#### Convolutional Matrix Factorization for Document Context-Aware Recommendation /AutoRec: Autoencoders Meet Collaborative Filtering

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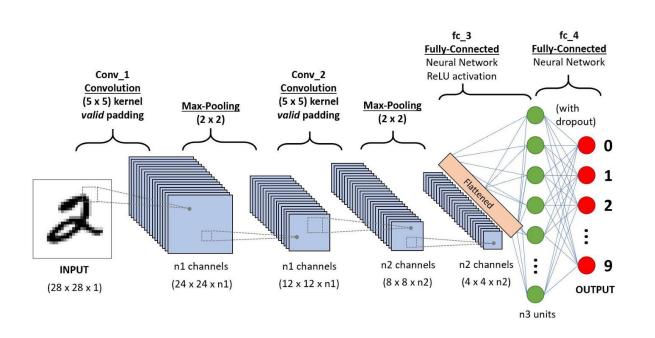
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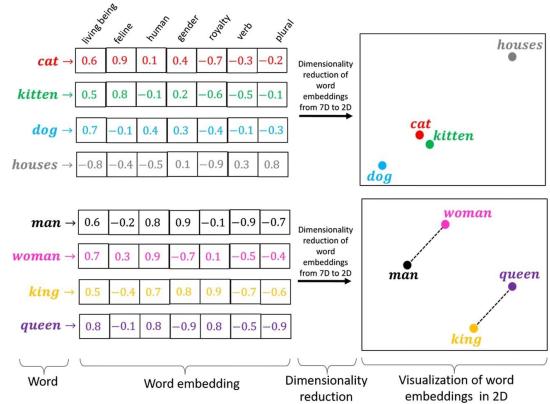
#### Introduction

- Exploding growth of the number of users and items in ecommerce services
  - Led to increase of the sparseness of user-item rating data
- Use auxiliary information such as demography of users, social networks, item description documents.
  - However, existing models ignores contextual information
  - This is because they use bag-of-words models
    - Ex)"people trust the man"
       "people betray his trust finally"

#### Introduction

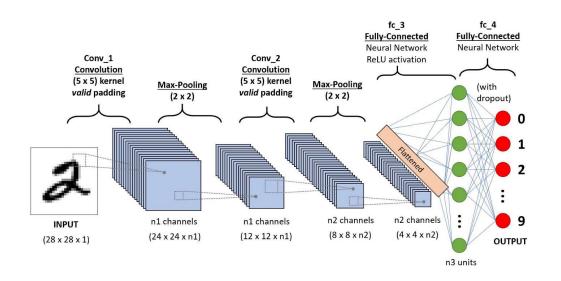
- Utilize Convolutional neural network(CNN) with natural language processing(NLP)
  - CNN captures local features of image or documents
  - Can also add pre-trained word embedding such as GloVe

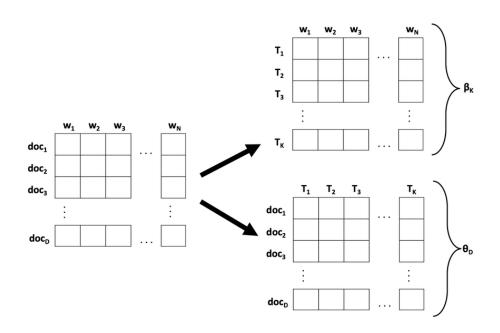




#### Introduction

- Add this CNN with PMF(Probabilistic Matrix Factorization)
  - Convolutional matrix factorization(ConvMF)





# Preliminary

- Matrix Factorization
  - Model-based methods (latent factor model)

$$\mathcal{L} = \sum_{i}^{N} \sum_{j}^{M} I_{ij} (r_{ij} - u_{i}^{T} v_{j})^{2} + \lambda_{u} \sum_{i}^{N} ||u_{i}||^{2} + \lambda_{v} \sum_{j}^{M} ||v_{j}||^{2}$$

