For $s \in \mathbb{Z}$ and $\mathbb{P}$ is the collection of all prime numbers, we have:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \in \mathbb{P}} \frac{p^s}{p^s - 1}. \tag{1}$$

The right hand side can be written as

$$\prod_{p \in \mathbb{P}} \frac{p^s}{p^s - 1} = \prod_{p \in \mathbb{P}} \frac{1}{1 - \frac{1}{p^s}} = \frac{1}{\prod_{p \in \mathbb{P}} 1 - \frac{1}{p^s}}$$

Recalling the method when finding the generating function of the Fibonacci number, we could divide the left hand side by the multiples of prime numbers. Assuming there is a function of prime numbers P_n that returns the n-th prime number.

$$\begin{aligned}
\zeta(s) &= \sum_{n=1}^{\infty} \frac{1}{n^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots . \\
\frac{1}{P_1^s}\zeta(s) &= \frac{1}{P_1^s}\left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots\right). \Longrightarrow \\
\frac{1}{2^s}\zeta(s) &= \frac{1}{2^s}\left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots\right) = \frac{1}{2^s} + \frac{1}{4^s} + \frac{1}{6^s} + \cdots . \\
\frac{1}{P_2^s}\zeta(s) &= \frac{1}{P_2^s}\left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots\right). \Longrightarrow \\
\frac{1}{3^s}\zeta(s) &= \frac{1}{3^s}\left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots\right) = \frac{1}{3^s} + \frac{1}{6^s} + \frac{1}{9^s} + \cdots . \\
\frac{1}{P_3^s}\zeta(s) &= \frac{1}{P_3^s}\left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots\right). \Longrightarrow \\
\frac{1}{5^s}\zeta(s) &= \frac{1}{5^s}\left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots\right) = \frac{1}{5^s} + \frac{1}{10^s} + \frac{1}{15^s} + \cdots .
\end{aligned}$$

Thus,

$$\begin{aligned}
\left(1 - \frac{1}{P_1^s}\right)\zeta(s) &= 1 + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \cdots . \text{ (No more multiples of 2)} \\
\left(1 - \frac{1}{P_2^s}\right)\left(1 - \frac{1}{P_1^s}\right)\zeta(s) &= 1 + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{11^s} + \cdots . \text{ (No more multiples of 3)} \\
&\cdots \\
\zeta(s)\left(1 - \frac{1}{P_1^s}\right)\left(1 - \frac{1}{P_2^s}\right)\cdots &= 1. \\
\zeta(s) &= \frac{1}{\left(1 - \frac{1}{P_1^s}\right)\left(1 - \frac{1}{P_2^s}\right)\cdots}.
\end{aligned}$$

Therefore, it is proved that

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \in \mathbb{P}} \frac{p^s}{p^s - 1}.$$