




## Agenda .

- 
- ① Allocate minimum No. of Pages
  - ② Aggressive cows
  - ③ Capacity to ship packages within  $k$  days
  - ④ Minimum limit of Balls in a bag

Binary Search }  
over sol<sup>n</sup>

Hard problems .

Basics Binary Search .

Allocate Minimum No. of Pages.

Books  $[] = \{ 34, 12, 67, 90 \}$       stud = 2

Requirements

- distribute these  $n$  Books among these  $M$  Students.
- such that Each student should get min<sup>m</sup> one Book.
- books should be distributed in contiguous manner.

min<sup>m</sup> of max<sup>m</sup> no. of pages read by a stud.

Books  $[] = \{ 34, 12, 67, 90 \}$

stud = 2

way 1

$s_1 \rightarrow 34 + 12 + 67 \rightarrow 113$   
 $s_2 \rightarrow 90$   
90

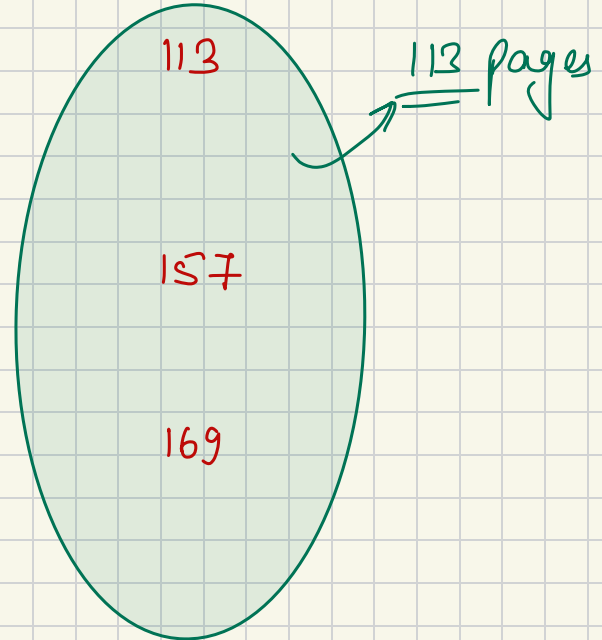
way 2

$s_1 \rightarrow 34 + 12 \rightarrow 46$   
 $s_2 \rightarrow 67 + 90 \rightarrow 157$

way 3

$s_1 \rightarrow 34 \rightarrow 34$   
 $s_2 \rightarrow 12 + 67 + 90 \rightarrow 169$