- Convolutional Neural Network
  - Convolution layer
  - Pooling Layer

## Preliminary

#### Convolutional Matrix Factorization

$$p(R|U, V, \sigma^2) = \prod_{i=1}^{N} \prod_{j=1}^{M} N(r_{ij}|u_i^T v_j, \sigma^2)^{I_{ij}}$$

$$p(U|\sigma_U^2) = \prod_i^N N(u_i|0, \sigma_U^2 I)$$

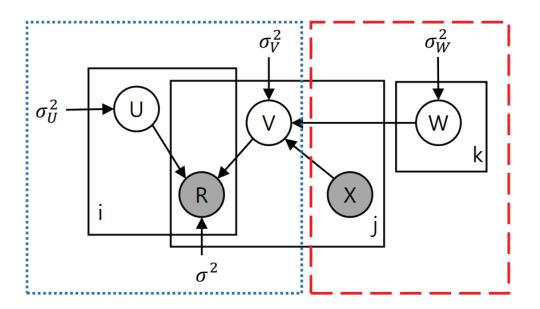


Figure 1: Graphical model of ConvMF model: PMF part in left (dotted-blue); CNN part in right (dashed-red)

- Convolutional Matrix Factorization
  - Internal weights W for CNN
  - Document item Xj
  - Gaussian noise epsilon

$$v_j = cnn(W, X_j) + \epsilon_j$$
$$\epsilon_j \sim N(0, \sigma_V^2 I)$$

$$p(W|\sigma_W^2) = \prod_k N(w_k|0, \sigma_W^2)$$

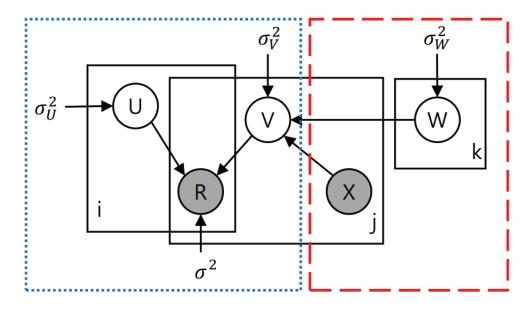


Figure 1: Graphical model of ConvMF model: PMF part in left (dotted-blue); CNN part in right (dashed-red)

$$p(V|W, X, \sigma_V^2) = \prod_{j}^{M} N(v_j|cnn(W, X_j), \sigma_V^2 I)$$

- Convolutional Matrix Factorization-CNN architecture
  - Embedding layer transforms raw document into a matrix represent document as concatenating word vectors

$$D = \left[ \begin{array}{ccccc} & | & | & | \\ \cdots & w_{i-1} & w_i & w_{i+1} & \cdots \\ | & | & | \end{array} \right]$$

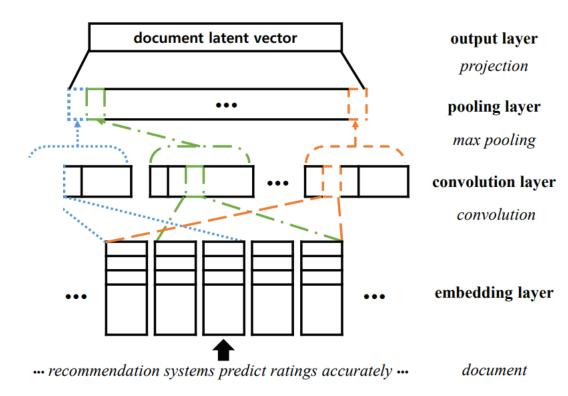


Figure 2: Our CNN architecture for ConvMF

- Convolutional Matrix Factorization-CNN architecture
  - Convolution layer
     Gets contextual feature

$$c_i^j = f(W_c^j * D_{(:,i:(i+ws-1))} + b_c^j)$$

$$c^{j} = [c_{1}^{j}, c_{2}^{j}, \ldots, c_{i}^{j}, \ldots, c_{l-ws+1}^{j}]$$

make multiple feature(j=1,2,...nc)

