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# **Social Recommendation Using Probabilistic Matrix Factorization**

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**&**

# **Recommender Systems with Social Regularization**

By Hao Ma, Dengyong Zhou, Chao Liu, Michael R. Lyu, Irwin King

**Presented by Kim Han**

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# Contents - SoReg

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1. Introduction
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# Introduction

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## Problem

### **Assume that,**

1. two users have rated at least some items in common  
→ data density  $< 1\%$
2. all the users are independent and identically distributed  
→ In reality, we are influenced by each other.

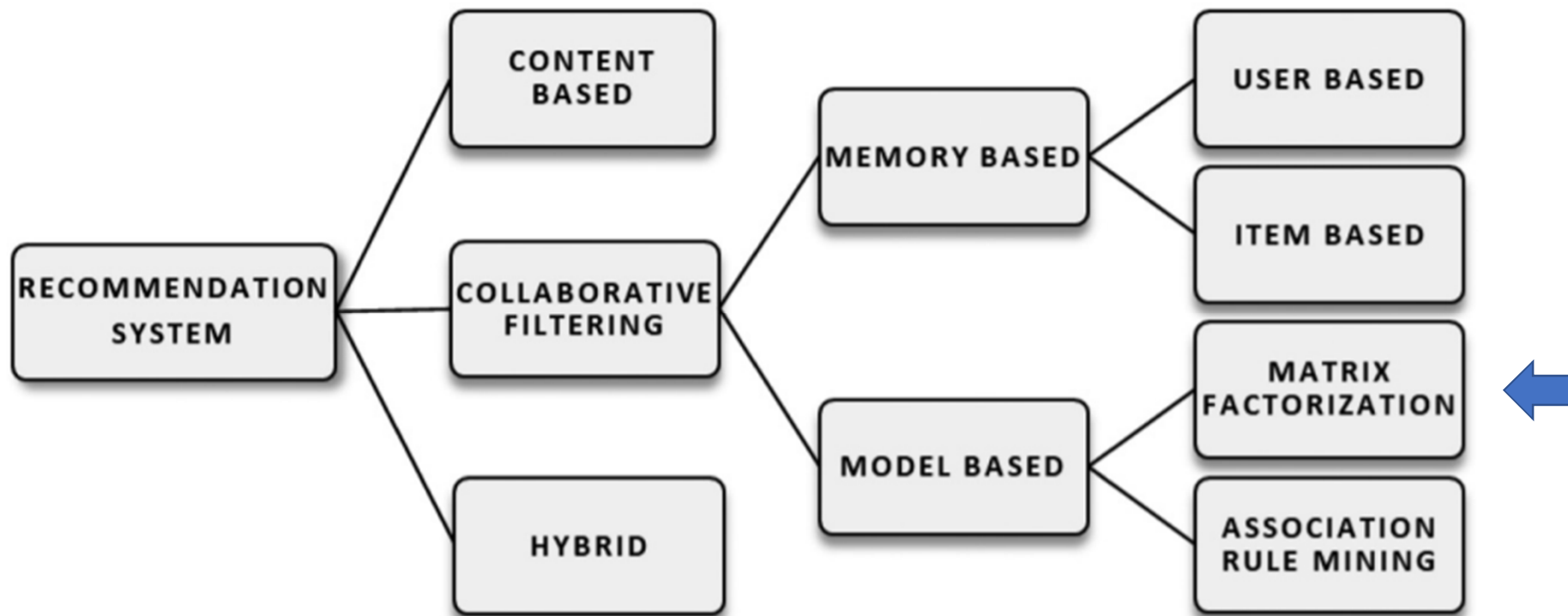
### **Hence,**

By using rating records + users' social network information,

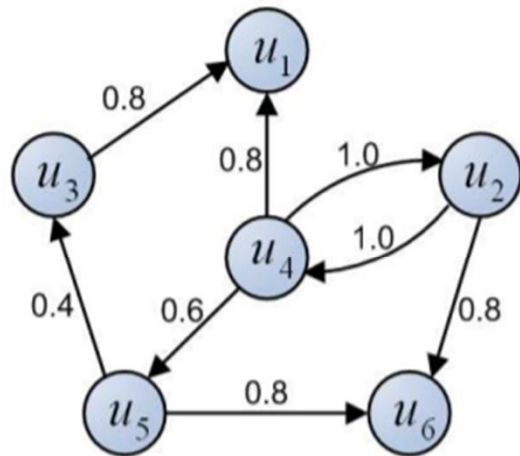
→ 1. more accurate and 2. personal taste recommendation system.

# Background

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# Framework



(a) Social Network Graph

$$\longrightarrow U^T Z$$

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$
$u_1$	5	2		3		4		
$u_2$	4	3			5			
$u_3$	4		2				2	4
$u_4$								
$u_5$	5	1	2		4	3		
$u_6$	4	3		2	4		3	5

(b) User-Item Matrix

$$\longrightarrow U^T V$$

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$
$u_1$	5	2	2.5	3	4.8	4	2.2	4.8
$u_2$	4	3	2.4	2.9	5	4.1	2.6	4.7
$u_3$	4	1.7	2	3.2	3.9	3.0	2	4
$u_4$	4.8	2.1	2.7	2.6	4.7	3.8	2.4	4.9
$u_5$	5	1	2	3.4	4	3	1.5	4.6
$u_6$	4	3	2.9	2	4	3.4	3	5

