

TESTING THE VALIDITY OF THE COUNTERFACTUAL

Jesse Leco

CORE CONCEPTS



The Selection Problem



Tests for Group Equivalence

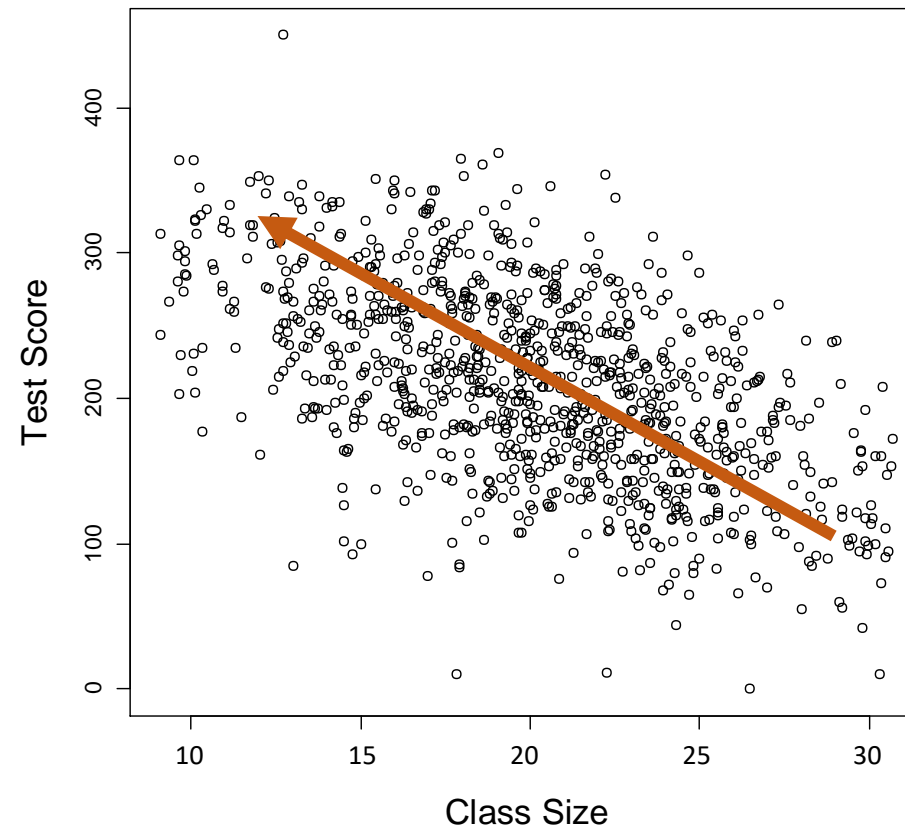
Omnibus Test
Bonferroni Correction



Tests for Nonrandom Attrition

NATURE GIVES US CORRELATIONS: THE SELECTION PROBLEM IN EVALUATION RESEARCH

If we reduce
classroom size we
should see student
performance
improve. Right?



THE MATH OF CITIES

Listen: 4:30 – 15:30

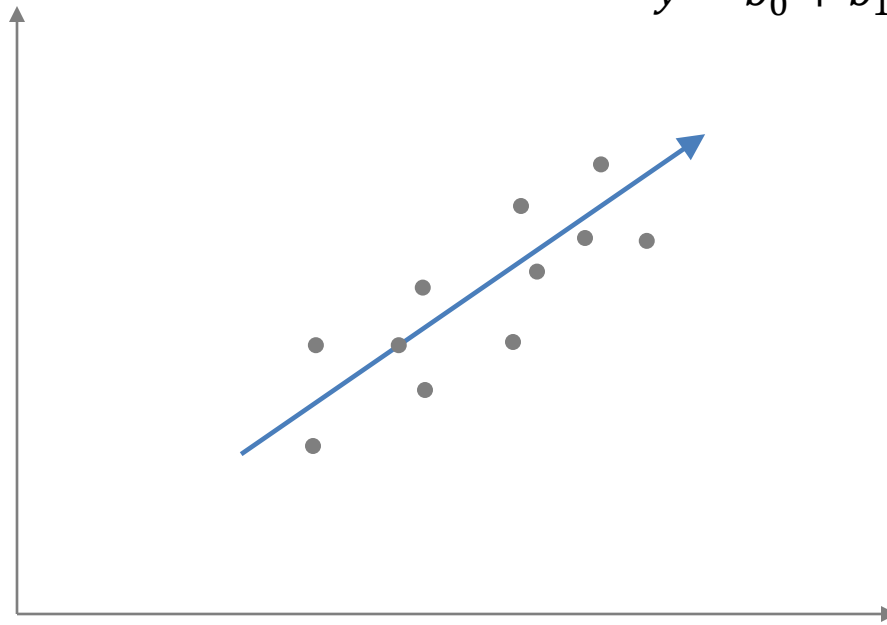
<http://www.radiolab.org/2010/oct/08/its-alive/>



THE MATH OF CITIES

Patents / Crime / Salary
(outcomes)

$$y = b_0 + b_1x$$

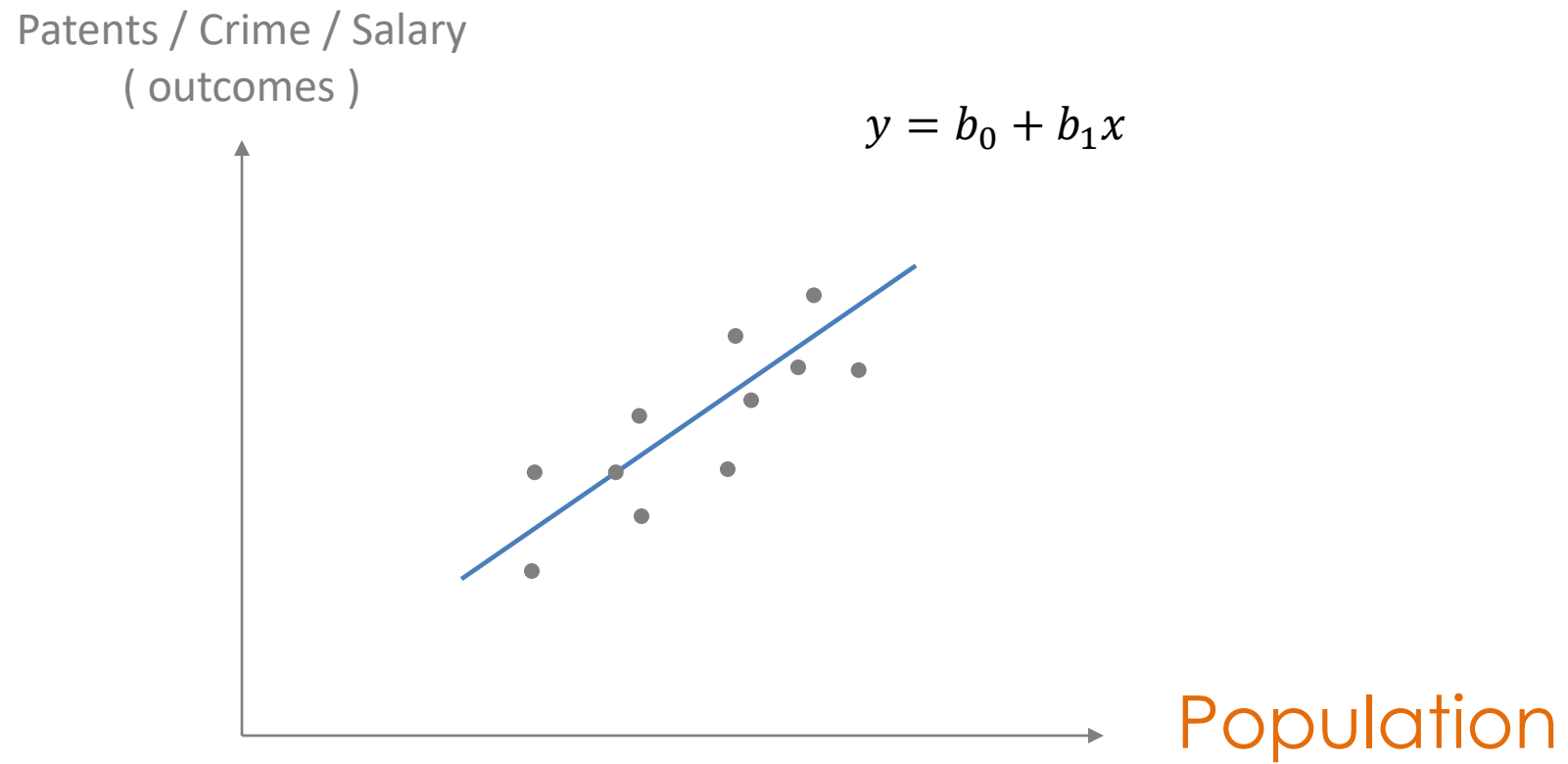


Walking Speed?

THE MATH OF CITIES

If you increase the
WALKING SPEED of a city,
would you increase the
number of patents
produced?

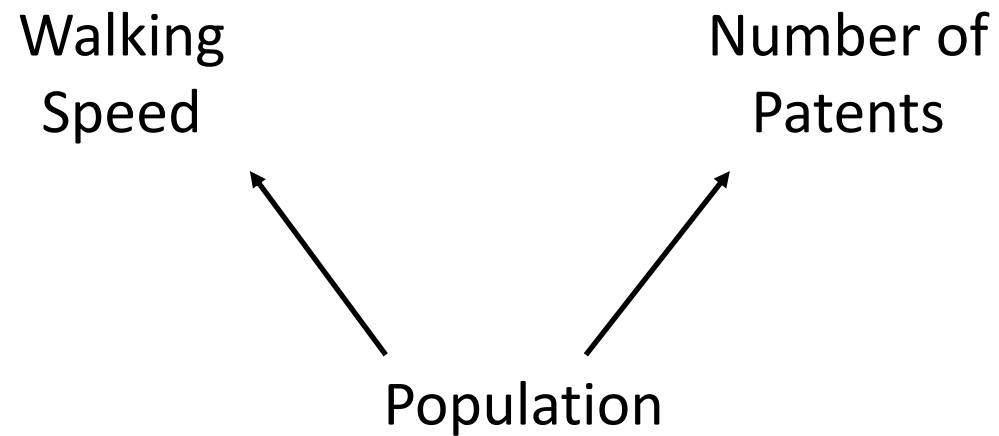
THE MATH OF CITIES



THE MATH OF CITIES

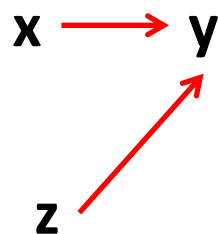
If you increase the
POPULATION of a city,
would you increase the
number of patents
produced?

