

2023/07/11

DSAIL summer-internship

STUDENT

유혜원

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# Collaborative Filtering for Implicit Feedback Datasets

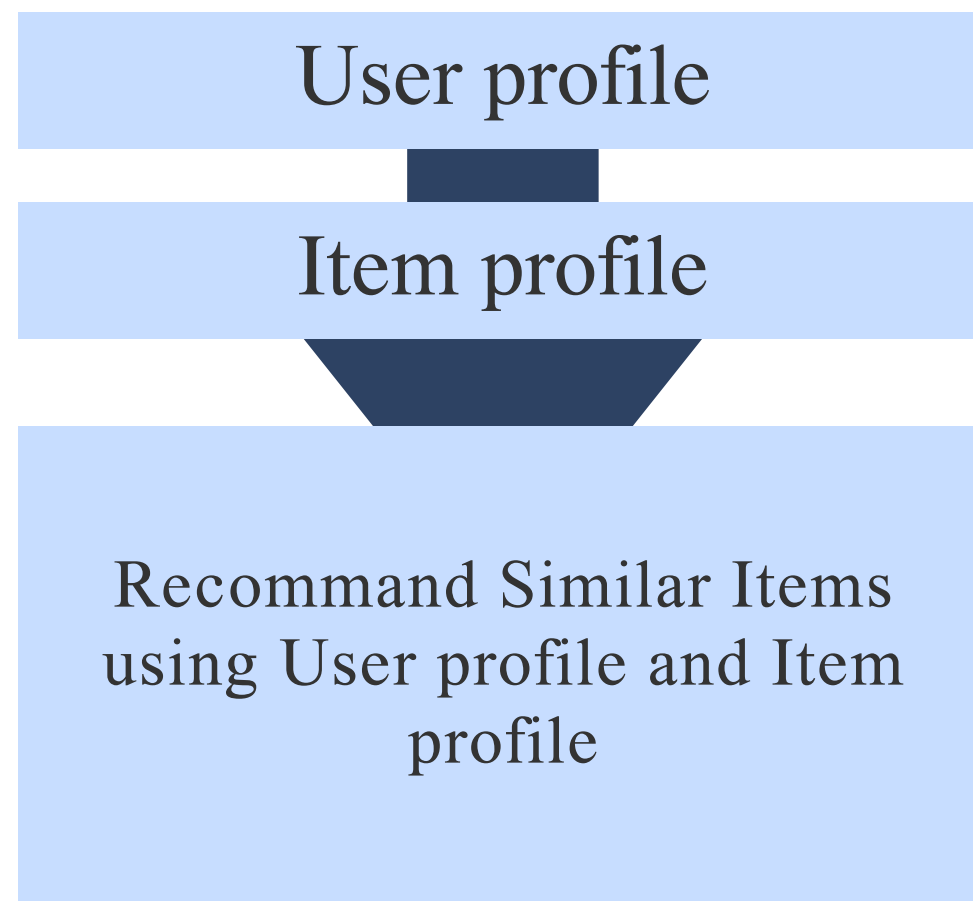
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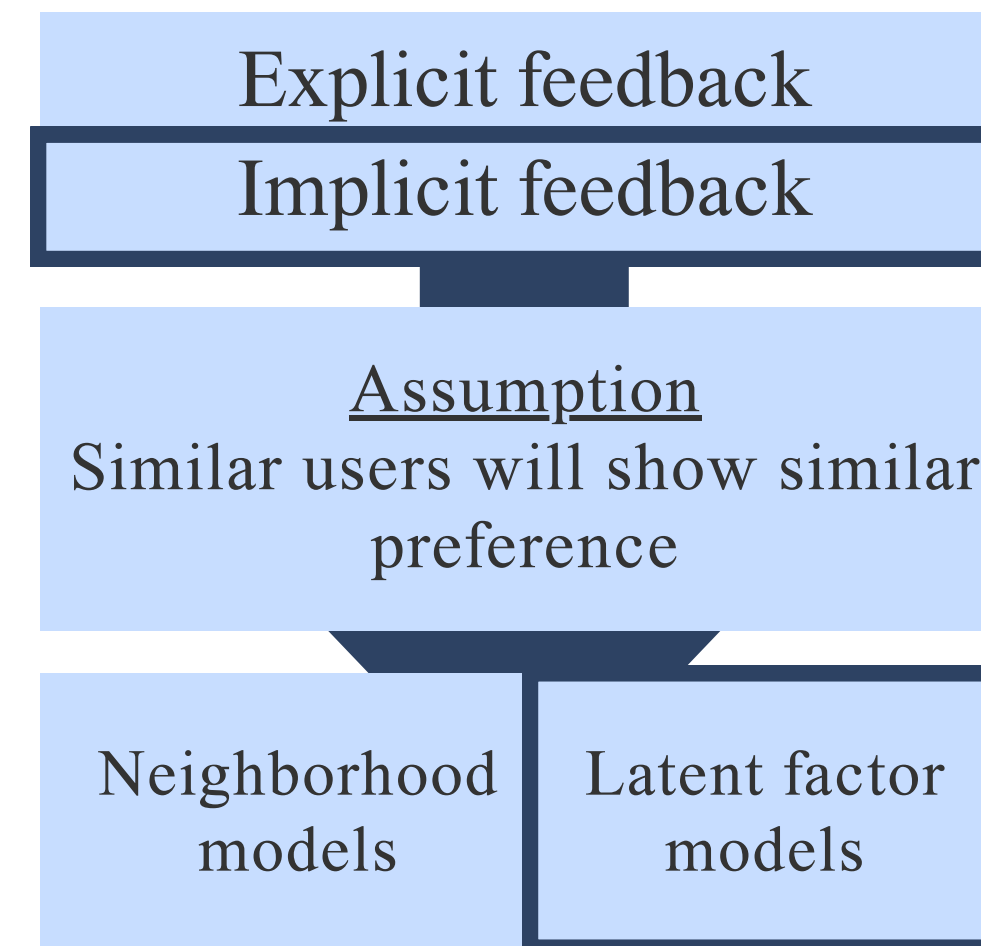
# I Research Background & Motivation

- Recommender systems provide users with personalized recommendations for products or services, which hopefully suit their unique taste and needs.

