



# Min<sup>m</sup> limit of Balls in a Bag.

int[] A = {<sup>0</sup>3, <sup>1</sup>2, <sup>2</sup>2, <sup>3</sup>4, <sup>4</sup>1, <sup>5</sup>4}

max Opt = 4

(1, 2)

opt = 1

{1, 2, 2, 2, 4, 1, 4}

penalty = 4

(1, 3) (2, 2)

opt = 2

{1, 2, 2, 2, 2, 2, 1, 4}

penalty = 4

(1, 3) (2, 2)

opt = 3

{1, 2, 2, 2, 2, 2, 1, 1, 3}

penalty = 3

(1, 2)

~~opt = 4~~

{1, 2, 2, 2, 2, 2, 1, 1, 1, 2}

~~penalty = 2~~

$$\text{int[] } A = \{ \overset{0}{3}, \overset{1}{2}, \overset{2}{2}, \overset{3}{4}, \overset{4}{1}, \overset{5}{4} \}$$

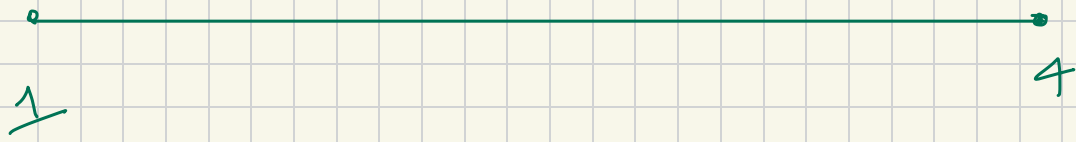
Case 1     $\text{maxOpt} = \infty$

$$\{ 1, 1, 1, \dots \}$$

$$\text{penalty} = 1$$

Case 2     $\text{maxOpt} = 0$

$$\text{penalty} = 4$$



$\text{int[] } A = \{ \overset{0}{3}, \overset{1}{2}, \overset{2}{2}, \overset{3}{4}, \overset{4}{1}, \overset{5}{4} \}$        $\text{maxLen} = 2$

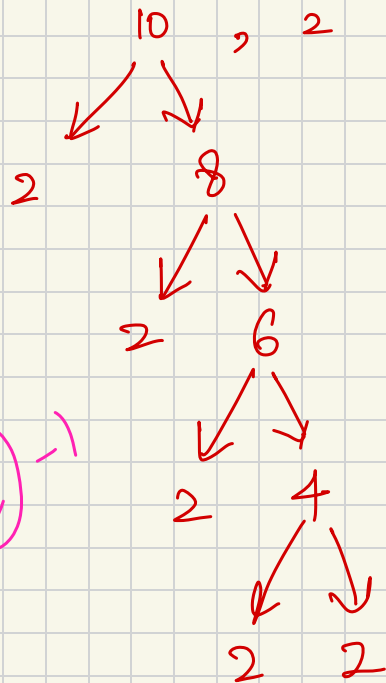
$(1, 2)$        $\nearrow$   $\nearrow$   $\nearrow$   $\nearrow$   $\nearrow$   $\nearrow$