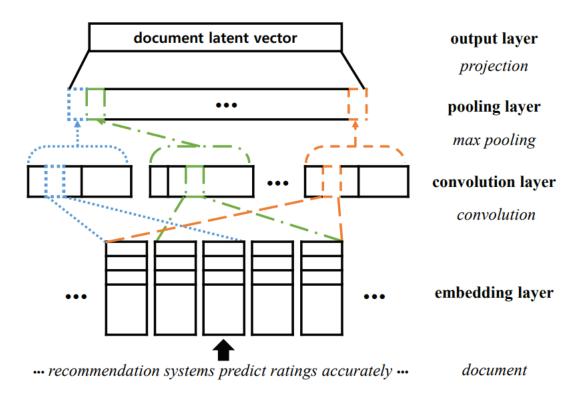


Figure 2: Our CNN architecture for ConvMF

- Convolutional Matrix Factorization-CNN architecture
  - Pooling layer

Extracts representative features from the convolution layer

deals with variable length of documents by pooling operation

->make fixed-length feature vector

$$d_f = [\max(c^1), \max(c^2), \ldots, \max(c^j), \ldots, \max(c^{n_c})]$$

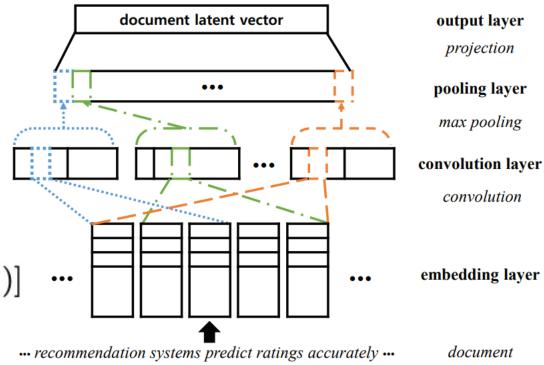


Figure 2: Our CNN architecture for ConvMF

- Convolutional Matrix Factorization-CNN architecture
  - Output layer
  - Project df to k-dimensional vector

$$s = \tanh(W_{f_2}\{\tanh(W_{f_1}d_f + b_{f_1})\} + b_{f_2})$$

where  $W_{f_1} \in \mathbb{R}^{f \times n_c}$ ,  $W_{f_2} \in \mathbb{R}^{k \times f}$  are projection matrices, and  $b_{f_1} \in \mathbb{R}^f$ ,  $b_{f_2} \in \mathbb{R}^k$  is a bias vector for  $W_{f_1}$ ,  $W_{f_2}$  with  $s \in \mathbb{R}^k$ .

$$s_j = cnn(W, X_j)$$

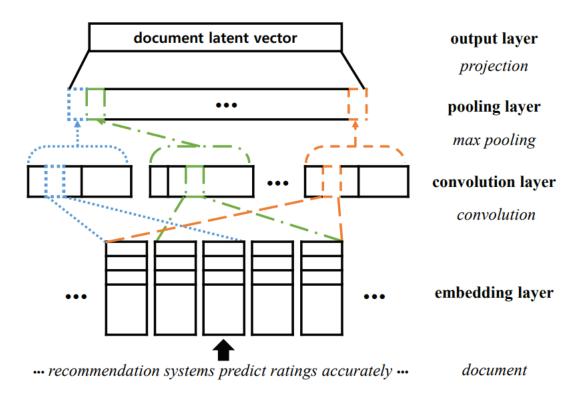


Figure 2: Our CNN architecture for ConvMF

- Optimization Methodology
  - Use maximum aposteriori(MAP)

$$\max_{U,V,W} p(U,V,W|R,X,\sigma^{2},\sigma_{U}^{2},\sigma_{V}^{2},\sigma_{W}^{2})$$

$$= \max_{U,V,W} [p(R|U,V,\sigma^{2})p(U|\sigma_{U}^{2})p(V|W,X,\sigma_{V}^{2})p(W|\sigma_{W}^{2})]$$

$$\mathcal{L}(U,V,W) = \sum_{i}^{N} \sum_{j}^{M} \frac{I_{ij}}{2} (r_{ij} - u_{i}^{T} v_{j})_{2} + \frac{\lambda_{U}}{2} \sum_{i}^{N} ||u_{i}||_{2}$$

$$+ \frac{\lambda_{V}}{2} \sum_{j}^{M} ||v_{j} - cnn(W,X_{j})||_{2} + \frac{\lambda_{W}}{2} \sum_{k}^{|w_{k}|} ||w_{k}||_{2},$$
(6)

where  $\lambda_U$  is  $\sigma^2/\sigma_U^2$ ,  $\lambda_V$  is  $\sigma^2/\sigma_V^2$ , and  $\lambda_W$  is  $\sigma^2/\sigma_W^2$ .