## Content based approach



## ✓ Collaborative Filtering



# I Research Background & Motivation

## Explicit feedback

Most convenient input in recommend system is the high quality explicit feedback which includes explicit input by users regarding their interest in products.

### Example

Netflix star rating,  
TV program thumbs-up/down

## Implicit feedback

Explicit feedback is not always available.

Thus, recommenders can infer user preferences from the more abundant implicit feedback, which indirectly reflect opinion through observing user behavior.

### Example

Purchase history, Browsing history, Search patterns



## II Characteristics of Implicit Feedback

Cannot use algorithms that were designed with explicit feedback directly

- No negative feedback

A user that did not watch a certain show might have done so because she dislikes the show or just because she did not know about the show or was not available to watch it

- Numerical value of implicit feedback does not indicate preference

Numerical values of implicit feedback describe the frequency of actions. A larger value is not indicating a higher preference but higher confidence level

- Implicit feedback is inherently noisy

Purchased items can be gifts for others, or the user can be disappointed with the product

- Evaluation of implicit feedback recommender requires appropriate measures

If we gather data on television show, it is unclear how to evaluate a show that has been watched more than once, or how to compare two shows that are on at the same time, and hence cannot both be watched by user

### III Model Description

#### Notation

for users:  $u, v$

for items:  $i, j$

→ observation:  $r_{ui}$

Ex) The number of times  $u$  purchased  $i$  or the time  $u$  spent on webpage  $i$

→ If no action was observed,  $r_{ui}$  is set to zero.

- similarity of item  $i$  and  $j$ :  $S_{ij}$

in Latent Factor models,

- user-factors vector:  $x_u \in R^f$
- item-factors vector:  $y_i \in R^f$

- cost function

$$\min_{x_*, y_*} \sum_{r_{u,i} \text{ is known}} (r_{ui} - x_u^T y_i)^2 + \lambda(\|x_u\|^2 + \|y_i\|^2)$$

### III Model Description

#### Notation

in suggesting model,

- preference of user  $u$  to item  $i$

$$p_{ui} = \begin{cases} 1 & r_{ui} > 0 \\ 0 & r_{ui} = 0 \end{cases}$$

- confidence in observing  $p_{ui}$

$$c_{ui} = 1 + \alpha r_{ui}$$

- cost function

$$\min_{x_*, y_*} \sum_{u, i} c_{ui} (p_{ui} - x_u^T y_i)^2 + \lambda \left( \sum_u \|x_u\|^2 + \sum_i \|y_i\|^2 \right)$$

→ Alternating Least Squares (ALS) optimization

[user-item matrix is too large to SGD optimization]

### III Model Description

#### ALS optimization

when two parameter exist, ALS minimizes two loss functions alternatively until they converge

- cost function

$$\min_{x_{\star}, y_{\star}} \sum_{u,i} c_{ui} (p_{ui} - x_u^T y_i)^2 + \lambda \left( \sum_u \|x_u\|^2 + \sum_i \|y_i\|^2 \right)$$

- 
- 1) fix item-factor  $y_i$
  - 2) optimize user-factor  $x_u$
  - 3) fix user-factor  $x_u$
  - 4) optimize item-factor  $y_i$
- } repeat until converge



### III Model Description

#### ALS optimization

- cost function

$$\min_{x_*, y_*} \sum_{u, i} c_{ui} (p_{ui} - x_u^T y_i)^2 + \lambda \left( \sum_u \|x_u\|^2 + \sum_i \|y_i\|^2 \right)$$

when one of the factors is fixed, the problem is turned into linear regression optimization

→ closed form solution exists

when calculate differentiation on  $x_u$

$$x_u = (Y^T C^u Y + \lambda I)^{-1} Y^T C^u p(u) \quad \longrightarrow$$

when calculate differentiation on  $y_i$

$$y_i = (X^T C^i X + \lambda I)^{-1} X^T C^i p(i)$$

$$Y^T C^u Y = \underbrace{Y^T Y}_{\text{independent of } u} + Y^T \underbrace{(C^u - I)}_{\text{only has } n_u \text{ non-zero elements}} Y$$

independent of  $u$

only has  $n_u$  non-zero elements

## IV Experiment settings & results in paper

### Data description

- collected from a digital television service
- about 300,000 set top boxes
- about 17,000 unique programs which aired during a four week period
- $r_{ui}$  : for each user  $u$ , and program  $i$ , represent how many times user  $u$  watched program  $i$  (show length was used as base units)
- train data: collected over 4 weeks
- test data: 1 week after train data is collected  $\longrightarrow r_{ui}^t$

## IV Experiment settings & results in paper

### Reflection of data characteristics

- Remove “easy” predictions from the test set corresponding to the shows that had been watched by that user during the training period.
- To make the test set more accurate, toggle to zero all entries with  $r_{ui}^t < 0.5$
- Employ the log scaling scheme with  $\epsilon = 10^{-8}$

$$c_{ui} = 1 + \alpha \log(1 + r_{ui}/\epsilon)$$

- Down-weight the second and subsequent shows after a channel tuning event.

$$\frac{e^{-(at-b)}}{1 + e^{-(at-b)}} r_{ui} \quad \text{Experimentally authors found } a = 2, b = 6$$

## IV Experiment settings & results in paper

### Evaluation methodology.

- Precision based metrics are not appropriate, as they require knowing which programs are undesired to a user.
- Recall based metrics are applicable.