max<sup>m</sup> Pages

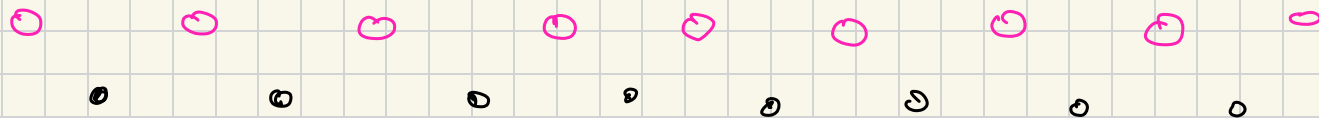


Brute Force 0

Books[] = {<sup>0</sup>34, <sup>1</sup>12, <sup>2</sup>67, <sup>3</sup>90}

stud = 2

2

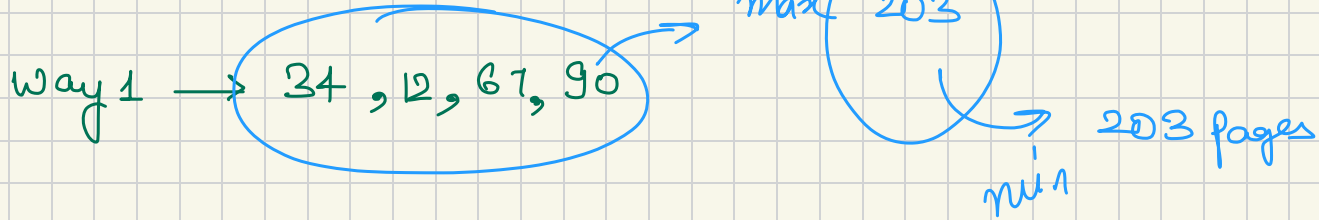


(N+1)  
P  
dividers

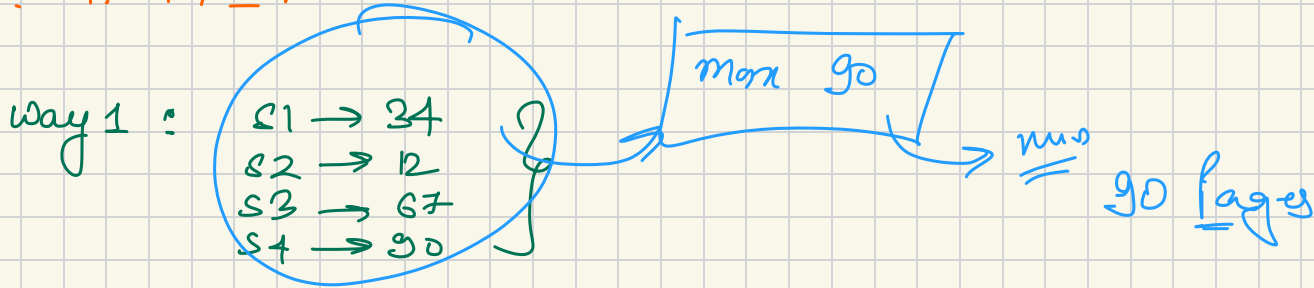
Books  $[] = \{ 34, 12, 67, 90 \}$

stud = 2

Case 1: if  $M = 1$



Case 2: if  $M = N$



$$1 \leq M \leq N$$

① 'step 1' define range

$$M = 2$$

~~90 pages~~

104

mid  
↓  
~~176~~  
~~171~~  
103

~~203 pages~~

~~175~~

115

pages = ~~176~~  
117

$$\text{Books}[] = \left\{ \underset{\uparrow}{34}, \underset{\uparrow}{12}, \underset{\uparrow}{67}, \underset{\uparrow}{90} \right\}$$

$$\text{Max}^m \text{Page} = 176$$

$$\left. \begin{array}{l} s_1 \rightarrow 34 + 12 + 67 \\ s_2 \rightarrow 90 \end{array} \right\}$$

$$\text{Books}[] = \left\{ \underset{\uparrow}{34}, \underset{\uparrow}{12}, \underset{\uparrow}{67}, 90 \right\}$$

$$\text{Max}^m \text{Page} = 117$$

$$s_1 \rightarrow 34 + 12 + 67$$

$$s_2 \rightarrow 90$$



Books  $[] = \{ 31, 12, 67, 90 \}$   
                   $\uparrow \quad \uparrow \quad \uparrow$

Max<sup>m</sup>Page = 103

$S1 \rightarrow 31 + 12$   
 $S2 \rightarrow 67$   
 $S3 \rightarrow 90$

# Aggressive Cows

stalls  $\Sigma = \{0, 3, 7, 9, 10, 4\}$

cows = 4 ↗ aggressive

max<sup>m</sup> the min<sup>m</sup> dist. b/w any two aggressive cows.