$\text{numOpt} = \cancel{0} \cancel{1} \cancel{2} 3$

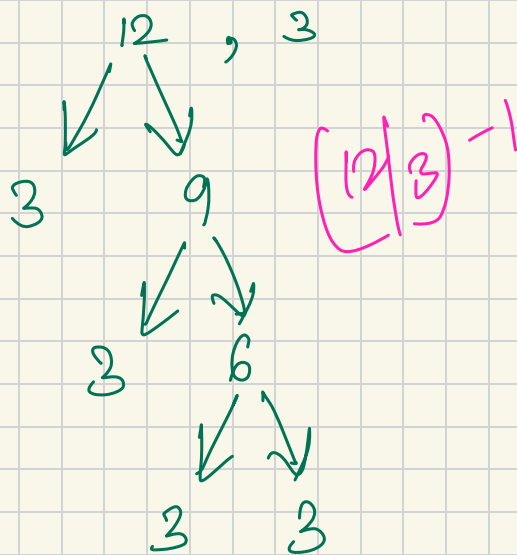
$\text{num} = x$

$\text{pen} = y$

$\text{new NumOfOpt} =$

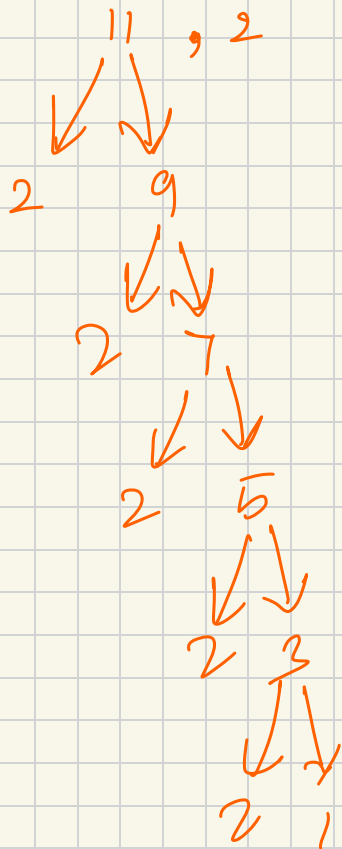


4opt



3opt

$$\text{maxOpt} = \left(\frac{x}{y}\right)^{-1} \checkmark$$



$x$  is divisible by  $y$

$$\left. \begin{array}{l} (x/y - 1) \\ (x/y) \end{array} \right\}$$

# Hashing (searching technique)

pool of Numbers  $\rightarrow$

{

$\swarrow$

}

int target

(1) Linear Search  $Tc: O(N)$   $Sc: O(1)$

(2) Binary Search  $Tc: O(\log N)$   $Sc: O(1)$   
(2)

} stores i/p arrays

✓ (3) Hashing  $Tc: O(1)$  ,  $Sc: O(1)$

} store i/p hash Table



Q/P 8, 3, 12, 16, 17, 4, 10 . . . . 50

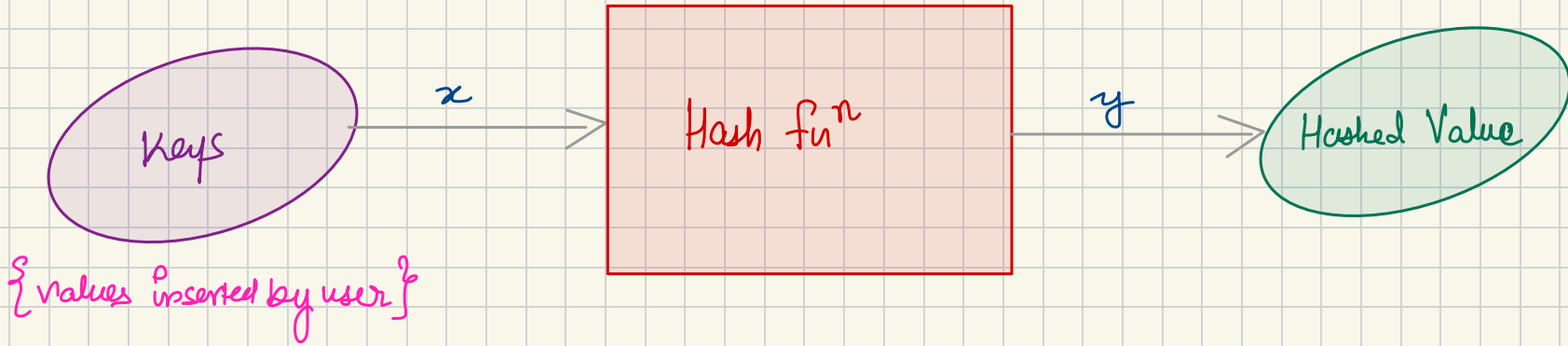
Boolean[] arr = {  
0 3 4 8 10 13 16 17 . . . . 50  
T T T T T T T T } T

```
search(int target)
{
    if (arr[target] == true)
        return "found";
    else
        return "not found";
}
```

TC:  $O(1)$   
SC:  $O(1)$

• high Memory consumption  
hashing was introduced

# Basic Mechanism of Hashing.



Hash fun<sup>n</sup>. { 2-step fun<sup>n</sup> }

Step 1  $g(x) = k$

key  $\swarrow$  integer  $\searrow$  }

Step 2  $f(k) = y$

integer  $\swarrow$  Hashed  $\searrow$

## Key Space

8 ✓

3 ✓

13 ✓

6 ✓

4

10

## Hash fun

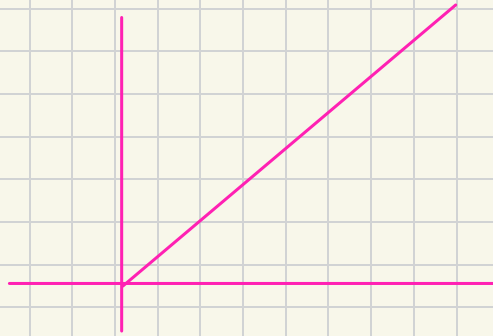
$$f(k) = k \quad (\text{one to one fun})$$