$rank_{ui}$  percentile-ranking of program  $i$  within the ordered list of all programs prepared for user  $u$

$$\overline{rank} = \frac{\sum_{u,i} r_{ui}^t rank_{ui}}{\sum_{u,i} r_{ui}^t} \quad \text{expected percentile ranking of a watching unit in the test period}$$

## IV Experiment settings & results in paper

### Evaluation results

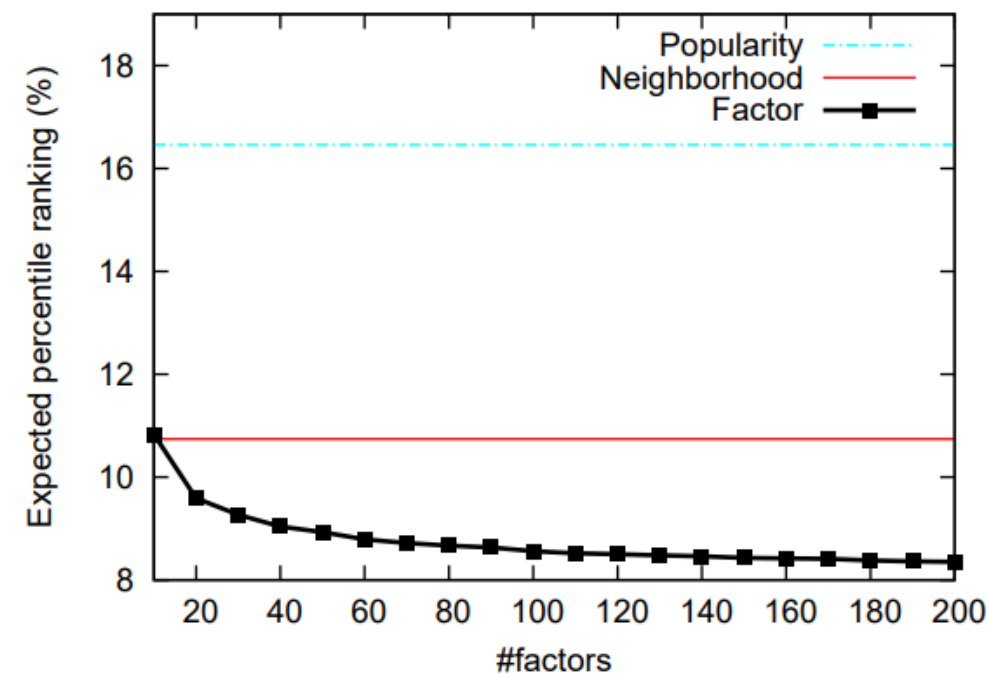


Figure 1. Comparing factor model with popularity ranking and neighborhood model.

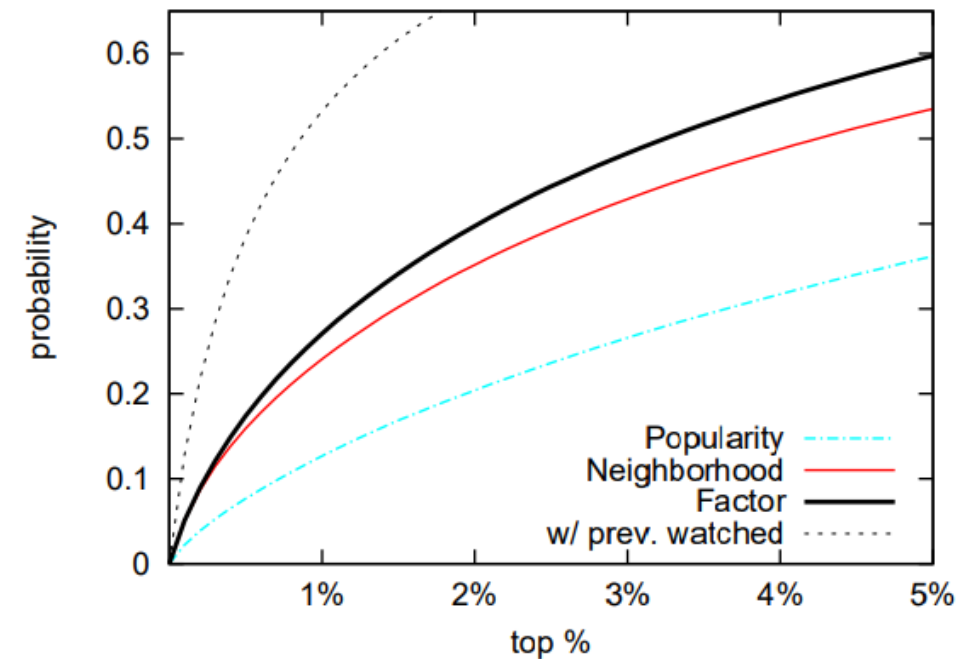


Figure 2. Cumulative distribution function of the probability that a show watched in the test set falls within top x% of recommended shows.