{		{		{	{		{	{		{
0	1	2	3	4	5	6	7	8	9	10

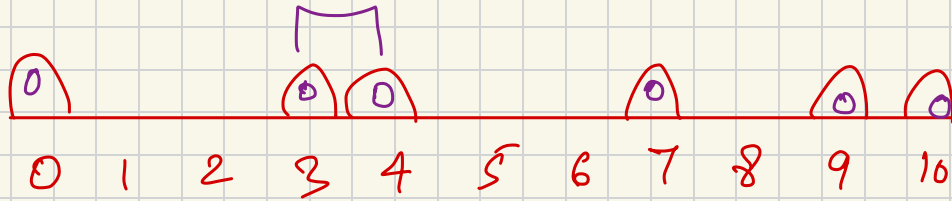
way 1 0                      0                      0

way 2 0                      0                      0

min - { non

${}^6C_4$  ways

Case 1      cows = N



min dist = 1 → max = 1

Case 2

cows = 2

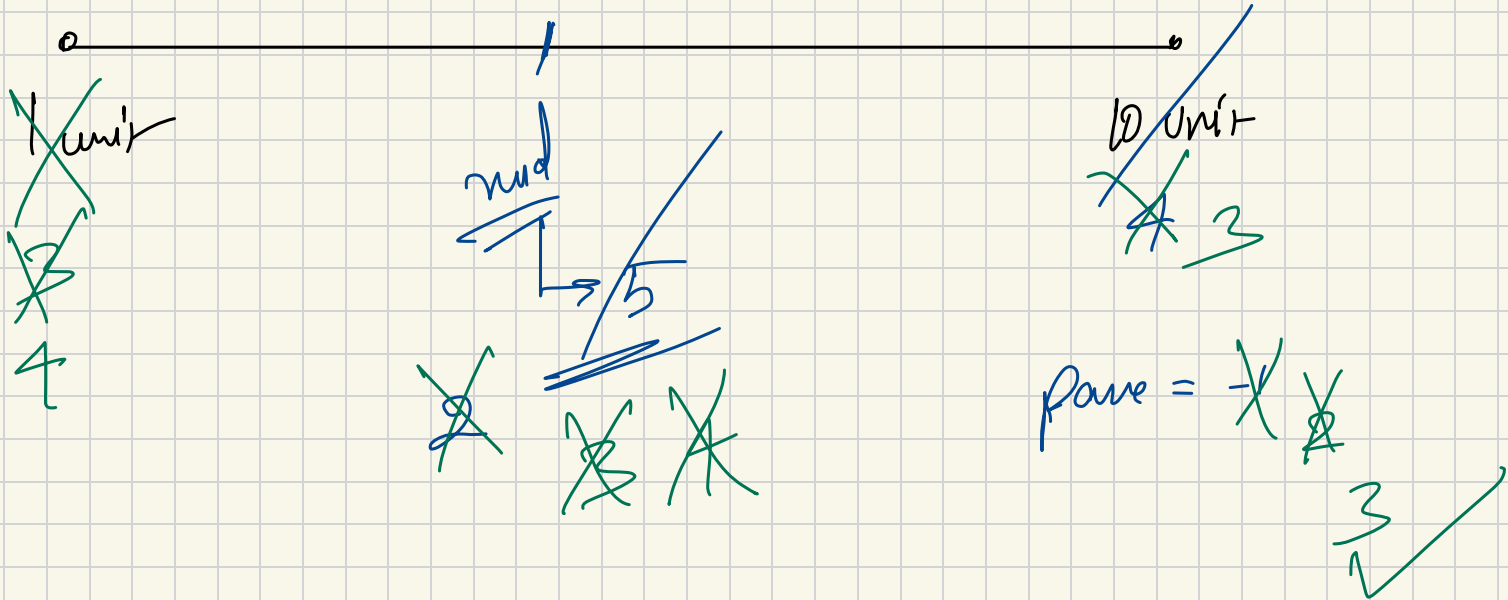
${}^6C_2$  way total