The problem with correlations:

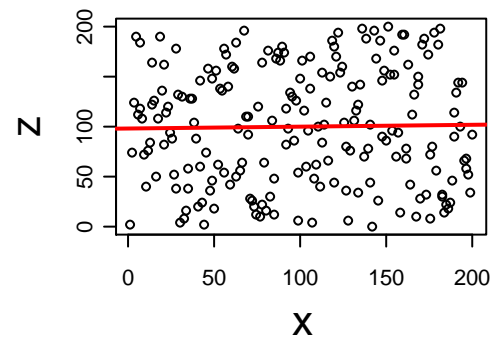
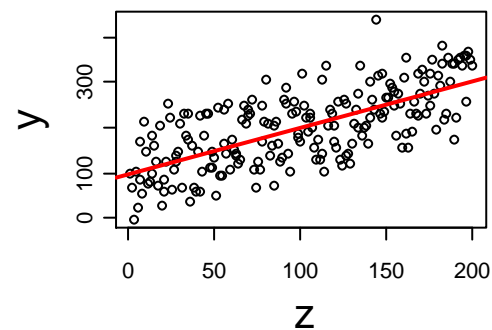
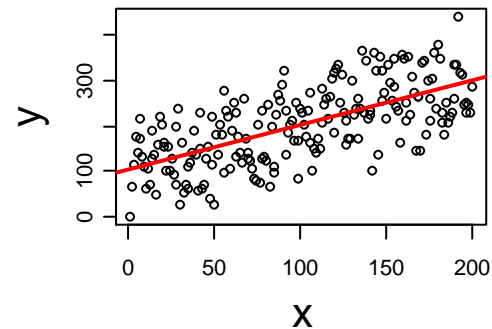


NATURE GIVES US CORRELATIONS

Example #1

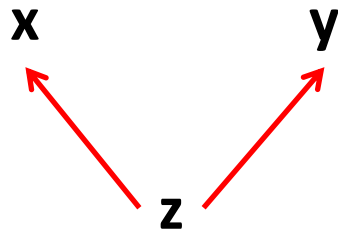


X and Z both
impact Y

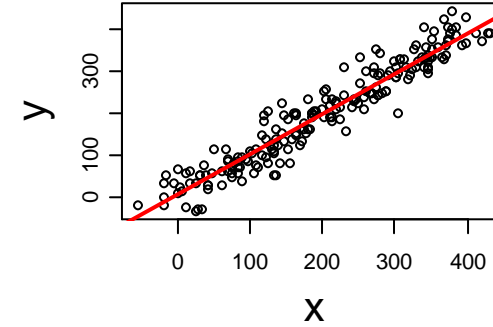


no correlation

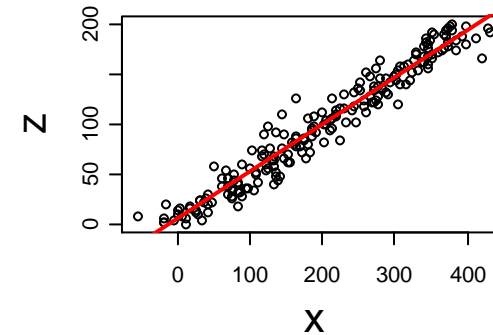
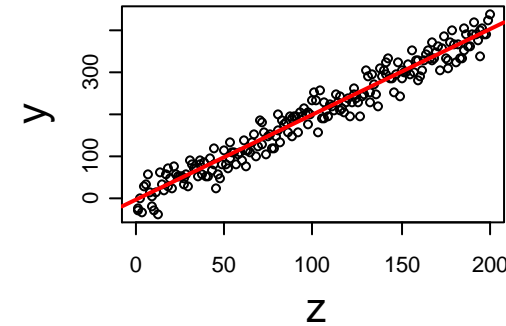
Example #2



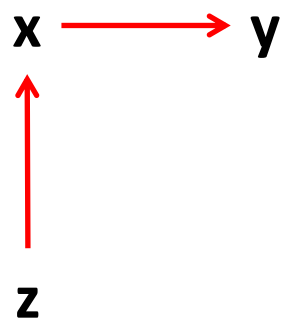
Z impacts
both X and Y



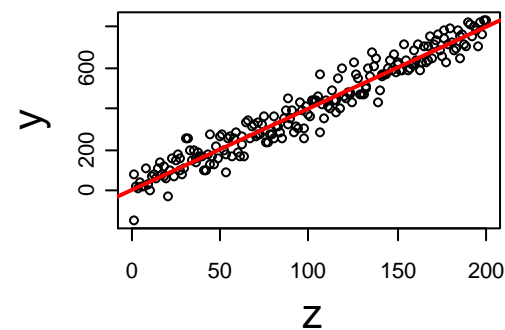
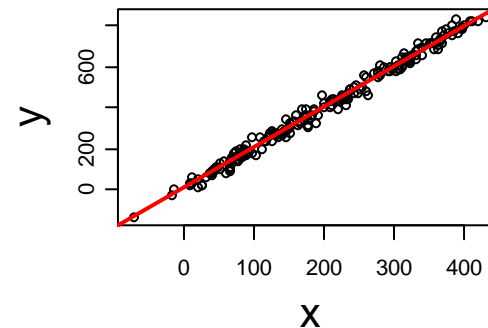
No causal
relationship but
high correlation



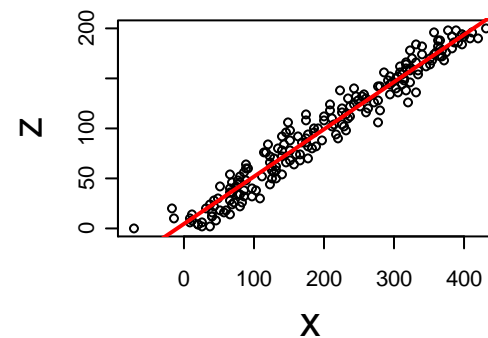
Example #3



Causal chain

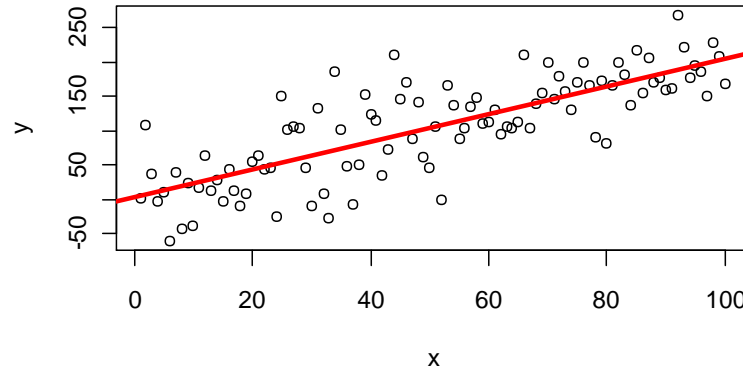


no direct causal
relationship but
highly correlated



Reverse causality

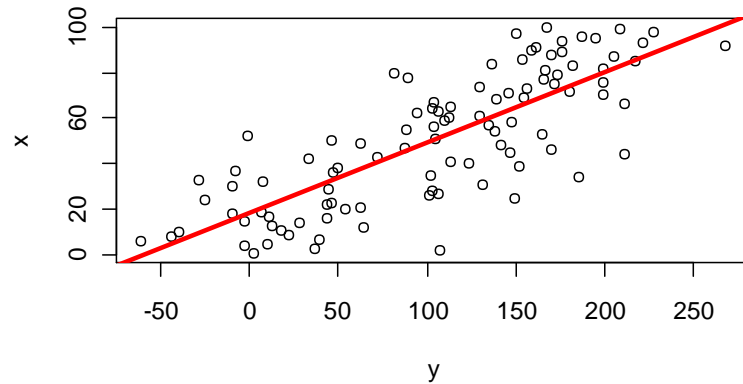
x → **y**



$$Y = b_0 + b_1X + e$$

Both models are highly significant, how do we know which one is the causal relationship?

x ← **y**



$$X = b_0 + b_1Y + e$$

THE PROGRAM EVALUATION FRAMEWORK

$$\text{HeartRate} = b_0 + b_1 \cdot \text{Caffeine} + \varepsilon$$

Treatment Effect =
typical dosage x slope b_1

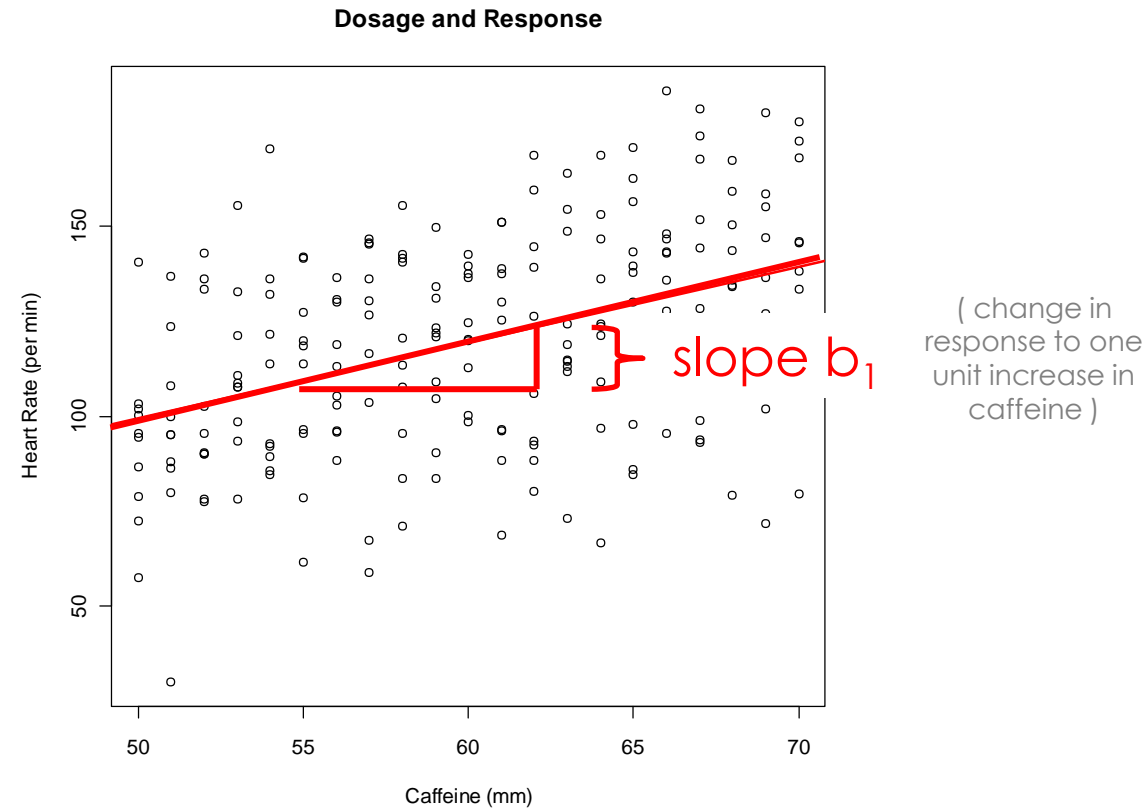
Slopes represent change in Y as a result of a one-unit change in X

Rarely is one-unit of X meaningful, so be sure to translate the slope to an appropriate dosage to generate a meaningful impact measure

If the treatment is one espresso:

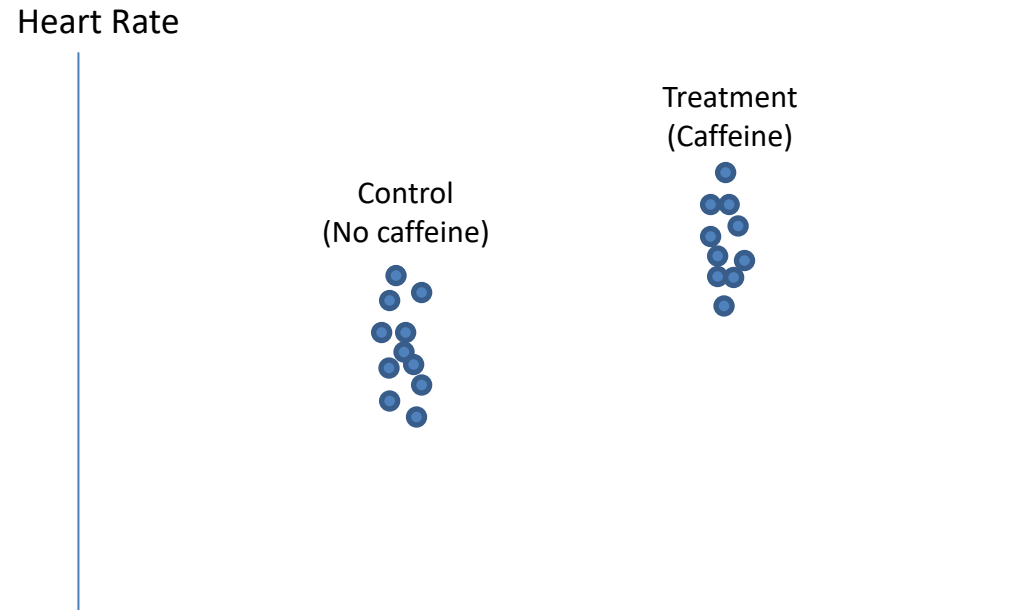
Espresso contains ~ 50mg caffeine
Heart rate increase = $(50)(b_1)$

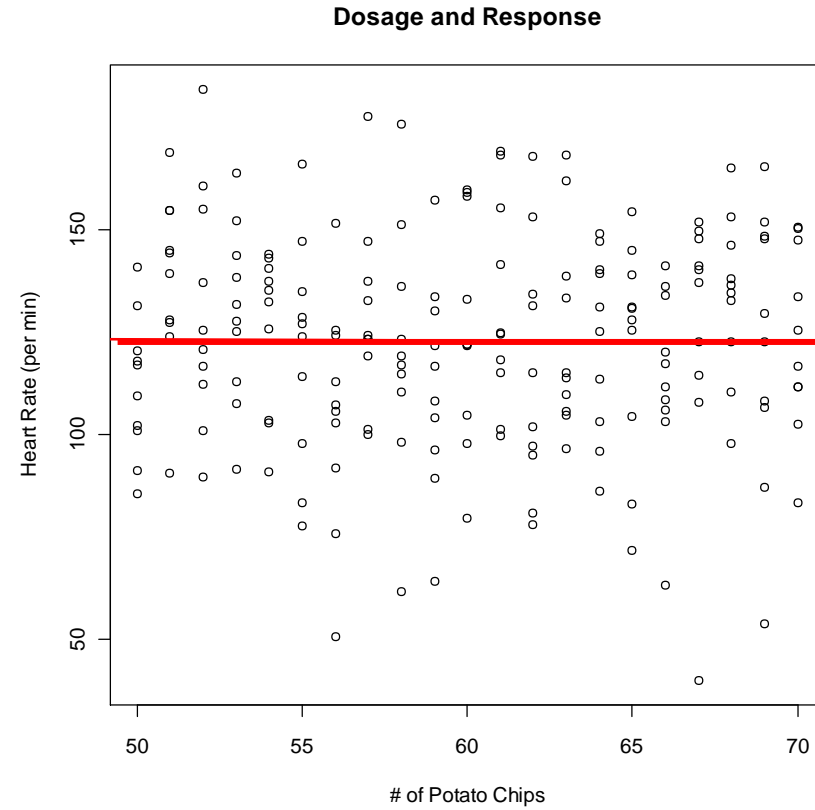
Effect Size = (dosage)(b_1)



DISCRETE TREATMENT CASE: MODEL IS THE SAME EXCEPT CAFFEINE IS A DUMMY NOT A LEVEL

$$\text{HeartRate} = b_0 + b_1 \cdot \text{Caffeine} + \varepsilon$$

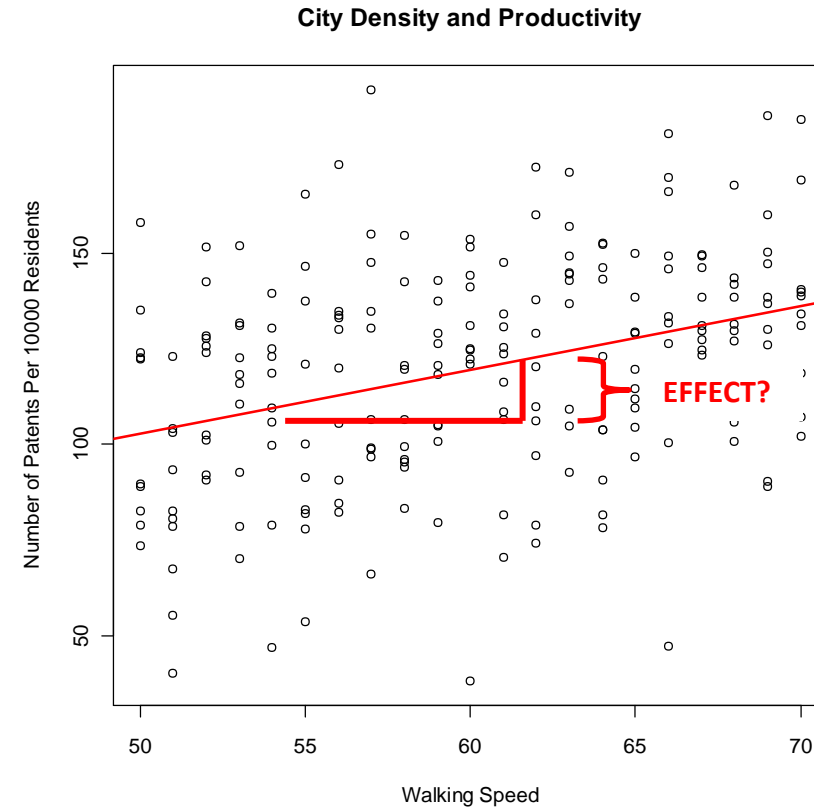
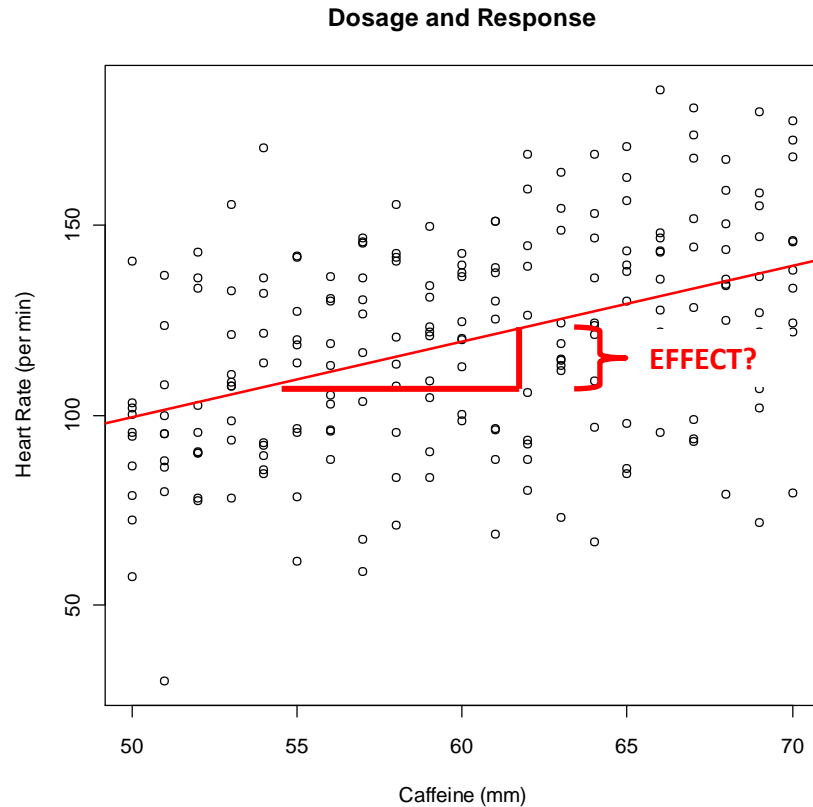




No effect of treatment
on outcome

HOW DO WE KNOW WHEN THE INTERPRETATION IS CAUSAL?

When is b_1 an impact, and when is it just a relationship in data?



THE SELECTION PROBLEM IN EVALUATION RESEARCH

Microfinance example of bias from selection INTO a study group

Number of each “type” of person in the study
(latent traits can be hard to observe thus hard to control for in a study)

	NOT Entrepreneurial	Entrepreneurial
No Loan	30	15
Takes a Loan	20	35

You are more likely to take a loan if you know you are good at business

Takes Loan?

$$NO: \frac{30 \cdot \$10 + 15 \cdot \$20}{45} = \$13.33$$

$$YES: \frac{20 \cdot \$10 + 35 \cdot \$20}{55} = \$16.37$$

Average weekly income AFTER loan period

	NOT Entrepreneurial	Entrepreneur
No Loan	\$10	\$20
Takes a Loan	\$10	\$20

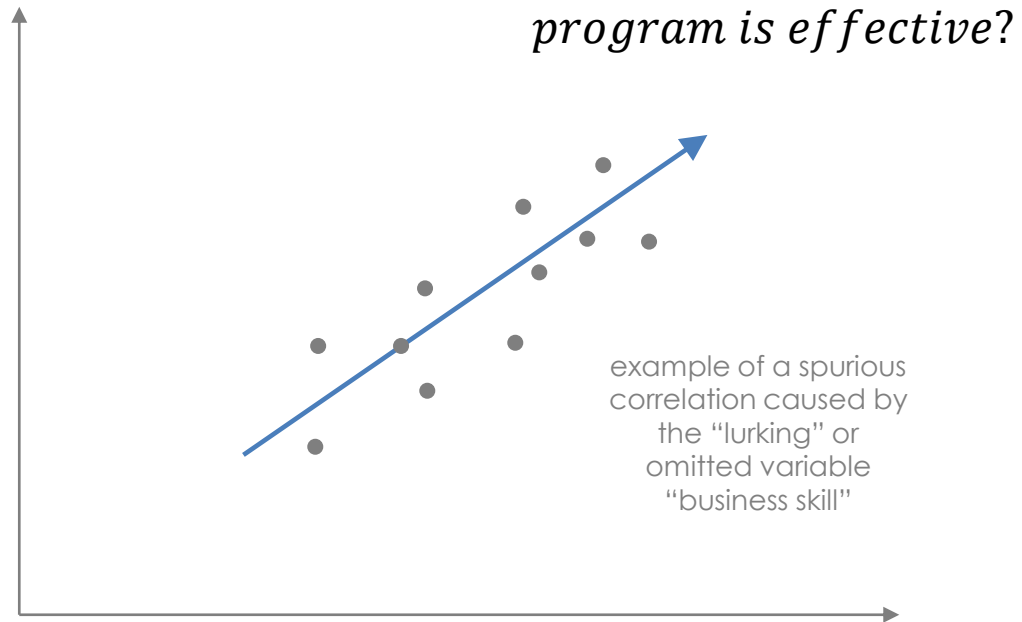
Income not impacted by the loan

The loan appears to have an impact!