$$r_{ij} \approx \mathbb{E}[r_{ij}|u_i^T v_j, \sigma^2]$$
$$= u_i^T v_j = u_i^T (cnn(W, X_j) + \epsilon_j)$$

- Optimization Methodology
  - Coordinate descent

$$u_i \leftarrow (VI_iV^T + \lambda_U I_K)^{-1}VR_i$$

$$v_j \leftarrow (UI_jU^T + \lambda_V I_K)^{-1}(UR_j + \lambda_V cnn(W, X_j))$$
(8)

- W cannot be optimized by an analytic solution as we can do for U and V
- We can observe that L can be interpreted as a squared error function

$$\mathcal{E}(W) = \frac{\lambda_V}{2} \sum_{j=0}^{M} \|(v_j - cnn(W, X_j))\|^2$$

$$+\frac{\lambda_W}{2}\sum_{k}^{|w_k|}\|w_k\|^2 + \text{constant}$$

#### Dataset

- ML-1m ML-10m AIV
- Users' rating +reviews on items/item description documents(AIV)
- Movielens-add IMDB data

Dataset	# users	# items	# ratings	density
ML-1m ML-10m	6,040 69,878	3,544 10,073	993,482 9,945,875	4.641% 1.413%
AIV	29,757	15,149	135,188	0.030%

Table 1: Data statistic on three real-world datasets

- Compared with various model(PMF,CTR,CDL)
- Evaluated with RMSE

	Dataset		
Model	ML-1m	ML-10m	AIV
PMF	0.8971	0.8311	1.4118
CTR	0.8969	0.8275	1.5496
CDL	0.8879	0.8186	1.3594
ConvMF	0.8531	0.7958	1.1337
ConvMF+	0.8549	0.7930	1.1279
Improve	3.92%	2.79%	16.60%

Table 3: Overall test RMSE

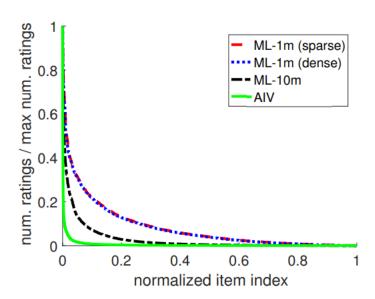


Figure 3: Skewness of the number of ratings for items on each dataset

	Ratio of training set to the entire dataset (density)						
Model	20% (0.93%)	30% (1.39%)	40% (1.86%)	50% (2.32%)	60% (2.78%)	70% (3.25%)	80% (3.71%)
PMF	1.0168	0.9711	0.9497	0.9354	0.9197	0.9083	0.8971
CTR	1.0124	0.9685	0.9481	0.9337	0.9194	0.9089	0.8969
CDL	1.0044	0.9639	0.9377	0.9211	0.9068	0.8970	0.8879
ConvMF	0.9745	0.9330	0.9063	0.8897	0.8726	0.8676	0.8531
Improve	2.98%	3.20%	3.36%	3.41%	3.77%	3.27%	3.92%

Table 4: Test RMSE over various sparseness of training data on ML-1m dataset

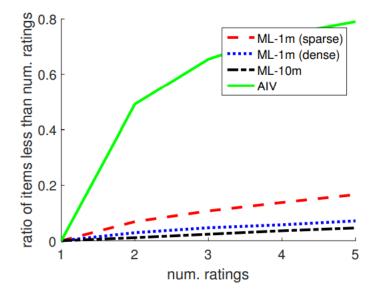


Figure 4: Ratio of items that have less than num. ratings (N) to each entire dataset