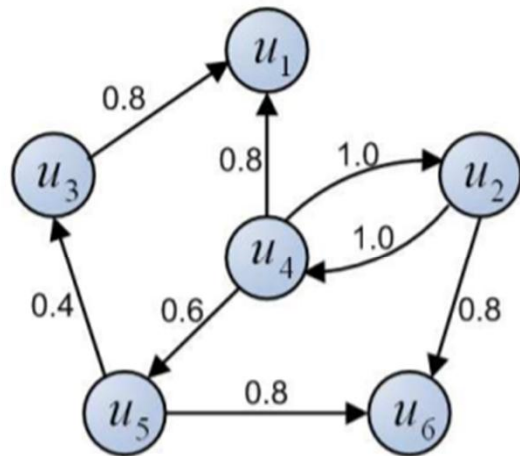
(c) Predicted User-Item Matrix

U - low-dimensional user latent feature space

Z - factor matrix in the social network graph

V - low-dimensional item latent feature space

# Framework



(a) Social Network Graph

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$
$u_1$	5	2		3		4		
$u_2$	4	3			5			
$u_3$	4		2				2	4
$u_4$								
$u_5$	5	1	2		4	3		
$u_6$	4	3		2	4		3	5

(b) User-Item Matrix

	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$
$u_1$	5	2	2.5	3	4.8	4	2.2	4.8
$u_2$	4	3	2.4	2.9	5	4.1	2.6	4.7
$u_3$	4	1.7	2	3.2	3.9	3.0	2	4
$u_4$	4.8	2.1	2.7	2.6	4.7	3.8	2.4	4.9
$u_5$	5	1	2	3.4	4	3	1.5	4.6
$u_6$	4	3	2.9	2	4	3.4	3	5

(c) Predicted User-Item Matrix

$$U = \begin{bmatrix} 1.55 & 1.22 & 0.37 & 0.81 & 0.62 & -0.01 \\ 0.36 & 0.91 & 1.21 & 0.39 & 1.10 & 0.25 \\ 0.59 & 0.20 & 0.14 & 0.83 & 0.27 & 1.51 \\ 0.39 & 1.33 & -0.43 & 0.70 & -0.90 & 0.68 \\ 1.05 & 0.11 & 0.17 & 1.18 & 1.81 & 0.40 \end{bmatrix}$$

$$V = \begin{bmatrix} 1.00 & -0.05 & -0.24 & 0.26 & 1.28 & 0.54 & -0.31 & 0.52 \\ 0.19 & -0.86 & -0.72 & 0.05 & 0.68 & 0.02 & -0.61 & 0.70 \\ 0.49 & 0.09 & -0.05 & -0.62 & 0.12 & 0.08 & 0.02 & 1.60 \\ -0.40 & 0.70 & 0.27 & -0.27 & 0.99 & 0.44 & 0.39 & 0.74 \\ 1.49 & -1.00 & 0.06 & 0.05 & 0.23 & 0.01 & -0.36 & 0.80 \end{bmatrix}$$

# Framework

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$$G = (V, E)$$

$$V = \{v_i\}_{i=1}^m$$

$$C = \{c_{ik}\} \quad c_{ik} \in (0, 1]$$

$$p(C|U, Z, \sigma_C^2) = \prod_{i=1}^m \prod_{k=1}^m \mathcal{N} \left[ \left( c_{ik} | g(U_i^T Z_k), \sigma_C^2 \right) \right]^{I_{ik}^C}$$

$$p(U|\sigma_U^2) = \prod_{i=1}^m \mathcal{N}(U_i|0, \sigma_U^2 \mathbf{I})$$

$$p(Z|\sigma_Z^2) = \prod_{k=1}^m \mathcal{N}(Z_k|0, \sigma_Z^2 \mathbf{I})$$

$$\begin{aligned} p(U, Z|C, \sigma_C^2, \sigma_U^2, \sigma_Z^2) &\propto p(C|U, Z, \sigma_C^2) p(U|\sigma_U^2) p(Z|\sigma_Z^2) \\ &= \prod_{i=1}^m \prod_{k=1}^m \mathcal{N} \left[ \left( c_{ik} | g(U_i^T Z_k), \sigma_C^2 \right) \right]^{I_{ik}^C} \\ &\times \prod_{i=1}^m \mathcal{N}(U_i|0, \sigma_U^2 \mathbf{I}) \times \prod_{k=1}^m \mathcal{N}(Z_k|0, \sigma_Z^2 \mathbf{I}) \end{aligned}$$



# Framework

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$$c_{ik}^* = \sqrt{\frac{d^-(v_k)}{d^+(v_i) + d^-(v_k)}} \times c_{ik}$$

$$p(C|U, Z, \sigma_C^2) = \prod_{i=1}^m \prod_{k=1}^m \mathcal{N} \left[ \left( c_{ik} | g(U_i^T Z_k), \sigma_C^2 \right) \right]^{I_{ik}^C}$$



$$p(C|U, Z, \sigma_C^2) = \prod_{i=1}^m \prod_{j=1}^n \mathcal{N} \left[ \left( c_{ik}^* | g(U_i^T Z_k), \sigma_C^2 \right) \right]^{I_{ik}^C}$$

# Framework

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$$g(x) = (x - 1) / (R_{\max} - 1)$$