Even though we know it didn't.

CORRELATION VERSION OF LAST EXAMPLE:

Income



Probability of
Taking a Loan

Selection
mechanism

probability
applies for
a loan

Income

Business
Skills

Business acumen
measured as a
level instead of a
type here

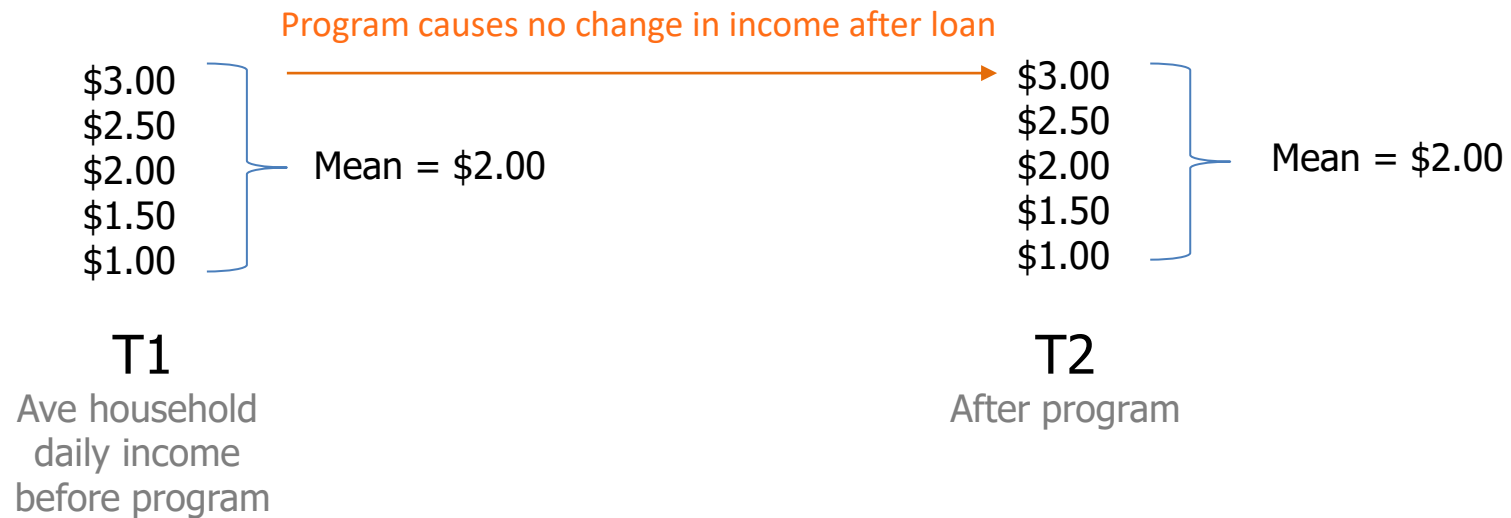
“Selection” Problem

Those that participate in the program
are different from those that do not participate.

This is the biggest problem in impact evaluation!

Microfinance example of bias from selection OUT OF a study group

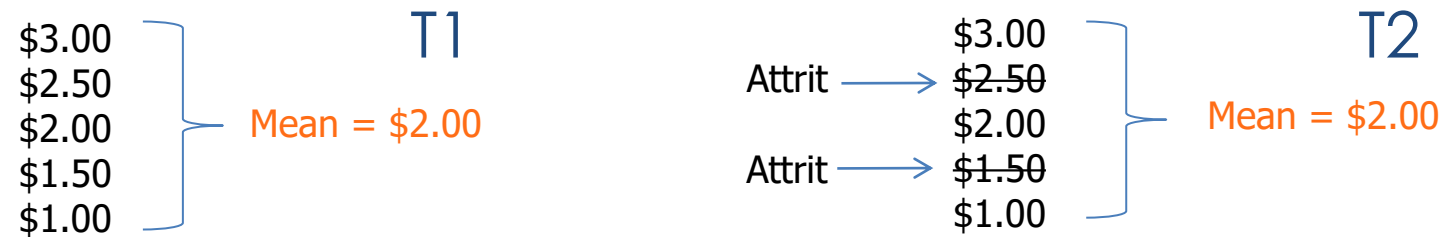
Reflexive design



$$T2 - T1 = 0$$

causal estimate is unbiased

Random Attrition Example

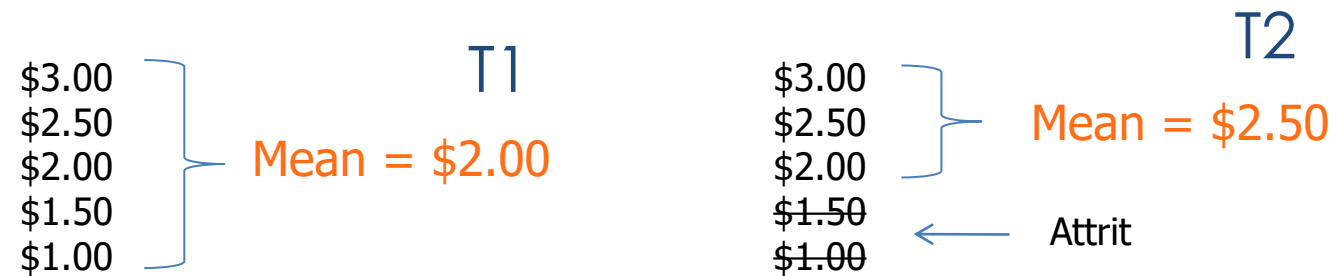


$$T2 - T1 = 0$$

Impact study accurately represents program effects
Program is not determined to be effective (no change)

(impact estimate is unbiased)

Non-Random Attrition Example



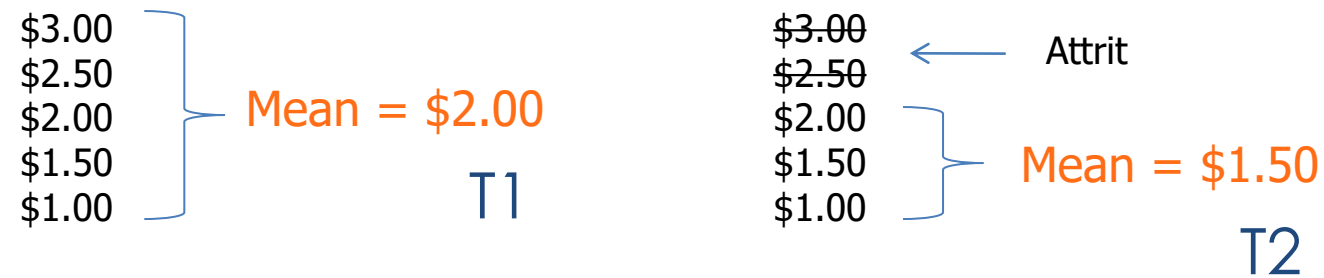
$$T2 - T1 > 0$$

We over-estimate program effects

Program appears to be effective

(impact estimate BIASED)

Non-Random Attrition Example



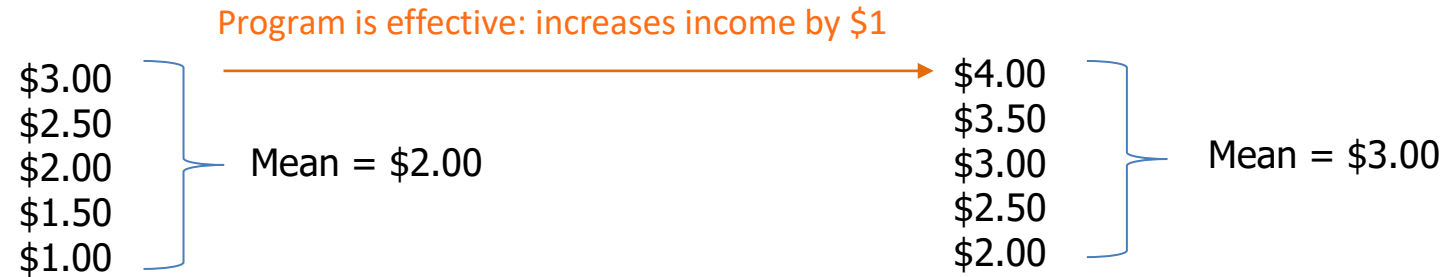
$$T2 - T1 < 0$$

We under-estimate program effects

Program appears to harm families

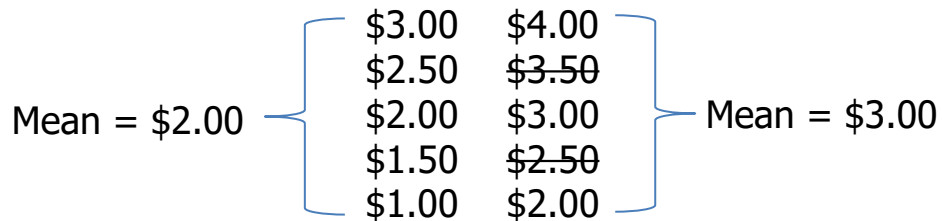
(impact estimate is BIASED)

What if the program has impact?



$$\text{True impact} = T2 - T1 = \$1.00$$

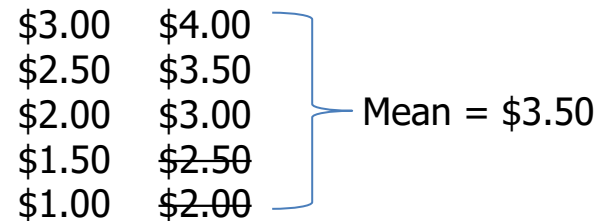
Random Attrition



$$T2 - T1 = \$1.00$$

(UNBIASED)

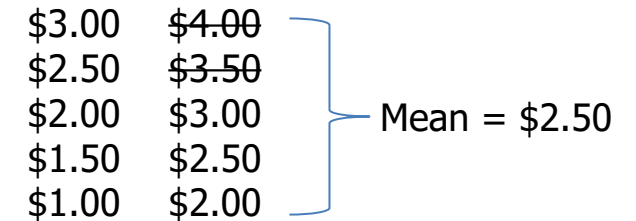
Non-Random Attrition A



$$T2 - T1 = \$1.50$$

(too big: BIASED)