	Dataset			
Model	ML-1m	ML-10m	AIV	
PMF	0.8971	0.8311	1.4118	
CTR	0.8969	0.8275	1.5496	
CDL	0.8879	0.8186	1.3594	
ConvMF	0.8531	0.7958	1.1337	
ConvMF+	0.8549	0.7930	1.1279	
Improve	3.92%	2.79%	16.60%	

Table 3: Overall test RMSE

Improve -0.22% 0.35% 0.51%

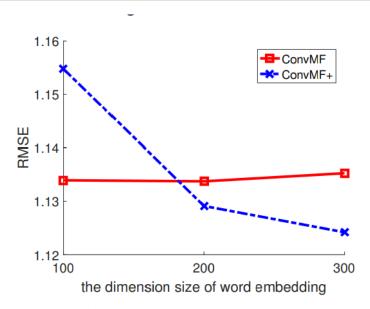


Figure 5: The effects of the dimension size of word embedding on Amazon dataset

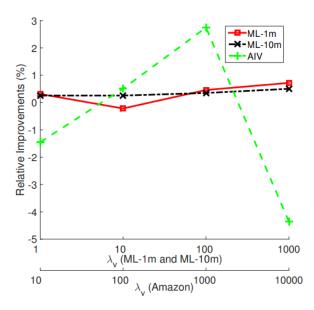


Figure 6: Relative improvements of ConvMF+ over ConvMF

Phrase captured by $W_c^{11}$	$\max(c^{11})$	Phrase captured by $W_c^{86}$	$\max(c^{86})$
people <b>trust</b> the man	0.0704	betray his <b>trust</b> finally	0.1009
Test phrases for $W_c^{11}$	$c_{test}^{11}$	Test phrases for $W_c^{86}$	c <sub>test</sub>
people believe the man	0.0391	betray his believe finally	0.0682
people faith the man	0.0374	betray his faith finally	0.0693
people <b>tomas</b> the man	0.0054	betray his <b>tomas</b> finally	0.0480

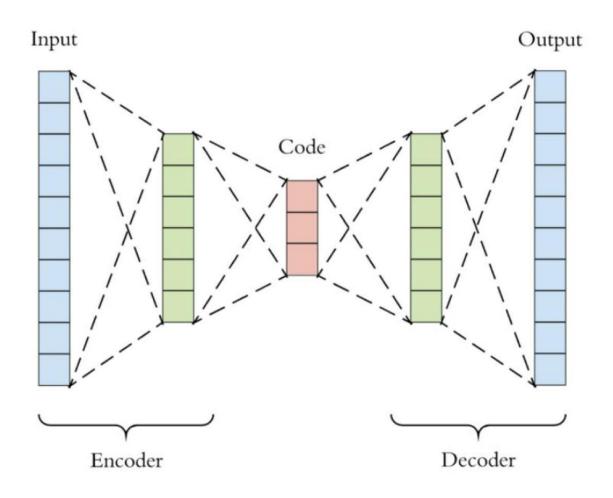
Table 5: Case study on two shared weights of ConvMF

#### Conclusion

#### ConvMF

- ConvMF significantly outperforms other competitors when dataset is extremely sparse
- The improvements of ConvMF over the competitors increase further even when dataset becomes dense
- Pre-trained word embedding model helps improve the performance of ConvMF when dataset is extremely sparse
- Best performing parameters verify that ConvMF well alleviates data sparsity
- ConvMF indeed captures subtle contextual differences

#### AutoRec



 $Encoder \; \phi: X \to F$ 

 $Decoder\ \varphi: F \to X$ 

 $\phi, arphi = argmin ||X - (\phi \circ arphi)X||_2^2$ 

#### AutoRec

$$\min_{\theta} \sum_{\mathbf{r} \in \mathbf{S}} ||\mathbf{r} - h(\mathbf{r}; \theta)||_2^2,$$

$$h(\mathbf{r}; \theta) = f(\mathbf{W} \cdot g(\mathbf{V}\mathbf{r} + \boldsymbol{\mu}) + \mathbf{b})$$