$$f(8) = 8$$

$$f(3) = 3$$

$$f(13) = 13$$

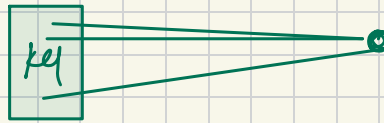
$$f(6) = 6$$



## Hash Table

0	
1	
2	
3	3
4	
5	
6	6
7	
8	8
9	
10	
11	
12	
⋮	
⋮	13

many to one relation

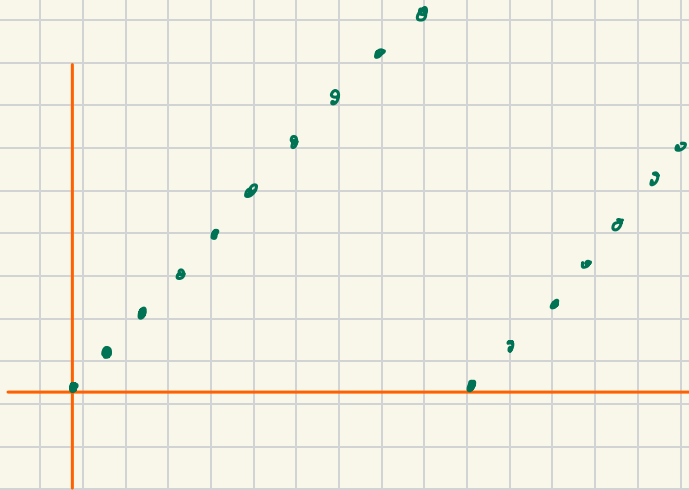


hashing fun!

$$f(k) = k \% 10$$

Range:  $[0, 9]$

→ Fixed hashed  
Table Size



Key = 12

hashed Value = 2

Key = 2

hashed Value = 2

## Key Space

8 ✓

3 ✓

13

6

4

10

## Hash fun<sup>n</sup>

$$f(k) = k \% 10$$

$$f(8) = 8 \% 10 = 8$$

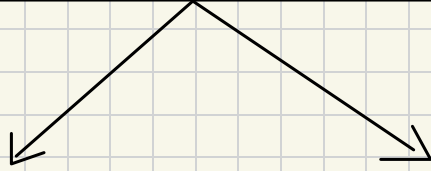
$$f(3) = 3 \% 10 = 3$$

$$f(13) = 13 \% 10 = 3$$

## Hash Table

0	
1	
2	
3	3 <u>collision!</u>
4	
5	
6	
7	
8	8
9	

## Methods to remove collision



### Open Hashing

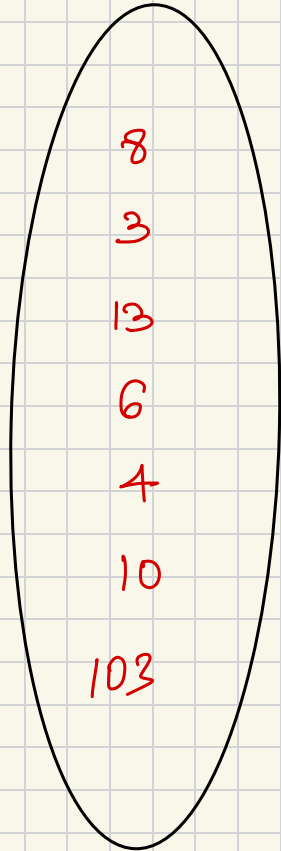
- Chaining

### Closed Hashing

- Linear Probing
- Quadratic Probing

# chaining

Key Space



Hash fun

$$f(k) = k \% 10$$

$$f(8) = 8 \% 10 = 8$$

$$f(3) = 3 \% 10 = 3$$

$$f(13) = 13 \% 10 = 3$$

$$f(6) = 6 \% 10 = 6$$

$$f(4) = 4 \% 10 = 4$$

$$f(10) = 10 \% 10 = 0$$

$$f(103) = 103 \% 10 = 3$$

search(103)

→ hashed value

Hash Table

0	10
1	
2	
3	3 → 13 → 103
4	4
5	
6	6
7	
8	8
9	

load factor

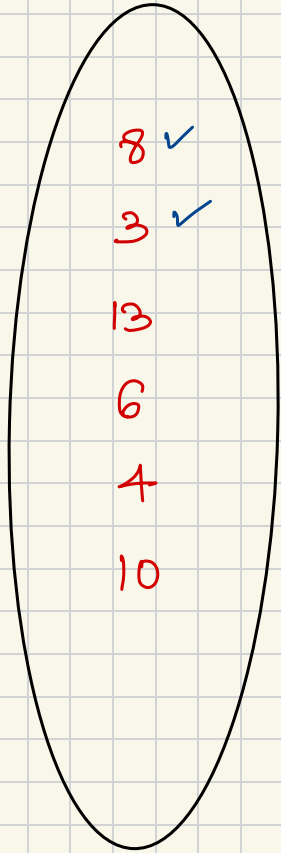
↳ 75% of Range

Hash fun<sup>n</sup> = key % Load Factor



# Linear Probing

Key Space



Hash fun

$$f'(k) = \{ f(k) + g(i) \} \% 10$$

$$f(k) = k \% 10$$

$$g(i) = 0, 1, 2, \dots$$

$$\begin{aligned} f'(8) &= \{ f(8) + g(0) \} \% 10 \\ &= (8 + 0) \% 10 = 8 \end{aligned}$$