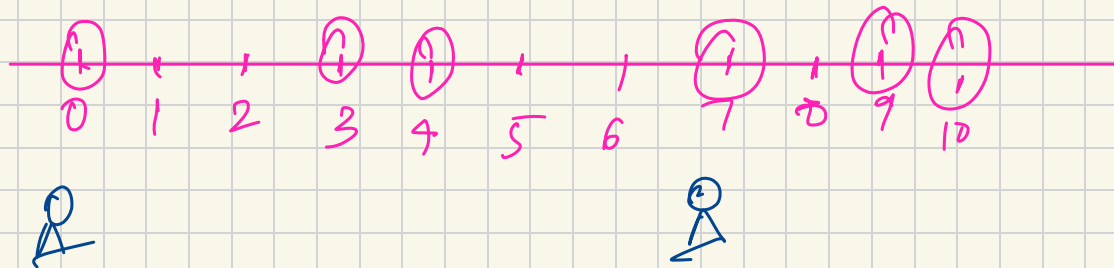
cows at first & last stall  
→ 10 units

$$\underline{2 \leq \text{cow} \leq N}$$

$$\text{cows} = 7$$

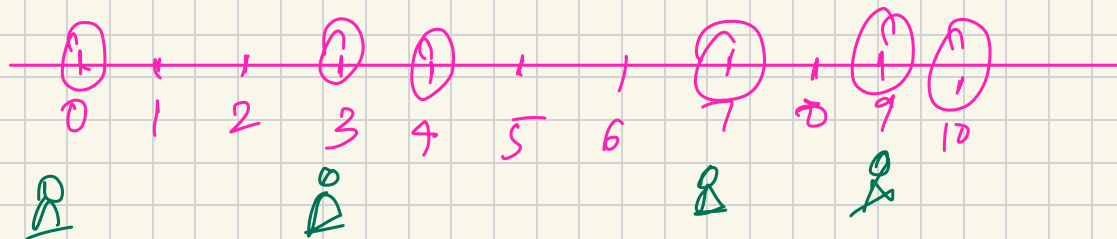
Range





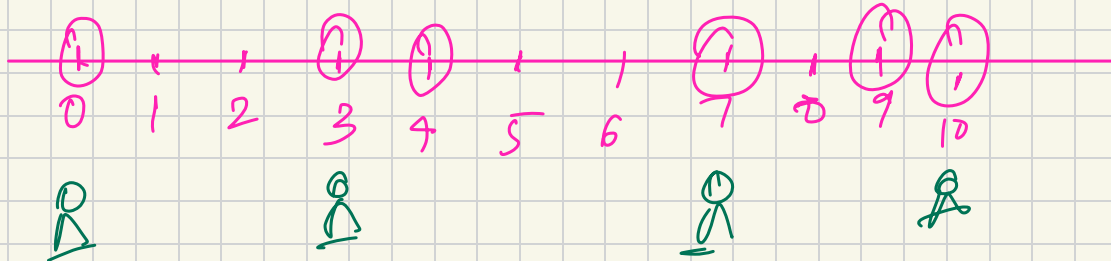
$$\min \text{Dist} = 5$$

$$\underline{\text{cost} = 2}$$



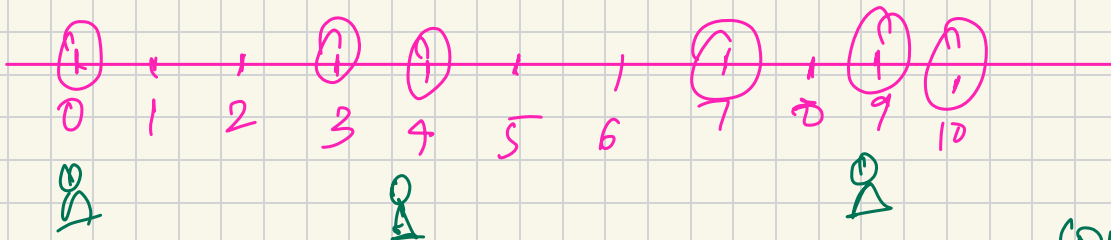
$$\min \text{Dist} = 2$$

