Data Structures for Task-based Priority Scheduling*

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Abstract

Many task-parallel applications can benefit from attempting to execute tasks in a specific order, as for instance indicated by priorities associated with the tasks. We present three lock-free data structures for priority scheduling with different trade-offs on scalability and ordering guarantees. First we propose a basic extension to work-stealing that provides good scalability, but cannot provide any guarantees for task-ordering in-between threads. Next, we present a centralized priority data structure based on k-fifo queues, which provides strong (but still relaxed with regard to a sequential specification) guarantees. The parameter k allows to dynamically configure the trade-off between scalability and the required ordering guarantee. Third, and finally, we combine both data structures into a hybrid, k-priority data structure, which provides scalability similar to the work-stealing based approach for larger k, while giving strong ordering guarantees for smaller k. We argue for using the hybrid data structure as the best compromise for generic, priority-based task-scheduling.

We analyze the behavior and trade-offs of our data structures in the context of a simple parallelization of Dijkstra's single-source shortest path algorithm. Our theoretical analysis and simulations show that both the centralized and the hybrid k-priority based data structures can give strong guarantees on the useful work performed by the parallel Dijkstra algorithm. We support our results with experimental evidence on an 80-core Intel Xeon system.

Keywords: Task-parallel programming, priority scheduling, k-priority data structure, work-stealing, parallel single-source shortest path algorithm

1 Introduction

Parallel tasks is a convenient parallel programming pattern for exposing independent work-units that can be scheduled over multiple processing elements. The popular work-stealing paradigm [4] is an efficient way to schedule such parallel work-loads of independent tasks, and forms the basis for well-known frameworks such as Cilk++ [12], Intel Threading Building Blocks (TBB) [11] and X10 [5]. Some task-parallel systems, like TBB and StarPU [3], support assigning priorities to tasks to influence the task execution order, with priorities typically restricted to a small number of discrete values. Some applications that rely on priority scheduling [16] resort to their own centralized scheduling scheme, based on a shared priority queue. However, it can be argued that shared priority queues are not necessarily a good solution for the priority scheduling

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problem [13]. Other schemes, that rely on decentralized priority queues, cannot provide any guarantees on the execution order of tasks in-between different threads [19, 20].

In this work we present three designs of lock-free data structures for priority scheduling, each with different trade-offs concerning scalability and scheduling guarantees. The designs include a priority work-stealing data structure, a centralized data structure inspired by k-fifo queues [10] with k-relaxed semantics, as introduced by Afek et al. [1], and a hybrid data structure combining both ideas. The designs support choosing the value of k per task, allowing kernels with different ordering requirements to coexecute. Using the single-source shortest path problem as an example, we show how the different approaches affect the prioritization and show how bounds on the number of examined nodes can be given. We argue that priority task scheduling allows for an intuitive and easy way to parallelize the otherwise hard to efficiently parallelize single-source shortest path problem. Experimental evidence supports the good scalability of the resulting algorithm.

The larger aim of this work is to understand trade-offs between priority guarantees and scalability in task scheduling systems. We show that ρ -relaxation is a valuable technique for improving scalability while still providing semantic guarantees. The lock-free, hybrid k-priority data structure shows that data structures can be implemented that have scalability on par with work-stealing, while at the same time providing strong priority scheduling guarantees, depending on the value used for k. Our theoretical results open up possibilities for even more scalable data structures due to further relaxations that do not influence the bounds. A C++ implementation of our data structures and applications is available for download as part of the open source task-scheduling framework Pheet [19, 20, 22]¹.

2 The model

The focus of this work is the presentation and evaluation of data structures for task scheduling with priorities. The model used is the async-finish model, which is well-known from X10 [5] and other task-based programming models, where new tasks can be spawned throughout the execution of a task. Tasks can be synchronized using finish regions. A finish region is a blocking synchronization primitive, where execution can only continue after all tasks transitively spawned inside the finish region have been executed.

We extend the task model to support priority scheduling. Our model of priority scheduling relies on a comparison operator between tasks, which can be specific to an application/algorithm. We call the comparison operator the *priority function* for the remainder of the paper. Our framework allows the programmer to store application-specific information alongside a task, to be used in the priority function. As an example, in the single-source shortest path application used for the evaluation in Section 5, each task represents a single node relaxation. The priority function for this application uses the length of the shortest path found so far for each node, and prioritizes nodes with smaller distance values, similar to Dijkstra's algorithm. The model was described in our previous work [19], which discusses programmability aspects with other example applications.

Our scheduling system relies on help-first scheduling [8], where newly spawned tasks are stored for later execution by any thread, and the current thread proceeds with the continuation. This can be contrasted with work-first scheduling, where the continuation is stored for later execution and the newly spawned task is executed by the current thread. Work-first scheduling has better space bounds for general task scheduling, but it is not feasible for priority scheduling since it relies on a fixed order in which tasks are executed (depth-first). Instead, a priority function has to be chosen for tasks that gives bounds on the number of concurrently available tasks. For many applications, like the single-source shortest path application used in this paper, the intuitive prioritization scheme inherently has bounds on the number of concurrently available tasks.

In our model, the task scheduling system has multiple threads of execution, each with its own supporting data structures. We use the term *place* to denote a single thread of execution and its supporting local data structures. Whenever a place is idle, it retrieves a task that has been stored for later execution and executes it until it is finished. The scheduling system terminates when all tasks have finished executing and no new

¹http://www.pheet.org

tasks were created.

2.1 Data structure model

In this paper we discuss three different approaches for how a priority data structure for storing tasks in our model can be implemented. All three data structures rely on the same interface to interact with the scheduling system. Each data structure consists of a *centralized*, global component that is shared by all places, and which is accessed by every place in the same manner. In addition, each place stores a separate, *local* component of the data structure. This allows for asymmetric access schemes, where the *owner* of the local component (the thread associated with the place) is the only thread allowed to perform specific operations, thereby allowing for simpler synchronization schemes.