$$\begin{aligned} f'(3) &= \{ f(3) + g(0) \} \% 10 \\ &= (3 + 0) \% 10 = 3 \end{aligned}$$

$$f'(13) = \{ f(13) + g(0) \} \% 10 = 3$$

$$f'(13) = \{ f(13) + g(1) \} \% 10 = 4$$

Hash Table

0	
1	
2	
3	3
4	13
5	
6	
7	
8	8
9	

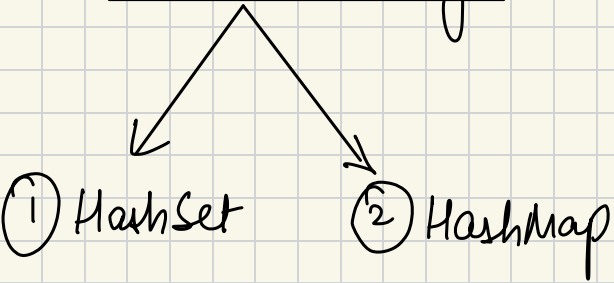
## Quadratic probing

$$f'(k) = \{ f(k) + g(i) \} \% \text{ L.P.}$$

$$f(k) = k \% \text{ L.P.}$$

$$g(i) = i^2 ; \quad 0, 1, 4, 9, 16, 25, \dots$$

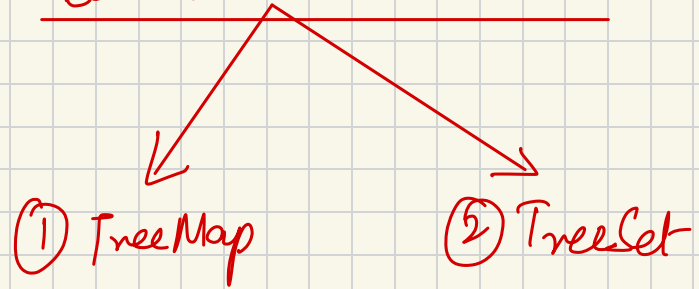
## Based on hashing



TC:  $O(1)$  Searching

TC:  $O(1)$  Insertion

## Based on Red Black Trees

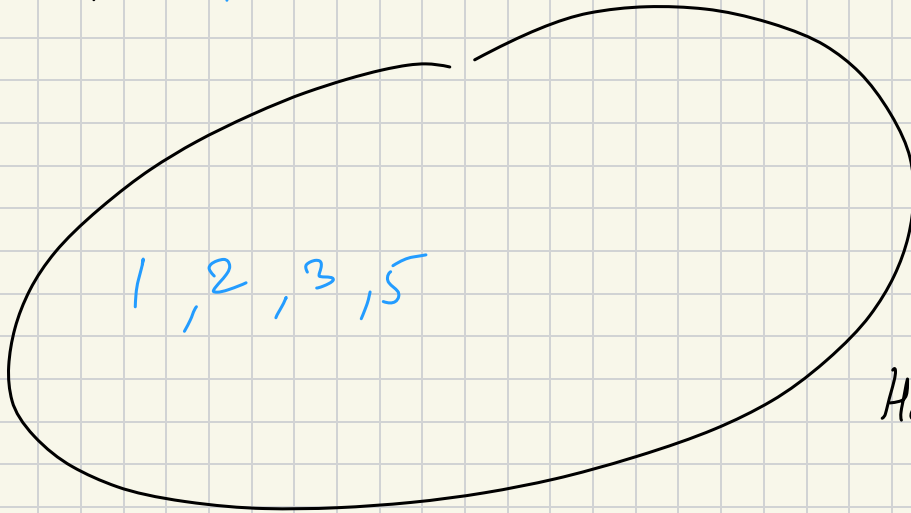


TC:  $O(\log_2 N)$  Searching, Insertion.

HashSet

→ set of unique entities

keySet = { 1, 2, 3, 3, 5, 2, 1, 3, 5 }



HashSet

Hash Map



stones (key, value) pair

HashSet



unique entities

Complete a shopping cart

Item

qty

unique entity

Hashing

lays → 2

eggs → 12

oranges → 2

yogurt → 4

HashSet<G> name\_hs = new HashSet<>();

HashMap<G> name\_hm = new HashMap<>();