$$\underline{\underline{\text{cost} = 4}}$$



$$\min Dist = 3$$

$$Covers = 4$$



$$\min Dist = 4$$

$$Covers = 3$$



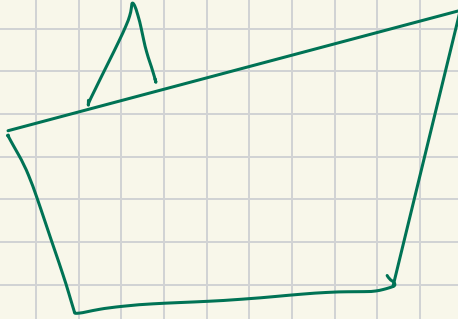
min →

✓ nearest stall will be the  $\text{max}^m$  threat

① compare with nearest stall.

Capacity to ship Packages within B days.

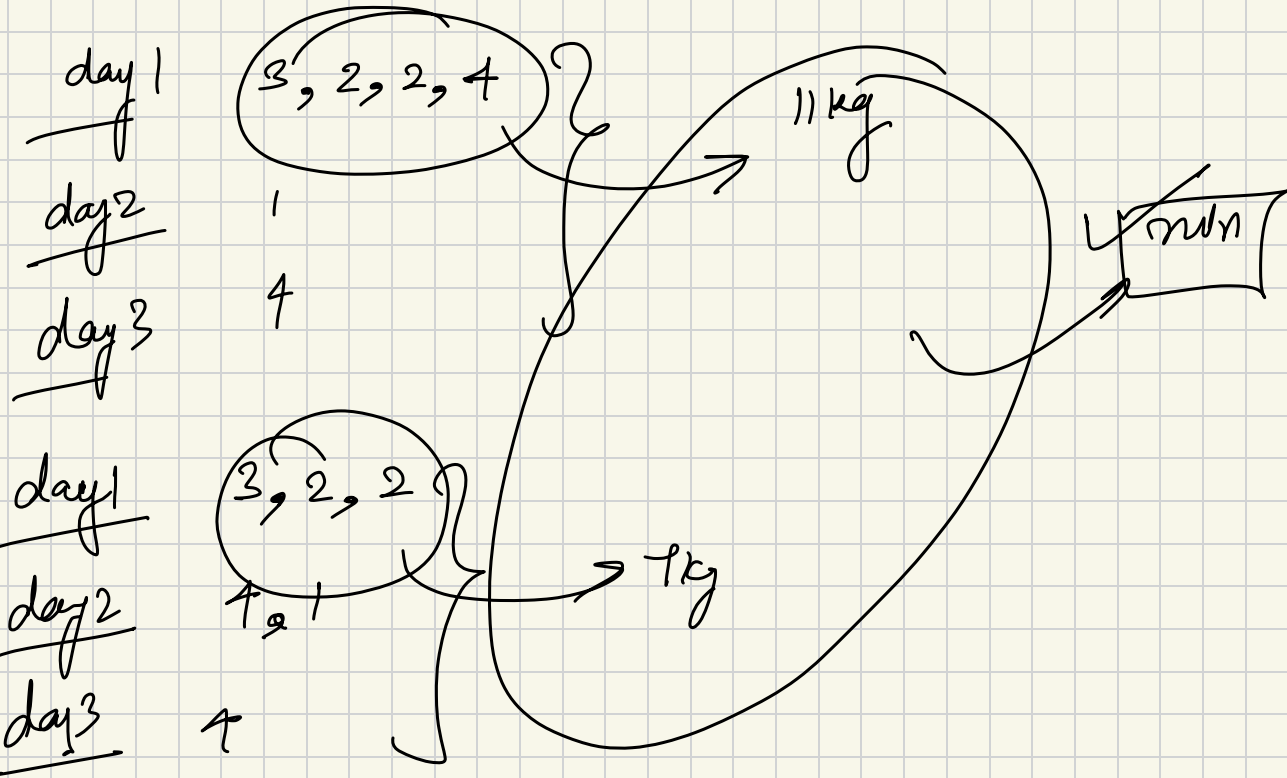
$\text{int[] } A = \{ 3, 2, 2, 4, 1, 4 \}$  day = 3



