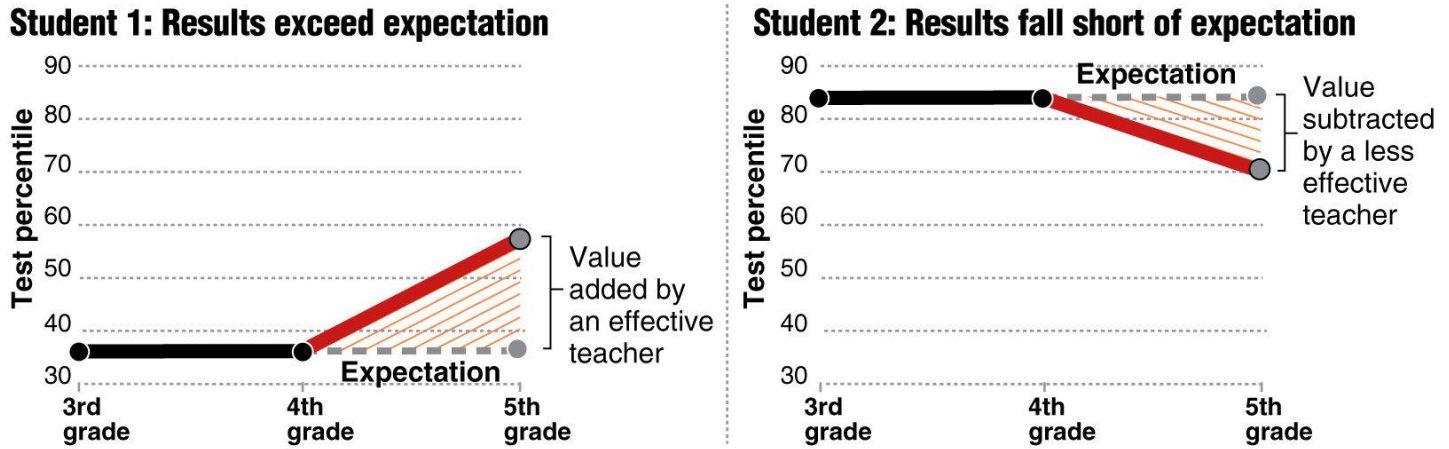
$$Y_{1i} = \zeta + \lambda Y_{0i} + \beta' X_i + \sum_{l(\text{district})} \sum_{k(\text{school})} \sum_{j(\text{class})} \left\{ \alpha_{jkl}^{\text{class}^*} I_{Cijkl} + \alpha_{kl}^{\text{school}^*} I_{Sikl} + \alpha_{l}^{\text{district}^*} I_{Dil} \right\} + \varepsilon_i$$

- Posttest
- Pretest
- Student Characteristics
- True Total Classroom Effect
- True Total School Effect
- True Total District Effect
- Stud. Error Term
- Classroom Participation Indicator
- School Participation Indicator
- District Participation Indicator

These will start to make more sense.

## What is 'value added'?

*"Value added" rates teachers based on their students' progress on standardized tests year after year. The difference between a student's expected growth and actual performance is the "value" a teacher adds or subtracts during the year.*




Source: California Standards Tests, Los Angeles Unified School District, Los Angeles Times reporting
Graphic: Los Angeles Times

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