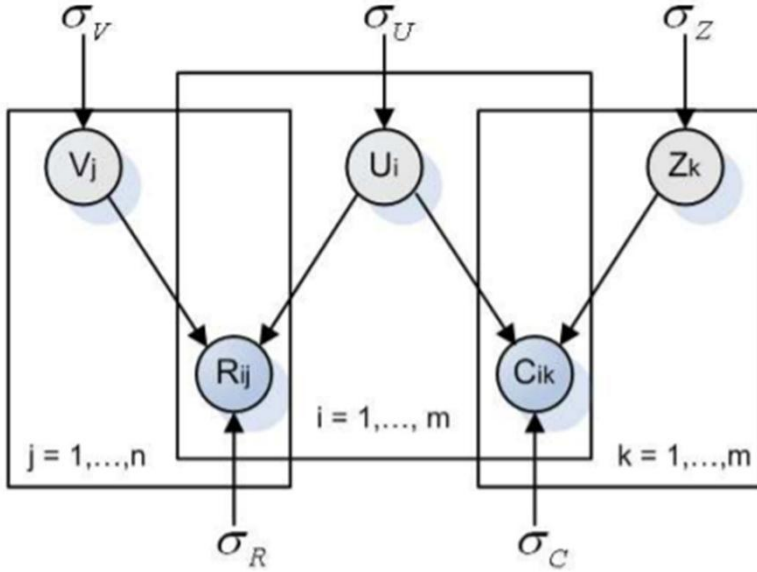
$$p(R|U, V, \sigma_R^2) = \prod_{i=1}^m \prod_{j=1}^n \mathcal{N} \left[ \left( r_{ij} | g(U_i^T V_j), \sigma_R^2 \right) \right]^{I_{ij}^R}$$

$$p(U|\sigma_U^2) = \prod_{i=1}^m \mathcal{N}(U_i | 0, \sigma_U^2 \mathbf{I})$$

$$p(V|\sigma_V^2) = \prod_{j=1}^n \mathcal{N}(V_j | 0, \sigma_V^2 \mathbf{I})$$

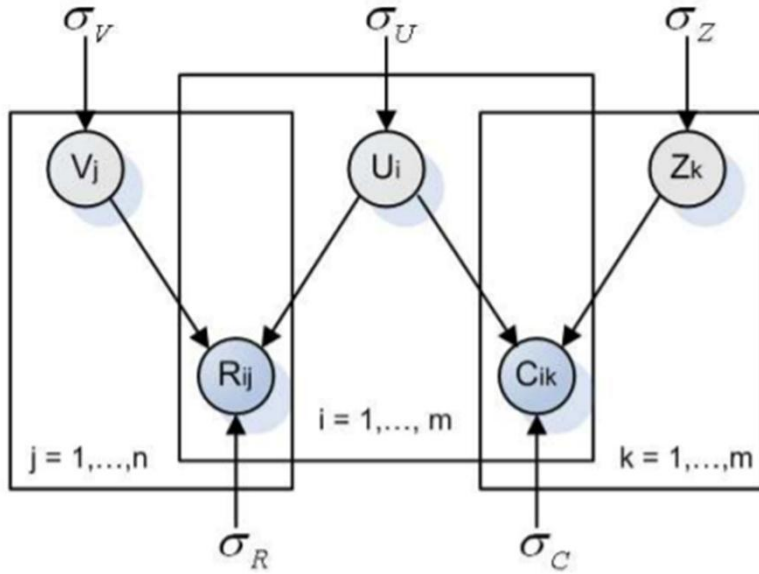
$$\begin{aligned} p(U, V | R, \sigma_R^2, \sigma_U^2, \sigma_V^2) & \propto p(R|U, V, \sigma_R^2) p(U|\sigma_U^2) p(V|\sigma_V^2) \\ & = \prod_{i=1}^m \prod_{j=1}^n \mathcal{N} \left[ \left( r_{ij} | g(U_i^T V_j), \sigma_R^2 \right) \right]^{I_{ij}^R} \\ & \times \prod_{i=1}^m \mathcal{N}(U_i | 0, \sigma_U^2 \mathbf{I}) \times \prod_{j=1}^n \mathcal{N}(V_j | 0, \sigma_V^2 \mathbf{I}) \end{aligned}$$

# Framework



$$\begin{aligned}
 \ln p(U, V, Z | C, R, \sigma_C^2, \sigma_R^2, \sigma_U^2, \sigma_V^2, \sigma_Z^2) = & \\
 & -\frac{1}{2\sigma_R^2} \sum_{i=1}^m \sum_{j=1}^n I_{ij}^R (r_{ij} - g(U_i^T V_j))^2 \\
 & -\frac{1}{2\sigma_C^2} \sum_{i=1}^m \sum_{k=1}^m I_{ik}^C (c_{ik}^* - g(U_i^T Z_k))^2 \\
 & -\frac{1}{2\sigma_U^2} \sum_{i=1}^m U_i^T U_i - \frac{1}{2\sigma_V^2} \sum_{j=1}^n V_j^T V_j - \frac{1}{2\sigma_Z^2} \sum_{k=1}^m Z_k^T Z_k \\
 & -\frac{1}{2} \left( \left( \sum_{i=1}^m \sum_{j=1}^n I_{ij}^R \right) \ln \sigma_R^2 + \left( \sum_{i=1}^m \sum_{k=1}^m I_{ik}^C \right) \ln \sigma_C^2 \right) \\
 & -\frac{1}{2} (m \ln \sigma_U^2 + n \ln \sigma_V^2 + m \ln \sigma_Z^2) + \mathcal{C}, \tag{8}
 \end{aligned}$$

# Framework



$$\lambda_C = \sigma_R^2 / \sigma_C^2, \quad \lambda_U = \sigma_R^2 / \sigma_U^2, \quad \lambda_V = \sigma_R^2 / \sigma_V^2$$