Non-Random Attrition B

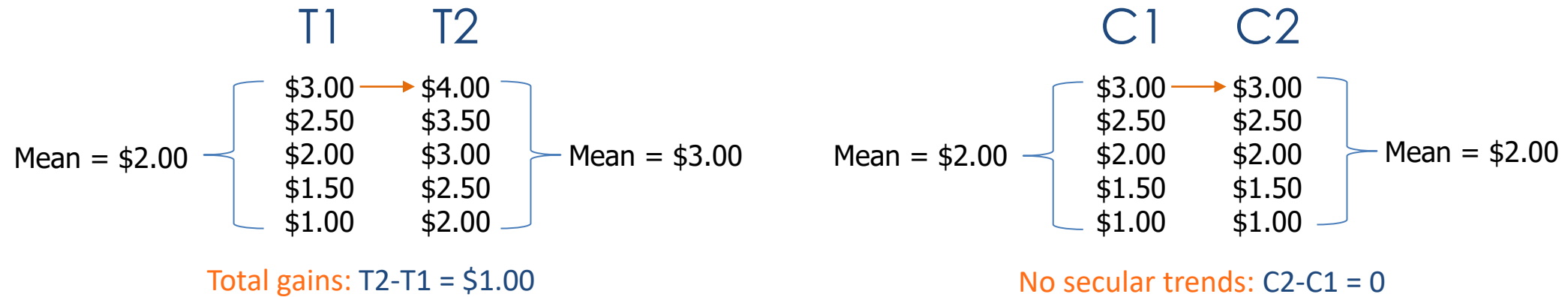


$$T2 - T1 = \$0.50$$

(too small: BIASED)

(note that a biased effect can still be non-zero or significant,
but the smaller sample size will also impact statistical power)

Attrition with two study groups



True impact: $(T2-T1) - (C2-C1) = \$1.00$

Random Attrition

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$(3.00-2.00)-(2.00-2.00) = \1.00
UNBIASED

Non-Random Attrition A

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$(3.50-2.00)-(2.50-2.00) = \1.00
UNBIASED

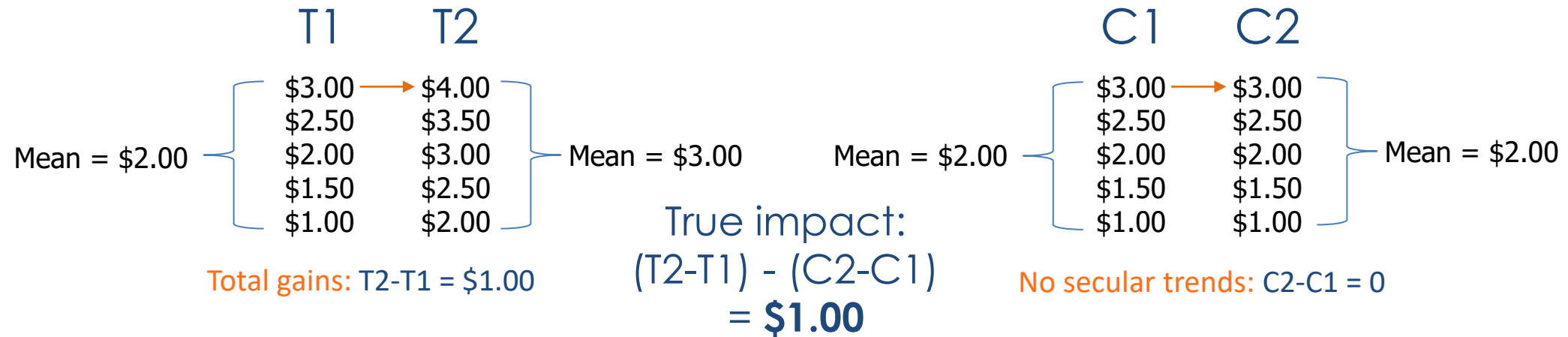
Non-Random Attrition B

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$(2.50-2.00)-(2.00-1.50) = \1.00
UNBIASED

The comparison group absorbs the impact of attrition similar to how it absorbs secular trend

Attrition with two study groups



Random Attrition

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$$T2 - T1 = (3.00 - 2.00) = \$1.00$$

UNBIASED

Non-Random Attrition A

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$$T2 - T1 = (3.50 - 2.00) = \$1.50$$

BIASED

Non-Random Attrition B

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$$T2 - T1 = (2.50 - 2.00) = \$0.50$$

BIASED

Note that we have no trend in the data so we can usually substitute the reflexive and diff-in-diff estimators ($T2 - T1 = [T2 - T1] - [C2 - C1]$), but when non-random attrition is present we cannot

Attrition with two study groups

Random Attrition

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$$(3.00 - 2.00) - (2.00 - 2.00) = \$1.00$$

UNBIASED

Non-Random Attrition A

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$$(3.50 - 2.00) - (2.50 - 2.00) = \$1.00$$

UNBIASED

Non-Random Attrition B

T1	T2	C1	C2
\$3.00	\$4.00	\$3.00	\$3.00
\$2.50	\$3.50	\$2.50	\$2.50
\$2.00	\$3.00	\$2.00	\$2.00
\$1.50	\$2.50	\$1.50	\$1.50
\$1.00	\$2.00	\$1.00	\$1.00

$$(2.50 - 2.00) - (2.00 - 1.50) = \$1.00$$

UNBIASED

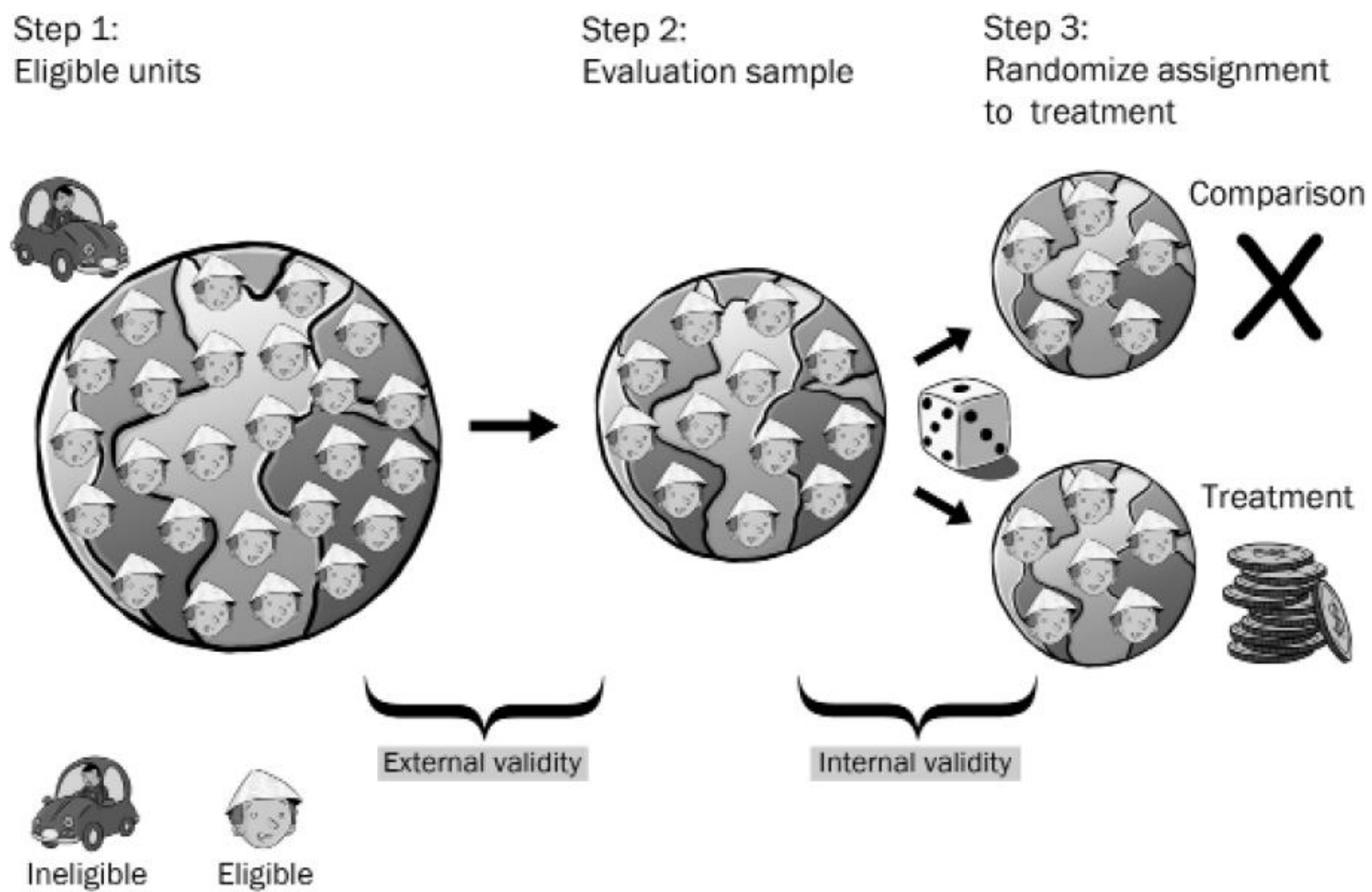
Note that this only works when the attrition process is the same in both groups (random cases, mostly high performers, or mostly low performers exiting from the treatment and study groups at the same rate)