The scheduling system interacts with the data structure using two functions, push and pop. Both functions are executed in the context of a specific place, therefore giving access to the local component of the priority data structure for the given place. The function push is called whenever a new task is spawned, and stores the new task in the data structure for later execution. The function pop returns a task and deletes it from the data structure. Each task that has been added to the data structure will be returned by pop exactly once. We allow pop to spuriously fail as long as another thread is making progress. The task returned by pop does not necessarily have to be the highest priority task. The guarantees on the ordering of tasks provided are specific to the data structure implementation and are discussed in Section 5.

2.2 ρ -relaxation

In order to improve the scalability of the proposed data structures, we adopt a ρ -relaxation scheme, as introduced by Afek et al. [1], which is a temporal property that allows certain items in the data structure to be *ignored*. We say an item is ignored whenever an item of lower priority is returned by a pop operation.

The centralized k-priority data structure presented in this work satisfies ρ -relaxation in the following sense: a pop operation is allowed to ignore the last k items added to the data structure, which, in the worst case, might be the top $\rho = k$ by priority.

On the other hand, pop operations for the hybrid k-priority data structure are allowed to ignore the last k items added by each thread, which implies that, being P the number of threads, up to $\rho = Pk$ items might be ignored in total.

3 Data structures

In this section we give a high level description of the priority data structures for task scheduling compared in this paper.

3.1 Work-stealing

One approach that we evaluate is to adapt work-stealing to priority scheduling, by using priority queues instead of standard deques, similar to an approach presented in previous work of the authors [19, 20]. This preserves the scalability of work-stealing, while imposing local prioritization on tasks. Due to the decentralized nature of work-stealing, where each thread is only aware of its own tasks, no global priority ordering can be established. Therefore, no guarantee can be given on the priority of tasks that are being executed.

Our implementation of work-stealing uses a local priority queue per place, which is used by the **push** and **pop** operations to store and prioritize tasks. When the **pop** operation is called on a place where the priority queue is empty, it chooses a random place and steals half the tasks from that place's priority queue. Stealing half the tasks allows tasks that are generated at one place to quickly spread throughout the system [9].

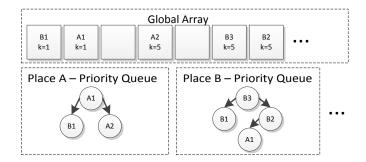


Figure 1: Centralized k-priority data structure. Each place maintains its own priority queue with references to items in the global array. The newest (rightmost) items in the global array are only visible to the place that created them.

3.2 Centralized k-priority data structure

The straightforward way to maintain strong guarantees on the priority of tasks is to use a data structure with the semantics of a centralized, global priority queue. Each push and pop operation directly communicates with the priority queue. It has been shown that a centralized, global priority queue exhibits a lot of congestion when used in a scheduling system [13], since all threads try to access the highest priority task. To reduce congestion, we use a ρ -relaxation scheme as described in Section 2.2.

3.3 Hybrid *k*-priority data structure

The hybrid k-priority data structure combines the work-stealing and the ρ -relaxation ideas into a single data structure. The main idea is that each place maintains its own, local priority queue, and that synchronization is only performed if either a place runs out of work, or if the guarantees provided by ρ -relaxation are violated.

4 Implementations

In this section we provide the implementations of both the k-priority data structures. Since the k-priority data structures constitute the main contribution of the paper, we omit the details of the work-stealing data structure.

4.1 Centralized k-priority data structure

The basic idea of the centralized k-priority data structure is to create a global priority ordering between all the tasks available in the system, while allowing each thread to miss up to k of the newest tasks, as long as each task is seen by at least one thread. To achieve such a ρ -relaxation, we split the data structure into two components. One component, a global, shared array, is used to share tasks between all threads and to maintain information about which tasks must be globally visible so as not to violate the k-requirement. Randomization is used to improve scalability when adding elements to the global array. The other component, which consists of local priority queues for each place (thread), is used to maintain the priority ordering of the tasks visible to each place. This is depicted in Figure 1. Any sequential implementation of a priority queue can be used for the local priority queues, since each priority queue is only accessed in the context of a single place, and therefore only by a single thread.

4.1.1 The push operation

The push operation stores the task, together with some additional information, in a structure which we call an *item*. For an item to be visible to all threads, it needs to be added to the global array. The items in the

Listing 1 Pseudocode for push in the centralized k-priority data structure.

```
void push(Place place, int k, Task task) {
   Item it = new Item(place, k, task);
    // Attempt until successful
    while(true) {
     int t = tail;
     // Choose a random offset at which to put item
     int offset = rand(0, k - 1);
10
     // try all indices in k-range starting at offset
11
     for(int i = offset; i < offset + k; ++i) {</pre>
12
        int pos = t + (i \% k);
13
       // A tag of -1 refers to a taken item. We store pos
        // in the tag field to omit the ABA problem
15
       it.tag = pos;
16
        // Try to put item into global array
17
        if(CAS(global_array[pos], null, it)) {
18
         // Item was succesfully put into array
19
         // Now put a reference into local priority queue
20
         ItemRef ref = new ItemRef(pos, it);
         place.prio_queue.push(ref);
22
         return;
23
     }}
24
25
     // No more free slot found, try updating tail
26
      // One thread will succeed, no need for checking which
     CAS(tail, t, t + k);
28
29 }}
```

global array are stored in an order close to sequential. A task may only be placed up to k positions away from its correct sequentially consistent position.

Pseudocode for the push operation is shown in Listing 1. There, we choose a random position in the range from tail to tail +k and try to put the item into the array at the chosen position, if the position has not yet been taken by another item. In case the position is taken, a linear search is performed inside the tail to tail +k range until a free position is found or all positions have been checked. If all positions are taken, tail can be updated to tail +k and the search restarted. This scheme for adding items to an array was inspired by the k-fife queues of Kirsch et al. [10].

As soon as the item has been added to the global array, a reference to it is added to the priority queue of the place at which it was created. This guarantees that at least one thread will attempt to execute this task next, if it has the globally highest priority.

