Integer partitioning function $p(n)$ can be represented as the generating function of the sequence of $p(n)$:

$$\sum_{n=0}^{\infty} p(n)x^n = \prod_{k=1}^{\infty} \frac{1}{1-x^k}. \tag{1}$$

The generating function of odd partition, denoted by $p_o(n)$, can be derived as following:

$$\begin{aligned}
\prod_{k=1}^{\infty} \frac{1}{1-x^k} &= \frac{1}{1-x} \cdot \frac{1}{1-x^2} \cdot \cdots \cdot \frac{1}{1-x^m} \cdot \cdots \\
&= \prod_{k=1}^{\infty} (x^0 + x^k + x^{2k} + \cdots) \\
&= (1 + x + x^2 + \cdots) \\
&\quad (1 + x^2 + x^4 + \cdots) \\
&\quad \cdots \\
&\quad (1 + x^m + x^{2m} + \cdots) \\
&\quad \cdots
\end{aligned}$$

The correspondence can be represented by using a simple example of $p(5)$.

$$\begin{aligned}
5 &= 1+1+1+1+1 && \Longrightarrow (x^1)^5 \text{ from } \frac{1}{1-x} \\
&= 2+1+1+1 && \Longrightarrow (x^2)^1 \text{ from } \frac{1}{1-x^2};\ (x^1)^3 \text{ from } \frac{1}{1-x} \\
&= 2+2+1 && \Longrightarrow (x^2)^2 \text{ from } \frac{1}{1-x^2};\ (x^1)^1 \text{ from } \frac{1}{1-x} \\
&= 3+1+1 && \Longrightarrow (x^3)^1 \text{ from } \frac{1}{1-x^3};\ (x^1)^2 \text{ from } \frac{1}{1-x} \\
&= 3+2 && \Longrightarrow (x^3)^1 \text{ from } \frac{1}{1-x^3};\ (x^2)^1 \text{ from } \frac{1}{1-x^2} \\
&= 4+1 && \Longrightarrow (x^4)^1 \text{ from } \frac{1}{1-x^4};\ (x^1)^1 \text{ from } \frac{1}{1-x} \\
&= 5 && \Longrightarrow (x^5)^1 \text{ from } \frac{1}{1-x^5}.
\end{aligned}$$

To express the partition function of the odd partition, we need to eliminate partitions that have even numbers in it, which means we won't count it into the odd partition if a in $(x^a)^b$ is even. Therefore, the generating function of the odd partition is

$$\begin{aligned}
\sum_{n=0}^{\infty} p_o(n)x^n &= \frac{1}{1-x} \cdot \frac{1}{1-x^3} \cdot \frac{1}{1-x^5} \cdot \cdots \\
&= \prod_{k=0}^{\infty} \frac{1}{1-x^{2n+1}}.
\end{aligned} \tag{2}$$

Similarly, we won't count another to the distinct partition if b in $(x^a)^b$ is larger than 1. Therefore, the expansions will only have their first two terms, and the generating function of the distinct partition is

$$\begin{aligned}
\sum_{n=0}^{\infty} p_d(n)x^n &= (1 + x + 0) \cdot (1 + x^2 + 0) \cdot \cdots \\
&= \frac{1-x^2}{1-x} \cdot \frac{1-x^4}{1-x^2} \cdot \frac{1-x^6}{1-x^3} \cdot \frac{1-x^8}{1-x^4} \cdot \cdots \\
&= \frac{1}{1-x} \cdot \frac{1}{1-x^3} \cdot \frac{1}{1-x^5} \cdot \cdots .
\end{aligned} \tag{3}$$

Therefore, it is proved that since the generating functions are the same, $p_o(n) = p_d(n)$.