RANDOMIZED CONTROL TRIALS (RCT'S): THE “GOLD STANDARD” FOR INTERNAL VALIDITY

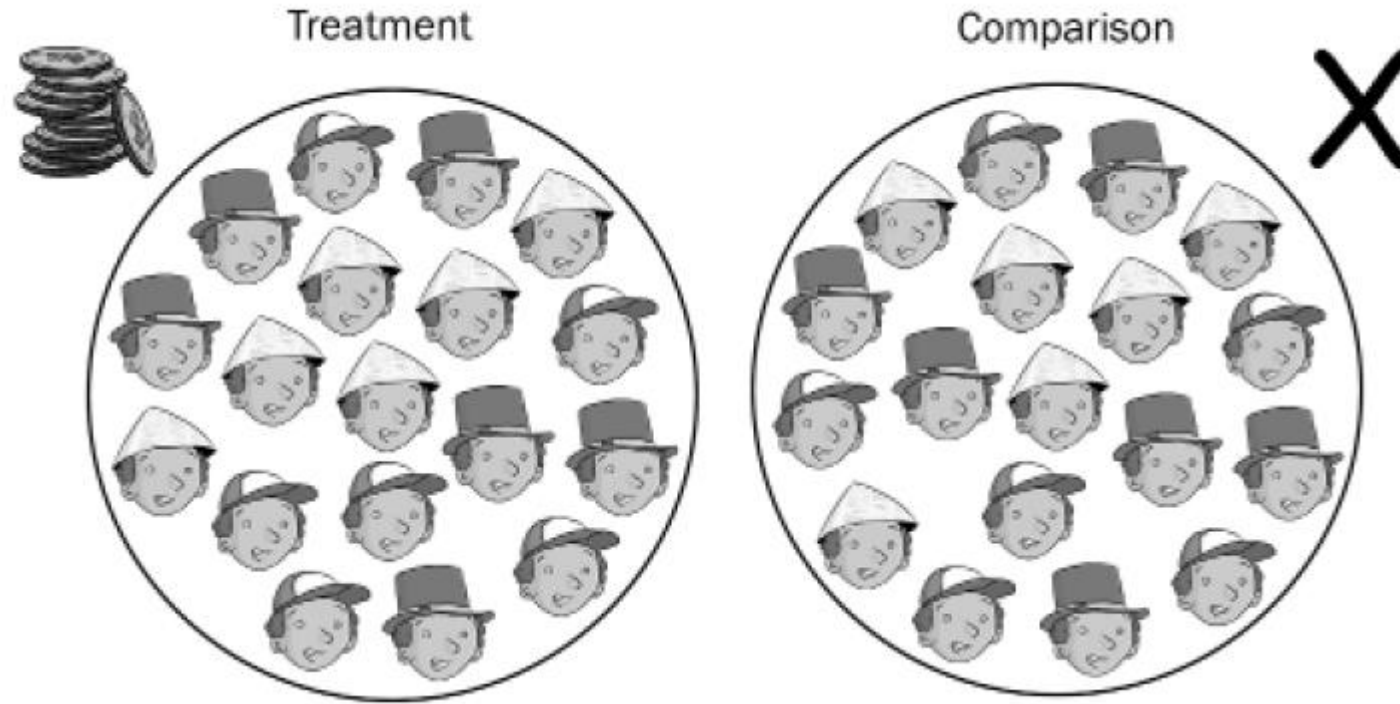
Gertler, P. J., Martinez, S., Premand, P., Rawlings, L. B., & Vermeersch, C. M. (2016). *Impact evaluation in practice*. The World Bank.

>> Chapter 4. Randomized Selection Methods

Figure 4.3 Steps in Randomized Assignment to Treatment



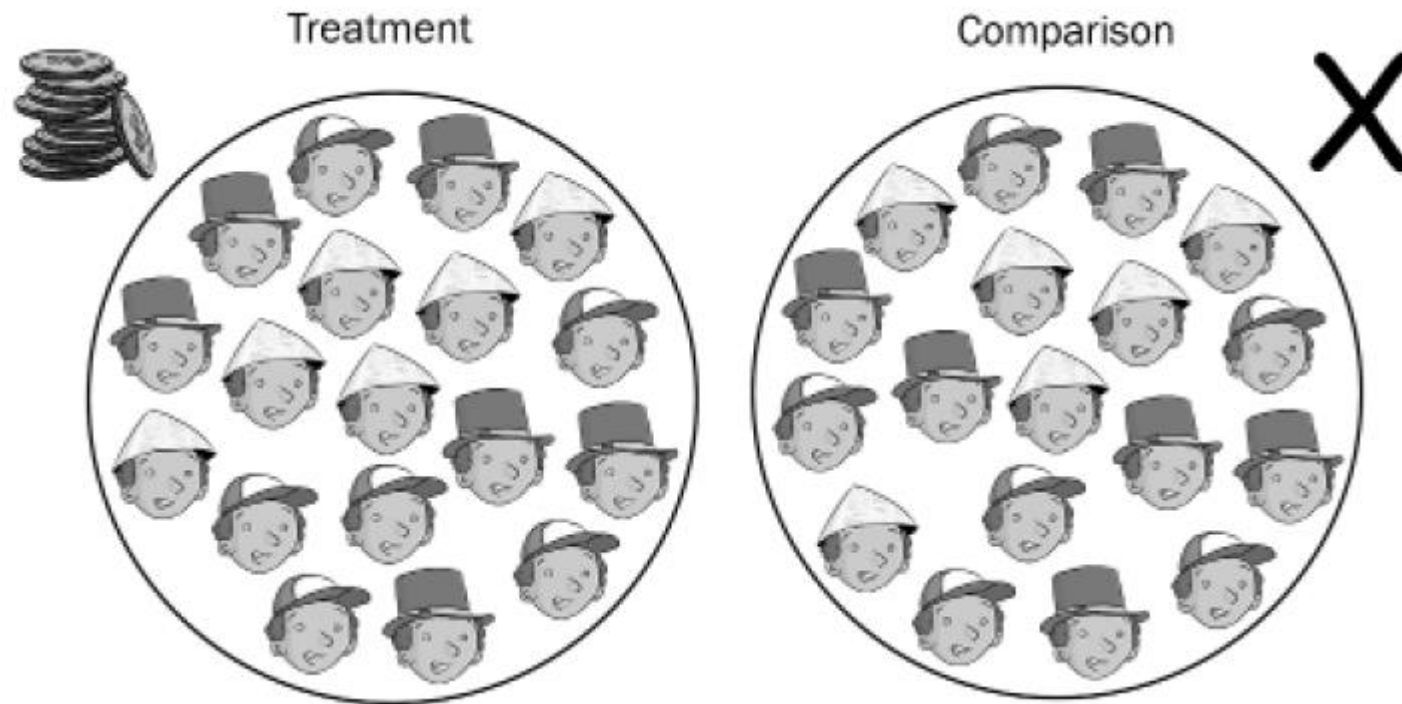
Our counterfactual framework is
valid / robust when the
groups only DIFFER BY THE TREATMENT
but are OTHERWISE “IDENTICAL”



When true, we can interpret the differences in group outcomes
after the treatment period to be caused by the treatment

How do we test the criteria:

groups only OTHERWISE "IDENTICAL" ???





Clap your hands if
your treatment
and control groups
have no significant
contrasts of
measured
participant
traits

“HAPPY” RANDOMIZATION

If we have a group of 100 people and we randomly assign them to two groups, 50 people each, how often would we expect the average weight of each group to differ?

If we have a group of 100 people and we randomly assign them to two groups, 50 people each, how often would we expect the average weight of each group to differ?

MATHEMATICALLY: ALWAYS !!!

weights will never be exactly identical

so what do we mean by “different”?

If we have a group of 100 people and we randomly assign them to two groups, 50 people each, how often would we expect the average weight of each group to differ?

STATISTICALLY:

TEST OF GROUP MEANS

Use a t-test and select a
level of confidence
that we are comfortable with

If we have a group of 100 people and we randomly assign them to two groups, 50 people each, how often would we expect the average weight of each group to differ?

STATISTICALLY:

TEST OF GROUP MEANS

What does $\alpha=0.05$ mean?
[--- 95% confidence interval ---]

If we have a group of 100 people and we randomly assign them to two groups, 50 people each, how often would we expect the average weight of each group to differ?

STATISTICALLY:

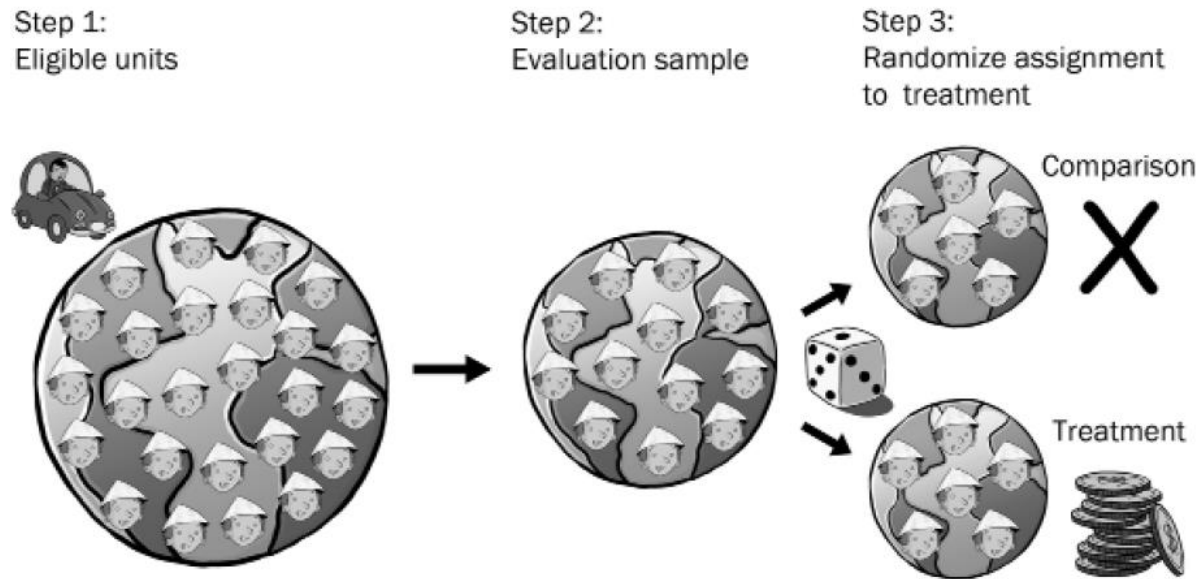
TEST OF GROUP MEANS

5 times out of 100 the two samples are drawn from the same population (or weight distribution), but we will still consider them to be different.



What does $\alpha=0.05$ mean?
[--- 95% confidence interval ---]

How often will randomization “fail”?



By definition of a test at a 95% confidence level, each measured characteristic like weight will differ

5 out of **100** times

“Unhappy Randomization”



Unhappy randomization is not failed randomization (process applied improperly), rather just bad luck of the draw

Table 4.1 Case 3—Balance between Treatment and Comparison Villages at Baseline

Household characteristics	Treatment villages (N = 2964)	Comparison villages (N = 2664)	Difference	t-stat
Health expenditures (\$ yearly per capita)	14.48	14.57	−0.09	−0.39
Head of household's age (years)	41.6	42.3	−0.7	−1.2
Spouse's age (years)	36.8	36.8	0.0	0.38
Head of household's education (years)	2.9	2.8	0.1	2.16*
Spouse's education (years)	2.7	2.6	0.1	0.006
Head of household is female = 1	0.07	0.07	−0.0	−0.66
Indigenous = 1	0.42	0.42	0.0	0.21
Number of household members	5.7	5.7	0.0	1.21
Has bathroom = 1	0.57	0.56	0.01	1.04
Hectares of land	1.67	1.71	−0.04	−1.35
Distance to hospital (km)	109	106	3	1.02

Source: Authors' calculation.
* Significant at the 5 percent level.

The most important table in every study: comparisons of treatment and control group characteristics

For the counterfactual to be **valid**, the groups can ONLY differ by the treatment, not by any measured traits.

Is this problematic?

What is the appropriate test for “identical” or equivalent groups?

We should observe no differences in measured traits.
Assume a 95% confidence interval.

Bonferroni Correction:

When we want to be 95% confident that two groups are the same, and we can measure those groups using a set of contrasts, then our decision rule is no longer to reject the null (that the groups are the same) if the p-value < 0.05 . A “contrast” is a comparison of means of any measured characteristic between two groups.