4.1.2 The pop operation

Pseudocode for the pop operation can be found in Listing 2. The pop operation checks whether tail has changed since the last time it was checked, and if so adds all the newly added tasks to the local priority queue. Each place maintains its own head index into the global array, to track which items have already been seen. Tasks that have been created by the same place can be omitted, since they were already added to the priority queue at the push operation. Next, the highest priority task is removed from the priority queue, and an attempt is made to mark the task as taken, by atomically setting the tag of the item to -1 using a compare-and-swap operation (CAS). Only one thread can succeed in updating the tag. In case of failure,

Listing 2 Pseudocode for pop in the centralized k-priority data structure.

```
1 Task pop(Place place) {
    // Check for new tasks in global array
    while(place.head < tail) {</pre>
      if(global_array[place.head].place \neq place) {
        ItemRef ref = new ItemRef(place.head, global_array[place.head]);
        place.prio_queue.push(ref);
    }}
    ItemRef ref;
    while(ref = place.prio_queue.pop()) {
10
      Task task = ref.it.task;
11
      // Take item atomically by setting tag to -1
12
      if(CAS(ref.it.tag, ref.tag, -1)) {
13
        // Success, return task
14
        return task;
15
16
      // Recheck for new tasks in global array again
17
      ... // (not shown)
18
    }
19
20
    // Priority queue is empty, try to find random task
21
    int offset = rand(0, k_{max} - 1);
22
    if(global_array[tail + offset] ≠ null &&
23
      global_array[tail + offset].k < offset) {</pre>
      Item it = global_array[tail + offset];
      Task task = it.task
26
      // Take item atomically by setting tag to -1
      if(CAS(it.tag, tail + offset, -1))
        return task;
29
    }
30
    return null;
31
32 }
```

the global array is rechecked for new tasks before trying again.

If the priority queue is empty, there can be up to k tasks stored after tail waiting for their execution, stored in the tail of the global array and its k subsequent positions. Our data structure allows for varying values for k per task, thereby making it necessary to specify a maximum value for k. We chose $k_{\text{max}} = 512$ for our implementation. Since there are at most k tasks stored after tail, no priority ordering needs to be guaranteed if there are no tasks before the tail, and a random position can be checked for a task to execute. If a task is found, the k value stored with the task is rechecked, to make sure not more than k tasks are ignored (tail might have been updated in the meantime). Since we allow for spurious failures on pop as long as another thread is making progress (executing a task), it is not necessary to exhaustively search for all tasks stored after tail, a random attempt suffices.

4.1.3 Additional implementation details

So far we have assumed that the global array used for storing tasks is unbounded. In practice, we implemented the global array as a linked list of arrays. Whenever an index is requested that is outside the bounds of the existing arrays, a new array is allocated and added to the end of the linked list using a single *compare-and-swap* operation.

Each array in the linked list can be deleted as soon as all tasks stored in the array have been executed

and the head indices of all places point to positions in arrays that are successors of the given array. The first condition can be lazily checked using a garbage collection scheme. We use the wait-free garbage collection scheme by Wimmer [18] for these purposes. The second condition can be checked by atomically decrementing a reference counter whenever a head index moves on to the next array. If the reference counter was initialized to the number of places in the beginning, it is guaranteed that no place will scan the array for new tasks once the counter reaches zero.

It is also necessary to clean up all the *items* used for storing tasks. For performance reasons we decided on a reuse scheme, where an item can be reused for a new task as soon as the previous task has been executed. The use of a tag for each item, which is initialized to the item's position in the global array, guards against the ABA-problem, since positions for items are strictly increasing. Also, since items may be reused directly after the compare-and-swap, the task has to be read out of the item before the compare-and-swap.

Both the head and tail indices in the data structure are strictly growing, therefore it is necessary to take a possible wraparound into account. We use 64-bit values, which ensures that wraparounds will only occur after a long time. Due to the long timespan between wraparounds, we consider it unlikely that an ABA problem will occur due to colliding indices.

4.1.4 Correctness

In this section we argue that the centralized k-priority data structure is lock-free and linearizable.

Theorem 1. The push operation is lock-free and linearizable.

Proof. All memory allocation is done using a wait-free memory manager [18]. The push operation tries to find an empty slot in the global array to insert its item. It searches k positions from a local copy of the tail index. If no empty slot is found, the tail is moved forward at least one step, either by the current thread or a concurrent thread. This is repeated until an empty slot is encountered. If no empty slot is found, or the CAS used to insert the item fails, another thread must have succeeded in inserting at least one item. This is in accordance with the lock-free property.

The tail can only be moved when all the slots before its new position are filled. So when an item is inserted, it is guaranteed to be at most k steps from the tail, since the tail cannot be moved before the successful insertion. Push operations can thus be linearized relative to each other at the point where they manage to insert their items into the array. The actual value of tail might differ from a sequential execution, but this does not affect the semantics.

Theorem 2. The pop operation is lock-free and linearizable.

Proof. The pop operation has to check the global array for new items. This can be done in a bounded number of steps if no other thread is making progress or adds new items. After reading the global array, the operation tries to acquire one of the tasks referenced in the priority queue. The size of the priority queue only grows when another thread is making progress and adds new items to the global array.

A push operation that inserts an item must be linearized before the pop operation that reads it. If the item is not in the k-relaxed part after the tail, the pop operation also needs to be linearized after the push operation that updated the tail to the value seen by the pop operation. Relative to each other, pop operations should be linearized at the point where they take the item using CAS.

All pop operations that observe a certain tail are linearized after the update of tail that stored the observed value and before the next. Only items after the tail will be ignored by any thread. At most k additional items can be stored after tail and therefore no more than k items will be ignored, regardless of the order in which items are popped from the data structure.

4.2 Hybrid k-priority data structure

The hybrid k-priority data structure consists of three components: (a) a global list storing tasks visible to all places, (b) one local task list per place, containing up to k tasks that are not guaranteed to be visible

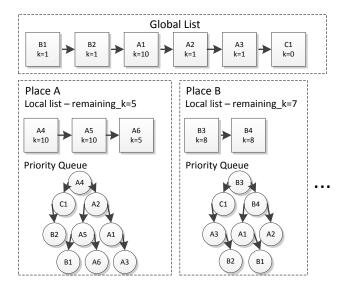


Figure 2: Hybrid k-priority data structure. Each place maintains its own priority queue with references to items. Each place adds new items to its local list as long as the ρ -relaxation guarantees are not violated. If adding a new item would violate these guarantees, the local list is appended to the global list, and a new local list is created.

to all places, and (c) one priority queue per place storing references to tasks in the global and local lists, ordered by priority. After more than k tasks have been added to the local task list, the place makes all local tasks globally visible by moving them to the global task list. A task can be referenced by multiple priority queues at the same time, which is required to guarantee that at most the k newest tasks by each thread are missed.