$$\lambda_Z = \sigma_R^2 / \sigma_Z^2$$



$$\lambda_U = \lambda_V = \lambda_Z$$

$$\begin{aligned} \mathcal{L}(R, C, U, V, Z) = & \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n I_{ij}^R (r_{ij} - g(U_i^T V_j))^2 + \frac{\lambda_C}{2} \sum_{i=1}^m \sum_{k=1}^m I_{ik}^C (c_{ik}^* - g(U_i^T Z_k))^2 \\ & + \frac{\lambda_U}{2} \|U\|_F^2 + \frac{\lambda_V}{2} \|V\|_F^2 + \frac{\lambda_Z}{2} \|Z\|_F^2, \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial U_i} &= \sum_{j=1}^n I_{ij}^R g'(U_i^T V_j) (g(U_i^T V_j) - r_{ij}) V_j \\ &\quad + \lambda_C \sum_{k=1}^m I_{ik}^C g'(U_i^T Z_k) (g(U_i^T Z_k) - c_{ik}^*) Z_k + \lambda_U U_i, \\ \frac{\partial \mathcal{L}}{\partial V_j} &= \sum_{i=1}^m I_{ij}^R g'(U_i^T V_j) (g(U_i^T V_j) - r_{ij}) U_i + \lambda_V V_j, \\ \frac{\partial \mathcal{L}}{\partial Z_k} &= \lambda_C \sum_{i=1}^m I_{ik}^C g'(U_i^T Z_k) (g(U_i^T Z_k) - c_{ik}^*) U_i + \lambda_Z Z_k, \end{aligned} \quad (10)$$

# Framework

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$$\frac{\partial \mathcal{L}}{\partial U} = O(\rho_R l + \rho_C l)$$

$$\frac{\partial \mathcal{L}}{\partial V} = O(\rho_R l)$$

$$\frac{\partial \mathcal{L}}{\partial Z} = O(\rho_C l)$$

Computational time : linear with respect to the number of observations in the two sparse matrices.

Thus, the approach can scale on large datasets

# Experiment

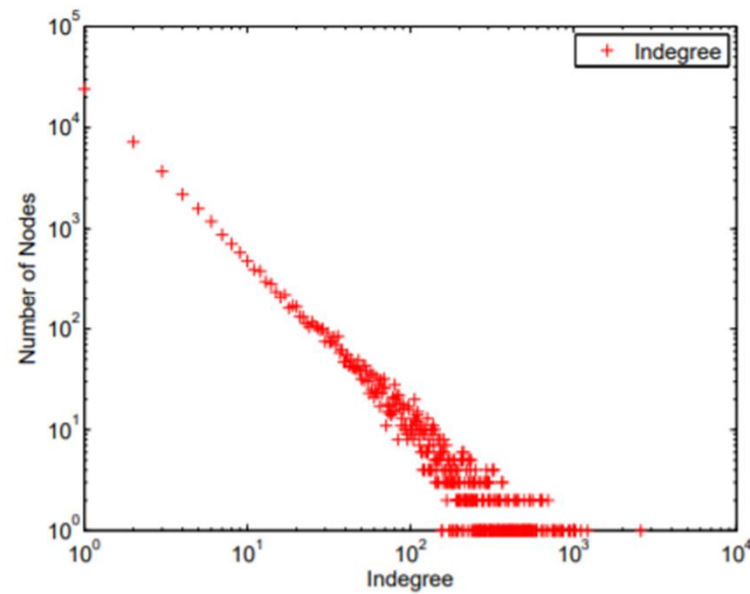
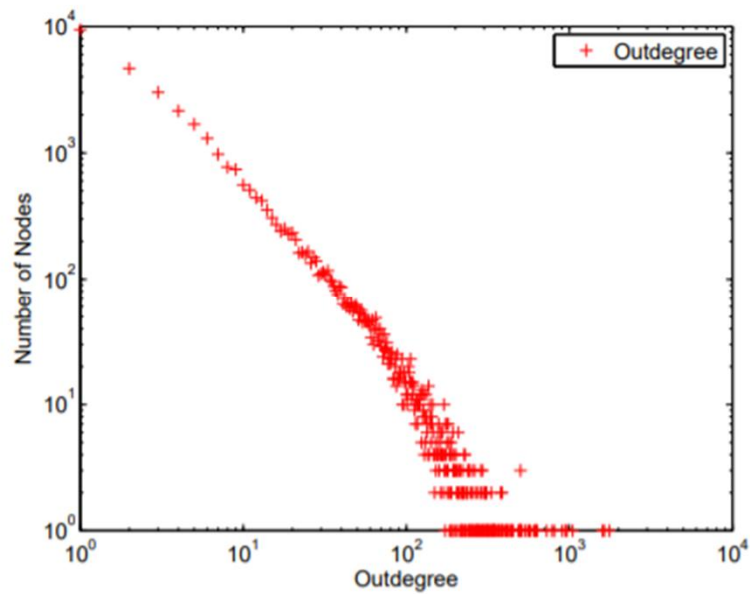
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1. How does our approach compare with the published state-of-the-art collaborative filtering algorithms?
2. How does the model parameter  $\lambda_C$  affect the accuracy of prediction?
3. What is the performance comparison on users with different observed ratings?
4. Can our algorithm achieve good performance even if users have no observed ratings?
5. Is our algorithm efficient for large datasets?

# Experiment

**Table 1:** Statistics of User-Item Rating Matrix of Epinions

Statistics	User	Item
Min. Num. of Rated	1	1
Max. Num. of Rated	1022	2018
Avg. Num. of Rated	16.55	4.76