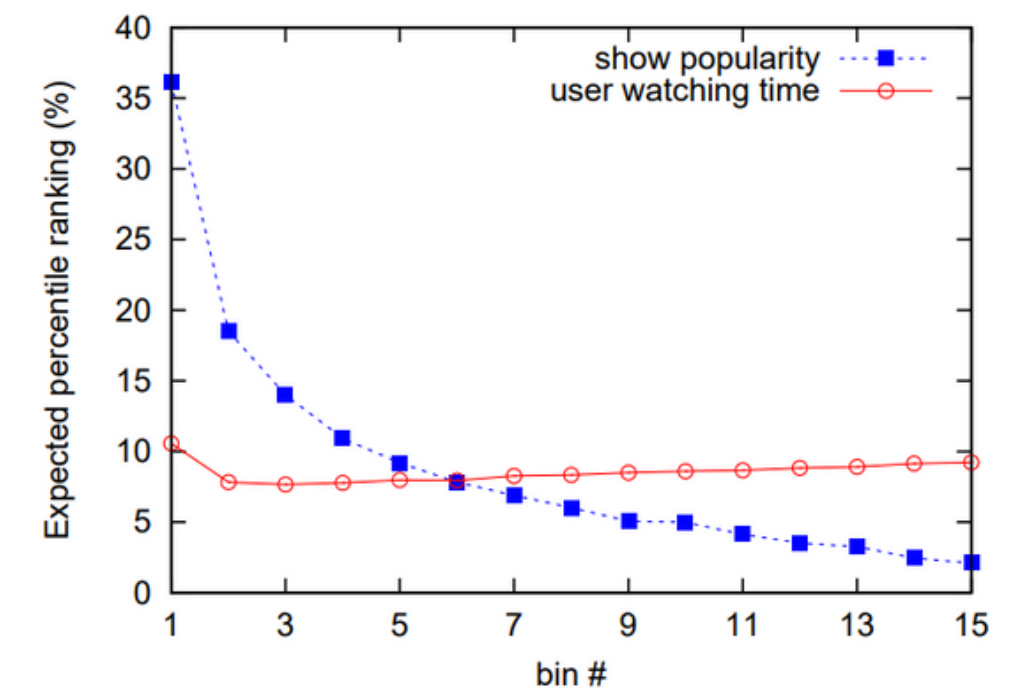


Figure 3. Analyzing the performance of the factor model by segregating users/shows based on different criteria.