4.2.1 The push operation

The push operation (see Listing 3) adds a new task into the data structure. Each task is associated with a specific value for k, which determines how many tasks are allowed to be added before the task has to be made public. The semantics of k are that no more than k tasks are allowed to be added to the local list of tasks before the given task must be published.

The push operation proceeds as follows: first, the task is inserted into the local list of the given place, and a reference is stored in the local priority queue of the place. Afterwards, a check is performed whether more tasks can be added without needing to publish any of the locally stored tasks, which is the case if the variable remaining k is greater than zero. If any of the locally stored tasks needs to be made available globally, the local list of tasks is appended to the global list. A new, empty local list is then created, which will be used in the next push operations.

4.2.2 The pop operation

The pop operation (see Listing 4) pops a reference to the highest-priority task from the local priority queue and tries to mark the task as taken by setting the taken flag with an atomic test-and-set operation. If it succeeds the task is returned. To make sure that no more than k tasks per place are ignored, the local priority queue has to be regularly updated with the newest additions to the global list. This is always done before a task is popped from the priority queue.

If the priority queue is empty after processing the global list, an attempt is made to find tasks stored locally at another place. This is called *spying*. Spying is related to stealing in work-stealing systems in that a remote place is selected semi-randomly among all places and searched for tasks that have not yet been

Listing 3 Pseudocode for push in the hybrid k-priority data structure.

```
void push(Place place, int k, Task task) {
    Item it = new Item(place, priority, task);
    // Place task in local list and priority queue
    place.local_list.add(it);
   place.prio_queue.push(new ItemRef(it));
    // All items need to be made globally visibly
    // to not violate the \rho-relaxation requirement
    remaining_k = min(remaining_k - 1, k);
    if(remaining_k == 0) {
10
     // Add local list to global list
11
     do {
12
       processGlobalList(place)
13
     } while(¬CAS(global_list.tail.next, null, local_list.head));
      // Create a new local list
15
     place.local_list = new List();
16
     remaining_k = \infty;
17
18 }}
_{
m 20} // Add references to unread items from
21 // the global list to the local priority queue
void processGlobalList(Place place) {
    while(place.iterator ≠ global_list.tail) {
     Item it = place.iterator.item()
      // Do not add local or already taken tasks
25
     if(it.place \neq place and \neg it.taken)
26
       place.prio_queue.push(new ItemRef(it));
     place.iterator = place.iterator.next;
29 }}
```

executed. The main difference is that tasks that are encountered during spying are not removed from the owner's local list of tasks. Instead, only references to the given tasks are stored in the priority queue. This is necessary to avoid breaking ρ -relaxation guarantees, but also greatly simplifies synchronization.

Spying is only required when less than k tasks are in flight at each place, so that k-prioritization is not violated if not all the victim's tasks are encountered during spying. Therefore the semantics of spy allows spurious failures, where the spying place does not see all the victim's tasks. This allows for a very lightweight synchronization scheme for spying. Spying may lead to tasks appearing in a single priority queue twice, but never more often, if the task was first encountered during spying, and was later made available globally. This does not affect the correctness however, since a task can only be executed once.

4.2.3 Additional implementation details

For efficiency reasons, our implementation of the hybrid k-priority data structure does not use linked lists, but instead uses a linked list of arrays, which can be implemented in a similar manner as for the centralized k-priority data structure, as described in Section 4.1.3.

Similarly, the memory management for *items*, which are used to store all the information about a task, can be taken over from the centralized data structure. Alternatively, items can be stored in-place in the linked list of arrays for higher efficiency. To guard against the ABA problem when an item is reused, we use a tag instead of an atomic flag to mark an item as taken, similar to the tag used in the centralized k-priority data structure. Since, contrary to the centralized data structure, an item has no global index at the time it is added to the data structure, each place maintains its own local indices, which are used to fill the tag field.

Listing 4 Pseudocode for pop in the hybrid k-priority data structure.

```
1 Task pop(Place place) {
   do {
2
      processGlobalList(place);
3
      // Try to take the highest priority task
      while(¬place.prio_queue.empty()) {
       Ref r = prio_queue.pop();
        if(\neg r.item.taken) {
          Task ret = r.item.task;
          if(TAS(r.item.taken))
            return ret;
10
11
       processGlobalList(place);
12
13
14
      // If the priority queue is empty, add references
15
      // to remote tasks from a pseudo-random place
16
      List vl = getRandVictim().local_list;
17
      foreach(Item it in vl) {
18
        if(it.place \neq place and \neg it.taken)
19
          place.prio_queue.push(new ItemRef(it));
20
21
    } while(¬place.prio_queue.empty());
22
    return null;
23
24 }
```

An offset is stored in each array in the linked list before it is linked to the global list, to allow other threads to calculate the offsets of the items.

Spying does not put any tasks into the local task list of the spying place (contrary to steal-half work-stealing), which makes the given place appear as being out of work for other spying places. To ensure a proper distribution of tasks throughout the whole system, each place stores a reference to its last successful spying victim. In case a victim is encountered with no local work, its last successful spying victim is checked instead.

4.2.4 Correctness

In this section we argue that the hybrid k-priority data structure is lock-free and linearizable.

Theorem 3. The push operation is lock-free and linearizable.

Proof. All memory allocation is done using a wait-free memory manager [18]. When there are less than k tasks in the local list, the entire push operation is done locally and is thus wait-free. When the local list has k tasks, it is added to the global list. This step requires making sure that the entire global list has been read and then adding the local list to the end. Adding the local list to the global list can fail if another place adds its list first, but this means another place made progress. Reading and adding to the global list is thus lock-free.