# Experiment

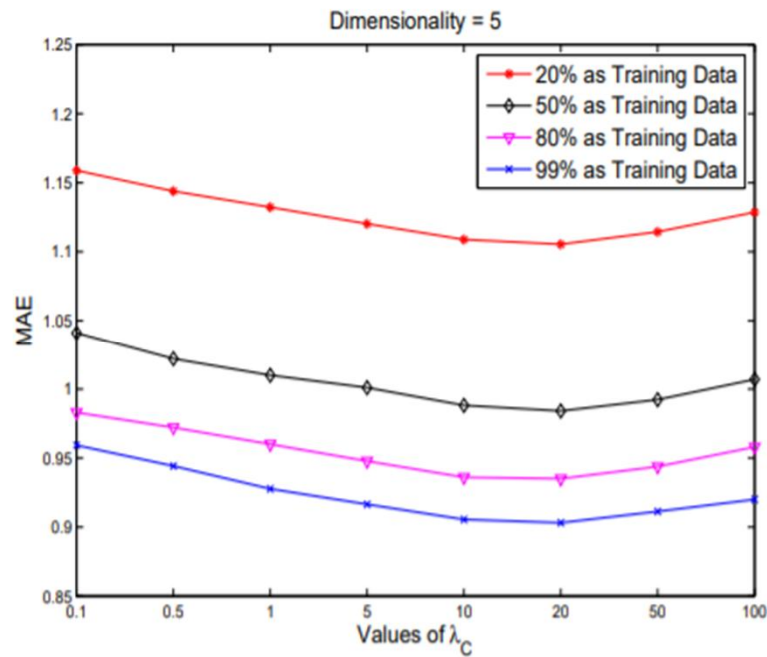
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Table 2: MAE comparison with other approaches (A smaller MAE value means a better performance)

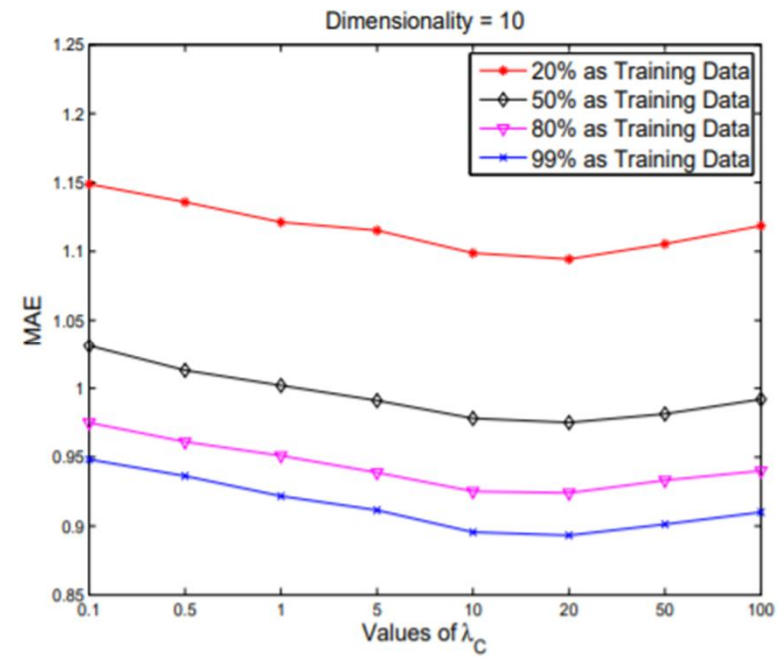
Training Data	Dimensionality = 5				Dimensionality = 10			
	MMMF	PMF	CPMF	SoRec	MMMF	PMF	CPMF	SoRec
99%	1.0008	0.9971	0.9842	<b>0.9018</b>	0.9916	0.9885	0.9746	<b>0.8932</b>
80%	1.0371	1.0277	0.9998	<b>0.9321</b>	1.0275	1.0182	0.9923	<b>0.9240</b>
50%	1.1147	1.0972	1.0747	<b>0.9838</b>	1.1012	1.0857	1.0632	<b>0.9751</b>
20%	1.2532	1.2397	1.1981	<b>1.1069</b>	1.2413	1.2276	1.1864	<b>1.0944</b>



# Experiment

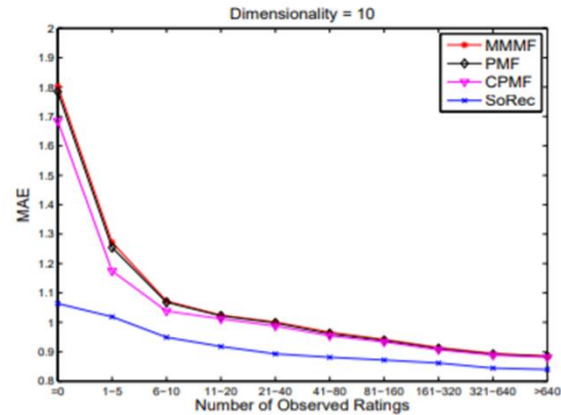


(a) Dimensionality=5

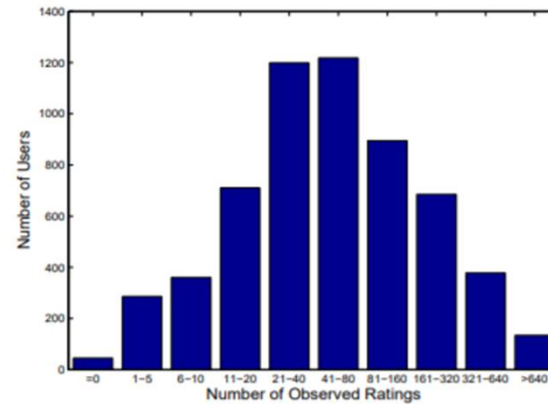


(b) Dimensionality=10

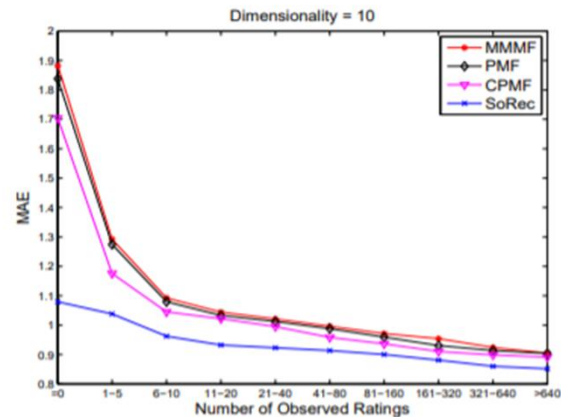
# Experiment



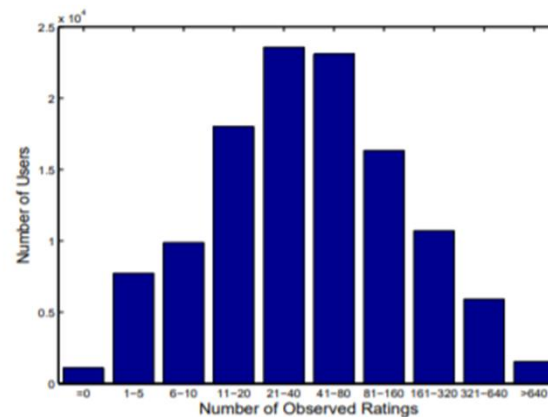
(a) Performance Comparison on Different User Rating Scales (99% as Training Data)