- number of factors : 100

# V Implementation

## Dataset

Original Dataset →

Music Recommendation Dataset

- Number of Users: 358868
- Number of Artists: 292363
- Observation Type: plays

Reduced Dataset

- Number of Users: 100
- Number of Artists: 7257
- Observation Type: plays

userID	artistNM	plays
1	bloc party	1677
1	radiohead	1287
1	the mars volta	1024

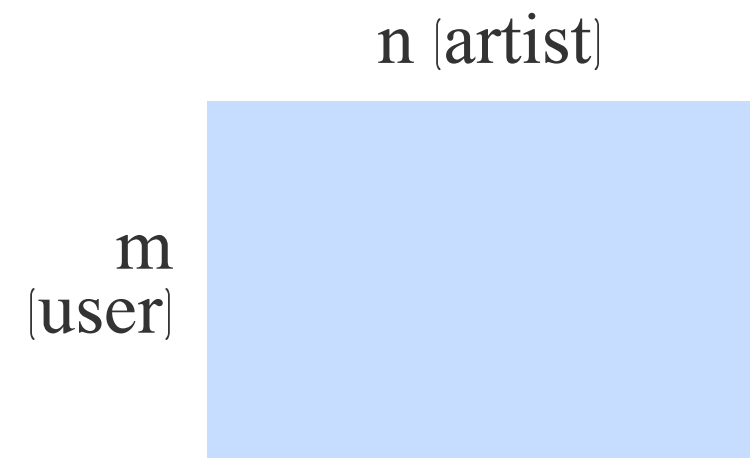
## Train/Test set Split

- Randomly separate train and test datasets within each user
- 80 : 20

## V Implementation

### Matrix construction

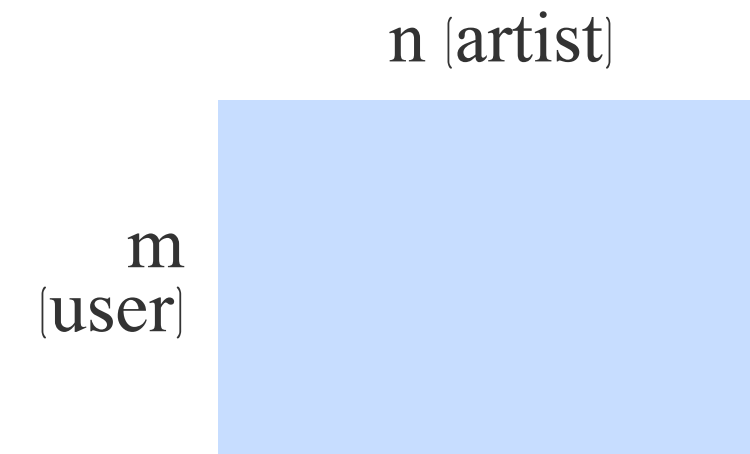
R matrix



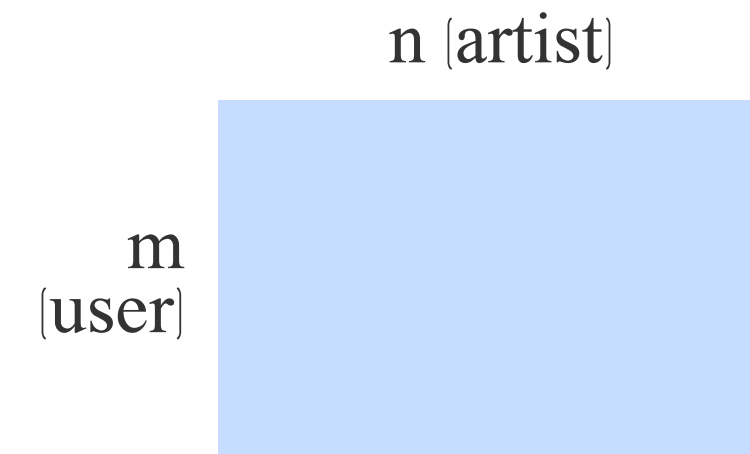
$$p_{ui} = \begin{cases} 1 & r_{ui} > 0 \\ 0 & r_{ui} = 0 \end{cases}$$



P matrix



C matrix



$$c_{ui} = 1 + \alpha \log(1 + r_{ui}/\epsilon)$$

use log scaling scheme  
(number of music plays  
varies over a large range)

## V Implementation

### Matrix construction

#### R matrix

```
def create_R_matrix(df, train_set):  
    # 모든 고유한 userID와 artistNM을 추출  
    all_userIDs = np.unique((df['userID'].unique()))  
    all_artistNMs = np.unique((df['artistNM'].unique()))  
  
    # 행렬 초기화  
    plays_matrix = pd.DataFrame(0, index=all_userIDs, columns=all_artistNMs)  
  
    # train set에 있는 데이터로 행렬 채우기  
    for _, row in train_set.iterrows():  
        userID = row['userID']  
        artistNM = row['artistNM']  
        plays = row['plays']  
        plays_matrix.loc[userID, artistNM] = plays  
  
    return plays_matrix
```

#### P matrix

```
def create_P_matrix(R_matrix):  
    P_matrix = np.copy(R_matrix)  
    P_matrix[R_matrix>0] = 1  
    return P_matrix
```

#### C matrix

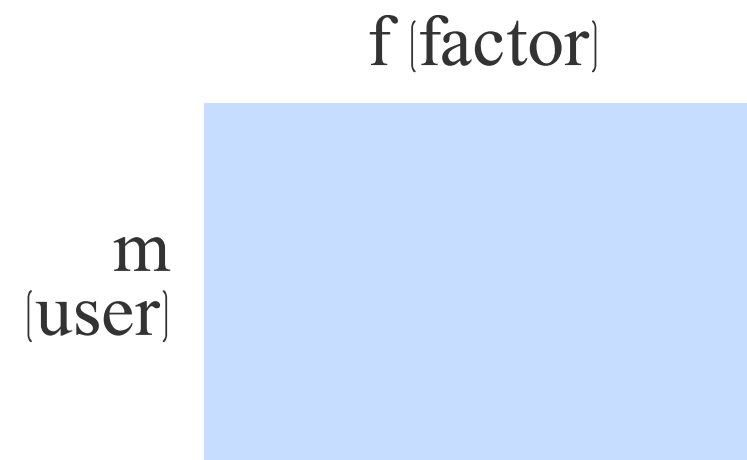
```
def create_C_matrix(R_matrix):  
    C_matrix = 1 + alpha_*np.log(1+R_matrix/epsilon_)  
    C_matrix = np.array(C_matrix)  
    return C_matrix
```



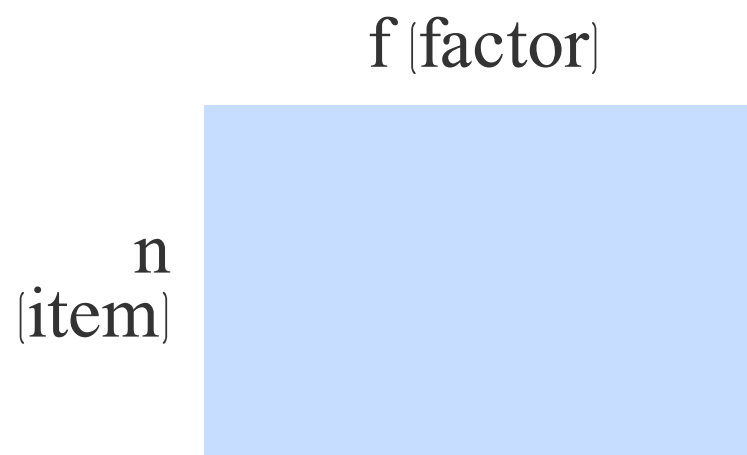
## V Implementation

### Matrix construction

User-factor matrix ( $X$ )



Item-factor matrix ( $Y$ )



```
# number of user and item
m = R_matrix.shape[0]
n = R_matrix.shape[1]

# user-factor matrix initialize
X = np.random.rand(m, n_factor) * 0.01
print(X.shape)

# item-factor matrix initialize
Y = np.random.rand(n, n_factor) * 0.01
print(Y.shape)
```