If we have a 5% chance of observing a p-value of less than 0.05 for each contrast, then the probability of observing at least one contrast with a p-value that small is greater than 5%! It is actually $n \times 0.05$ (minus prob of observing multiple < 0.05 at same time) where n is the number of contrasts.

So if we want to be 95% confident that the groups are different (not just the contrasts), we have to adjust our decision rule to α/n .

For example, if we have 10 contrasts, then our decision rule is now $0.05/10$, or 0.005 . The p-value of at least one contrast must be below 0.005 for us to conclude that the groups are different.

```
x1 <- rbinom( 10000, 6, 0.05 )  
table( x1 ) / 10000  
y1 <- rbinom( 10000, 6, 0.05/6 )  
table( y1 ) / 10000
```

Test for Group Equivalence

TABLE 2

Background Characteristics of Students in Treatment and Control Groups
(Total numbers of cases in parentheses)

Characteristic	All students in the study			All students with scores three or four years after application		
	Choice students	Control students	p value ^a	Choice students	Control students	p value ^a
→ Math scores before application	39.7 (264)	39.3 (173)	.81	40.0 (61)	40.6 (33)	.86
→ Reading scores before application	38.9 (266)	39.4 (176)	.74	42.1 (60)	39.2 (33)	.35
→ Family income	10,860 (423)	12,010 (127)	.14	10,850 (143)	11,170 (25)	.84
→ Mothers' education 3 = some college 4 = college degree	4.2 (423)	3.9 (127)	.04	4.1 (144)	3.8 (29)	.15
→ Percent married parents	24 (424)	30 (132)	.17	23 (145)	38 (29)	.11
→ Parents' time with children 1 = 1-2 hours/week 2 = 3-4 hours/week 3 = 5 or more	1.9 (420)	1.8 (130)	.37	1.9 (140)	1.7 (27)	.26
→ Parents' education expectations of children 4 = college 5 = graduate school	4.2 (422)	4.2 (129)	.85	4.2 (142)	3.7 (27)	.01

a. The tests of significance are suggestive of the equivalence of the two groups. Technically, tests of significance should be done at each point of random assignment, but the number of cases at each point is too few for such tests to be meaningful.

seven
contrasts
reported

Smallest p-value in table

0.04 > 0.0071

New alpha = (0.05 / 7) = 0.0071

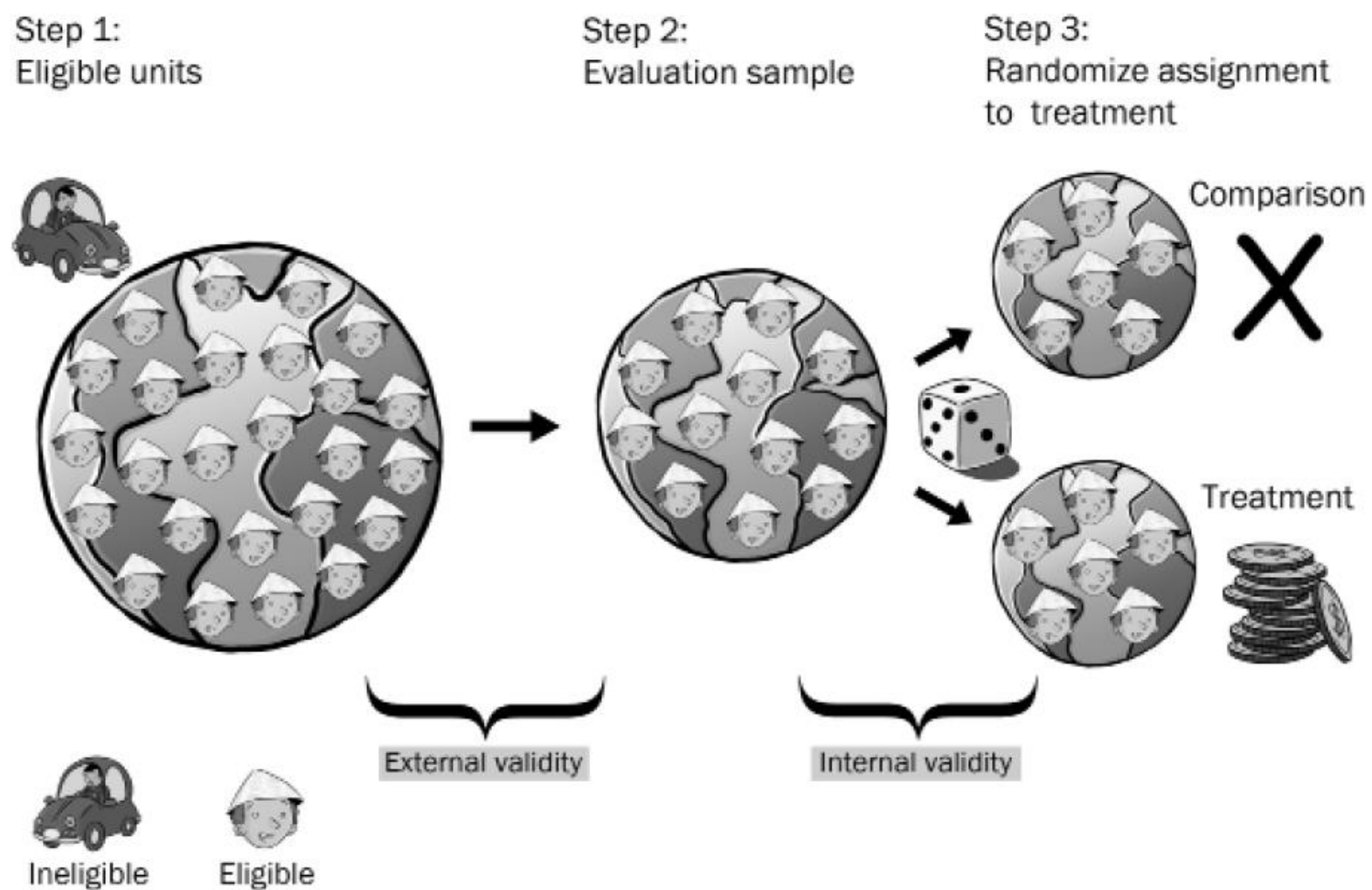
Do not reject :: Groups are equivalent

RCT versus Natural Experiments:

- 1. RCT assumes complete control over the assignment process**
- 2. Natural Experiments often utilize randomization:**
 - Charter School lotteries
 - Vietnam draft
- 3. Quasi-Experimental techniques can use other methods to create group equivalence (for example, matching)**

ATTRITION

Figure 4.3 Steps in Randomized Assignment to Treatment



Tests for Selection-Into Study Group

TABLE 2

Background Characteristics of Students in Treatment and Control Groups
(Total numbers of cases in parentheses)

Characteristic	All students in the study			All students with scores three or four years after application		
	Choice students	Control students	p value ^a	Choice students	Control students	p value ^a
Math scores before application	39.7 (264)	39.3 (173)	.81	40.0 (61)	40.6 (33)	.86
Reading scores before application	38.9 (266)	39.4 (176)	.74	42.1 (60)	39.2 (33)	.35
Family income	10,860 (423)	12,010 (127)	.14	10,850 (143)	11,170 (25)	.84
Mothers' education 3 = some college 4 = college degree	4.2 (423)	3.9 (127)	.04	4.1 (144)	3.8 (29)	.15
Percent married parents	24 (424)	30 (132)	.17	23 (145)	38 (29)	.11
Parents' time with children 1 = 1–2 hours/week 2 = 3–4 hours/week 3 = 5 or more	1.9 (420)	1.8 (130)	.37	1.9 (140)	1.7 (27)	.26
Parents' education expectations of children 4 = college 5 = graduate school	4.2 (422)	4.2 (129)	.85	4.2 (142)	3.7 (27)	.01

a. The tests of significance are suggestive of the equivalence of the two groups. Technically, tests of significance should be done at each point of random assignment, but the number of cases at each point is too few for such tests to be meaningful.

Non-Random Attrition

If the people that leave a program or study are different than those that stay, the calculation of effects will be biased.

The Fix:

Examine characteristics of those that stay versus those that leave.

Non-Random attrition tests for selection OUT of the study group

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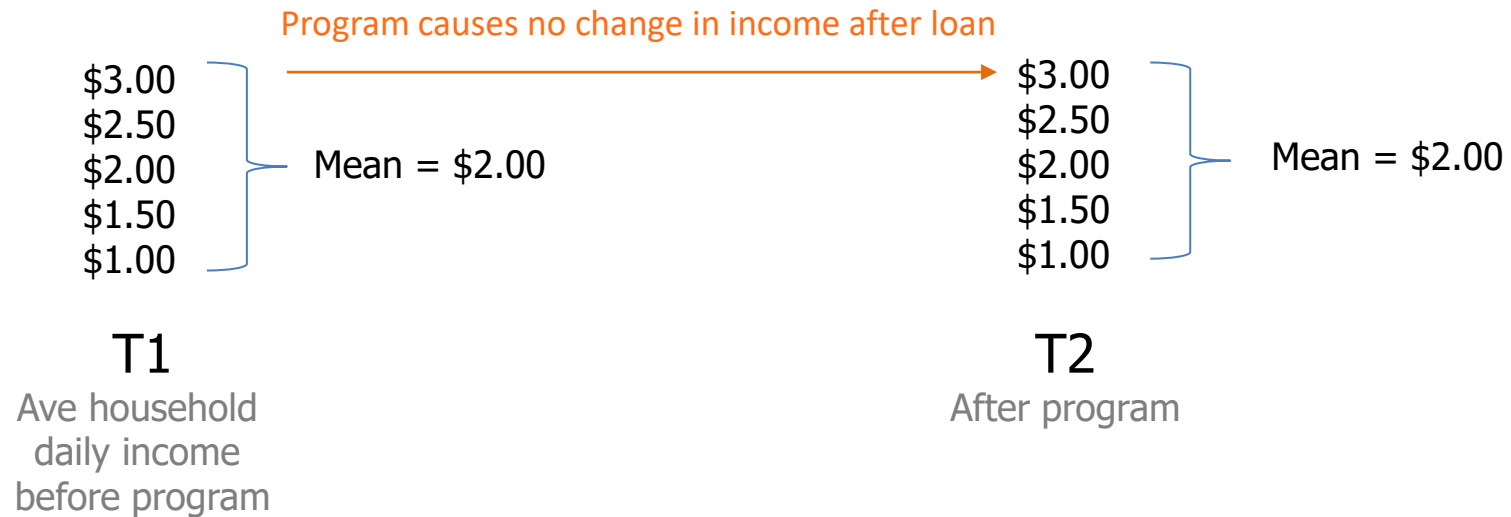
Do group traits differ after attrition occurs?

Attrition is natural, the question is whether it is random (will not change the groups) or non-random (will change the groups)

Can also be tested by comparing traits of those that stay to those that leave.