A push operation for a task which is taken before being added to the global list, has its linearization point where it is added to the local list. Before this point the task is not visible to any remote place, while after the point it can be spied and taken by any place. For tasks which are taken after being globally announced, the push operation is linearized at the point where the local list was atomically added to the global list. The pop operation always reads all new tasks when reading from the global list, so push operations can be linearized in any order relative to each other.

Listing 5 Pseudocode for a single node relaxation in our parallel single-source shortest path algorithm.

```
void relaxNode(Graph graph, int node, int distance) {
   int d = graph[node].distance;
   if(d \neq distance) {
      // Dead task, distance has already been improved in the meantime
     return;
   }
   for(int i = 0; i < graph[node].num_edges; ++i) {</pre>
     int new_d = d + graph[node].edges[i].weight;
     int target = graph[node].edges[i].target;
10
     int old_d = graph[target].distance;
11
12
     // Check if path through this node is shorter
13
     while(old_d > new_d) {
14
       // Try to update distance value
       if(CAS(&(graph[target].distance), old_d, new_d)) {
16
         spawn(new_d, // priority, smaller is better
17
           relaxNode, // Function used for task
18
           graph, target, new_d));
19
         break:
20
       old_d = graph[target].distance;
22
23 }}}
```

Theorem 4. The pop operation is lock-free and linearizable.

Proof. At certain points the pop operation needs to make sure it has read the entire global list. The global list can only grow if another place is making progress, which makes reading the list lock-free. Multiple places may try to acquire the same task, but only one will successfully take it. The number of already taken tasks can only grow if another place is making progress. If the priority queue is empty and the global list has been read, an attempt is made to spy on the local list of another place. The length of the remote local list is bounded by k, making the spying wait-free.

A successful pop operation has to be linearized relative to other pop operations at the point where the task was atomically marked as taken. Relative to push operations, the pop has to be linearized at the point where the global list was last read. At this point the place has a snapshot that contains all but at most (P-1)k tasks, where P is the number of places. If instead the pop was linearized when the task was taken, another push operation could have added new tasks to the global list before that, causing the place to miss more than the allowed Pk tasks.

An unsuccessful pop operation is linearized at the point where the global list was last read. At this point both the global and the local lists were empty. The final spying part is allowed to fail even though there might be tasks at other places. \Box

5 Evaluation

The goal of this section is to show how ρ -relaxation can help to give bounds beyond what can be achieved with work-stealing for the execution of a parallel application. For this we base our evaluation on the well-understood single-source shortest path problem. We derive theoretical bounds, which are also later verified through simulation. Furthermore, we show that the bounds are applicable to all variations of ρ -relaxation presented in this paper. In addition we provide experiments that show the practical performance gains of our approach.

5.1 Application

We base our evaluation on a simple and well-understood example application that profits from priorities, and consider the single-source shortest path problem (SSSP) [2]. We focus on a simple parallelization of Dijkstra's algorithm. Dijkstra's algorithm maintains a tentative distance value for each node in the graph. At each iteration, a node relaxation is performed, where the tentative distance values of the neighboring nodes are decreased if the path through the relaxed node is shorter. At termination, the distance from the source node is available for each node. A priority queue is used to decide the order in which nodes are relaxed; the priority ordering guarantees that each node is relaxed exactly once.

Our parallel version relaxes multiple nodes in parallel. Due to the parallelization some node relaxations might be performed prematurely, when a node is not yet *settled*, which means that its distance value is not final. These nodes will have to be re-relaxed when their distance values are updated. Premature relaxations are therefore *useless work*.

In our parallel implementation, each node that has to be relaxed corresponds to a task in the scheduling system. These tasks are prioritized using the distance value of the node, as in Dijkstra's algorithm. For the sake of comparability to other works on single-source shortest paths, we will use the terms *node* and *relax* instead of *task* and *execute* throughout this section. Instead of a priority queue, we let our scheduling system choose the next node (task) to relax. Pseudocode for the SSSP tasks is given in Listing 5.

We diverge from Dijkstra's algorithm whenever a better distance value is found for an active node in the priority queue. Instead of updating the priority using a decrease key operation, we reinsert the node into the priority queue. The previous instance of the same node, with an old distance value as priority, is lazily removed as soon as it is noticed. Our scheduling data structures have been implemented to recognize such nodes lazily, and automatically remove them when recognized. For more discussion on dead task elimination, see [19, 20].

The goal of this evaluation is to show that, using k-priority data structures, the amount of useless work generated is small compared to the actual work, and that bounds can be given on the amount of useless work generated.

5.2 Theoretical analysis

For the theoretical analysis we use a simplified model of task-parallel computations: the system operates on a global pool of nodes (tasks), which are ordered by their tentative distance value. Execution occurs in temporal phases and, in each phase, up to P nodes with the lowest tentative distance values are relaxed. We assume an *ideal priority queue*, in which all nodes are visible to all places at the beginning of each phase. We are interested in upper bounding the amount of useless work that is performed during each phase. Similar bounds have previously been obtained for Δ -stepping and other SSSP algorithms [14, 15].

5.2.1 Formal model

We are given an undirected graph G=(V,E) (with n=|V| and m=|E|), a source node $s\in V$ and a positive weight function $\lambda:E\to\mathbb{R}^+$. For each temporal step t we maintain a partition of V into two subsets: $V=A_t\cup B_t$, of sizes α_t and β_t ($\forall t\ \alpha_t+\beta_t=n$). The set $A_t=\{a_t(1),a_t(2),\ldots,a_t(\alpha_t)\}$ contains the active nodes, $B_t=\{b_t(1),\ldots,b_t(\beta_t)\}$ the inactive nodes. For each node $v\in V$ we also keep a tentative distance $\delta_t(v)\in\mathbb{R}\cup\{\infty\}$. Let $d_t(i)=\delta_t\left(a_t(i)\right)$, we assume the nodes in A_t to be ordered by d_t , with ties broken arbitrarily, i.e., $\forall i\in\{1,\alpha-1\}$ $d_t(i)\leq d_t(i+1)$. Initially (t=0) we have $A_0=\{s\}$, $B_t=V\setminus\{s\}$, $\delta_0(s)=0$ and $\delta_0(v\neq s)=\infty$. In each phase (up to) P active nodes $\Phi_t=\{a_t(1),\ldots,a_t(P)\}$ with lowest d_t are selected and relaxed, so that at the end of the phase the tentative distance of a generic node $w\in V$ is

$$\delta_{t+1}(w) = \min \left\{ \delta_t(w), \min_{v \in \Phi_t} \left\{ \delta_t(v) + \lambda(v, w) \right\} \right\} .$$

Any node (whether active or inactive) which had its tentative distance updated is moved into A_{t+1} , relaxed nodes which were not updated are moved into B_{t+1} , all the other nodes remain in their former sets for the

next time phase. The algorithm terminates, at some time $\tau < n$, when there are no more active nodes, i.e., $A_{\tau} = \emptyset$ and $B_{\tau} = V$, with the nodes reachable from s having a finite distance.