(b) Distribution of Testing Data (99% as Training Data)

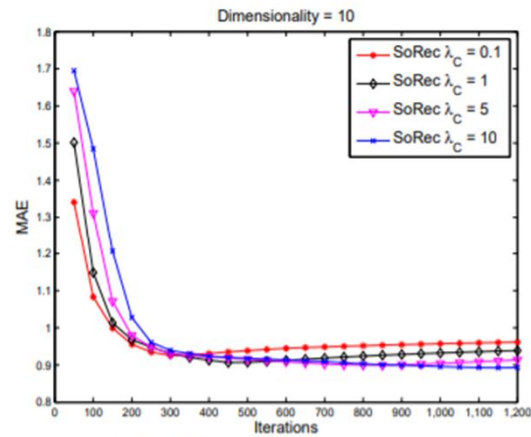


(c) Performance Comparison on Different User Rating Scales (80% as Training Data)

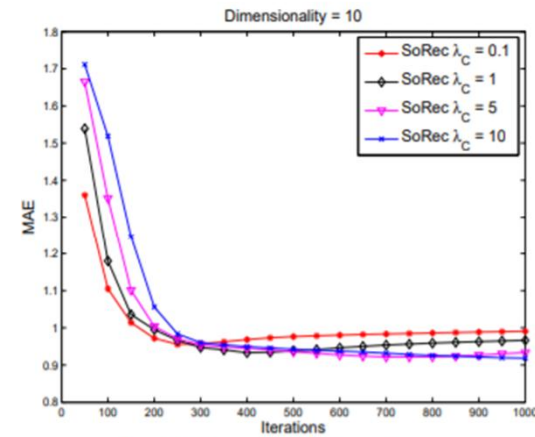


(d) Distribution of Testing Data (80% as Training Data)

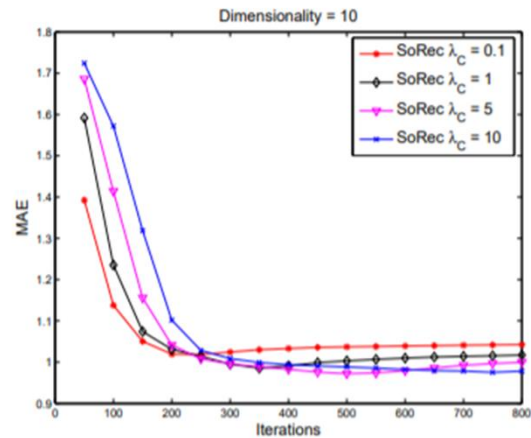
# Experiment



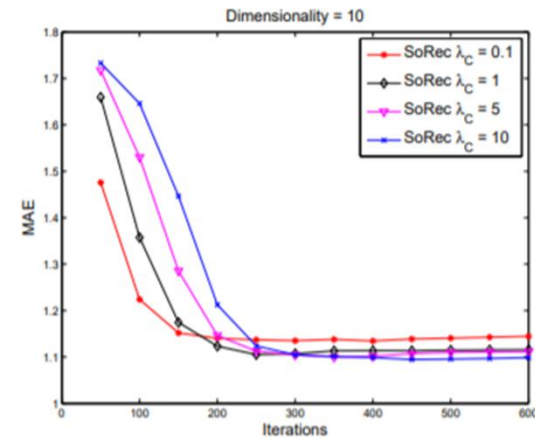
(a) 99% as Training Data



(b) 80% as Training Data



(c) 50% as Training Data



(d) 20% as Training Data

# Conclusion

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## **Conclusion**

- \* Experimental results: the approach outperforms the other state-of-the-art collaborative filtering algorithms
- \* Complexity analysis: it is scalable to very large datasets.
- \* Can also be used to predict connections on social network.

## **Future work**

- \* Distrust information
- \* Diffusion process

# Introduction

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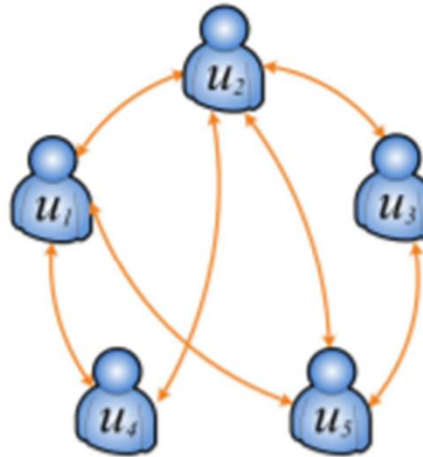
## Problem

1. “trust relationships” are different from “social friendships”
2. assumption that users have similar tastes with other users they trust
3. online users spend more and more time on social network with real friends

# Framework



(a) Real World Social Recommendation



(b) Social Network

	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$
$u_1$	1		2	3	
$u_2$		3			1
$u_3$		4		5	
$u_4$	5			4	
$u_5$		2	5		4