 $\theta = \{W, V, \mu, b\}$  :parameter

 $W \in \mathbb{R}^{m \times k}, V \in \mathbb{R}^{k \times m}$ : Weight matrix

 $\mu \in \mathbb{R}^k$ ,  $b \in \mathbb{R}^m$ : bias

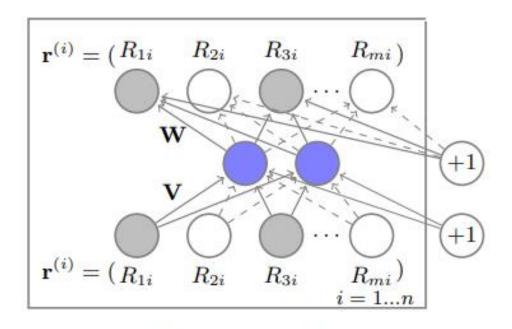


Figure 1: Item-based AutoRec model. We use plate notation to indicate that there are n copies of the neural network (one for each item), where  $\mathbf{W}$  and  $\mathbf{V}$  are tied across all copies.

$$\min_{ heta} \sum_{i=1}^n ||r^{(i)} - h(r^{(i)}; heta)||_O^2 + rac{\lambda}{2} \cdot (||W||_F^2 + ||V||_F^2)$$

#### AutoRec

	ML-1M	ML-10M		
U-RBM	0.881	0.823		
I-RBM	0.854	0.825		
U-AutoRec	0.874	0.867		
I-AutoRec	0.831	$\bf 0.782$		
(a)				

$f(\cdot)$	$g(\cdot)$	RMSE
Identity	Identity	0.872
Sigmoid	Identity	0.852
Identity	Sigmoid	0.831
Sigmoid	Sigmoid	0.836
	(b)	

	ML-1M	ML-10M	Netflix
BiasedMF	0.845	0.803	0.844
I-RBM	0.854	0.825	-
U- $RBM$	0.881	0.823	0.845
LLORMA	0.833	0.782	0.834
I-AutoRec	0.831	0.782	0.823

(c)