## V Implementation

### Alternating Least Square (ALS) training

Loss function 
$$\min_{x_\star, y_\star} \sum_{u,i} c_{ui} (p_{ui} - x_u^T y_i)^2 + \lambda \left( \sum_u \|x_u\|^2 + \sum_i \|y_i\|^2 \right)$$

```
def loss_function(C, P, pred, X, Y, lambda_):  
    predict_error = np.square(P-pred)  
    confidence_error = np.sum(C*predict_error)  
    regularization_error = lambda_*(np.sum(np.square(X))+np.sum(np.square(Y)))  
    total_loss = confidence_error + regularization_error  
    return np.sum(predict_error), confidence_error, regularization_error, total_loss
```

Optimizer 
$$x_u = (Y^T C^u Y + \lambda I)^{-1} Y^T C^u p(u) \quad y_i = (X^T C^i X + \lambda I)^{-1} X^T C^i p(i) \longrightarrow Y^T C^u Y = Y^T Y + Y^T (C^u - I) Y$$

```
def user_optimization(X, Y, C_matrix, P_matrix, m, n, n_factor, lambda_):  
    yT = np.transpose(Y)  
    yTy = np.matmul(yT, Y)  
    for user in range(m):  
        Cu = np.diag(C_matrix[user])  
        yT_Cu_y = yTy + np.matmul(np.matmul(yT, Cu - np.identity(n)), Y)  
        lambda_l = lambda_ * np.identity(n_factor)  
        yT_Cu_pu = np.matmul(np.matmul(yT, Cu), np.transpose(P_matrix[user]))  
        X[user] = np.linalg.solve(yT_Cu_y+lambda_l, yT_Cu_pu)
```

```
def item_optimization(X, Y, C_matrix, P_matrix, m, n, n_factor, lambda_):  
    xT = np.transpose(X)  
    xTx = np.matmul(xT, X)  
    for item in range(n):  
        Ci = np.diag(C_matrix[:,item])  
        xT_Ci_x = xTx + np.matmul(np.matmul(xT, Ci - np.identity(m)), X)  
        lambda_l = lambda_ * np.identity(n_factor)  
        xT_Ci_pi = np.matmul(np.matmul(xT, Ci), np.transpose(P_matrix[:,item]))  
        Y[item] = np.linalg.solve(xT_Ci_x+lambda_l, xT_Ci_pi)
```

## V Implementation

### Alternating Least Square (ALS) training training function

```
def train(n_iter, X, Y, C_matrix, P_matrix, m, n, n_factor, lambda_):
    # result save
    predict_errors = []
    confidence_errors = []
    regularization_list = []
    total_losses = []

    for i in range(n_iter):
        print(f'processing on {i+1}th iteration')
        if i != 0:
            user_optimization(X, Y, C_matrix, P_matrix, m, n, n_factor, lambda_)
            item_optimization(X, Y, C_matrix, P_matrix, m, n, n_factor, lambda_)
        predict = np.matmul(X, np.transpose(Y))
        predict_error, confidence_error, regularization, total_loss = loss_function(C_matrix, P_matrix, predict, X, Y, lambda_)
        predict_errors.append(predict_error)
        confidence_errors.append(confidence_error)
        regularization_list.append(regularization)
        total_losses.append(total_loss)

        print(f'{i+1}th iteration is done')

    fin_predict = np.matmul(X, np.transpose(Y))
    print('final predict')
    print([fin_predict])
    return fin_predict, predict_errors, confidence_errors, regularization_list, total_losses
```