Microfinance example of bias from selection OUT OF a study group

Reflexive design



$$T2 - T1 = 0$$

causal estimate is unbiased

Random Attrition Example



$$T2 - T1 = 0$$

Impact study accurately represents program effects
Program is not determined to be effective (no change)

Non-Random Attrition Example

\$3.00
\$2.50
\$2.00
\$1.50
\$1.00

Mean = \$2.00

\$3.00
\$2.50
\$2.00
\$1.50
\$1.00

Mean = \$2.50

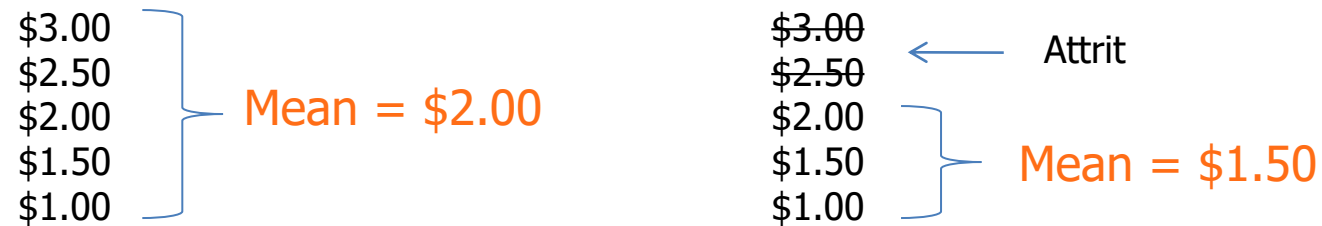
← Attrit

$$T2 - T1 \neq 0$$

We over-estimate program effects

Program appears to be effective

Non-Random Attrition Example



$$T2 - T1 \neq 0$$

We under-estimate program effects

Program appears to harm families

WHAT DO WE MEAN BY “TREATED”?

HOW NON-COMPLIANCE
CHANGES OUR
MEASURE OF
EFFECTS

One of The Physicists Behind The Higgs Boson Has Made an Algorithm to Replace The Pill

It's up to 99.5% effective at stopping pregnancy.

Those more effective methods are ones that don't require people to remember to take a pill, put on a condom, or record their temperature daily, such as intrauterine contraception or implants.

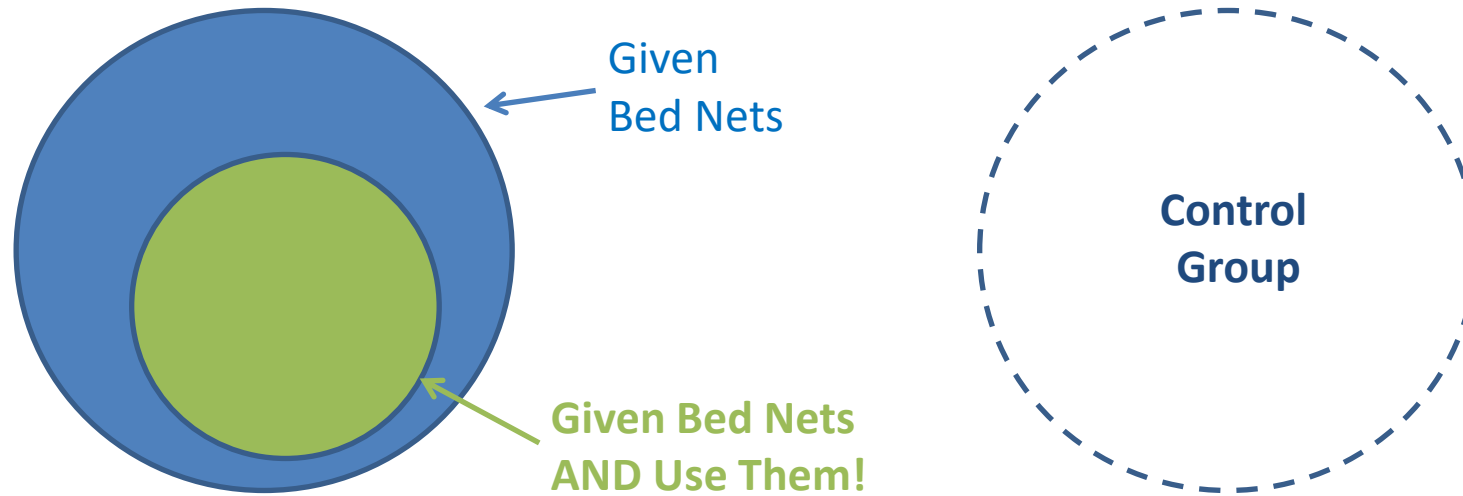
That's because human error can mess with things quite a lot. In fact, the UK's National Health Service (NHS) explained that **when the app was used perfectly all the time**, only five out of every 1,000 women would fall pregnant every year - a rate slightly better than the pill (**99.5 percent**).

But for "**typical use**" - **where the app isn't used entirely correctly every day** - it's more likely that seven out of every 100 women would experience accidental pregnancies, which is around **93 percent** efficiency.

What is the TRUE effectiveness of the app?

99.5 percent, or 93 percent?

Estimation of the counter-factual:

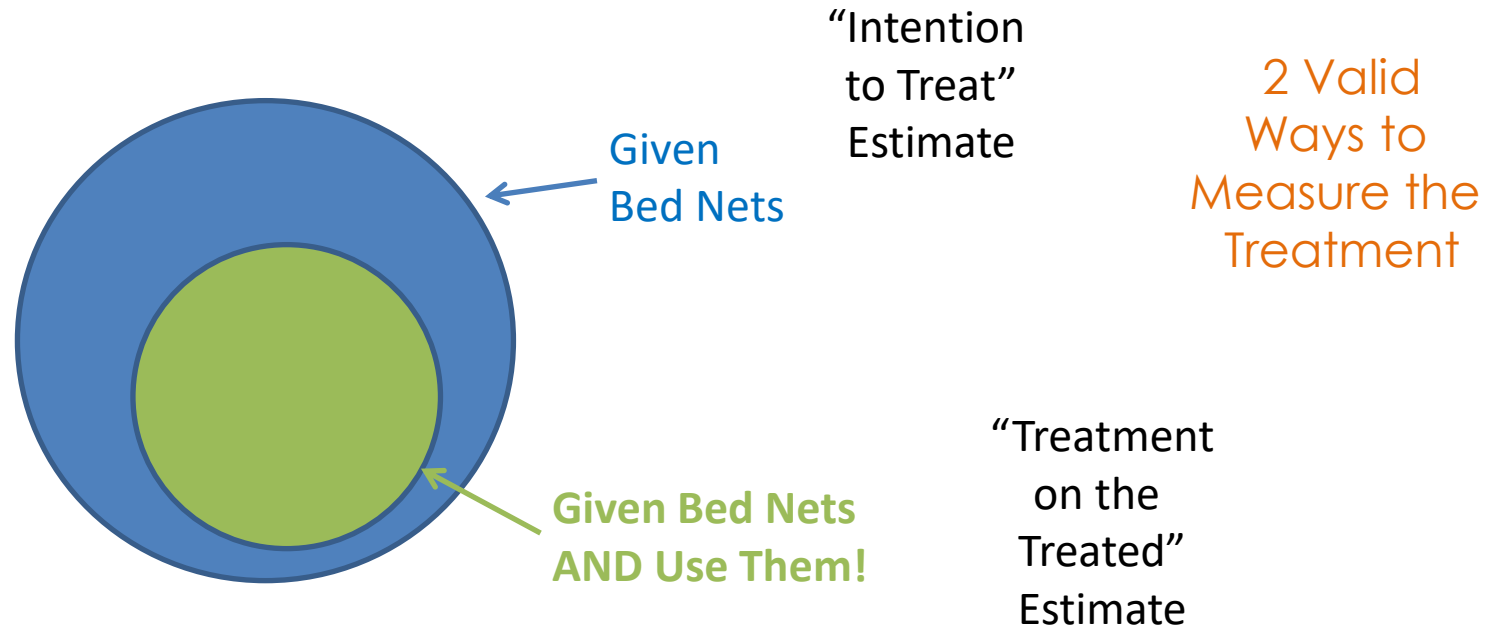


$$\text{Program Effect} = T2 - C2$$

Is Group T (for treatment) those that are GIVEN bed nets, or those that USE them?

TERMINOLOGY:

- “Average” treatment effects
 - Treatment on the Treated (TOT) Effects
 - Intention to Treat (ITT) Effects

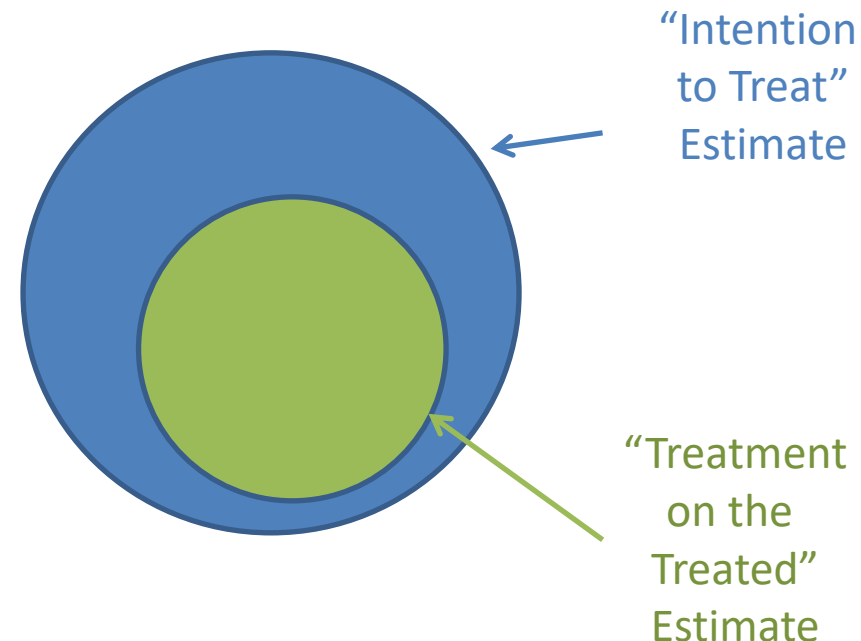


WHY DO WE NEED TWO MEASURES?

THE TOT MEASURE is the optimistic or best-case scenario. It tells us how effective the program or intervention is when followed with HIGH FIDELITY.

THE ITT MEASURE is the more cynical or realistic version. We never expect that programs work exactly as designed. The ITT is closer to a measure of how the laboratory equipment works once it's in the field. It is also a better estimate of how much change we can expect at the population level.

The difference between the TOT and the ITT tells us how many gains can be made by improving program implementation! So both are useful and important!



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Which is treatment on the treated effect?

Which is the intention-to-treat effect?

NOTE !!!

These two measures are **NOT** about attrition.

They are about **refusing the treatment** or failing to follow the properly-prescribed treatment regiment (or one school burning down during the study).

Non-compliance means participants refuse to receive or comply with the treatment. But they do not refuse to participate in the study – **we can still measure their performance in the second time period.**

Attrition means people left the study. You can accept the treatment and still attrit. Or you can refuse treatment and refuse participation. Either way, **attrition means we can't measure the outcome in the post-treatment study period.**