$\min^m \left\{ \max^m \text{ Capacity of the ship} \right\}$



$\text{int}[1] A = \{ 3, 2, 2, 4, 1, 4 \}$  day = 3



$$\log_2(10^4) = 4 \times 3.1 \approx 13 \text{ steps}$$

↓  
Range

$$\log_2(10^8) = 8 \times 3.1 = \underline{24 \text{ steps}}$$

# Binary Search over an infinite array

infinite sorted array

{ 1, 3, 10, 11, 15, 17, 20, 25, . . . . . }

target = 1014

Brute Force

↳ Linear Search

$O(10^6)$

↳ Index of Element.

$\{ \overset{0}{1}, \overset{1}{3}, \overset{2}{10}, \overset{3}{11}, \overset{4}{15}, \overset{5}{17}, \overset{6}{20}, \overset{7}{25}, \overset{8}{30}, \overset{9}{35}, \dots, \overset{16}{103}, \dots \}$   
 $\uparrow$   $\uparrow$   
 $lo$   $hi$

target = 90

~~Range [0, 1]  $\rightarrow$  x 2~~

Range [2, 2]  $\rightarrow$  1  $\rightarrow 2^0$

Range [3, 4]  $\rightarrow$  2  $\rightarrow 2^1$

Range [5, 8]  $\rightarrow$  4  $\rightarrow 2^2$

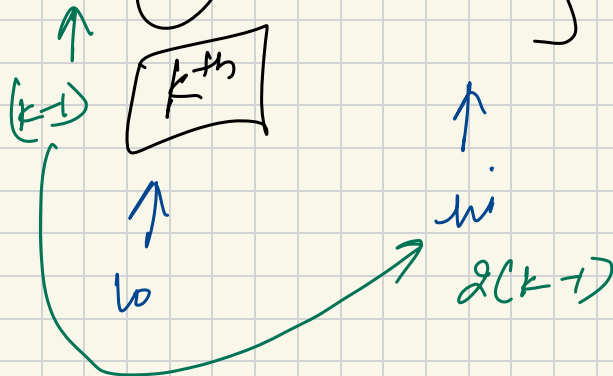
Range [9, 16]  $\rightarrow$  8  $\rightarrow 2^3$

$\left\{ \begin{array}{l} \text{target} \geq \text{arr}[lo] \text{ \& \& } \\ \text{target} \leq \text{arr}[hi] \end{array} \right.$

$$lo = hi + 1$$

$$hi = 2 \times hi$$

$\{a, b, c, d, \dots, \textcircled{e}, \dots, \dots\}$



$$\left. \begin{aligned} l_0 &= k^{\text{th}} \\ l_1 &= 2(k-1)^{\text{th}} \end{aligned} \right\}$$

$\rightarrow \begin{matrix} \log k \\ O_2 \end{matrix}$   $\rightarrow$  time complexity to search in the array

$$\begin{aligned} \text{total Ele} &= 2(k-1) - k \\ &= (k-2) \text{ Elements} \end{aligned}$$

$$1^{\text{st}} \text{ Range} = 1$$

$$2^{\text{nd}} \text{ Range} = 2$$

1

1

1

1

1

1

$$p^{\text{th}} \text{ Range} = 2^{p-1}$$

$$2^{p-1} = k-2$$

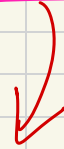
$$p-1 = \log_2 (k-2)$$

$$p = \log_2 (k-2) + 1$$

$$\boxed{p \approx \log_2 k}$$

Overall TC

$$\log_2 k + \log_2 k = TC: \underline{O(\log_2 k)}$$



I can find an Ele

$$\underline{\underline{TC: \log_2(k)}}$$

~~$\log_2 1000$~~   
 $\log_2 100$