# V Implementation

## Evaluation Metric

### Expected Percentile Ranking

$$\overline{rank} = \frac{\sum_{u,i} r_{ui}^t rank_{ui}}{\sum_{u,i} r_{ui}^t}$$

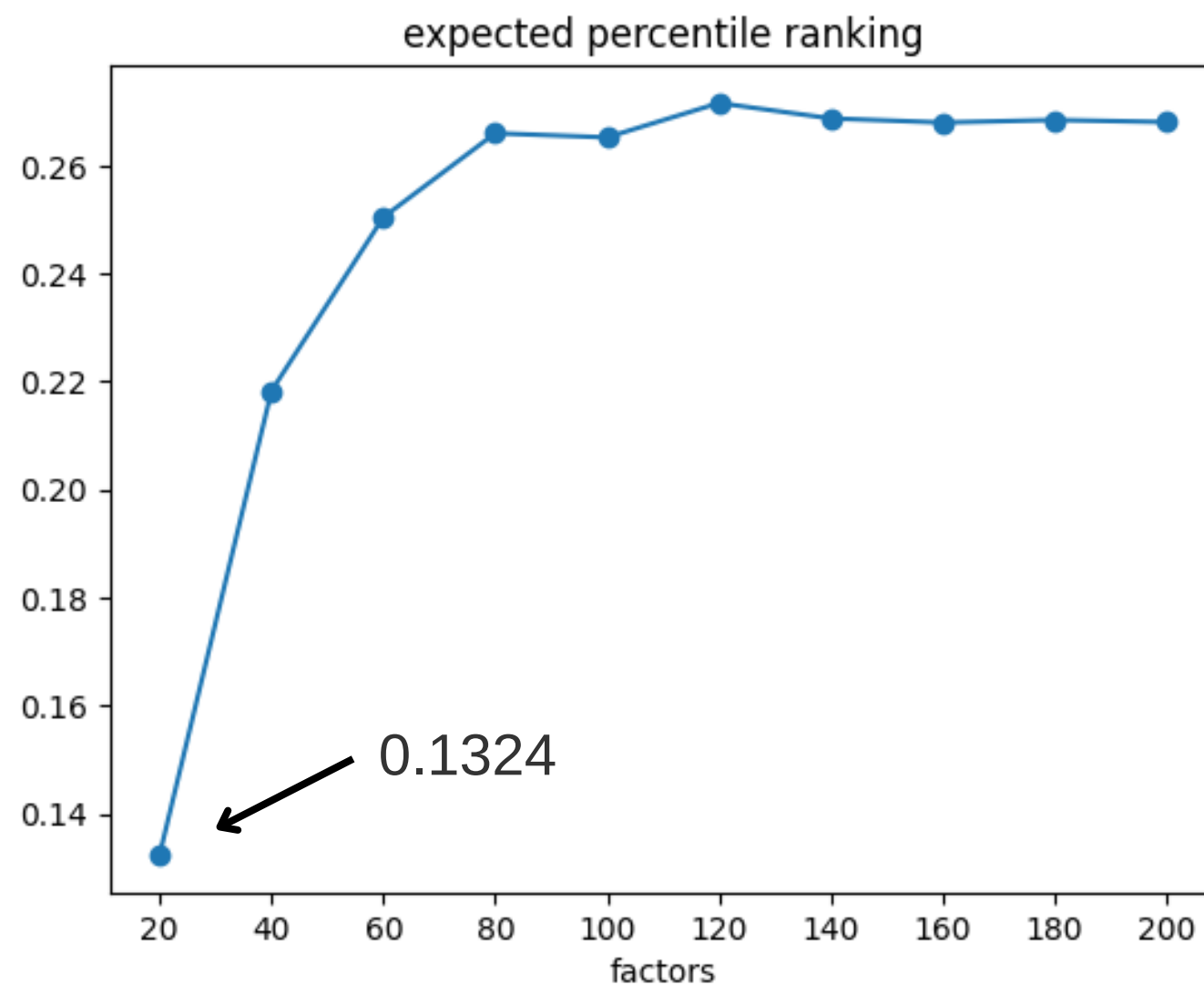
```
def calculate_rank(predict_df, R_matrix, train_set, test_set):  
  
    # 1. predict_df에서 0.5 미만 값을 갖는 경우 모두 0으로 변환  
    predict_df[predict_df < 0.5] = 0  
  
    # 2. rank_matrix 구성 (size는 predict_df와 동일)  
    rank_matrix = predict_df.copy()  
  
    # predict_df에서 train에 포함된 데이터의 경우 값을 모두 0으로 처리  
    train_indices = [(row, col) for row, col in train_set[['userID', 'artistNM']].values]  
    rank_matrix[train_indices] = 0  
  
    # user(row)별로 값이 0이 아닌 데이터들에 대해 percentile rank를 계산하여 값 변환  
    for row in rank_matrix.index:  
        nonzero_indices = rank_matrix.columns[rank_matrix.loc[row] != 0].tolist()  
        if len(nonzero_indices) > 0:  
            tmp_col_list = rank_matrix.loc[row, nonzero_indices].tolist()  
            tmp_qua_list = calculate_percentile_ranks(tmp_col_list)  
            rank_matrix.loc[row, nonzero_indices] = np.array(tmp_qua_list)
```

```
# 3. R_test_matrix 구성  
R_test_matrix = R_matrix.copy()  
  
# R_matrix에서 train에 포함된 데이터의 경우 값을 모두 0으로 처리  
train_indices = [(row, col) for row, col in train_set[['userID', 'artistNM']].values]  
R_test_matrix[train_indices] = 0  
  
# 4. rank_bar 계산 (expected percentile ranking)  
rt = np.array(R_test_matrix)  
rank = np.array(rank_matrix)  
rt_rank = rt * rank  
numerator = np.sum(rt_rank)  
denominator = np.sum(rt)  
rank_bar = numerator / denominator  
print(rank_bar)  
return rank_bar
```