(c) User-Item Rating Matrix

$$R \approx U^T V$$

U - low-dimensional user latent feature space

~~Z - factor matrix in the social network graph~~

V - low-dimensional item latent feature space

# Framework

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$$\frac{1}{2} \|R - U^T V\|_F^2$$



$$\min_{U, V} \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n I_{ij} (R_{ij} - U_i^T V_j)^2$$



$$\min_{U, V} \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n I_{ij} (R_{ij} - U_i^T V_j)^2 + \frac{\lambda_1}{2} \|U\|_F^2 + \frac{\lambda_2}{2} \|V\|_F^2$$

# Framework – Social Regularization

$$\begin{aligned}
 \min_{U,V} \mathcal{L}_1(R, U, V) &= \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n I_{ij} (R_{ij} - U_i^T V_j)^2 \\
 &+ \frac{\alpha}{2} \sum_{i=1}^m \left\| U_i - \frac{1}{|\mathcal{F}^+(i)|} \sum_{f \in \mathcal{F}^+(i)} U_f \right\|_F^2 \\
 &+ \frac{\lambda_1}{2} \|U\|_F^2 + \frac{\lambda_2}{2} \|V\|_F^2, \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 \min_{U,V} \mathcal{L}_1(R, U, V) &= \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n I_{ij} (R_{ij} - U_i^T V_j)^2 \\
 &+ \frac{\alpha}{2} \sum_{i=1}^m \left\| U_i - \frac{\sum_{f \in \mathcal{F}^+(i)} \text{Sim}(i, f) \times U_f}{\sum_{f \in \mathcal{F}^+(i)} \text{Sim}(i, f)} \right\|_F^2, \\
 &+ \frac{\lambda_1}{2} \|U\|_F^2 + \frac{\lambda_2}{2} \|V\|_F^2. \quad (8)
 \end{aligned}$$



$$\begin{aligned}
 \frac{\partial \mathcal{L}_1}{\partial U_i} &= \sum_{j=1}^n I_{ij} (U_i^T V_j - R_{ij}) V_j + \lambda_1 U_i \\
 &+ \alpha \left( U_i - \frac{\sum_{f \in \mathcal{F}^+(i)} \text{Sim}(i, f) \times U_f}{\sum_{f \in \mathcal{F}^+(i)} \text{Sim}(i, f)} \right) \\
 &+ \alpha \sum_{g \in \mathcal{F}^-(i)} \frac{-\text{Sim}(i, g) \left( U_g - \frac{\sum_{f \in \mathcal{F}^+(g)} \text{Sim}(g, f) \times U_f}{\sum_{f \in \mathcal{F}^+(g)} \text{Sim}(g, f)} \right)}{\sum_{f \in \mathcal{F}^+(g)} \text{Sim}(g, f)}, \\
 \frac{\partial \mathcal{L}_1}{\partial V_j} &= \sum_{i=1}^m I_{ij} (U_i^T V_j - R_{ij}) U_i + \lambda_2 V_j. \quad (9)
 \end{aligned}$$



# Framework – Social Regularization

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$$\frac{\beta}{2} \sum_{i=1}^m \sum_{f \in \mathcal{F}^+(i)} \text{Sim}(i, f) \|U_i - U_f\|_F^2,$$

$$\begin{aligned} \min_{U, V} \mathcal{L}_2(R, U, V) &= \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^n I_{ij} (R_{ij} - U_i^T V_j)^2 \\ &+ \frac{\beta}{2} \sum_{i=1}^m \sum_{f \in \mathcal{F}^+(i)} \text{Sim}(i, f) \|U_i - U_f\|_F^2 \\ &+ \lambda_1 \|U\|_F^2 + \lambda_2 \|V\|_F^2. \end{aligned} \quad (11)$$

# Framework – Social Regularization

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$$Sim(i, f) = \frac{\sum_{j \in I(i) \cap I(f)} R_{ij} \cdot R_{fj}}{\sqrt{\sum_{j \in I(i) \cap I(f)} R_{ij}^2} \cdot \sqrt{\sum_{j \in I(i) \cap I(f)} R_{fj}^2}}, \quad : \text{VSS}$$

$$Sim(i, f) = \frac{\sum_{j \in I(i) \cap I(f)} (R_{ij} - \bar{R}_i) \cdot (R_{fj} - \bar{R}_f)}{\sqrt{\sum_{j \in I(i) \cap I(f)} (R_{ij} - \bar{R}_i)^2} \cdot \sqrt{\sum_{j \in I(i) \cap I(f)} (R_{fj} - \bar{R}_f)^2}}, \quad : \text{PCC} \quad f(x) = (x + 1)/2$$

# Experiment

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**Table 1: Statistics of User-Item Matrix of Douban**

Statistics	User	Item
Min. Num. of Ratings	1	1
Max. Num. of Ratings	6,328	49,504
Avg. Num. of Ratings	129.98	287.51

**Table 2: Statistics of Friend Network of Douban**

Statistics	Friends per User
Max. Num.	986
Avg. Num.	13.07

**Table 3: Statistics of User-Item Matrix of Epinions**

Statistics	User	Item
Max. Num. of Ratings	1960	7082
Avg. Num. of Ratings	12.21	7.56