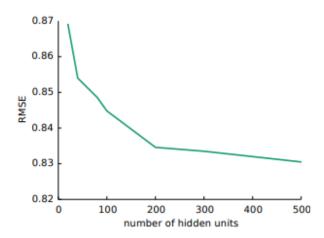


Figure 2: RMSE of I-AutoRec on Movielens 1M as the number of hidden units k varies.

```
class ConvMF(tf.keras.Model):
   def __init__(self, num_users, num_items, latent_dim, num_filters, a,b,c,d,e):
       super(ConvMF, self).__init__()
       self.num users = num users
       self.num items = num items
       self.latent_dim = latent_dim
       self.user latent matrix = tf.Variable(
           initial_value=tf.random.normal(shape=(num_users, latent_dim), mean=0.0, stddev=0.01)
       self.item latent matrix = tf.Variable(
           initial_value=tf.random.normal(shape=(num_items, latent_dim), mean=0.0, stddev=0.01);
       self.item bias = tf.Variable(
           initial_value=tf.zeros(shape=(num_items, 1));
```

```
#change to word vector
self.word_embedding_matrix = np.random.randn(10, 18)
self.genre_cony = tf.keras.layers.Conv1D(
    filters=num_filters,
        kernel_size=2,
        activation='relu',
)
self.genre_max_pool = tf.keras.layers.MaxPool1D(
    pool_size=2,
        strides=2
)
self.genre_flatten = tf.keras.layers.Flatten()

self.fc_layer1 = tf.keras.layers.Dense(units=latent_dim, activation='tanh')
self.fc_layer2 = tf.keras.layers.Dense(units=latent_dim, activation='tanh')
```

```
def call(self, inputs):
    inputs = tf.reshape(inputs, [-1, 7])
   user latent factors = tf.nn.embedding lookup(self.user latent matrix, inputs[:, 0])
   item_latent_factors = tf.nn.embedding_lookup(self.item_latent_matrix, inputs[:, 1])
   item bias = tf.nn.embedding lookup(self.item bias, inputs[:, 1])
    inputs = tf.constant(inputs)
    result_matrix = self.word_embedding_matrix[:, [inputs[:, 2], inputs[:, 3], inputs[:, 4], inputs[:, 5], inputs[:, 6]]]
   user_item_matrix = tf.matmul(user_latent_factors, item_latent_factors, transpose_b=True)
    feature map = self.genre conv(result matrix)
    feature map = self.genre max pool(feature map)
    feature_map = self.genre_flatten(feature_map)
   out = self.fc_layer1(feature_map)
   out = self.fc layer2(out)
   predicted_rating = tf.reduce_sum(out, axis=1, keepdims=True) + item bias
    return predicted_rating
def sentence to vector(sentence):
   genres = ['Action', 'Adventure', 'Animation', "Children's", 'Comedy', 'Crime', 'Drama', 'Romance', 'Thriller',
   genre_to_num = {genres[i]: i + 1 for i in range(len(genres))}
   vector = [0] * len(sentence.split())
   for i, word in enumerate(sentence.split()):
        if word in genre to num:
            vector[i] = genre to num[word]
   return vector
```

```
for epoch in range(100):
    for row in train_data.values:
       user_id, item_id, rating, genre = row[1], row[0], row[2], row[4]
        genre = sentence_to_vector(genre)
        genre = pad_sequences([genre], maxlen=5, padding='post', value=0)
       genre = genre[0]
       with tf.GradientTape() as tape:
           inputs = tf.stack([user_id, item_id, genre[0]_genre[1]_genre[2]_genre[3]_genre[4]], axis=-1)
           predictions = model(inputs)
            loss = loss_fn(rating, predictions)
        grads = tape.gradient(loss, model.trainable variables)
        optimizer.apply_gradients(zip(grads, model.trainable_variables))
```

```
Epoch 0: lest loss = 1.5095343589/82/15
   grads = tape.gradient(loss, model.trainable variables)
                                                                                                 Epoch 1: Test loss = 1.4805691242218018
   optimizer.apply gradients(zip(grads, model.trainable variables))
                                                                                                 Epoch 2: Test loss = 1.461288571357727
                                                                                                 Epoch 3: Test loss = 1.443916916847229
avg loss = 0.0
                                                                                                 Epoch 4: Test loss = 1.4289628267288208
for row in test data.values:
                                                                                                Epoch 5: Test loss = 1.4151443243026733
   user id, item id, rating, genre = row[1], row[0], row[2], row[4]
                                                                                                 Epoch 6: Test loss = 1.4030861854553223
   genre = sentence to vector(genre)
                                                                                                 Epoch 7: Test loss = 1.391825556755066
   genre = pad sequences([genre], maxlen=5, padding='post', value=0)
                                                                                                 Epoch 8: Test loss = 1.3811259269714355
   genre = genre[0]
   inputs = tf.stack([user_id, item_id, genre[0], genre[1], genre[2], genre[3], genre[4]], axis=-1)
                                                                                                Epoch 9: Test loss = 1.371309757232666
   predictions = model(inputs)
                                                                                                 Epoch 10: Test loss = 1.3623532056808472
   avg loss += loss fn(rating, predictions)
                                                                                                 Epoch 11: Test loss = 1.352412462234497
avg loss /= len(test data)
                                                                                                 Epoch 12: Test loss = 1.344534<u>8739624023</u>
print("Epoch {}: Test loss = {}".format(epoch, avg loss))
                                                                                                 Epoch 13: Test loss = 1.3376710414886475
                                                                                                Epoch 14: Test loss = 1.331301212310791
                                                                                                Epoch 15: Test loss = 1.3255774974822998
                                                                                                Epoch 16: Test loss = 1.3208509683609009
                                                                                                Epoch 17: Test loss = 1.3166801929473877
```

```
Epoch 19: Test loss = 1.3098766803741455
Epoch 20: Test loss = 1.3079893589019775
Epoch 21: Test loss = 1.3051671981811523
Epoch 22: Test loss = 1.3021999597549438
Epoch 23: Test loss = 1.2093050813674927
Epoch 25: Test loss = 1.2971255779266357
Epoch 26: Test loss = 1.2955207824707031
Epoch 27: Test loss = 1.2941497564315796
Epoch 28: Test loss = 1.2914210557937622
Epoch 29: Test loss = 1.2911709547042847
```

Epoch 18: Test loss = 1.3129180669784546