## V Implementation

### Experiment Results

Expected Percentile Ranking change by number of factors



- Setting

`n_factor : range(20,201,20)`

`n_iter : 10`

`lambda : 150`

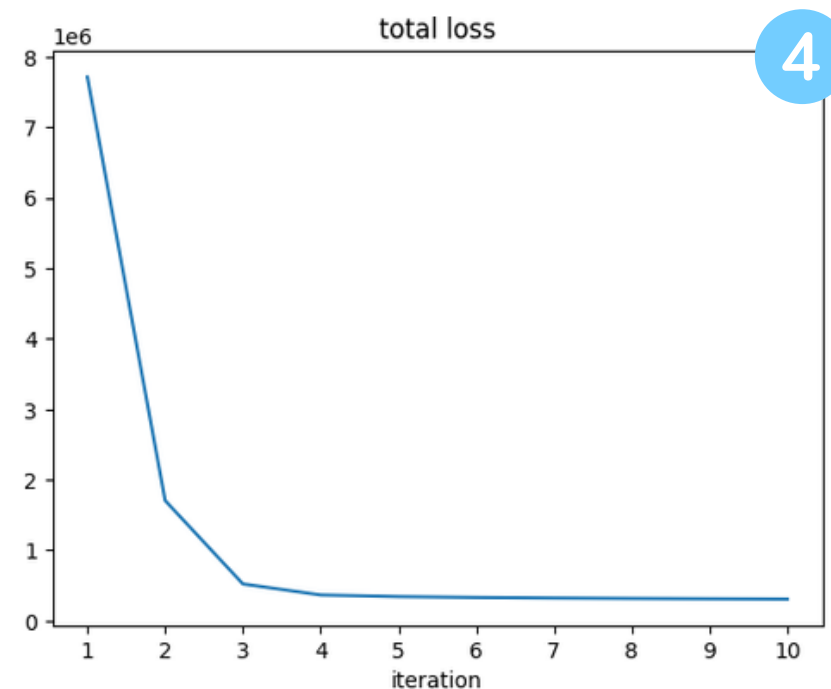
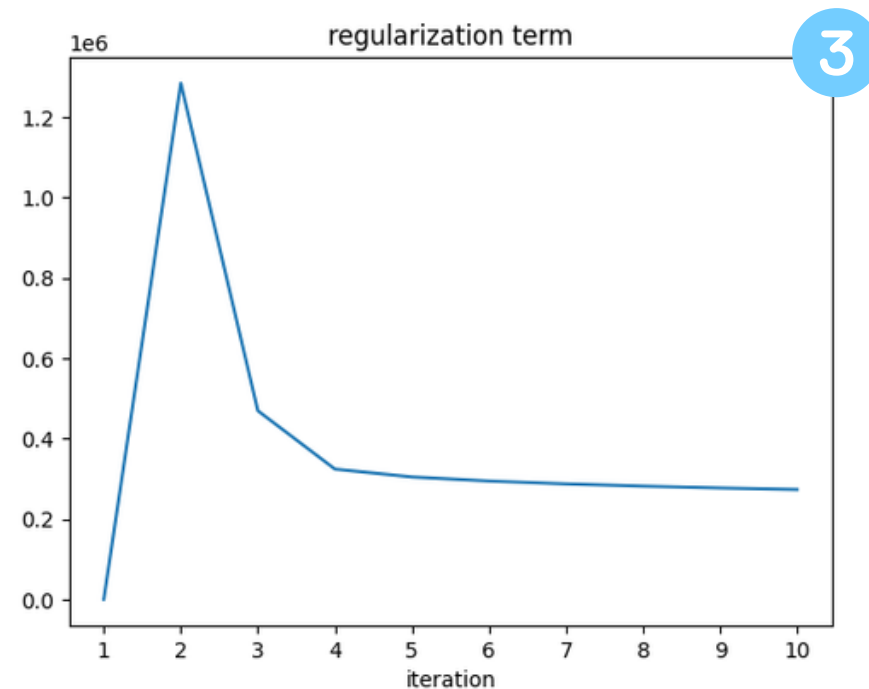
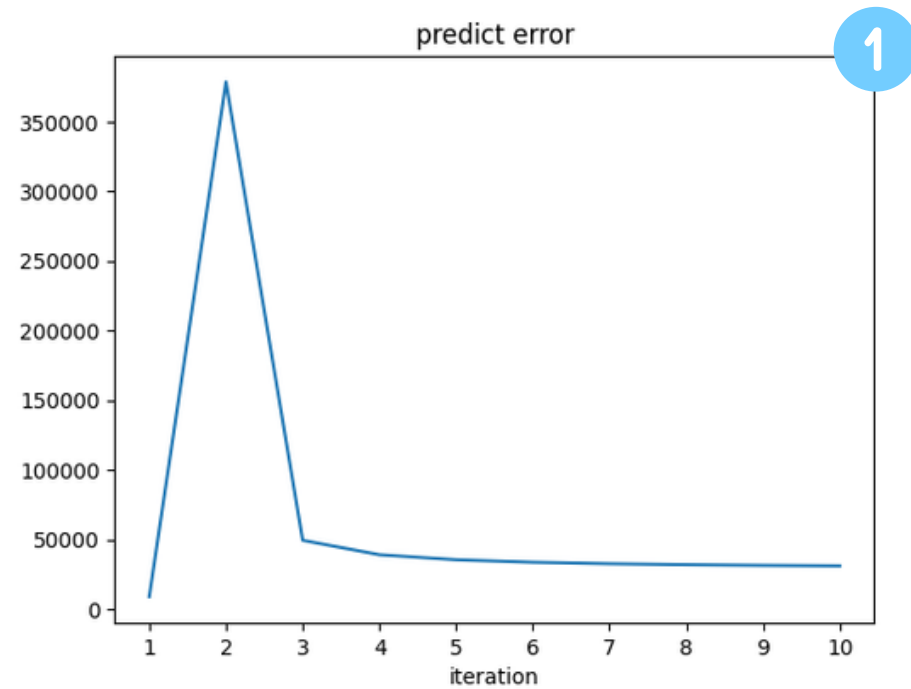
`alpha: 40`

`epsilon : 10**(-8)`

- Ranking increases as the number of factors increases
- Contrary to the original paper
- Due to the decrease in the number of data, it can be interpreted that a small number of factors are sufficient to represent the data

# V Implementation

## Experiment Results



$$\min_{x_*, y_*} \underbrace{\sum_{u,i} c_{ui} \underbrace{(p_{ui} - x_u^T y_i)^2}_{\text{1}}} + \lambda \left( \underbrace{\sum_u \|x_u\|^2}_{\text{2}} + \underbrace{\sum_i \|y_i\|^2}_{\text{3}} \right)$$

4

- Fast converge on this dataset  
→ when iteration 3~4

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Thank you  
for listening