We restrict our analysis to Erdős-Rényi random graphs [6, 7] of parameters n and p, i.e., graphs with n nodes, for which each of the $\binom{n}{2}$ possible edges has independent probability p to occur. Furthermore, we assign, independently for each edge, a weight uniformly distributed between 0 and 1: $\forall e \in E, \lambda(e) \in \mathcal{U}]0,1]$. We assume the source node s to be chosen uniformly at random in V. In order to ensure, w.h.p., the connectedness of the graph, we also assume $p > \frac{(1+\epsilon)\ln n}{n}$ for some $\epsilon > 0$.

5.2.2 Useless work

We say that a node is settled at time t when its tentative distance is equal to its final distance. Every time that a node which is not settled is relaxed, useless work is performed, since the node will need to be relaxed again when its tentative distance is going to be updated (Dijkstra's algorithm only relaxes nodes which are settled, thus performing only useful work, but, on the other hand, it is hard to parallelize because of its dependencies). The following theorem (proof in Section 5.2.3) bounds the useless work W_t performed by our algorithm as a function of d_t .

Theorem 5. Let W_t be the useless work performed at time t by our algorithm, using an ideal priority queue, and let $h_t(i,j) = d_t(j) - d_t(i)$. We can bound W_t from above as:

$$W_t \le \sum_{j=1}^{P} \left[1 - \prod_{i=1}^{j-1} \prod_{L=1}^{n-1} \left(1 - \frac{\left(p \, h_t(i,j) \right)^L}{L!} \right)^{\frac{(n-2)!}{(n-1-L)!}} \right] .$$

Remark 1. A simpler (but weaker) form of this bound can be obtained by substituting $h_t(i,j)$ with $h_t^* = \max_{i,j} h_t(i,j) = h_t(1,P)$.

5.2.3 Proofs and lemmata

In order to simplify the analysis, we assume the following property to hold when the number of nodes n is large. The property has been experimentally validated using the simulator presented in Section 5.4.

Conjecture 1. Throughout the execution of the ideal priority queue SSSP algorithm, for all values of $t \in \mathbb{N}$, $1 \le i < j \le P$ and $h \in]0,1]$, the probability that there is a path of weight less than h between $a_t(i)$ and $a_t(j)$ is bounded from above by the probability that such a path exists in a random graph, between two (uniformly) random nodes.

Lemma 1. Let $h \in]0,1]$ and let $\pi^L = (\pi_0, \pi_1, \dots, \pi_{L-1}, \pi_L)$ be a path in G chosen uniformly at random among the paths of length L, such that the subpaths $\pi' = (\pi_0, \dots, \pi_{L-1})$ and $\pi'' = (\pi_{L-1}, \pi_L)$ both have weights smaller than h. Let $f^L(\lambda)$ be the probability density function associated with the total weight $\lambda(\pi^L) = \sum_{i=1}^L \lambda(\pi_{i-1}, \pi_i)$. We can write f^L as

$$f^L(\lambda) = \begin{cases} \frac{\lambda^{L-1}}{h^L} & \lambda \in]0, h] \\ \frac{1}{h} - \frac{(\lambda - h)^{L-1}}{h^L} & \lambda \in]h, 2h] \\ 0 & otherwise \end{cases}.$$

Proof. The proof is by induction on L.

Base case. For L=1, since the edge weight is uniformly distributed between 0 and 1, we clearly have

$$f^{1}(\lambda) = \begin{cases} \frac{1}{h} & \lambda \in]0, h] \\ 0 & \text{otherwise} \end{cases}.$$

INDUCTION. We assume now that the inductive hypothesis holds for all values $l \leq L$. Let f_h^l be the probability density function obtained by conditioning its weight $\lambda(\pi^l)$ to be smaller than h, i.e.,

$$f_h^l(\lambda) = \begin{cases} \frac{l\lambda^{l-1}}{h^l} & \lambda \in]0, h] \\ 0 & \text{otherwise} \end{cases}.$$

We have $\lambda(\pi^L) = \lambda(\pi') + \lambda(\pi'')$, where π' and π'' are subpaths of length L and 1. Since $\lambda(\pi')$ and $\lambda(\pi'')$ are independent, the density function f^{L+1} can be obtained by convolution of f_h^L and f_h^1 :

$$f^{L+1} = f_h^L * f_h^1 \qquad \qquad \Rightarrow \qquad \qquad f^{L+1}(\lambda) = \begin{cases} \frac{\lambda^L}{h^{L+1}} & \lambda \in]0, h] \\ \frac{1}{h} - \frac{(\lambda - h)^L}{h^{L+1}} & \lambda \in]h, 2h] \\ 0 & \text{otherwise} \end{cases} ,$$

which concludes the induction.

Corollary 1. The probability that a (uniformly random) path π^L has $\lambda(\pi^L) < h$, conditioned to $\lambda(\pi') < h$ and $\lambda(\pi'') < h$, is equal to $\frac{1}{L}$.

Proof. Just integrate f^L between 0 and h.