**Table 4: Statistics of Trust Network of Epinions**

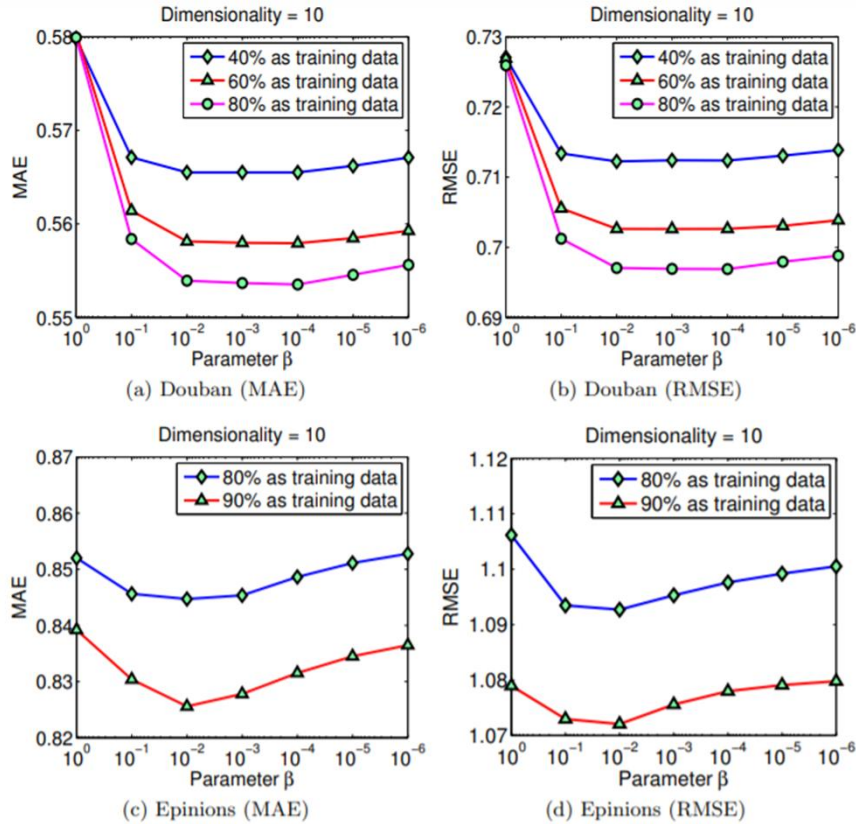
Statistics	Trust per User	Be Trusted per User
Max. Num.	1763	2443
Avg. Num.	9.91	9.91

# Experiment

**Table 5: Performance Comparisons (Dimensionality = 10)**

Dataset	Training	Metrics	UserMean	ItemMean	NMF	PMF	RSTE	SR1 <sub>vss</sub>	SR1 <sub>pcc</sub>	SR2 <sub>vss</sub>	SR2 <sub>pcc</sub>
Douban	80%	MAE	0.6809	0.6288	0.5732	0.5693	0.5643	0.5579	0.5576	0.5548	<b>0.5543</b>
		Improve	18.59%	11.85%	3.30%	2.63%	1.77%				
		RMSE	0.8480	0.7898	0.7225	0.7200	0.7144	0.7026	0.7022	0.6992	<b>0.6988</b>
		Improve	17.59%	11.52%	3.28%	2.94%	2.18%				
	60%	MAE	0.6823	0.6300	0.5768	0.5737	0.5698	0.5627	0.5623	0.5597	<b>0.5593</b>
		Improve	18.02%	11.22%	3.03%	2.51%	1.84%				
		RMSE	0.8505	0.7926	0.7351	0.7290	0.7207	0.7081	0.7078	0.7046	<b>0.7042</b>
		Improve	17.20%	11.15%	4.20%	3.40%	2.29%				
	40%	MAE	0.6854	0.6317	0.5899	0.5868	0.5767	0.5706	0.5702	0.5690	<b>0.5685</b>
		Improve	17.06%	10.00%	3.63%	3.12%	1.42%				
		RMSE	0.8567	0.7971	0.7482	0.7411	0.7295	0.7172	0.7169	0.7129	<b>0.7125</b>
		Improve	16.83%	10.61%	4.77%	3.86%	2.33%				
Epinions	90%	MAE	0.9134	0.9768	0.8712	0.8651	0.8367	0.8290	0.8287	0.8258	<b>0.8256</b>
		Improve	9.61%	15.48%	5.23%	4.57%	1.33%				
		RMSE	1.1688	1.2375	1.1621	1.1544	1.1094	1.0792	1.0790	1.0744	<b>1.0739</b>
		Improve	8.12%	13.22%	7.59%	6.97%	3.20%				
	80%	MAE	0.9285	0.9913	0.8951	0.8886	0.8537	0.8493	0.8491	0.8447	<b>0.8443</b>
		Improve	9.07%	14.83%	5.68%	4.99%	1.10%				
		RMSE	1.1817	1.2584	1.1832	1.1760	1.1256	1.1016	1.1013	1.0958	<b>1.0954</b>
		Improve	7.30%	12.95%	7.42%	6.85%	2.68%				

# Experiment



**Table 6: Similarity Analysis (Dimensionality = 10)**

Dataset	Training	Metrics	SR2 Sim=1	SR2 Sim=Ran	SR2 <sub>vss</sub>	SR2 <sub>pcc</sub>
Douban	80%	MAE	0.5579	0.5592	0.5548	0.5543
		RMSE	0.7034	0.7047	0.6992	0.6988
	60%	MAE	0.5631	0.5643	0.5597	0.5593
		RMSE	0.7083	0.7098	0.7046	0.7042
	40%	MAE	0.5724	0.5737	0.5690	0.5685
		RMSE	0.7195	0.7209	0.7129	0.7125
Epinions	90%	MAE	0.8324	0.8345	0.8258	0.8256
		RMSE	1.0794	1.0809	1.0744	1.0739
	80%	MAE	0.8511	0.8530	0.8447	0.8443
		RMSE	1.1002	1.1018	1.0958	1.0954

# Conclusion

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## **Conclusion**

- \* Two general algorithms are proposed that imposed social regularization using PCC and VSS
- \* Quite generic method also can be applied to trust aware recommendation problems
- \* Comparison

## **Future work**

- \* Categorical cluster
- \* Clicking behavior and Tagging record

**감사합니다.**