Proof (Theorem 5). Let $1 \le i < j \le \alpha_t$ and let $\pi_t^L(i,j) = (\pi_0 = a_t(i), \pi_1, \dots, \pi_L = a_t(j))$ be a path between $a_t(i)$ and $a_t(j)$ of length L; we denote the weight of $\pi_t^L(i,j)$ as

$$\lambda \left(\pi_t^L(i,j) \right) = \sum_{k=0}^{L-1} \lambda(\pi_k, \pi_{k+1}) .$$

A node $a_t(j)$ is not settled if and only if there exists i < j such that there exists a path $\pi_t^L(i,j)$ with $\lambda\left(\pi_t^L(i,j)\right) < d_t(j) - d_t(i)$. Note that the non-existence of a particular path with weight less than $h_t(i,j)$ does not decrease the probability for another different path not to exist. Therefore, being $h_t(i,j) = d_t(j) - d_t(i) \le 1$, the probability $q_t(j)$ that $a_t(j)$ is settled can be bounded as

$$q_{t}(j) \geq \prod_{i=1}^{j-1} \prod_{L=1}^{n-1} \Pr\left[\nexists \pi_{t}^{L}(i,j) : \lambda \left(\pi_{t}^{L}(i,j) \right) < h_{t}(i,j) \right]$$
$$= \prod_{i=1}^{j-1} \prod_{L=1}^{n-1} \left(1 - r_{t}^{L}(i,j) \right) ,$$

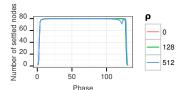
where $r_t^L(i,j)$ is the probability that a path $\pi_t^L(i,j)$, with weight less than $h_t(i,j)$, exists. Assuming that we are relaxing the first P nodes of A_t , we can compute the expected value of the useless work performed at time t as $W_t = \sum_{j=1}^{P} (1 - q_t(j))$. Let $\tilde{r}_t^L(i,j)$ be the probability that a particular path $\pi_t^L(i,j)$ exists, with weight less than $h_t(i,j)$; we can bound $r_t^L(i,j)$ as

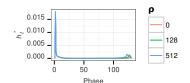
$$r_t^L(i,j) \le 1 - \left(1 - \tilde{r}_t^L(i,j)\right)^{\frac{(n-2)!}{(n-1-L)!}}$$

Note that there exists a path $\pi_t^L(i,j)$ with weight less than $h_t(i,j)$ if and only if the two subpaths $\pi' = (\pi_0, \dots, \pi_{L-1})$ and $\pi'' = (\pi_{L-1}, \pi_L)$ exist, their weights are smaller than $h_t(i,j)$, and so is the sum of their weights. Because of Conjecture 1 and Corollary 1 we have $\tilde{r}_t^1(i,j) = p h_t(i,j)$, which finally implies

$$\tilde{r}_{t}^{L}(i,j) = \frac{\tilde{r}_{t}^{L-1}(i,j)\tilde{r}_{t}^{1}(i,j)}{L} = \frac{\left(\tilde{r}_{t}^{1}(i,j)\right)^{L}}{L!},$$

$$r_{t}^{L}(i,j) \leq 1 - \left(1 - \frac{\left(p h_{t}(i,j)\right)^{L}}{L!}\right)^{\frac{(n-2)!}{(n-1-L)!}}.$$





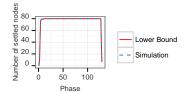


Figure 3: From left to right: nodes settled per phase; difference between biggest and smallest tentative distance of nodes relaxed per phase; comparison between the theoretical bound and the simulation. (n = 10000, P = 80, p = 50%)

5.2.4 k-priority data structures

We can adapt our theoretical framework to support k-priority data structures, which allow that up to ρ of the newest tasks may not be visible to all places, and may therefore not be executed even though they would have been with the ideal data structure. For the centralized k-priority data structure $\rho = k$, for the hybrid one $\rho = Pk$. The bound of Theorem 5 can be adapted by changing the sum over all j's to only the j's corresponding to nodes $a_t(j)$ which have been actually relaxed $(\sum_{j=1}^P \to \sum_{j \in R_t})$, with $R_t = \{j : a_t(j) \text{ has been relaxed}\}$. Similarly to the previous case, a simpler form of this bound can be obtained by substituting $h_t(i,j)$ with h_t^* , defined as the difference between the largest and smallest tentative distance of nodes relaxed at time t, which implies $h_t^* \leq \max_{i,j} h_t(i,j) = h_t(1, P + \rho)$.

5.3 Weakening the requirements ρ -relaxation

All our current k-priority data structure implementations rely on a temporal formulation of ρ -relaxation (see Section 2.2), allowing only the last k items added to the data structure to be ignored (k items per thread for the hybrid k-priority data structure). This means that after k push operations a thread will make all its newest tasks globally available, regardless of how many of the k new tasks have already been executed. As it turns out, our model does not require the temporal formulation of ρ -relaxation, and relies on a weaker structural formulation instead that requires a pop operation never to ignore more than ρ items regardless of their age.

This result opens up possibilities for priority queues that achieve similar bounds but do not need to maintain the temporal property. We believe that this will lead to priority queues with even better scalability than the priority queues presented in this work, and first results with such data structures look promising.

5.4 Simulation

We have used a simulator to bridge between the findings in our theoretical model and the experiments in Section 5.5. The simulator helped understand why ρ -relaxation gives such strong guarantees and was a valuable tool to shape our theoretical analysis. The simulator uses the phase-wise execution model used in the theoretical analysis and allows us to vary the parameters P and ρ . The simulator stores all active nodes in a single array sorted by distance value. Execution proceeds in phases, where in each phase the first P nodes from the array are relaxed. At the end of each phase the array is updated with all new active nodes.

If $\rho > 0$, newly created active nodes are marked with a sequence id. To ensure randomness, nodes created in a single phase are shuffled first before assigning sequence id's. The nodes with the ρ highest sequence id's are stored separately from the sorted array of nodes. These nodes represent the nodes that might be ignored due to the ρ -relaxation. An exception is made if a node has the lowest distance value of all nodes. This node is guaranteed to be relaxed in the next phase, and is therefore added to the array of active nodes. A deterministic tie-breaking scheme is used to ensure that only one node has the lowest distance value of all at any time. In case that less than P nodes are available in the array, a random selection of all other active nodes is relaxed by the other places.

5.4.1 Simulation results

We ran our simulator in a setting that closely resembles the setup used in the experiments in Section 5.5. We use exactly the same 20 random graphs used in the experiments and report the mean. The number of places, P, is set to 80, which corresponds to the 80 cores of the machine used in our experiments. We use three values for ρ : 0, which represents an ideal priority data structure, 128 and 512.

The first graph in Figure 3 depicts the number of nodes settled in each phase throughout the simulation. It can be seen that for most of the execution almost all nodes that are relaxed are already settled. Non-settled nodes are only encountered in the first phases. For higher ρ some variation can also be observed towards the end when a significant amount of nodes is not visible to all places. Throughout most of the execution almost all of the nodes that are relaxed are already settled.

The middle graph in Figure 3 shows h_t^* , the difference between the largest and smallest distance value of nodes relaxed in each phase. After only a few iterations, all of the nodes that are relaxed have distance values close to each other, and the distance values only grow a bit at the end of the execution, a bit more with higher ρ . It is easy to see the close relationship between distance values and nodes settled per phase.

Finally, the last graph in Figure 3 gives a comparison between the theoretical lower bound and the number of settled nodes in the simulation. It can be seen that the calculated theoretical lower bound on the number of settled nodes, and the number of nodes settled in the simulation are very close.

5.5 Experiments

The data structures have been evaluated on an 80-core Intel Xeon system with 1 TB of memory. Figure 4 shows the average total execution time and number of spawned tasks for 20 undirected graphs, each with 10000 nodes, an edge probability of 50% and uniformly distributed random edge weights. The k value is set to 512.

Ideally, a parallel implementation of single-source shortest paths relaxes each node exactly once. This is the case if nodes are only relaxed when they are settled. For our input graphs with n = 10000, this means that if more than 10000 nodes were relaxed, some useless work was performed. As can be seen in Figure 4, close to no useless work is generated by any of the data structures, with exception of work-stealing. With random stealing and only local prioritization to go by, it generates more than twice the number of necessary tasks. This shows up in the total execution time, which is higher for work-stealing.

The parallel implementations are compared to a sequential implementation of Dijkstra's algorithm (only shown for one thread). Due to the small task granularity, the overhead for parallel execution on all data structures is relatively high, but for two or more threads the execution times drop below the sequential time. The algorithm scales very well for up to 10 threads. For more threads the algorithm becomes memory bandwidth bound. On the hybrid k-priority data structure some more speedup can still be achieved up until 40 threads.

Figure 5 shows, for the same graphs as in the previous figure, the total execution time and number of spawned tasks for different k values. The number of places is fixed at 80. Here it can be seen that the centralized k-priority data structure works best for k in the range of 32 to 128. For higher k the cost of the sometimes required linear search outweighs the gains.

The hybrid k-priority data structure shines with larger k where it exhibits scalability similar to work-stealing. While the wasted work is higher than for the centralized data structure, it is still bounded, and therefore low with the right values chosen for k.

The minimum k required to match work-stealing performance in the hybrid data structure is dependent on task granularity. The more fine-grained tasks are, the higher the minimum required k to match work-stealing. It is interesting to note that even with really high values for k, which result in no global synchronization, the wasted work is still half of the wasted work in work stealing. This comes due to the use of spying, which allows a task to be visible to multiple place unlike stealing, where a task is only seen by one place. We found k = 512 to be the best compromise between scalability and priority guarantees for the hybrid data structure on the given machine.

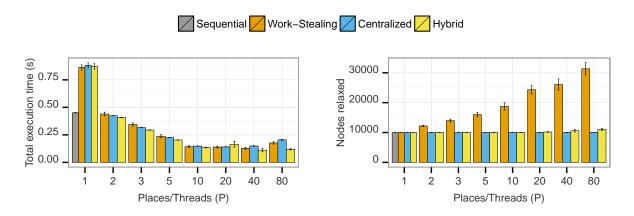


Figure 4: Total execution time and number of nodes relaxed for varying P (n = 10000, k = 512, p = 50%).

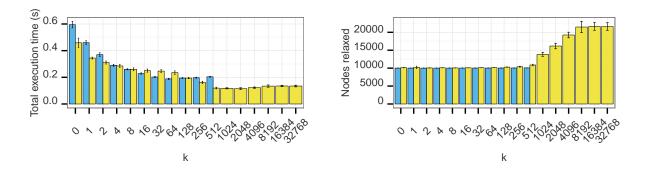


Figure 5: Total execution time and number of nodes relaxed for varying k (n = 10000, P = 80, p = 50%).

6 Conclusion

We have developed lock-free data structures that can be used in task scheduling systems to support priority scheduling. Each of these data structures provides different trade-offs between scalability and guarantees concerning the execution order of tasks. Our hybrid, k-priority data structure allows to adjust the trade-offs using the parameter k, enabling the programmer to dynamically chose between a scalable data structure with performance comparable to work-stealing, and a centralized data structure with strong semantic guarantees.

We evaluated all three data structures analytically, experimentally and using a simulation. Using the single-source shortest path algorithm as an example, we showed that, compared to work-stealing, ρ -relaxation can provide a significant reduction of useless work performed for the single-source shortest path algorithm, even with relatively large values for k. Nonetheless, the limits to scalability become visible in cases with very small task granularities, where most of the time is spent on synchronization.

In future work we plan to explore additional data structures that further reduce the bottlenecks while maintaining the flexibility of the hybrid, k-priority data structure. We expect that k-relaxed data structures that rely on the weaker, structural formulation of ρ -relaxation as described in Section 5.3 will exhibit better scalability than the data structures presented in this work, due to the reduced need for synchronization. First results with structurally relaxed k-priority data structures look promising.

This work also shows how extensions to the task model, like priority queues, can help to create simple and efficient parallel versions of algorithms that are otherwise hard to parallelize, like Dijkstra's single-source shortest path algorithm. In the future we plan to provide additional scheduler data structures useful for specific problems, allowing for an even more general use of task schedulers for algorithm design. As an

example, we plan to provide k-relaxed Pareto priority queues with guarantees that can then be used for parallelization of a multi-objective shortest path search [17].

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